Application of the Trace Coherence to HH-VV PolInSAR TanDEM-X Data for Vegetation Height Estimation

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Abstract—This paper investigates for the first time the inclusion of the operator Trace Coherence (TrCoh) in polarimetric and interferometric SAR (PolInSAR) methodologies for the estimation of biophysical parameters of vegetation. A modified inversion algorithm based on the well-known Random Volume over Ground (RVoG) model, which employs the TrCoh, is described and evaluated. In this regard, a different set of coherence extrema is used as input for the retrieval stage. Additionally, the proposed methodology improves the inversion algorithm by employing analytical solutions rather than approximations. Validation is carried out exploiting single-pass HH-VV bistatic TanDEM-X data, together with reference data acquired over a paddy rice area in Spain. The added value of the TrCoh, and the convenience of the use of analytical solutions are assessed by comparing with the conventional PolInSAR algorithm. Results demonstrate that the modified proposed methodology is computationally more effective than current methods on this data set. For the same scene, the steps required for inversion are computed in 6 minutes with the conventional method, while it only takes 6 seconds with the proposed approach. Moreover, vegetation height estimates exhibit a higher accuracy with the proposed method in all fields under evaluation. The root mean square error reached with the modified method improves by 7 cm with respect to the conventional algorithm.

Index terms—Agriculture, height, inversion, PolInSAR, Trace Coherence.

I. INTRODUCTION

POLARIMETRIC SAR Interferometry (PolInSAR) [1] can be used for the retrieval of vegetation parameters and it often relies on the inversion of physical models, e.g. [2]–[6], among many others. Such physical models are used to relate the PolInSAR measurements (i.e. interferometric complex coherences acquired at different polarimetric channels) to scene biophysical parameters (e.g. vegetation height). In the literature, the most widely used model for this purpose is the Random Volume over Ground (RVoG) [7]–[10].

One of the most critical steps in current RVoG inversion algorithms is the line fit to the coherences represented on a complex plane and the selection of the extreme coherences associated with minimum and maximum ground contributions [11]–[13]. This process requires understanding of the coherence region (i.e. locus of points in the polar plot), which contains all possible coherence values. A traditional way considers the border of the coherence region and the identification of coherences with extreme phase values. This is the starting point of the inversion algorithm, from which the rest of the model parameters are retrieved [2], [11]–[14]. Consequently, the accuracy of the final estimates is strongly determined by this selection.

In this paper, we investigate the use of the operator Trace Coherence (TrCoh), originally derived in [15], as a key part of the PolInSAR inversion strategy for the estimation of vegetation parameters, with focus on the vegetation height. This operator provides the centre of mass of the coherence region, which is the location of maximum accumulation of coherences in the complex plane. This centre of mass is calculated by an approximation of the integral of the PolInSAR coherences over all possible scattering mechanisms. Therefore, the TrCoh synthesizes information of the physical target observed by the SAR system as a single entity. Moreover, it represents a coherence independent of the selection of a specific scattering mechanism.

In this work, a modified PolInSAR inversion methodology which employs the TrCoh is proposed for the first time. We use the rationale that, in order to improve the fitting of the data with the model assumptions, the RVoG line needs to cross the point of maximum accumulation of coherences, i.e. the TrCoh. This new line produces a different set of coherence extrema, which is employed as input for inversion. In addition, the proposed methodology refines the conventional PolInSAR inversion algorithm for dual-pol data by introducing analytical solutions, instead of approximations. The complementary information provided by the TrCoh and the convenience of the use of analytical solutions are investigated and assessed to improve current PolInSAR parameter retrieval strategies. Our investigation exploits HH-VV dual-pol TanDEM-X [16] data of an area in Sevilla (Spain), in which ground-truth data over rice fields are available.

This paper is organised as follows. Section II provides a theoretical introduction to PolInSAR with the purpose of presenting the mathematical formalism behind the inversion strategy. In Section III, the proposed dual-pol inversion based on the TrCoh is detailed. Then, Section IV describes the test
site and the available data set. In Section V, the results of the inversion methodology are presented and analysed with the validation data. At last, Section VI discusses the results and draws the final conclusions.

II. POLARIMETRIC SAR INTERFEROMETRY

PolInSAR processing exploits the variation of interferometric coherence with polarization to characterise the physical target observed by the radar [1], [10]. In the case of HH-VV dual-pol TanDEM-X data [16], each partial target is expressed as a scattering vector

\[ \vec{k}_1 = [S_{HH}^1, S_{VV}^1]^T, \quad \text{and} \quad \vec{k}_2 = [S_{HH}^2, S_{VV}^2]^T, \]

(1)

where \( S_{HH}^n \) represents the complex scattering amplitude at the \( n \)-th end of the spatial baseline. With partial targets (i.e. targets composed by different scattering mechanisms), we need to extract second-order statistics. Typically, this is described by means of the covariance \([C]\) or coherency \([T]\) matrix according to a lexicographic or Pauli basis, respectively. Thus, the covariance matrix is defined as:

\[ [C] = \langle \vec{k}_n \cdot \vec{k}_n^* T \rangle = \begin{bmatrix} [C_{11}] & [\Omega_{12}] \\ [\Omega_{12}]^T & [C_{22}] \end{bmatrix}, \]

(2)

where superscript \( * \) denotes conjugate, \( T \) transposition, and \( \langle \cdot \rangle \) spatial averaging or multi-looking. In (2), \([C_{11}]\) and \([C_{22}]\) are \( 2 \times 2 \) polarimetric matrices, whereas \([\Omega_{12}]\) is a \( 2 \times 2 \) matrix containing polarimetric and interferometric information.

To form interferograms, scattering vectors \( \vec{k}_1 \) and \( \vec{k}_2 \) are converted into scalars employing unitary complex vectors \( \vec{\omega} \), which specify the selected polarimetric combination, \( S_n(\vec{\omega}) = \vec{\omega}^* T \vec{k}_n \) [1], [11]. The interferometric combination of both scalars results in the PolInSAR coherence:

\[ \gamma(\vec{\omega}) = \frac{\vec{\omega}^* T [\Omega_{12}] \vec{\omega}}{\sqrt{\left(\vec{\omega}^* T [C_{11}] \vec{\omega}\right) \left(\vec{\omega}^* T [C_{22}] \vec{\omega}\right)}} \]

(3)

The representation in the polar plot of all the possible PolInSAR coherences varying with the projection vector is the coherence region (CoRe) [17], i.e. \( \{ \gamma(\vec{\omega}), \vec{\omega} \in C^2, ||\vec{\omega}|| = 1 \} \). The estimation of vegetation parameters by means of PolInSAR is based on its geometrical interpretation. A common technique to derive mathematical properties from the CoRe consists in employing a simplified PolInSAR matrix \([A]\) [11], [18], [19], defined as

\[ [A] = [C]^{-1/2} [\Omega_{12}] [C]^{-1/2}, \text{ with } [C] = ([C_{11}] + [C_{22}])/2. \]

(4)

For dual-pol data, the field of values of any \( 2 \times 2 \) matrix \([A]\) is an ellipse [20], [21]. Fig. 1 shows an example of this, where different PolInSAR signatures of four selected scenes are observed. From \([A]\), the characteristic parameters of the CoRe can be obtained by means of a Schur decomposition (see [11, eq. (6.31)]):

\[ [A] = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow [A] = [U_2]^* T \begin{bmatrix} \lambda_1 & \delta \\ 0 & \lambda_2 \end{bmatrix} [U_2], \]

(5)

where

\[ 2a = \sqrt{\lambda_1 - \lambda_2}^2 + |\delta|^2, \]
\[ 2b = |\delta|, \]
\[ 2g_m = \frac{1}{2} (\lambda_1 + \lambda_2). \]

(6)

In (6), \( \lambda_1 \) and \( \lambda_2 \) are the ellipse’s foci, \( a \) and \( b \) are the semi-major and semi-minor axis, respectively, and \( g_m \) is the centre of the ellipse (i.e. mean coherence).

III. DUAL-pol INVERSION BASED ON THE TRACE COHERENCE

The proposed inversion algorithm which employs the TrCoh with PolInSAR TanDEM-X data is shown in Fig. 2. The general methodology with TanDEM-X data was first developed for forest scenes in [12]. Then, different methods were proposed to retrieve crop vegetation parameters (e.g. [23]–[25]). Later on, in [13], the RV oG model was adapted for the specific properties of rice scenes (i.e. double-bounce dominant ground contribution). Now, the algorithm is modified according to the inclusion of the TrCoh. To this end, key points of the new algorithm are:

- The formulation of the correction of non-volumetric decorrelation sources [16] for the covariance matrix and the TrCoh is presented for the first time.
- Thanks to the elliptical form of the dual-pol CoRe (see eq. (5) and (6)), the analytical expression of the extreme coherences associated with the minimum and maximum ground contributions is derived.
- A new approach to exploit the TrCoh for vegetation height estimation is proposed.
- Given the expressions of the CoRe and the TrCoh, the selection of the new set of coherences employed for inversion is also derived analytically.

Following the block diagram in Fig. 2, each of these key steps is detailed in the next sections.

A. Covariance De-noising

In bistatic TanDEM-X acquisitions, after range spectral filtering [26], the main non-volumetric decorrelation contribution is due to additive noise, \( \gamma_{SNR} [16] \). Therefore, each polarization channel needs to be compensated by the signal-to-noise (SNR)
zero mean variables is proximal to zero. Thus, there is no way to remove the effect of noise on $[\Omega_{12}]$.

In second place, the correction of quantization effects is performed as follows:

$$
[\hat{C}_{11}]_{nf} = [\hat{C}_{11}] \cdot \gamma_{\text{BAQ}} \\
[\hat{C}_{22}]_{nf} = [\hat{C}_{22}] \cdot \gamma_{\text{BAQ}} \\
[\Omega_{12}]_{nf} = [\Omega_{12}].
$$

In the case of the BAQ correction (9), multiplying the decorrelation term, i.e. $\gamma_{\text{BAQ}} \approx 0.965$, by the $[\hat{C}_{11}]$ and $[\hat{C}_{22}]$ matrices is equivalent to divide the coherence in (3) by it, as it is commonly performed [13].

Since the noise removal of the covariance matrix is the starting point of the inversion methodology (see Fig. 2), the remaining parameters necessary for inversion are obtained from the de-noised covariance matrix. As demonstrated later in Sections V and VI, this has a direct impact on both the accuracy and the computational efficiency of the algorithm.

### B. Trace Coherence De-noising

The main novelty of this paper is the use of the TrCoh in the PolInSAR retrieval algorithm. This parameter, as introduced by Marino in [15], is defined as:

$$
\gamma_{tr} = \frac{\text{Trace}([\Omega_{12}])}{\sqrt{\text{Trace}([\hat{C}_{11}]) \text{Trace}([\hat{C}_{22}])}},
$$

where the operator Trace(.) represents the sum of the diagonal elements of a matrix. As mentioned in the introduction, the TrCoh provides an approximation to the centre of mass of the CoRe. By integrating over all possible realizations of the CoRe, the TrCoh combines, compacts and synthesizes large amount of information about the natural scene. Hence, the TrCoh is a unique PolInSAR parameter with which the entire physical target is characterised in average, without being associated with a specific scattering mechanism (e.g. extreme coherences provided by the minimum and maximum ground contributions). This independence of the selection of the scattering mechanism makes the TrCoh also independent of the polarimetric basis. Therefore, matrices $[\hat{C}_{11}], [\hat{C}_{22}]$, and $[\Omega_{12}]$ in (10) may be obtained from the covariance $[\hat{C}]$ or coherency $[\hat{T}]$ matrices indistinctly. In the proposed methodology, the noise free TrCoh, i.e. compensated by SNR and BAQ decorrelation terms, is directly obtained from the de-noised covariance matrices (9) using (10).

Alternatively, the de-noised TrCoh can be computed independently of the de-noised covariance matrix, i.e. directly from the measurements $[\hat{C}]$. In that case, the compensation of the SNR decorrelation for the TrCoh would be performed based on traces:

$$
\text{SNR}_1 = \frac{\text{Trace}([\hat{C}_{11}])}{\text{Trace}([N_{1}])}, \quad \text{SNR}_2 = \frac{\text{Trace}([\hat{C}_{22}])}{\text{Trace}([N_{2}])}.
$$

Please note that in (11), the noise of the partial target is not associated with a specific scattering mechanism. The $\gamma_{\text{SNR}}$ would be then computed as usual [12]:

$$
\gamma_{\text{SNR}} = \frac{1}{\sqrt{\left(1 + \frac{1}{\text{SNR}_1}\right) \left(1 + \frac{1}{\text{SNR}_2}\right)}}.
$$
With (12), the noise would be removed from the TrCoh by $\hat{\gamma}_{tr} = \gamma_{tr}/\gamma_{SNR}$. Finally, the noise free TrCoh would be obtained after compensation of BAQ effects: $\tilde{\gamma}_{trd} = \hat{\gamma}_{tr}/\gamma_{BAQ}$.

In a reduced dual-pol observation space (i.e. TanDEM-X data), the CoRe is contained in an ellipse (6). However, the real CoRe of quad-pol is not bound to be an ellipse. This means that some of the coherence values can only be observed using quad-pol, and the dual-pol CoRe is a subset of the full region. Obviously, once dual-pol data are acquired, we are only observing a subregion, and there is no way to know what the full region is. Trying to quantify the differences of dual- and quad-pol region on different target types is outside the purpose of this work. However, in Fig. 3 we show a preliminary test which suggests that the dual-pol region using HH-VV on crops is a decent approximation of the full region, and the best among all the dual-pol combinations in the linear basis. Therefore, we assume that the TrCoh in dual-pol HH-VV data is a good approximation for the TrCoh in quad-pol data over rice fields.

Fig. 3. CoRe an density map of a quad-pol system compared with the equivalent dual-pol CoRe. TrCoh of each polarimetric channel is represented as a coloured dot. Data corresponds to a maize sample of around 1.8 m height measured at 9 GHz (X-band) in the EMSL, JRC-Ispra.

Examples that offer an empirical physical interpretation of the TrCoh with the dual-pol data exploited in this study are shown in Fig. 4. The CoRe and its density map are represented for scenes with different shapes and distribution of points. In all cases, the TrCoh is close to the centre of mass of the CoRe. More validation examples about this feature are provided by Marino in [15]. Please note the centre of mass is the average of all the possible coherence values, but it is not the mode (i.e. peak) of the density function.

C. Line Fit and Estimation of the Coherence Extrema

A crucial step in the inversion algorithm is the line fit to the CoRe and the identification of the coherence extrema [2]. These coherences are provided by the two scattering mechanisms which maximise the separation of the phase centre [1], [11]. Among the different methods to obtain the coherence extrema [28]–[30], this process is usually performed in two steps [12], [13]. First, starting from the coherency matrix, the border of the CoRe is generated by means of an eigenproblem [17]. Then, the extreme coherences with minimum and maximum phase are identified. These coherences correspond to those with minimum and maximum ground contribution, associated with the projection vectors $\omega^*_\text{min}$, $\omega^*_\text{max}$, respectively. They are the coherences furthest and closest to the topographic phase.

In this paper, we propose to obtain the coherence extrema, i.e. the points which maximise the phase separation, from the analytical expression of the coherence loci. For dual-pol data, this corresponds to the locus of points defined by an ellipse in the complex plane, as in (5) and (6). The solution is straightforward. The starting point is the expression of the ellipse (6) in the complex plane. In Cartesian coordinates, $\gamma_{\text{cell}} = x + iy$:

$$
\begin{align*}
  x &= a \cos \phi \cos \theta - b \sin \phi \sin \theta + \Re(g_m) \\
  y &= a \sin \phi \cos \theta - b \cos \phi \sin \theta + \Im(g_m),
\end{align*}
$$

(13)

where $a$ and $b$ are the semi-major and semi-minor axis, $g_m$ the centre (as in (6)), $\phi$ the rotation angle, and $\theta$ the independent variable. From (13), the phase corresponds to $\varphi = \arctan(y/x)$. Thus, the minimum and maximum phases, i.e. $\varphi_{\text{min}}$ and $\varphi_{\text{max}}$, are provided by the critical angle

$$
\theta_c = \pm \arccos\left(\frac{pm - qn}{t}\right) - \arctan\left(-\frac{a}{k}\right),
$$

(14)
in which,
\[ n = a \cos \phi, \quad m = -b \sin \phi, \quad k = q \Re(g_m) - m \Im(g_m), \]
\[ p = a \sin \phi, \quad q = b \cos \phi, \quad s = n \Im(g_m) - p \Re(g_m), \]
\[ t = \text{sgn}(k) \sqrt{k^2 + s^2}. \]  

(15)

From (14) and (15), the coherence points of minimum and maximum phase are obtained. These points are the coherence extrema: \( \gamma(\tilde{\omega}_{\text{min}}) \) and \( \gamma(\tilde{\omega}_{\text{max}}) \).

In the end, the line fit to the CoRe is performed by connecting the computed extreme coherences. These two points are of special interest for the estimation of the ground topography, which is computed as in [12]. The intersection of the line defined by the extreme coherences and the unit circle, moving from \( \gamma(\tilde{\omega}_{\text{min}}) \) to \( \gamma(\tilde{\omega}_{\text{max}}) \), provides the estimation of the topographic phase, i.e. \( \phi_0 \).

Fig. 5 shows the difference between the CoRe obtained by means of an approximation [17], and analytically using the expression of the ellipse (6). Although the approximation is usually accurate enough, specific scenes where the difference is noticeable have been selected for illustration purposes.

Fig. 5 (a) corroborates that the dual-pol ellipse is distorted when computing it from the approximation. An extreme case is shown in the CoRe on the left, in which the approximated CoRe (i.e. orange dots) shrinks to a straight line, whereas the analytical CoRe (i.e. blue line) remains an ellipse. The approximated CoRe is formed by discrete, not continuous, points. In addition, the distribution of these points is not uniform but more concentrated at the more angular extremes.

A consequence of this is displayed in Fig. 5 (b). While the analytical extreme coherences correspond to the points of minimum and maximum phase, the approximated coherences, located at the area of low density of points, are slightly different. In this case, the difference between the analytical and approximated solutions leads to an important mismatch in the estimation of the topographic phase. Since the topographic phase is the first parameter to be estimated (see Fig. 2), a bias in its estimation results in a bias in the estimation of the rest of model parameters, i.e. \( h_v, \sigma, \mu_{\text{min}}, \mu_{\text{max}} \).

D. Coherence Loci of the RVoG Model

The RVoG model describes the scene as a two-layer medium comprised of a vegetation layer of randomly oriented scatterers, and a second layer of impenetrable ground surface [7]–[10]. While the scattering from the volume is distributed according to a vertical reflectivity function \( f(z) \) (e.g. exponential [12]), the scattering from the ground is located at a fixed point \( z_0 \). Assuming a dominant direct ground contribution [31], the PolInSAR coherence can be expressed as follows:

\[ \tilde{\gamma}(\kappa_Z, \tilde{\omega}) = e^{i\phi_0} \left( \tilde{\gamma}_V + \frac{\mu(\tilde{\omega})}{1 + \mu(\tilde{\omega})} (1 - \tilde{\gamma}_V) \right) \]

(16)

\[ = e^{i\phi_0} \left( \tilde{\gamma}_V + F (1 - \tilde{\gamma}_V) \right). \]

In (16), \( \kappa_Z \) is the vertical wavenumber which is driven by the effective spatial baseline [7], [8], \( \phi_0 = \kappa_Z z_0 \) is the topographic phase, and \( \tilde{\gamma}_V \) is the coherence of the vegetation volume alone, without any ground contribution. The term \( \mu(\tilde{\omega}) \) is the ground-to-volume ratio at a specific polarization. Thus, the polarization dependence is isolated in a single term \( F \) (i.e. with \( F \in [0,1] \)). From (16), it is derived that the coherence loci of the RVoG model are a straight line in the complex plane moving along the ground-to-volume amplitude ratio, \( \mu(\tilde{\omega}) \) [9]–[11]. In practice, the visible segment of this line extends from \( \gamma(\tilde{\omega}_{\text{min}}) \) to \( \gamma(\tilde{\omega}_{\text{max}}) \), which are the extreme coherences taken as input for inversion. The use of these two coherence points as input for inversion is justified in the fact that they are provided by the scattering mechanisms which yield the largest physical separation in the scene [1], [11].

The methodology proposed in this paper not only takes advantage of the information provided by the extreme coherences, but it also benefits from the added value provided by the TrCoh. Accordingly, the coherence loci of the RVoG model (16) is redefined as the straight line which crosses the
topographic phase $\phi_0$ and the TrCoh $\gamma_{tr}$:

$$
\tilde{\gamma}(\kappa_Z, \tilde{\omega}) = e^{i\phi_0} + F(\gamma_{tr} - e^{i\phi_0}).
$$

(17)

Hence, the coherences taken as input for inversion correspond to the crossings of the redefined RVoG line (17) with the ellipse which represents the border of the CoRe. Expressing the ellipse (6) as

$$
|h - \lambda_1| + |h - \lambda_2| = 2a,
$$

(18)

the system of equations formed by (17) and (18) is solved through the following quadratic equation:

$$
AF^2 + BF + C = 0 \Rightarrow F = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}
$$

with

$$
A = 4 \left( t_2 - 4a^2|\gamma_1 - \gamma_2|^2 \right)
$$

$$
B = 4 \left( t_1 t_2 - 8a^2(\Re(\gamma_1 - \lambda_2)\Re(\gamma_2 - \gamma_1) + \Im(\gamma_1 - \lambda_2)\Im(\gamma_2 - \gamma_1)) \right)
$$

$$
C = t_1^2 - 16a^2|\gamma_1 - \lambda_2|^2,
$$

and

$$
t_1 = |\gamma_1 - \lambda_1|^2 - |\gamma_1 - \lambda_2|^2 - 4a^2
$$

$$
t_2 = \Re(\gamma_2 - \gamma_1)\Re(\lambda_2 - \lambda_1) + \Im(\gamma_2 - \gamma_1)\Im(\lambda_2 - \lambda_1).
$$

(19)

In (19), $\gamma_1 = e^{i\phi_0}$ and $\gamma_2 = \gamma_{tr}$. The crossings $\gamma_{min}$ and $\gamma_{max}$ are obtained substituting the term $F$ in (17). This new set of coherences is employed for inversion.

Fig. 6 shows the CoRe of a selected scene with its characteristic parameters. The traditional RVoG coherence loci as a straight line defined by the extreme coherences and the topographic phase (in black) is compared with the redefined line crossing the TrCoh (in red). Although we can get different shapes and orientations of the CoRe depending on the physical properties of the volume and the underlying ground for real data [9] (as demonstrated in Fig. 1), the results shown here are very representative. The new coherences taken as input for inversion, $\gamma_{min}$ and $\gamma_{max}$, are consistently of higher magnitude compared to those provided by the extreme phases, i.e. $\gamma(\tilde{\omega}_{min})$ and $\gamma(\tilde{\omega}_{max})$. In principle, higher coherence magnitude translates into more accurate estimates. This supports the use of the new set of coherences for inversion.

At the end, the numerical inversion of the RVoG model is carried out by an iterative minimisation of the distance between the measured complex coherences, i.e. $\gamma_{min}$ and $\gamma_{max}$, and the modelled ones (16) [13]:

$$
\min_{h_v, \sigma, \mu_{min}, \mu_{max}} \left\| \begin{array}{l}
\gamma_{max} e^{-i\phi_0} - \tilde{\gamma}(\kappa_Z, h_v, \sigma, \mu_{max}) \\
\gamma_{min} e^{-i\phi_0} - \tilde{\gamma}(\kappa_Z, h_v, \sigma, \mu_{min})
\end{array} \right\|.
$$

(20)

This process yields the final vegetation estimates, i.e. $h_v$, $\sigma$, $\mu_{min}$, $\mu_{max}$.

E. Interpretation of the Data and the RVoG Model

This section provides additional considerations to help understand the interpretation of the data (i.e. measurements) and the RVoG model (i.e. actual physical scene).

As described in Section III-D, the RVoG model is employed to provide an analytical expression of the interferometric coherence as a function of some physical parameters of

the natural scene observed by the radar. It can be shown analytically that the coherence loci of the RVoG model follow a straight line in the complex plane (16). This line follows a physical criterion by definition. The aim of the inversion is to find out how to extract the RVoG line from the (measured) data available. Nevertheless, at the moment of extracting that line there exist different options.

One possibility consists in defining the RVoG line as that which crosses the measured coherence points with maximum phase separation [12]. This criterion assumes that these coherences are associated with the highest and lowest scattering centres of the scene along the vertical coordinate. Following this criterion, and moving along the RVoG line, the point with maximum ground contribution (i.e. $\mu \rightarrow \infty$) is assumed to be the topographic phase. However, the criterion to retrieve the line from the data could be different. It could be based on maximising either the coherence magnitude (see [11, Section 6.2.1 and 6.2.2]) or the geometrical separation of the coherences (see [11, Section 6.2.3]). In that case, with dual-pol data, the maximum separation is given by the major axis of the ellipse. Another possible criterion could be to exploit directly the coherences of the measured polarimetric channels (i.e. $\gamma_{HH}$ and $\gamma_{VV}$) [9]. All these strategies provide different inversion results.

An additional prove of this is the methodology proposed in this paper, in which the RVoG line is defined by the TrCoh and the topographic phase (17). The coherence points employed for inversion are then the crossings of that line and the elliptical CoRe.

The key point is that the interpretation of the actual RVoG line is independent of the polarimetric mode used. That is, it is independent of the subspace observed by the available polarisation channels in dual-pol. In other words, while the CoRe depends of the type of polarimetric data acquired (as shown in Fig. 3), the RVoG model represents the physical scene, which is the same regardless of the subspace measured by the dual-pol mode. Fig. 3 not only highlights the limits of a dual-pol observation with respect to quad-pol, but it also manifests the independence of the RVoG model with respect to the subspace observed by the dual-pol mode. Following,
e.g. the criterion of maximum phase separation, the observed CoRe from different dual-pol combinations leads to different estimates of the topographic phase, and thus, to different estimates of the remaining model parameters. This supports the use of the TrCoh in defining the inversion methodology. In Fig. 3, we can observe that the centre of mass for dual co-pol and quad-pol are very close to each other. This allows us a way to retrieve some information that is similar to the full space and, therefore, more similar to reality. Thus, the TrCoh is the point which best describes the physical target.

IV. Test Site and Data Set

A. Test Site and Ground Campaign Data

An area of rice fields is selected for the evaluation of crop height estimates. The test site, as shown in Fig. 7, covers an area of 30 km $\times$ 30 km located in Sevilla, SW of Spain (37.1 N, 6.15 W). As opposed to other cultivation practices [32], [33], the parcels are kept flooded during the whole rice growth cycle (from May to October), and not only in the early vegetative stage.

![Fig. 7. Coherence amplitude of the HH channel acquired on June 26 2015, showing the location of the study area in Sevilla, Spain. The test fields of the ground campaign are highlighted in orange.](image)

In 2015, the local association of rice farmers gathered agricultural descriptors of four rice fields on a weekly basis. This includes phenological information according to the BBCH (Biologische Bundesanstalt, Bundesforschungsamt und Chemische Industrie) [34] scale and above-water vegetation height. This study area has been widely employed for rice crop monitoring in former studies, e.g. [31]–[37].

B. TanDEM-X Data and SAR Processing

Dual-pol TanDEM-X SAR data acquired during the Science Phase of this mission are exploited. The data cover most of the phenological cycle of the monitored crops, from May to September 2015. Specific characteristics of the employed time-series are summarized in Table I. On account of the correlation between the incidence angle ($\theta_0$) and the height of ambiguity (HoA) [38], each time-series provides a different level of interferometric sensitivity to the vertical distribution of scatterers within the vegetation volume.

The SAR data processing starts from the standard Coregistered Single-Look Slant-range Complex (CoSSC) product of each acquisition. The large spatial baselines that characterise the input data (i.e. 2–3 km) lead to a notable geometrical decorrelation derived from changes in the wavenumber. Hence, range spectral filtering is applied to compensate for this decorrelation. Multi-looking is then performed by means of a $21 \times 21$ boxcar filter. Flat Earth and topographic phase contributions are removed before forming the interferograms, so they are actually differential interferograms. In this case, a 5 m grid size DEM provided by the Spanish National Centre for Geographic Information (CNIG) is used in the processing.

V. Results

This section presents the results of the proposed methodology, with a focus on the vegetation height. The assessment is performed by comparing these results to those obtained following the conventional PolInSAR method [12], [13]. Fig. 8 shows the temporal evolution of the vegetation height at four monitored fields for 22.7 degrees of incidence. The available time-series at this incidence angle is especially suitable for crop monitoring, since as represented in Table I, it is associated with the largest $\kappa_Z$ and shortest HoA, i.e. $\approx 2.48$ rad/m and 2.53 m, respectively. This ensures the highest interferometric sensitivity of the entire data set. In Fig. 8, the accuracy of the vegetation height estimates of the PolInSAR conventional method (denoted as M1, in red) and the proposed one (denoted as M2, in green) is analysed by direct comparison with the reference ground-truth data (in black). Although both methodologies perform relatively well, estimates obtained with the proposed approach based on the TrCoh are closer to the reference data in all cases.

Focusing now on a single field, Field 1, Fig. 9 shows the temporal evolution of the vegetation height for incidences of 30 (top) and 39 degrees (bottom). Similar results as those in Fig. 8 are observed. Vegetation height estimates retrieved with the proposed approach are more accurate than those of the
conventional method. Nevertheless, as one moves to shallower incidences, the improvement seems less noticeable. Hence, at 39 degrees a clear overestimation is still present at most dates, even with the proposed method. Although not shown here, analogous results are obtained for the remaining fields.

A common behaviour in both methodologies is that at the early season (DoY < 180), when plants are very small (i.e. below ∼30 cm), there is a lack of interferometric sensitivity to the vertical distribution of scatterers in the scene, which leads to overestimated vegetation heights [11], [13], [33]. The flooded ground reflects the incident signal in a specular direction, yielding low returns to the SAR antenna. At 22.7 degrees, the overestimation is mainly restricted to the first dates. However, at 30 and 39 degrees, due to the associated large HoA and reduced vertical wavenumber (see Table I), this effect is visible through the entire crop season. This explains the overestimation observed at these shallower incidences.

A quantitative comparison between average inversion height per field and date against ground-truth data is presented in Fig. 10. Previous comments on the trends observed in the estimated heights can also apply to the results in Fig. 10. For very short vegetation, a lack of interferometric sensitivity together with small residual non-volumetric decorrelation yield important height errors. Due to the very low signal returns at these dates, even after the compensation of the coherence, i.e. by SNR and BAQ corrections (see Sections III-A and III-B), the signal levels are still not enough to ensure a reliable inversion. Hence, there will always be some decorrelation that cannot be compensated. On this basis, larger effective spatial baselines than the ones provided in the current data set (see Table I) should be used to provide suitable short vegetation height estimates. In accordance with this criterion, statistics presented in Fig. 10 are computed for the range of values in which field measurements are above 25 cm (i.e. from the second available TanDEM-X acquisition onwards). Thus, the analysis is performed over \( n = 28 \) measurements. A root-mean-squared error (RMSE) of 23 cm and a determination coefficient \( R^2 \) of 0.48 are obtained with the conventional PolInSAR method. On the other hand, an RMSE of 16 cm and an \( R^2 \) of 0.58 are achieved with the proposed approach. Again, these numbers refer to rice plants taller than 25 cm.

VI. DISCUSSION AND CONCLUSIONS

This paper presents a modified PolInSAR inversion methodology based on the operator TrCoh. Key points of the proposed inversion methodology are reviewed to evaluate the added value of the TrCoh and the benefits of the use of analytical solutions:

- The formulation of the correction of non-volumetric decorrelation sources of the covariance matrix and the TrCoh leads to a direct and simple application. Higher coherences provide lower overestimation.
- The exploitation of the elliptical form of the dual-pol CoRe for the computation of the coherence extrema speeds up the inversion and provides more accurate results not based on approximations.

Fig. 8. Temporal evolution of the vegetation height. Estimates obtained following the conventional PolInSAR method (red) are compared with estimates obtained with the proposed approach (green), and with the ground-truth data (black). Columns from left to right show estimates of the four monitored rice fields in Sevilla, for an incidence of 22.7 degrees. Average results computed for all pixels inside each field are presented. Error bars denote standard deviation.

Fig. 9. Temporal evolution of the vegetation height: Field 1. Estimates obtained following the conventional PolInSAR method (red) are compared with estimates obtained with the proposed approach (green), and with the ground-truth data (black). Results are presented for an incidence of 30 degrees (top) and 39 degrees (bottom). Average results computed for all pixels inside each field are presented. Error bars denote standard deviation.

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The inclusion of the TrCoh in the PolInSAR retrieval algorithm is analysed for the first time. As a result, the coherence loci of the RVoG model is forced to cross the TrCoh. This produces a line which is more stable, since it is more related to the bulk of the distribution of coherences in the polar plot.

Thanks to the expressions of the CoRe and the TrCoh, the analytical selection of the new set of coherences used for inversion is faster than other methods based on approximations [17].

The suitability of using the TrCoh for inversion is justified in an improved accuracy of the resulting vegetation height estimates. An RMSE of 16 cm is obtained with the proposed method, which is an improvement of 7 cm with respect to the conventional PolInSAR method. Regarding the coefficient $R^2$, the accuracy improves by 10% with the proposed approach. We interpret this is because we constrain the line to pass through the area of most points in the CoRe (capturing more of the target). At the same time, this justifies why we obtain more accurate results with the TrCoh than following other criteria based only on the CoRe, such as employing the vertices of the major axis of the ellipse for inversion. Although not included here, results following this latter approach are far worse than those based on the TrCoh (or on the maximum phase separation, like the conventional PolInSAR method). This leads us to conclude that the selection of the measured coherences employed for inversion, as modelled by the RVoG, needs to focus on the most significant representation of the physical target, which is not represented by the dual-pol ellipse axes.

In addition, the use of analytical solutions enhances the computational efficiency. A 16 cores computer with 160 GB RAM, and a scene of $1500 \times 300$ pixels corresponding to Field 1 (e.g. on 15th June) are considered to quantify the improvement. Since the minimisation is a common step (see Fig. 2), the comparison between the conventional and the proposed method is performed without considering this final stage. Thus, the conventional algorithm, with $N = 72$ points to simulate the CoRe, takes around 6 minutes 4 seconds, whereas the same required steps with the proposed approach are performed in 6 seconds. As the number of points used in the conventional method to approximate the CoRe increases, so does the computational cost. For instance, employing $N = 720$ points would be required to avoid important mismatches, as the one observed in Fig. 5 (b). Such a larger number can help prevent the important bias observed in the extreme coherences computed with the approximate method. The density of points of the approximated border of the CoRe would be higher, and thus the approximated extreme coherences would be closer to their actual values. In that case, the computational cost increases up to 8 minutes.

Overall, the modified PolInSAR parameter retrieval strategy based on the TrCoh yields more accurate height estimates in a more efficient and analytical manner. Moreover, the methodology presented here can apply to other scenarios, e.g. crops with non-flooded ground conditions or forest scenes.

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Fig. 10. Correlation plots of the vegetation height estimates with respect to the ground-truth data. Method 1 corresponds to the conventional RVoG inversion, whereas Method 2 corresponds to the proposed approach. Statistics are computed for the range of values in which field measurements are above 25 cm (i.e. from the second available TanDEM-X acquisition onwards).