Post-Keynesian Models of Economic Growth: Open Systems

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Abstract  The closed systems nature of neoclassical models of economic growth – guaranteeing automatic equality between planned savings and investment which, in turn, ensures stability of such models – is achieved by assuming away the existence of uncertainty inherent in economic systems. Once the role of Keynesian uncertainty is acknowledged, the assumption of automatic equality between ex-post savings and ex-ante investment becomes untenable. This paper attempts to show that once this possibility of planned savings and investment inequality is incorporated in an otherwise essentially neoclassical model of economic growth, its closed system nature disappears and the model metamorphoses itself into an open system.

Keywords: open systems, closed systems, growth and instability, Harrodian instability, technical progress function, Keynesian uncertainty

JEL Classifications: B41, O40, E12,

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1. Introduction

The purpose of this paper is to take a look at growth theory in terms of whether or not it is developed as a closed or open system. We then suggest that the system underlying the Keynesian and post-Keynesian models of economic growth are open systems contrasted with the neoclassical growth models, which are presented and analysed in a closed system environment. What the paper attempts to show is that, in the presence of Keynesian uncertainty and the possibility of planned saving and investment inequality, the economic system can only be represented as an open system.

In Section 2, we introduce the concept of open system which is the basis of the rest of the paper. Section 3 presents, following Sen (1970), a description of the reappearance of Harrodian instability in a neoclassical growth model, with a Cobb-Douglas production function, in the presence of an independent investment function. Section 4 presents the role of history in the production function underlying Klador’s technical progress function. Finally, Section 5 presents the concluding observations.

2. Open systems

We start with the observation, ‘(a)n open system, ……….has interactions with the outside world. In the real world instances of perfect isolations are rare’ (Chick, 2004: 5) following on to, ‘(i)t is quite clear that not only are all parts of the economic system interconnected to a greater or lesser degree but the economic system is embedded in and connected with politics, philosophy, history, values, all the elements of social life. Ontologically then, the economy is unequivocally an open system.’(Chick, 2004:.5).
Since the elements of social life go through many changes and transformations over time, their interaction with it implies that the underlying economic system (or economic structure, if you prefer) will also undergo many changes as time passes. The arguments presented in this paper are based upon the understanding of an economy as an open system, where the nature of an open system is as outlined in the above quotations from Chick (2004)

In the growing crop of recent literature on the nature of economic systems and definition of open and closed systems, one may, perhaps, be forgiven for having a sensation not far removed from that of a blind person trying to ‘see’ an elephant. It depends on the viewer’s perspective. For the most comprehensive definition of open and closed systems please see Chick and Dow (2005). However, judging by the conditions for open systems as presented in Table 1 of Chick and Dow (2005), any one of which can make the system open, we can conclude that these conditions, both for the ‘real world systems’ and their ‘implications for theoretical systems’ can be seen to be satisfied in the post-Keynesian models of economic growth. With reference to Tables 1 and 2 in Chick and Dow, one may argue that when, once a closed system is specified, it starts to run on auto pilot. This we observe in neoclassical models of economic growth. The system always converges to equilibrium and any disequilibrium gets automatically corrected without interventions from outside. Also, any change in the system is imposed from outside as technological progress in the form of exogenously determined manna from heaven. In the context of growth economics, an open system is interpreted as the account of the economic system as in the first three chapters of the Wealth of Nations in the sense described by Kaldor (1972: Section II). Following the spirit of Kaldor and Allyn Young,
in an open system, ‘the actual state of an economy during any one ‘period’ cannot be predicted except as a result of the sequence of events in previous periods which led up to it.’ (Kaldor, 1972: 1244).

In closed systems like the neoclassical general equilibrium system, history or social interactions do not matter. As Kaldor (1972) writes, ‘(t)he very notion of ‘general equilibrium’ carries the implication that it is legitimate to assume that the operation of economic forces operate in an environment that is ‘imposed’ on the system in a sense other than being just a heritage of the past – one could almost say an environment which in its most significant characteristics is independent of history.’ (Kaldor, 1972:.1244). He then goes on to observe (in the same paragraph) that ‘Continuous economic change on these assumptions can only be conceived as some kind of ‘moving equilibrium’ through the postulate of an autonomous (and unexplained) time rate of change in the exogenous variables of a kind that is consistent with ‘continuous equilibrium’ through time…’.

One of the main objections made by Kaldor is to the underlying assumption of the general equilibrium analysis of the automatic equality between planned saving and investment. He pointed out that Keynes postulated that ‘…in one particular market, the market for savings, the price is not, or need not be, ‘market clearing’ (owing to liquidity preference), and if it is not, there is another mechanism that of the multiplier, to bring about equality in that market …..But that mechanism operates by varying the amount of production in general. It leads to a situation that is not resource constrained.’ (Kaldor, 1975: 350; emphasis and parentheses in the original). Looked at from this point of view, the condition which is relevant for our discussion relates to the assumption of the economic agents behaviour in the post-Keynesian and neoclassical growth models.
In an open system, agents and their interactions may change and the structure and agency are interdependent. In closed systems, the nature of atomistic economic agents is treated as if constant (Chick and Dow, 2005: 366-367). Such an assumption of agents’ behaviour is what guarantees equality between planned savings and investment. In the context of Keynesian uncertainty, however, an assumption of economic agents running on an ‘auto pilot’ is hard to justify. Planned saving need not and does not equalise with planned investment all the time. This paper concentrates on the implication of planned saving and investment inequality in models of economic growth.

3. Stability problem due to Harrod

We start from, what is known in the literature as, the First Harrod Problem – that although steady state growth at full employment is possible in a model of economic growth, such a ‘Golden Age’ (Robinson, 1969: 99-100) is highly improbable. The steady state growth with full employment implies equality of the actual rate of growth \( G_a \), \textit{warranted} rate of growth, \( G_w \), (defined as ‘that overall rate of advance which, if executed, will leave entrepreneurs in a state of mind in which they will be prepared to carry on a similar advance’ Harrod (1948: 82)) and the natural rate of growth \( G_n \) (the rate of growth of labour). What is the reason behind attainment of such Golden Age being highly improbable? We know that \( G_a \) is the ratio between the marginal propensity to save \( s \), and the actual capital-output ratio \( v \) and \( G_w \) is the ratio between the marginal propensity to save and \( v_r \), the capita-output ration \textit{required} by the entrepreneurs. Now \( s, v \) (and \( v_r \)) as well as the natural rate of growth, \( G_n \) are all determined independent of each other. For this reason steady-state rate of growth with full employment can only be attained through some happy accident. The First Harrod Problem is only the first step towards the more
serious Second Harrod Problem which Harrod regarded as his main theme. The Second Harrod Problem states that the warranted rate of growth is fundamentally unstable in the sense that divergence of actual rate of growth, $G_a$, from the warranted rate, $G_w$, is not only not self-correcting but, *left to itself*, would produce even larger divergences over time (Harrod, 1948).

Solow (1956) proposed a way out of the problem of instability one encounters in the model of economic growth due to Harrod. Solow claimed that the Harrod-Domar\(^1\) model studied long run behaviour of the economy, which is the domain of neoclassical analysis (‘the land of margin’), with tools of short run analysis such as the multiplier, accelerator and fixed capital coefficient. He proposed a ‘model of long-run growth which accepts all the Harrod-Domar assumptions *except that of fixed proportions*’ (Solow, 1956:162; emphasis added). In this model Solow could demonstrate that the steady state growth with full employment can be achieved in a model of economic growth, constructed using the neoclassical general equilibrium methodology.

However, Solow (1956) implicitly assumed away existence of uncertainty as presented by equation 1 of his model, as well as his arguments leading up to the equation, that savings – a proportion of income, is always equal to the investment at every instant of time. This assumption of investment as an accommodating variable along with that of a flexible capital-labour ratio and an exogenously given rate of growth of labour guaranteed long run stable equilibrium. Not surprisingly, growth in this general equilibrium model comes from the exogenously determined rate of growth of the efficiency of labour. As Hahn and Matthews (1964:789-790) have pointed out, ‘In its

\(^1\) Though Solow (1956) refers to Harrod-Domar, the problem of instability addressed in it is due to Harrod.
basic form the neo-classical model depends on the assumption that it is always possible and consistent with equilibrium that investment should be undertaken of an amount equal to full-employment savings. The mechanism that ensures this is as a rule not specified.’

The rate of interest that ensures planned investment is equal to full employment saving is adjusted in one of the three possible interventions: (a) through the operation of Say’s Law, in the absence of money or when demand for money is interest inelastic; (b) through adjustment of the price level to influence the rate of interest via its effect on the real money balances or (c) through the use of appropriate monetary policy (Hahn and Matthews, 1964, p.790). The last of these three options is due to Meade (1961). The implication of this policy suggestion, due to Meade, is that there has to be some form of intervention by the monetary authorities to ensure the stability of the neoclassical models of economic growth. The implication of this last observation is that the neoclassical growth model can no longer be considered as a closed system. Otherwise, following Hahn and Matthews (1964:790), ‘The familiar Keynesian difficulties therefore arise….’

Commenting on the instability problem due to Harrod Joan Robinson (1961:360) pointed out that:

‘As the statement of ex ante equilibrium conditions, it (the familiar formula \( g = \frac{s}{v} \)) fails to isolate independent variables; \( s \), the ratio of annual net saving to annual net income, is strongly influenced by the ratio of profits to income, which in turn is strongly influenced by the ratio of annual net investment to the value of capital, that is, by \( g \) itself; \( v \), the ratio of value of capital to annual net income, is influenced both through the prices of capital goods and through the choice of technique, by the ratio of profit, which is a function of \( s \) and \( g \). All the formula
can say is that, if growth is going on under equilibrium conditions at the rate $g$, then $s/v$ is equal to it.’ (emphasis added).

She went on to add:

‘Harrod…….. did not want to throw away the General Theory and make savings govern investment…What he shows is that, if we write down a function for the inducement to invest (whether in terms of the accelerator, or of expected profits, of the supply of finance, or just of the animal spirits of the managers of firms) generating a desired rate of growth, and a set of identical conditions (the labour supply, the flow of new investment and so forth) providing a ‘natural’ or better, a physically possible rate of growth, and, furthermore, postulate equilibrium with full employment, we have overdetermined our system.’ (Robinson, 1961:360-61).

What are the ways out of this problem of an overdetermined system? Robinson (1961) suggested three ways out;

(i) Give up the idea of equilibrium and exhibit an economy blundering on from one situation to another (as happens in the history of the world we live in) following no simple predictable path. In other words, learn to live with the problem envisaged by Harrod.

(ii) Introduce a functional connection between the desired and the possible growth rate so that the one determines the other.

(iii) Give up the desired rate of growth and simply assume that actual growth goes on, in equilibrium conditions, with continuous full employment of available labour.
Solow (1956) is the neoclassical ‘solution’; the way out (iii) in the list above. In other words, the assumption of savings and investment equality leaves out the role of the expected rate of growth completely and consequently, the role of the ‘animal spirits’ of entrepreneurs which guides investment decisions. Thus, there is no role for an independent investment function in the neoclassical growth models. Investment is always equal to saving which then leads to the familiar formula $g = s/v$.

What will happen if we reintroduce the role of an independent investment function in a neoclassical growth model while retaining all the other assumptions of the model – constant returns to the scale, a smooth twice-differentiable production function that satisfies the Inada conditions (Inada, 1964), the marginal productivity theory of distribution as well as the flexible capital labour ratio? Sen (1970) did just that. He started with the well behaved neoclassical production function

$$Y = e^{\alpha K} L^{1-\alpha} \quad (3.1)$$

(where, $Y$, $K$, and $L$ stand for output, capital and labour respectively.)

Which leads us to

$$\frac{Y}{K} = \frac{r}{\alpha} \quad (3.1')$$

where $r$, commodity (assumed = money) rate of interest is equal to $\frac{\delta Y}{\delta K}$ or marginal productivity of capital. Since $r$ and $\alpha$ are given, $Y$ and $K$ must grow at the same proportional rate. Therefore Harrod’s warranted rate, $G_w = s/v$, turns out to be

$$G_w = \frac{rs}{\alpha} \quad (3.2)$$

Sen (1970) then introduced an independent investment function based on an expected rate of growth, which is not necessarily equal to the warranted rate. A set of neoclassical
entrepreneurs, given the expected (exponential) rate of growth of \( j \) over time plan to invest enough to make an expected rate of profit equal to the own rate of interest.

Using (3.1) we get:

\[
r = \frac{\bar{Y} e^{\mu t} \alpha}{K_t}
\]  

(3.3)

\( \bar{Y} \) = current income

From (3.3)

\[
K_t = \frac{\bar{Y} e^{\mu t} \alpha}{r}
\]

From the multiplier relationship, we have

\[
Y_s = \frac{1}{s} \frac{dK_t}{dt} = \bar{Y} e^{\mu t} \frac{j}{rs} \alpha
\]  

(3.4)

But from (3.2) \( G_w = \frac{rs}{\alpha} \)

So if \( j = \frac{rs}{\alpha} \) (i.e., expected rate = warranted rate) actual income is equal to the expected income. Let the actual rate of growth is given by \( Y_s = \bar{Y} e^{\alpha t} \). Now, if \( j > \frac{rs}{\alpha} \) we have

\( j < G_w \) and if \( j < \frac{rs}{\alpha} \) we have \( j > G_w \)

Thus, in whatever direction one might err in, one would feel that the error lies in the other direction causing the second Harrod problem to come back.

Before we end this section, it will be interesting to point out that Solow himself was not completely unaware of the possibility that investment may not always be an accommodating variable. In a small section, entitled ‘Uncertainty etc.’, at the end of his 1956 article he wrote:
‘No credible theory of investment can be built on the assumption of perfect foresight and arbitrage over time. There are only too many reasons why net investment should be at times insensitive to current changes in the real return to capital, at other times oversensitive. All these \textit{cobwebs} and some others have been brushed aside throughout this essay. \textit{In the context, this is perhaps justifiable.’} (Solow, 1956:93-94; emphasis added).

It would be interesting to speculate about the ‘context’ which ‘justifies’ these ‘cobwebs’ to ‘be brushed aside’

4. Kaldor’s technical progress function

Kaldor (1957) introduced the concept of technical progress function, which is a relationship which incorporates both shift of and movement along a production function. The technical progress function ‘permits the desired rate of growth to bring the possible rate into equality with itself’ (Robinson, 1961:361). The general form of the technical progress function is:

\[
\frac{\dot{y}}{y} = f \left( \frac{\dot{k}}{k} \right) \quad f' > 0, f'' < 0 \quad (4.1)
\]

and the specific form he selected was

\[
\frac{\dot{y}}{y} = \alpha \frac{\dot{k}}{k} + \beta \quad (4.1')
\]

This choice of the linear form created some unnecessary confusion and, for a time caused attention to be diverted away from the true nature of the technical progress function in (4.1). Black (1962) integrated (4.1’) and, not surprisingly, got a Cobb-Douglas production function complete with manna from heaven shift factor. Solow
(1967) was the first to point out that in considering (4.1) above one should not ignore the history (i.e. path of) capital accumulation. Writing (4.1) in difference equation form and solving it one gets:

\[ y_i = y_0 \prod^{t}_{k=1} g\left( \frac{k_i}{k_{i-1}} \right) \quad i=1,2,\ldots,t \]

which captures the history of capital accumulation up to the current period in determining the level of production at any given period.

It can be shown (Ghosh, 1985; Ghosh and Banerjee, 1993) that the history of capital accumulation becomes relevant when investment plans are not realised in the sense that when planned investment \( (I^p) \) is not equal to planned savings \( (S^p) \).

Following Stein (1969) they wrote that under situation like this the actual investment \( (I^a) \) can be represented as a convex combination of planned investment and planned savings. This can be written as:

\[ I^a = \Omega I^p + (1 - \Omega)S^p, \quad 0 < \Omega < 1 \quad (4.2) \]

Here \( \Omega \) is a constant which reflects the institutional framework. When \( \Omega = 1 \), means that both savings and investment plans in the society has been realised. Remembering that \( \dot{K} = I^a \) and incorporating (4.2) into (4.1'), one gets:

\[ \frac{\dot{y}}{y} = \alpha \Omega \left[ \frac{k^p}{k} + n - \frac{s^p}{v} \right] + \alpha \left[ \frac{s^p}{v} - n \right] + \beta \quad (4.3) \]

Here, \( S^p = s^pY \) and \( n \) is the natural rate of growth. It is to be noted that both \( s^p \) and \( v \) has a time dimension.
Upon integration (4.3) gives us

\[ y(t) = A k^{\omega(4.4)} \exp \left[ \phi t + \int_0^t \frac{s^p(t)}{v(t)} (\alpha - \alpha \Omega) dt \right] \]  

(4.4)

Where \( A \) is a constant of integration and \( \phi = \beta - n \alpha (1 - \Omega) \).

The expression under the integral sign provides the history of economic growth from the initial period to the present time. When \( \Omega = 1 \), in other words when planned investment is equal to planned savings we get from (4.4):

\[ y(t) = A e^{\phi t} k^\alpha \]  

(4.5)

where \( \phi' = \beta \)

which is the familiar Cobb-Douglas production function.

That the sufficient condition for (4.4) to collapse to a C-D production function when \( \Omega = 1 \) is obvious. However, what is the sufficient condition for this to happen? The necessary and sufficient condition for (4.4) to collapse to a C-D production function is

\[ \left( \frac{s^p(t)}{v(t)} (\alpha - \alpha \Omega) \right) \]  

to be constant. Since \( \alpha \) is assumed to be a constant from the beginning, assuming further that \( \Omega \) is also a constant but not necessarily equal to unity, the condition then becomes \( \frac{s^p}{v} = \) constant. This, however, is a familiar condition which is the simple equilibrium relation \( -\frac{s}{v} = n \). As Pasinetti (1974:125-126) pointed out, this equilibrium condition simply states that if \( s \) and \( n \) are constant, the equilibrium output

\[ 2 \] It is to be noted that since \( \dot{k}^p \) is small compared to the existing stock of capital \( k(t) \), integration of \( \frac{\dot{k}^p}{k} \) with respect to \( t \) can be written as \( \log k(t) \) without any loss of generality.
capital ratio, if it exists, is determined by the natural rate of growth divided by the over-all propensity to save independently of anything else and therefore, independent of the shape of the production function.

We now have the condition that if the economy is in equilibrium then \( \frac{s^p}{v} = n \) implies that equation (4.4) collapses to equation (4.5). However, when the economy is in growth equilibrium all relevant variables grow at the same rate. This, in turn, implies that all plans, including savings and investment plans, are realised; which means \( \Omega = 1 \). Thus \( \Omega = 1 \) is both the necessary and sufficient condition for the production function underlying the technical progress function to collapse into a C-D production function. However, if the economy is not in growth equilibrium, the whole history of capital accumulation, i.e., the expression under the integral sign in (4.4), becomes relevant for the determination of the level of income at any given time period. In other words, when all variables are growing at the same constant rate over time the economic system can be represented by any system including a closed system. However, outside equilibrium, where history, politics, socio economic policies shape agents’ behaviour it is only an open system which can represent a true description of the economic system.

Kaldor himself ‘always wanted to give the technical progress function a “disequilibrium interpretation”’ though he by no means agreed that an underlying production function must necessarily exist. The purpose of our exercise in this section, however, was not to search for the existence of a production function underlying the technical progress function. Our objective was to demonstrate that in the presence of

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Kaldor’s personal communication, available from the author on request  
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uncertainty and hence the absence of automatic equality between savings and investment, the technical progress function demonstrates the role of initial condition as well as past history in determination of economic growth.

5. Conclusions

Srinivasan (1985: 40) while describing the two theorems of welfare economics, that (i) a competitive equilibrium is a Pareto optimum and (ii) associated with any Pareto optimum is set of prices that will sustain it as competitive equilibrium, observed that ‘(a) curious facet of this neoclassical gem is that it is ‘institution free’ in that it does not explicitly refer to any state’. If this world view is accepted one can find very little justification for analysing an economy from an open systems perspective. However, this claim of a system being institution free begs the question, can such systems represent the reality of economic life in general and that of economic growth in particular. Above, we have demonstrated that even in an economic model where we start with the iconic well behaved Cobb-Douglas production function and borrow all the mathematical techniques from the rulebook of the neoclassical economics, the closed system nature of the neoclassical general equilibrium world disappear as soon as, following Keynes, we recognise that planned saving and investment in an economy are not necessarily always in equilibrium. In a world of liquidity preference, where perfect certainty is not the order of the day, where entrepreneurs speculate – in other words, where uncertainty rules, we would expect saving and investment to diverge and that is when the true nature of the real world economic system – an open system – becomes obvious.

The discussion above may lead to the question – is the disequilibrium nature of the economy presented in sections 3 and 4 are coincidental. The answer is ‘probably not’.
As it has been observed ‘(c)losure precludes openness to history and creativity. Open systems are path-dependent and non-ergodic and may exhibit neither event regularities nor unique equilibrium.’ (Chick, 1998:1856).

As Loasby (2003: 294) has suggested, rather than contrasting openness and closure, it is more fruitful to think in terms of dimensions and degrees of closure. He observes that “(c)ompletely closed models are obviously limiting cases in terms of degree, but human cognitive powers requires a drastic restriction on the number of dimensions in order to secure closure – as is illustrated by the practice of economists. Partial closure is necessary for any exploration of openness: we have to close our minds to many possibilities in order to pay attention to a few ….Routines, institutions, organizational design and the acceptance of authoritative pronouncements – only a small proportion of them from normal superiors – all contribute to such closures” (Loasby, 2003: 294). If the neoclassical world view that economic systems can be analysed as closed systems is accepted, are there any roles for the policy makers in economics? The logic of the closed systems leads to the conclusion that once the equilibrium is achieved in such a system (which, in any case, is automatic) it can be left to run on auto-pilot given that the system is inherently stable. The role of policy interventions becomes relevant only in the context of open systems. In context of the Post-Keynesian models of economic growth, as with the passage of time these routines, institutions, organisational design etc change, the connections within a closed system changes and demands another (temporary) closure for the policy makers to work on. Only open systems can accommodate uncertainty, structural social and sociological changes and hence their impacts on the economic system and the implications of such uncertainty and changes for
economic policy making and in the words of Joan Robinson (quoted in section 3 above) ‘(g)ive up the idea of equilibrium and exhibit an economy blundering on from one situation to another (as happens in the history of the world we live in) following no simple predictable path.’. In other words, move from one temporarily closed system to another one which becomes more relevant for analysing the economy at that specific period of time given the nature of technological progress, social and sociological conditions and expectations prevailing at that particular point in history.

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