The Evaluation of Empirical Coal Pillar Strength Formula Based on Uncertainty Criterion

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ABSTRACT

Several empirical equations to estimate coal pillar strength have been presented in academic studies. The development processes of these equations are similar and are usually obtained by fitting the mathematical function (curve) to field data. One of the best criteria to evaluate the quality of fitting for such equations is the correlation coefficient R^2, which has limited applicability. It is necessary to calculate the correlation coefficient access to the initial data for which the equation is presented; this is impossible for many coal pillar strength formulas. This paper presents a new approach based on the analysis of uncertainty amplitude to compare the coal pillar strength. This approach utilizes a combination of parameters such as Mean Squared Error (MSE), Root Mean Squared Error (RMSE), function type and degrees of freedom. The confidence level of constants is subsequently formed and the correlation coefficient becomes more comprehensive. Therefore, for an effective comparison, the efficiency and accuracy of coal pillar strength formula can be used.

Keywords: Coal Pillar Strength, Correlation Coefficient, MATLAB, Uncertainty Criterion

INTRODUCTION

The use of coal pillars for roof stability in underground coal mining is an old approach whereby roof stability depends on the bearing capacity of the coal pillar. If the applied load exceeds the coal pillar strength limit, the pillar fails and the possibility of roof collapse is increased (Brady and Brown, 2004). In addition, as the pillar dimensions increase, a reduction in mining recovery occurs because the quantity of coal remaining to be used as a pillar is increased (Oraee et al., 2008). The fluctuation of the pillar dimensions threatens the safety of mining and reduces the economic efficiency; therefore, the selection of an appropriate formula for coal pillar design plays a significant role in safety, economics and operational performance (Hosseini et al., 2010).

In recent years, significant research has been carried out on the determination of coal pillar strength, and various formulae have been introduced. The majority of these formulae is empirical and is based on shape, effective dimensions and the laboratory testing of cubic coal samples (Hosseini et al., 2010; Mark, 1990; Su and Hasenfus, 1999; Peng, 2006). While some limits persist, the experimental nature of these equations reduces their complexity. Determining the amplitude with accuracy is one of the most important of these limitations (Hosseini, 2007).

Based on the basic principles of statistical engineering, the correlation coefficient (R^2) is one of the best criteria for evaluation of such equations (Bird, 2003). Although the correlation coefficient is a well known statistical indicator that has many advantages, it also has some limitations. It is necessary to access the initial data in order to compare the equations of coal pillar design using the correlation coefficient. On many coal pillar strength formulae, however, such access is not possible. Therefore, due to this limitation and other similar cases, using the correlation coefficient on its own is inadequate and other mathematical and statistical criteria are needed for evaluating these equations.

COAL PILLAR STRENGTH FORMULA

Although the pillar-bearing capacity depends on various parameters, the width-to-height ratio (W/H), which acts as an indicator for pillar dimension, and the uniaxial compressive strength of intact coal play a significant role in pillar strength (Peng, 2008). Therefore, in many of the equations, the pillar strength is calculated based on these two critical parameters. Many equations that are used to estimate the coal pillar strength have been presented hitherto. Based on available field data in this research, Bunschinger (Equation 1), Bieniawski (Equation 2), Holland (Equation 3) and Oraee-Hosseini (Equation 4) are four well known equations that were selected for comparison and evaluation. The respective equations are given below (Hosseini, 2007; Peng, 2008; Oraee et al., 2009a):

$$\sigma_p = \sigma_1 (0.778 + 0.222 \frac{W}{H}) \quad (1)$$

$$\sigma_p = \sigma_1 (0.64 + 0.36 \frac{W}{H}) \quad (2)$$

$$\sigma_p = \sigma_1 \sqrt{\frac{W}{H}} \tag{3}$$

$$\sigma_p = \sigma_1 \exp(-0.43 + 0.668\sqrt{\frac{W}{H}}) \tag{4}$$

where σ_p is the pillar strength, σ_1 is the uniaxial compressive strength of a cubical specimen, W and H are width and height of pillar, respectively.

EVALUATION OF COAL PILLAR STRENGTH FORMULA

The equations of coal pillar strength are obtained based on field data. Therefore, the quality of these equations can be examined from the statistical engineering viewpoint. One of the most common statistical criteria for evaluation of these equations is the correlation coefficient (R^2), which is calculated through *Equation 5* (Bird, 2003):

$$R^2 = 1 - \frac{\sum[\sigma_{p_i} - f((\frac{W}{H})_i)]^2}{\sum(\sigma_{p_i})^2 - (\sum\sigma_{p_i})^2 / n} \tag{5}$$

where σ_{pi} and $(W/H)_i$ are the pillar strength and width to height ratio of pillar of *i*th element in field data of *n* data pair.

In the correlation coefficient, the efficiency of each coal pillar strength formula is determined based on errors of regression related to both the type of function and quality of raw data. The correlation coefficient assumes that the coal pillar strength is only related to the width-to-height ratio and the uniaxial compressive strength of coal. While other parameters, such as roof and floor condition, and the geomechanical properties of coal have an important role in coal pillar bearing capacity disregarded in coal pillar strength formulae. Therefore, a more efficient criterion for the evaluation of coal pillar strength equations is required. In fact, with determination of certainty of amplitude, the risk of error and uncertainty are denoted.

Erroneous data can be obtained in the field due to measurement conditions (Hosseini, 2007). Thus, adjusting for the uncertainty, the effects of input data on the equation should be studied where the constants of each coal pillar strength equation are considered as a variable which may have different (diverse) values.

In fact, the amplitude of each constant should be determined in terms of their lower and upper bounds. On the other hand, if $(X_1,..., X_n)$ is a series of field data, the $R(X_1,..., X_n)$ as upper bound function, the $L(X_1,..., X_n)$ as lower bound function should be determined. The probability of a function placed between upper and lower bounds are calculated by *Equation 6* (Bird, 2003):

$$P[L(X_1,..., X_n) < f(\sigma_3) < R(X_1,..., X_n)] = 1 - \alpha \tag{6}$$

where α is the percentage of allowable error of estimation and $(1-\alpha)$ is the confidence level. The uncertainty of the function (equation of coal pillar strength) by the distance of upper bound and lower bounds will be shown.

DETERMINATION OF UNCERTAINTY

A dataset for the evaluation of the coal pillar strength formula has been created using field data (Hosseini, 2007; Oraee et al., 2009b). Subsequently, uncertainty is studied by drawing the curve of the equation of each coal pillar strength formula using the *Curve Fitting* toolbox of MATLAB (MATLAB, 2010) and fitting this data. The fitting curves of coal pillar strength formula (Equations 1 to 4) on the data are shown in Figure 1. These curves are drawn using the coal pillar strength formula based on adjusting the constant coefficients.

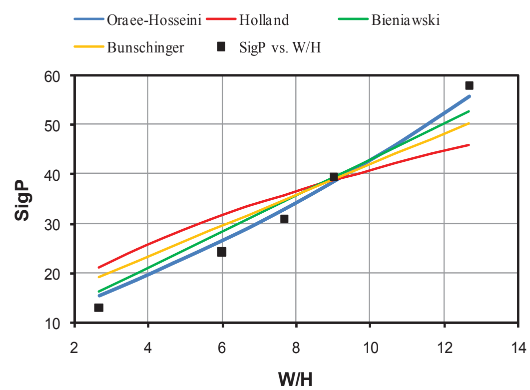

Figure 1. The curve fitting of coal pillar strength formula on the filed data. The vertical axis is, and the horizontal axis is.

In order to determine the uncertainty, the upper and lower prediction bounds of each coal pillar strength formula are drawn. Prediction bounds define the lower and upper values of the associated interval, and define the width of the interval. The prediction is based on an existing fit to the data. Additionally, the bounds can be simultaneous and measure the confidence for all predictor values, or they can be nonsimultaneous and measure the confidence only for a single predetermined predictor value. Simultaneous bounds measure the confidence that a new observation lies within the interval regardless of the predictor value. The nonsimultaneous prediction bounds for a new observation at the predictor value *x* are given by:

$$P_{n,o} = y \pm t\sqrt{s^2 + xSx^T} \tag{7}$$

where s^2 is the mean squared error, *t* depends on the confidence level, and is computed using the inverse of Student's *t* cumulative distribution function, and *S* is the covariance matrix of the coefficient estimates, $(X^T X)^{-1}S^2$. Note that *x* is defined as a row vector of the design matrix or Jacobian evaluated at a specified predictor value. The simultaneous prediction bounds for a new observation and for all predictor values are given by:

$$P_{s,o} = y \pm f\sqrt{s^2 + xSx^T} \tag{8}$$

where f depends on the confidence level, and is computed using the inverse of the F cumulative distribution function. The nonsimultaneous prediction bounds for the function at a single predictor value x are given by:

$$P_{n,f} = y \pm t\sqrt{xSx^T} \tag{9}$$

The simultaneous prediction bounds for the function and for all predictor values are given by:

$$P_{s,f} = y \pm f\sqrt{xSx^T} \tag{10}$$

Figures 2 to 5 show the uncertainty amplitude of each equation with 95% precision and field data of coal pillar strength.

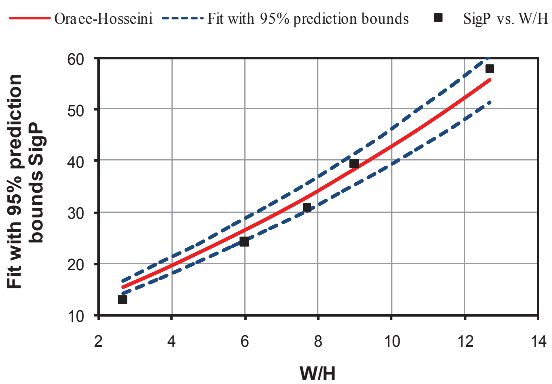

Figure 2. The curve of Oraee-Hosseini formula with 95% prediction bounds and field data.

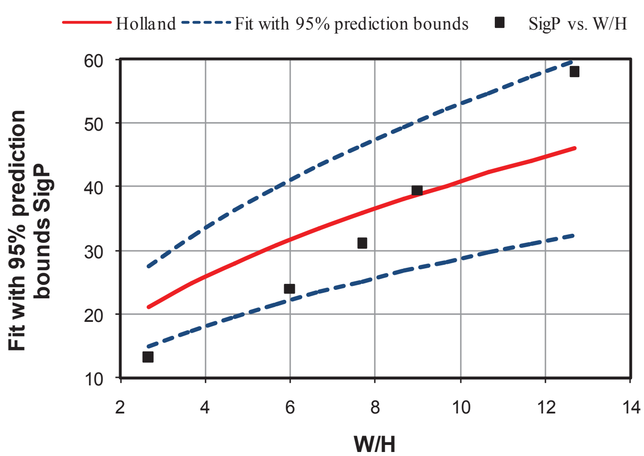

Figure 3. The curve of Holland formula with 95% prediction bounds and field data.

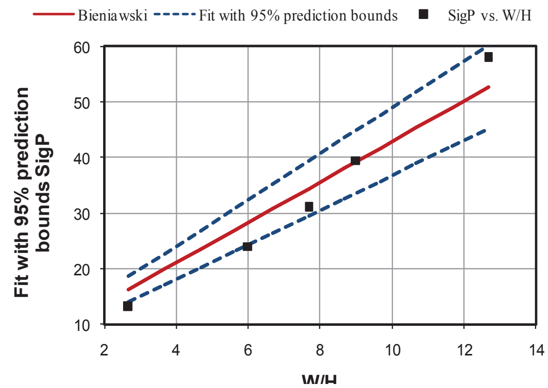

Figure 4. The curve of Bieniawski formula with 95% prediction bounds and field data.

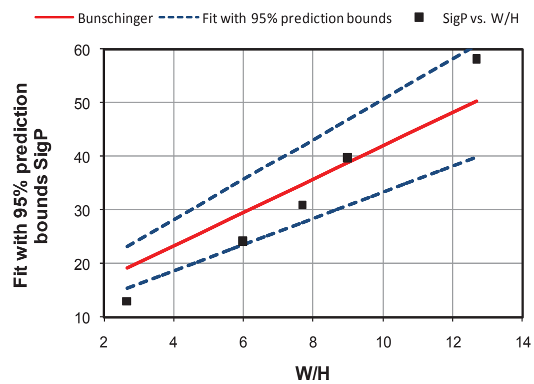

Figure 5. The curve of Bunschinger formula with 95% prediction bounds and field data.

RESULTS AND DISCUSSION

As shown above, the variation trend of the prediction bound for the various formulae are different. The Holland formula has the most uncertainty, while the Bunschinger formula has the second highest uncertainty. The Oraee-Hosseini formula, however, shows a lower uncertainty. The Holland and Oraee-Hosseini formulae have linear equations, whereas the equations of Bunschinger and Bieniawski formulae are nonlinear. With regards to the data utilized, certain advantages based on linear or nonlinear equations have not been accessed.

Overall, the uncertainty of each coal pillar strength formula is shown by the distance of the upper and lower bounds from the fitting curves. The closer the upper and lower bounds, the higher the confidence level of coal pillar strength formula will become, and vice-versa. To understand the uncertainty of each coal pillar strength formula more clearly, the trends of the distance between the upper and lower prediction bounds with 95% precision for various coal pillar strength formula are depicted in Figure 6.

As generally observed in all of the above formulae, uncertainty increases with the increasing width-to-height ratio of the pillar. Also, the increasing rates of uncertainty for all formulae are almost identical. Based on the uncertainty criterion, the Oraee-Hosseini formula is ranked first and Bieniawski, Bunschinger and Holland are ranked second to fourth, respectively. Of course, this would be related to the nature and properties of the field data; most of the data that is used in this research was collected at the Tabas coal mine in Iran. It should be noted that the Oraee-Hosseini coal pillar strength formula is presented based on the Tabas field data.

CONCLUSION

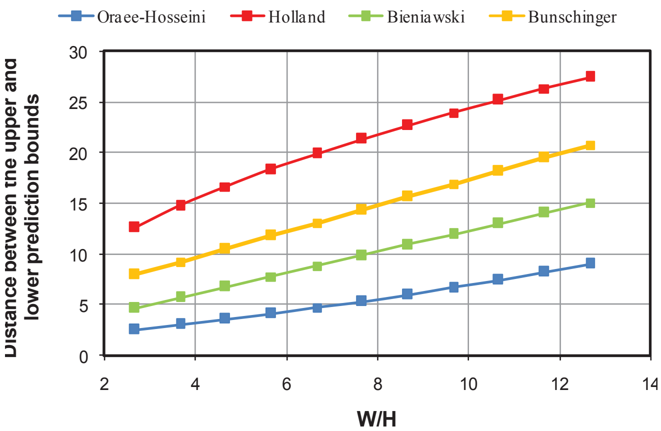

Figure 6. The trends of the distance between the upper and lower prediction bounds.

Some of coal pillar strength formulae in this paper are compared based on uncertainty. According to the results, whilst considering the limited field data of the coal pillar strength, the prediction of Oraee-Hosseini coal pillar formula is more suitable than other failure criteria. The prediction certainty and confidence of coal pillar formulae are decreased as the width-to-height ratio of the coal pillar is increased. Therefore, according to field characteristics, operational conditions and geomechanical properties, the appropriate coal pillar strength formula for each area, such as the equation with the highest certainty, can be selected. In short, the uncertainty can be used as a guide to select the appropriate coal pillar strength formula.

Using the correlation coefficient is the conventional method of evaluating equations fitted on data. However, the results of this study show that the use of uncertainty in the selection of the coal pillar strength formula as a new approach is a more applicable criterion. The calculation is usually impossible since the correlation coefficient requires the initial data, and that is complicated to obtain. The uncertainty criterion for each new data point can be used in a simple way, and the new approach that is presented in this paper is applicable to any underground coal mine.

REFERENCES

Bird, J. (2003). *Engineering Mathematics*, Elsevier Science, Newnes, London, UK 531.

Brady, B.H.G., Brown, E.T.. (2004). *Rock Mechanics for Underground Mining*. London, UK: George Allen & Unwin.. 628.

Hosseini, N. (2007). *Modelling of Pillars in Longwall Method Using Advanced Numerical Techniques*. *M.Sc. Thesis*. IAU, South of Tehran branch, 235.

Hosseini, N., Oraee, K., Gholinejad, M. (2010). *Optimization of Chain Pillars Design in Longwall Mining Method*. In: 29th International Conference on Ground Control in Mining (ICGCM). Morgantown, West Virginia, USA. 67-70.

Mark, C.. (1990). *Pillar Design Methods for Longwall Mining*. USBM Information Circular 9247. Washington, D.C. 1-53.

MATLAB version 7.11.0.584 (2010) Help and demos, for MATLAB reference, curve fitting toolbox.

Oraee, K., Hosseini, N., Gholinejad, M. (2008). *3D Strain Softening Modelling of Coal Pillars in a Deep Longwall Mine*. In: Proceedings of Seventeenth International Symposium on Mine Planning and Equipment Selection (MPES 2008). Beijing, China. 761-767.

Oraee, K., Hosseini, N., Gholinejad, M. (2009a). *Coal Pillar Strength Based on the Ground Reaction Curve – A New Approach*. In: Proceedings of the 28th International Conference on Ground Control in Mining. Morgantown, West Virginia, USA. 21-24.

Oraee, K., Hosseini, N., Gholinejad, M. (2009b). *Estimation of Coal Pillar Strength by Finite Difference Model*. In: Proceedings of the 2009 Coal Operators' Conference. Wollongong, Australia. 53-60.

Peng, S. S. (2006). *Longwall Mining, Second edition*. Society for Mining, Metallurgy, and Exploration, Inc. (SME), Department of Mining Engineering, West Virginia University, USA, 621.

Peng, S. S. (2008). *Coal Mine Ground Control, Third edition*. Society for Mining, Metallurgy, and Exploration, Inc. (SME), Department of Mining Engineering, West Virginia University, USA, 764.

Su, D. W. H., Hasenfus, G. (1999). *Coal Pillar Strength and Practical Coal Pillar Design Considerations*. In: Proceedings of the Second International Workshop on Coal Pillar Mechanics and Design. Pittsburgh, Pennsylvania, USA. Department of the Interior, Bureau of Mines. 155-163.