Multiscale stock-bond correlation: Implications for risk management

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Abstract

This paper examines the multiscale return correlation between the stocks and government bonds of different maturities returns in 25 countries. The analysis reveals that developed markets correlations are generally negative at the first time-scale and move in a positive direction at higher scales. This contrasts with emerging markets, where the correlation tends to be positive throughout. Thus, the results support a greater flight-to-safety effect in developed markets. Further evidence highlights the ability of the correlation to produce portfolios with a lower VaR. Results support this at longer time-scales and for both developed and emerging markets. The results here demonstrate the importance of accounting for time-scales in modelling the stock-bond correlation and in constructing portfolios.

Keywords: Correlation; Wavelet; Flight-to-Safety; Value-at-Risk.

JEL Classification G01; G12; G15; C58; C63

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1. Introduction.

An inverse correlation between bond and stock markets is a phenomenon of notable interest to financial market participants, being associated with a fight-to-safety effect (see, for example, Ilmanen, 2003; Gulko, 2002). Baur and Lucey (2009) show that such flight-to-safety behaviour tends to spread simultaneously across countries. An open question for investors is how the correlation varies across time horizons and to what extent this variation is important in managing portfolio risk.

Dimic et al. (2016) find that stock and bond correlations vary significantly over different time horizons in ten emerging countries.[1] They also find that global stock and bond market uncertainties play a major role in explaining the stock and bond correlation in these countries. Sakemoto (2018) finds that a rise in uncertainty can lead to 'fear' among investors, resulting in flight-to-safety. Indeed, Skintzi (2019) finds evidence of such an effect in the Euro-area during periods of financial distress. Further studies consider the effect of the 2007-2009 global financial crisis in conditioning the correlation. Mustafa et al. (2015) report investor flight to Islamic and conventional corporate bonds throughout this crisis period, although does not consider time-scale effects on the correlation.

For European markets, Kim et al. (2006) find that economic integration typically increases the stock-bond correlation. Cappiello et al. (2006) examine asymmetries in the daily stock-bond correlation in European, North American and Australasian markets. They find evidence of flight-to-safety in European markets before and after the establishment of monetary union. Using daily stock and 10-year government bond returns for 23 countries, Baele et al. (2020) find that flight-to-safety is a country specific phenomenon rather than a global one and is related to uncertainty (e.g., higher VIX and TED spread), but decreases in the level of investor sentiment and appreciations of certain currencies including the U.S dollar.

[1] Their study comprises of only emerging markets and employs a continuous wavelet approach. Our study is based on a sample of 25 developed and emerging markets and utilises the (more advanced) stationary wavelet approach.

This paper examines the stock and bond return correlation across twenty developed and five emerging markets using a maximum overlap discrete wavelet transform (henceforth, MODWT) to consider any variation across time-scales over the period from 1991 to 2016. This wavelet approach has several advantages over alternative wavelet and filtering methods. First, it reveals information from the series across time and frequency without losing any observations. Second, the decomposition using this approach distributes the variance of the series across the time-scales without any loss. Third, this method does not need the series to be stationary. As noted by Conlon et al. (2018), wavelet analysis is appropriate for the study of financial time series, as it helps at separating long-run and short-run movements.

This study is closely related to, and based on, the work of Kim and In (2007) and Lehkonen and Heimonen (2014). Kim and In (2007) employ a wavelet correlation approach and examine the correlation between stocks and long-term government yields in G7 markets. They find that stock and bond returns do not move together over different time-scales. Their study, however, does not examine the correlation across market states, nor the implications for investors. In contrast, we consider a Value-at-Risk (VaR) exercise in order to examine such implications. Lehkonen and Heimonen (2014) examine daily BRIC and developed stock market return series. They employ an asymmetric Dynamic Conditional Correlation-General Autoregressive Conditional Heteroscedasticity (ADCC-GARCH) model and find that the stock market co-movement depends on several factors including regional factors, the level of development and the time-scale of the return series. Their findings indicate that countries in the BRIC region cannot be considered as a homogenous group in terms of stock co-movements and for the purpose of diversification.

We investigate the stock-bond correlations over short- and long-term horizons time-scales and across recessionary and expansionary periods. This allows for the examination of potential flight-to-safety periods, while also considering attendant implications for investors. Our results suggest the following. The variance of stock returns is greater during expansions relative to contractionary periods. In comparison, the position with regard to bonds is less clear, with differences across markets and maturities. The sign of the wavelet-based correlation differs across the time horizons. In developed markets the corelation has a tendency to be negative at the first time-scale and the move in a positive direction at longer time-scales. For emerging markets, however, there is greater evidence of a positive correlation throughout.

In seeking to examine the implications of our results, we consider the role of the time-varying correlation in a VaR exercise. The results indicate that VaR, an indicator of portfolio risk, improve when accounting for the time-varying stock and bond correlation, especially at lower frequencies. This, therefore, emphasises the importance of considering time-scales when managing stock-bond portfolios.

The remainder of this paper is organised as follows. Next section summarises the related literature. Sections 3 and 4 describe the data and the methodology respectively. Section 5 discusses the main findings while Section 6 empirically outlines the implications of the findings and Section 7 concludes.

2. Literature Review.

Within financial markets, the wavelet approach is used to examine the dynamic relation among variables. For example, using data for wholesale and retail managed fund returns, In et al. (2008) show that as time-scales increase, the Sharpe ratio for each fund increases, reaching an extreme at the 32-64 month horizon. In a time-scale examination of portfolio allocation, Kim and In (2010) discover that with an increase in investor horizon, allocation decisions move towards risky assets for identical risk tolerance levels. In examining the role of scaling in portfolio formulation, Kim et al. (2010) test the wavelet approach with both the Fama and French three factor model and the capital asset pricing model (CAPM). In general, their results show that at long time horizons, stronger relations exist between risk factors and the market return, especially for large stocks. In an analysis of seven Gulf equity markets under the CAPM, Masih et al. (2010) note that beta, as a measure of systematic risk, has a tendency to increase at higher time-scales.

Regarding the role of time-scales in volatility, Gençay et al. (2010) document that when high time-scales reveal a low realized volatility regime, it is typically followed by low volatility at shorter time-scales. However, the opposite relation is not supported in a high volatility regime. Based on their result, Gençay et al. (2010) propose what they term the ‘asymmetric in information flow between volatilities across scales’. The volatility scaling approach is utilised by Sun et al. (2011) investigating the effect of four representative macroeconomic news releases on the volatility of high tick exchange rate data. Their results indicate that for the first wavelet, intraday clustering is more apparent after the news release compared to before. Conlon and Cotter (2012) investigate the appropriateness of wavelet decomposition for hedging. Decomposing cash and future returns (West Texas Crude oil, the S&P 500 index and British Pound/U.S. Dollar exchange rate), the study indicates that hedging effectiveness approaches the maximum level at longer-time horizons. Interestingly, the hedge ratio is exactly one at the 12-day horizon.

Recent studies also apply wavelets to explore relations among financial or economic variables (see, for example, Rua, 2012) and the interdependencies between different financial markets (e.g., Kiviaho et al., 2014; Alzahrani et al., 2014; El. Alaoui et al., 2015; Bekiros, 2016; Ftiti et al., 2015; Dewandaru et al., 2016). Gallegati et al. (2011) indicate that a negative relation between U.S. quarterly wage inflation and the unemployment rate appears to be more evident at short time horizons over the period 1948-2009. Estimating the Phillips curve regression indicates greater stability in the sub-period 1948-1993, but not subsequently, which they argue is due to a change in wage setting behaviour following low inflation levels after the mid-1990s. Using continuous wavelet analysis, Rua (2012) investigates the relation between aggregate M3 money growth and inflation in the Euro area. The results suggest a weaker relation at low time frequencies. The findings of both Rua (2012) and Gallegati et al. (2011) demonstrate the importance of using time-scales to investigate economic relations.

To explore causality between monthly U.S. dollar exchange rates and the oil price, Benhmad (2012) uses the Maximal Overlap Discrete Wavelet Transform (MODWT). The results document both linear and nonlinear bidirectional relations at the 32-64 month time horizon and higher, while the oil price is found to Granger cause the dollar at the shorter 2-4 and 8-16 month time-scales. The interdependence between daily U.S. dollar exchange rates and oil is studied by Robredo and Rivera-Castro (2013). Their results indicate that this relation is more stable over time-horizons before the 2008 crisis and less so afterwards.

Using a sample that covers the dot-com crash, the financial crisis and its subsequent effects, Martín-Barragán et al. (2015) find that the relations between oil price and four developed stock markets fluctuate the greatest at longer time-scales during major financial shocks. In contrast, Ftiti et al. (2015) show that co-movement between oil and stock prices in G7 countries is greatest at short- and intermediate time-scales. Bekiros et al. (2016) focus on the S&P 500 and eleven commodity markets. Their results show clear evidence that co-movements between these markets vary over the time-scales with an increase in this relation in the post financial crisis period.

Bekiros and Marcellino (2013) examine causality between three foreign exchange rate returns and report different causality characteristics across time-scales. Benhmad (2013) examines the correlation between the S&P 500 and other international stock market returns using the wavelet correlation in a rolling regression framework (with a window size of 250 days). Benhmad (2013) finds that correlation dynamics are more evident during a crisis and fluctuates significantly across scales. The same conclusions are reached by Kiviaho et al. (2014) using a different wavelet approach. This confirms that correlations among stock markets are functions of time-scales and have a tendency to increase during crisis periods. Kiviaho et al. (2014) also document evidence that the effects of some macroeconomic factors impact co-movements differently over time horizons. In wavelet-based causality relations, Alzahrani et al. (2014) study the lead-lag relation between the oil spot and future markets in the U.S. Using daily data, the results show that bidirectional causality exists between the two markets across all different time horizons. Rahim and Masih (2016) use both continuous wavelet and MODWT methods and document evidence of varying levels of independence across the time-scales in Malaysian Shari'ah portfolios.

3. Data and Descriptive Statistics.

Daily data for Government bond yields and stock returns, denominated in U.S. dollars, are obtained from Datastream. The sample comprises 20 developed and 5 emerging markets.[2] The sample period ranges from June 24, 1991 to November 30, 2016 totalling 6638 observations. The exceptions are for Norway and Portugal, where the sample starts on December 1, 1992 and January 1, 1996 (totalling to 6262 and 5458), respectively. Table 1.A presents summary statistics for the stock returns and long-term bond yields for developed markets, calculated as logarithmic difference. As shown in Panel A of Table 1.A, the mean stock returns is almost always positive and varies slightly across the 20 developed markets. Notably, Belgium has the highest mean, followed by Austria, Italy and Finland, with all other developed markets relatively similar. As is typical with financial markets, the return standard deviation for the 20 markets is larger than the mean return, and is greatest for Finland and lowest for both Denmark and New Zealand.

Panel B of Table 1.A presents the descriptive statistics for the Government bonds returns and reveals a number of points. First, the 2-year government bonds tend to exhibit a similar mean of 0.0001. Exceptions to this include Australia, Austria, Denmark, Finland, Japan, Netherland, Sweden and Switzerland with a negative average return of -0.0001. In terms of the

[2] The reason for selecting fewer emerging markets is the availability of government bond data, especially in ensuring a sufficient history.

standard deviation, the 2-year bond of Portugal has the highest value (0.0013) while those of Germany and Japan are the lowest (0.0003). Second, across the markets, the average return on the 5-year government bond is 0.0001, except for Belgium, which has a negative average return of -0.0001. The standard deviation of the US is highest (0.0027) while that of Japan and Netherland is lowest (0.0007). Third, Australia and Japan have the highest average return on the 10-year government bond of 0.0003 and 0.0002 respectively, while all other countries have the same average return of 0.0001. In addition, the standard deviation is highest for the U.S. 10-year government bond (0.0046) and lowest for Switzerland (0.001).

Table 1.B presents the descriptive statistics of stock returns and bond returns for the emerging markets. For stock returns, all markets, except Greece, show a positive mean return over the sample period. At the same time, Greece exhibits the highest standard deviation with South Africa presenting the lowest. Regarding the return on the 2-year, 5-year and 10-year government bonds, all show a positive rate of return except the Czech Republic on the 2-year bond and Greece across all maturities. Greece also appears with the highest standard deviation for the two, five and 10-year government bond.

In general, when comparing developed with emerging markets, we note that the stock market return is broadly similar across markets, with Belgium having the highest average return. However, the standard deviation of stock returns is generally higher for emerging markets although Finland had the highest standard deviation (0.0077). This pattern is broadly repeated when comparing government bonds. Although, again, there is some evidence that the standard deviation is higher for emerging markets.

4. Empirical Methodology.

We use the wavelet approach, for which a key advantage is its ability to examine co-movement between assets while considering both the time and frequency domains in an integrated framework. We use the stationary (MODWT) wavelet transform to decompose each return series on a time-scale-by-scale basis. The process starts by using a scaling index j and translation index k that both contribute to the decomposition process, which is mainly based on the mother wavelet:

$$\phi_{k,j} = \frac{1}{2^{j/2}} \phi \left(\frac{t-2^j k}{2^j}\right) \tag{1}$$

Which integers to one at a given time- scale j (i.e. $\int \phi(\mathrm{t}) d_j = 1$). A father wavelet is given by:

$$\psi_{k,j} = \frac{1}{2^{j/2}} \psi \left(\frac{t-2^j k}{2^j}\right) \tag{2}$$

Which integers to zero such as $\int \psi(\mathrm{t}) d_j = 0$, where 2^j is a measure of the scale, or width, of the functions $\phi_{k,j}$ and $\psi_{k,j}$.

Our decomposition proceeds by incorporating both mother and father wavelets in a linear combination through a high- and low-pass filter. Using the low-pass filter, the return series R_n for the number of observations n is decomposed into sub-series. These approximation elements, A_n, capture events that are long in time and rarely occur with respect to the frequency. The high-pass filter, on the other hand, creates more detailed components, D_j, that are short in time and high in frequency. The overall process is described as follows:

$$V_{i,t} = \phi A_n + \sum_{j=1}^{n} \Psi D_j \tag{3}$$

To examine the correlation between stocks and bonds across time scales, the variance and covariances must first be estimated. Using the detailed coefficients, wavelet-based variances at given time-scale $\lambda_j \equiv 2^{j-1}$ can be calculated as follows:

$$\tilde{\sigma}_s^2(\lambda_j) = \frac{1}{\tilde{N}_j} \sum_{t=L_{j-1}}^{N-1} \tilde{D}_{j,t}^2, \tag{4}$$

Where s denotes either the stock (x) or bond (y) return series. According to Gençay et al. (2001), the above calculation also relaxes the requirement of dyadic sample size and makes it subjective to the length N. Additionally, $L_j = (2^j - 1)(L-1) + 1$ denotes the length of the scale

λ_j wavelet filter, while $\tilde{N}_j = N - L_j + 1$ is the number of coefficients to be unaffected by the boundary. As a result, a larger variance is obtained if this difference is also large. The wavelet-variance estimator is also unbiased when it is applied to a stationary time series and it produces a zero mean for $\tilde{D}_{j,t}$ at any time-scale with the differencing embedded within the filter (Percival and Walden, 2000).

After calculating the variances of the stock and bond return series, the unbiased MODWT-based estimator of covariance can be given by:

$$Cov_{X,Y}(\lambda_j) = \frac{1}{\tilde{N}_j} \sum_{t=L_{j-1}}^{N} \tilde{D}_{j,t}^{X} \tilde{D}_{j,t}^{Y} \quad (5)$$

The wavelet-based correlation estimator is then obtained by dividing the covariance over the product of the squared root of the *x* and *y* variances as such:

$$\tilde{\rho}_{X,Y}(\lambda_j) = \frac{Cov_{X,Y}(\lambda_j)}{\tilde{\sigma}_x(\lambda_j)\tilde{\sigma}_y(\lambda_j)} \quad (6)$$

Percival and Walden (2000) emphasise that the wavelet variance estimator must be unbiased to preserve the same degree of variance in the time series when decomposing over the subsequent time-scales. In another step, Percival and Walden (2000) define a random confidence interval, which, in turn, must vary when the variance, covariance or correlation is estimated across scales. That is, at p=5% significance level, the (1-p) ×100% confidence interval with its lower and upper components can then be given by:

$$[\nu_s^2(\lambda_j) - \Phi^{-1}(1-p)\sqrt{\mathrm{var}(\nu_s^2(\lambda_j))}, \nu_s^2(\lambda_j) + \Phi^{-1}(1-p)\sqrt{\mathrm{var}(\nu_s^2(\lambda_j))}] \quad (7)$$

With the assumption that for $\Phi^{-1}(1-p)$ being the $(1-p)$ percentage point is for the Gaussian distribution and this holds throughout the analysis.

Regarding the selection of the appropriate filter, we follow previous work, for example by Daubechies (1992) and Percival and Walden (2000), among others, and based on their choice for mother wavelet, use the Daubechies least asymmetric with the length of 8 (D8, hereafter). This filter is asymmetric and has the property of generating optimally parallel wavelet coefficients within a given time series. Further, Kim and In (2010) prove that this element is sufficient in representing volatile time series. An important element in the decomposition process, is the number of resolution levels to be used. For our purposes, we decompose the return series at six levels (J=6).[3] Moreover, among alternative wavelet transforms, the approach here keeps the same number of observations as the original series each time the decomposition is performed. Hence, we undertake the analysis using all the observations, while allowing the variance of the decomposed series to change over time-scales.

5. Empirical Results.

5.1. Difference in Wavelet-based variance across economic states

We begin by examining the differences in stock and bond variances across time-scales before considering the nature of their correlation. We estimate the variances over different time-scales, as noted in Section 4, and illustrate how the variance changes across regimes of behavior. Specifically, the variance on each time-scale during an economic expansion is subtracted from its counterpart during the recessions. For the US, we use the NBER recession indicator while the OECD database is used to define the economic states of the other countries. The results from the analysis on the developed and emerging markets is reported in Table 2.

Panel A.1 of Table 2 reveals that for the stock market, significant differences in return variance typically arise at the longest horizon of 64-128 days. Moreover, where the differences are significant, the value is generally negative indicating a higher variance in an expansionary period (exceptions to that include Denmark, Germany and Norway). Panels A.2 and A.3 report the results of the same exercise for the 2-year and 10-year bond markets respectively. Here again, greater significance is found at the long horizon. For the 2-year bond market, the sign of the difference in variance is split equally between positive and negative values. For the 10-year

[3] Scales between 5 and 7 are usually considered appropriate in the task of decomposition irrespective of the frequency of data at hand. For instance, Kim and In (2005) use up to 7 scales with monthly data, while Galagedera and Maharaj (2008) decompose their daily return data at 6 scales.

bond, as with the stock return, the difference is typically negative, indicating higher variance in an expansionary period.

The results for the emerging markets broadly reflect those of the developed markets. That is, the significant variance differences typically arise at the longer time horizons and that they are mostly negative for the stock and 10-year bond return variance but are more positive for the shorter bond. The results also indicate that the variance differences are smaller in emerging markets compared to developed markets. Given the generally higher variance in emerging markets, this is likely an indication that variance remains high regardless of the economic state, whereas developed markets are more likely to exhibit low variance regimes. In comparing the two groups of bonds, the variance difference is generally lower for the 2-year bond compared to the 10-year one. This finding is likely to reflect the lower level of risk associated with a shorter bond.

5.2. Graphical analysis for the wavelet unbiased correlation between stock and bond return

Panels A.1, A.2 and A.3 of Figure 1 illustrate the MODWT-based wavelet correlation between stock returns and 2-year, 5-year and 10-year bond returns, respectively, for the developed markets across our sample period. The sold line indicates the wavelet correlation between stock return and bond return of different maturities over time-scales up to the sixth scale. The upper and lower dotted lines indicate the 95% confidence intervals.

In Panel A.1, which shows the stock return and 2-year bond return correlation, the figure reveals a predominantly negative relation in 14 out of the 20 developed markets up to the fifth scale, while from the sixth wavelet scale, there is greater evidence of a positive relation, although it is mixed and with the majority of these correlations not significant. The remaining markets reveal a positive correlation in all time-scales except for Ireland, which has a negative correlation in fifth and sixth scale. Similar results are found for the correlation between the stock and 5-year bond returns in Panel A.2 of Figure 1. Here, 12 of the 20 markets exhibit a negative relation until the fifth time-scale. All other countries show a positive relation across all time-scales except, again, for Ireland, which shows an erratic pattern switching between negative and positive relation over the different time-scales. This pattern is largely repeated again in Panel A.3 for the stock and 10-year bond returns, with a negative relation over the first scales before switching to a more positive one at longer scales but again with limited statistical significance.

Figure 1 Panels B.1, B.2 and B.3 presents the correlation between stock returns and 2-year, 5-year and 10-year bond returns, respectively, for the emerging markets. Unlike the correlations in Panel A, most of the emerging markets have a positive correlation on the first time-scale. This finding becomes clearer once we consider bond maturities beyond the 2-year bond. Moreover, this correlation has a tendency to increase at higher time-scales, strengthening the positive correlation, with the exception of Greece.

Overall, while the developed market correlations are generally negative (with some exceptions) at the first time-scale, they move towards a positive direction at higher scales. Emerging market correlations are typically positive at the first time-scale and likewise move in a positive direction. In terms of statistical significance, for the developed markets, correlations are more likely to be significant at the shorter horizons than the longer horizons. While the same is broadly true for the emerging markets, there is greater significance at the longer horizons especially for the longer maturity bond correlations.

5.3.The difference in wavelet-based correlation across sub-periods

Figure 2 Panels A.1-A.2 and B.1-B.2 compare the wavelet-based stock and 2-year and 10-year bond returns correlation during contractions and expansions for the developed and emerging markets, respectively. Panel A presents the results for developed markets and shows that a negative correlation is evident in most of the markets at all the time-scales during recessions. For expansionary periods, there is greater evidence of a positive correlation, although for some markets, the evidence is more mixed. This includes for the UK and US, for which the predominant evidence is of a negative correlation. These finding are broadly consistent across bond maturity, although there is greater evidence of a positive correlation during expansions with the 10-year bond. In contrast, the results in Panel B, suggest that correlation are positive for emerging markets in both contractionary and expansionary periods. Although for Greece there is notable evidence of a negative correlation during an expansion.

The evidence of a negative correlation during a recession for most of the developed markets is consistent with a flight-to-safety argument. For example, Baele et al. (2020), who use daily stock and bond data, find that flight-to-safety events are country specific and tend to coincide with increases in implied volatility, the TED spread and decreases in consumer sentiment indicators. Our evidence for a negative correlation in developed, but not emerging markets, appears consistent with this argument. A further explanation is the resilience of emerging markets, notably to the global financial crisis, which results in a smaller flight-to-safety, and the higher GDP growth relative to the developed markets (Kose and Prasad, 2011).

6. Portfolio Risk Management.

Several studies indicate the importance of considering the wavelet decomposition in risk management (see, for example, Fernandez, 2006; Rua and Nunes, 2012; Mensi et al., 2018; Meng and Huang, 2019). Following these studies, we use value-at-risk (VaR), as a well-known risk management measurement, to examine the performance of stock and bond portfolios across time-scales. Specifically, VaR shows the maximum loss of a portfolio at a pre-determined confidence level over a given period of time. The VaR of a portfolio comprising *n* number of assets at the (1-α) confidence level can be given by:

$$VaR = I_0 \phi^{-1}(1-\alpha)\sigma_P \tag{8}$$

Where I_0 relates to the value of the portfolio, $\phi(.)$ is the standard normal-based cumulative distribution function and σ_P is the standard deviation of the portfolio return. For a portfolio consisting one stock and one government bond (n=2), the variance can be calculated as follows:

$$\sigma_p^2 = \omega_i^2\sigma_i^2 + \omega_j^2\sigma_j^2 + \sum_{i}^{n}\sum_{i \neq j}^{n} 2.\omega_i\,\omega_j \operatorname{cov}(r_i, r_j) \tag{9}$$

Where ω_i and ω_j are the weights of the stock and bond in the portfolio respectively, σ_i^2 and σ_j^2 are their corresponding estimated return variances and $\operatorname{cov}(r_i, r_j)$ is the covariance between the returns of the two assets.

To illustrate the role of time-scales within risk management, we calculate the portfolio variance and return using a 40/60 (stock/bond) weighting.[4] In examining the effect of wavelet analysis on VaR, we follow Meng and Huang (2019) and Mensi et al. (2018), among others. Specifically, we first estimate the VaR at a given time-scale assuming no co-movement (i.e., $\operatorname{cov}(r_i, r_j)$ =0) between stock and bond returns. Second, we estimate the correlation and thus allow it to take a non-zero value. To examine the effect of co-movement at each time-scale, we calculate the ratio of VaR without the zero restriction (henceforth, VaR_c) to that with the zero correlation restriction, VaR_u. Thus, we can test whether estimating the correlation across different time-scales affects the VaR estimate at each scale, such that if the $\frac{VaR_c}{VaR_u}$ ratio is less (more) than one, the portfolio incorporating the time-varying corelation improves performance over the zero correlation portfolio.

Table 3 reports the VaR ratios for portfolios comprising the 10-year bonds along with the stock returns.[5] Again, the analysis is performed over expansionary and contractionary periods. Several interesting observations are revealed in the table. First, the VaR deviates from one in almost all markets regardless of the market state meaning that stock-bond co-movement

[4] We consider alternative 60/40 and 50/50 portfolios with similar results.
[5] The analysis with the alternative bond maturities qualitatively similar results to that with the 10-year bond. Therefore, these are omitted to save the space and available upon request.

affects the portfolio loss both over the sample and across the time-scales. Second, the VaR ratios are generally less during expansions relative to recessions, although there are exceptions to this, notably for the emerging markets as well as Australia and Portugal. Third, the VaR ratios decrease at longer frequencies and this is clearly evident at time-scales five and six.[6] For example, the ratio is smallest at the 64-128 days horizon during expansions for 12 countries (Australia, Austria, Belgium, Ireland, Japan, Spain, Sweden, Norway, Greece, Hungary and Czech Republic). Moreover, the VaR ratio is also at the lowest level for the horizon of 32-64 during expansions for Canada, Denmark, Finland, Italy, New Zealand, UK, South Africa and Poland. For recessionary periods, the lowest ratio tends to be found at the 32-64 time-scale across the majority of the countries.

7. Summary and Conclusion.

Understanding the nature of the stock and bond correlation is important given the key role it plays in portfolio formation. While this relation receives a large degree of attention in the literature, we contribute by, primarily, examining how the correlation varies over time-scales using a Wavelet approach. Further, we contribute by considering the correlation for 25 developed and emerging markets and using three bond maturities over the period from 1991 to 2016, whereas exiting research typically focuses on a more limited set of markets.

An initial examination of volatility over different time-scales reveals differences in volatility between expansionary and recessionary periods, with the difference lower for the correlation involving the 10-year bond return compared with bonds at higher maturities. Our analysis for the correlation across time-scales documents that developed market correlations are generally negative at the first time-scale but move in a positive direction at longer time-scales (although often insignificantly so). In contrast, for emerging markets, the correlation is

[6] This evidence confirms those of Mensi et al. (2018) and Meng and Huang (2018) who reached the same conclusion in their portfolio analysis. although, in comparison with these papers, we observe greater differences between the VaR ratios across market states at higher scales

positive even at the first time-scale. In addition, we find more evidence for flight-to-safety behaviour during recessions for developed markets compared to emerging markets. A key element of the correlation relates to portfolio management. We, therefore, examine the ability of the wavelet correlations to improve VaR modelling. Our results indicate VaR improvement across time-scales with the wavelet correlations, with the decrease in portfolio loss at the highest time-scales of 32-64 and 64-128 days.

The results in this paper should enable both investors and regulators to better understand the interrelation between stock returns and government bond return and the changes over different time horizons. Notably the results reveal the nature of changing correlations across time-scales and that it can improve portfolio risk performance.

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Table 1.A Description of the data and Summary Statistics- Developed Markets.

#	Country	Sample starting date (Day/Month/Year)	Mean	Std. Deviation
	Panel a. Stock markets			
1	**Australia**	24/06/1991	0.0001	0.0040
2	**Austria**	24/06/1991	0.0003	0.0046
3	**Belgium**	24/06/1991	0.0009	0.0045
4	**Canada**	24/06/1991	0.0001	0.0042
5	**Denmark**	24/06/1991	0.0001	0.0033
6	**Finland**	24/06/1991	0.0002	0.0077
7	**France**	24/06/1991	0.0001	0.0053
8	**Germany**	24/06/1991	0.0001	0.0051
9	**Ireland**	24/06/1991	0.0001	0.0053
10	**Italy**	24/06/1991	0.0003	0.0059
11	**Japan**	24/06/1991	0.0000	0.0056
12	**New Zealand**	24/06/1991	0.0001	0.0033
13	**Netherlands**	24/06/1991	0.0001	0.0051
14	**Norway**	01/12/1992	0.0001	0.0059
15	**Portugal**	01/01/1996	0.0001	0.0049
16	**Spain**	24/06/1991	0.0001	0.0055
17	**Sweden**	24/06/1991	0.0001	0.0062
18	**Switzerland**	24/06/1991	0.0001	0.0045
19	**U.K**	24/06/1991	0.0001	0.0040
20	**U.S.**	24/06/1991	0.0001	0.0050
	Panel b. Government bonds			
	Country	**Used Gov. bonds data**	**Mean**	**Std. Deviation**
1	**Australia**	2-year 5-year 10-year	-0.0001 0.0001 0.0003	0.0006 0.0012 0.0022
2	**Austria**	2-year 5-year 10-year	-0.0001 0.0001 0.0001	0.0003 0.0008 0.0014
3	**Belgium**	2-year 5 year 10-year	0.0001 -0.0001 0.0001	0.0004 0.0009 0.0015
4	**Canada**	2-year 5-year 10-year	0.0001 0.0001 0.0001	0.0005 0.0011 0.0017
5	**Denmark**	2-year 5-year 10-year	-0.0001 0.0001 0.0001	0.0005 0.0013 0.0015
6	**Finland**	2-year 5-year 10-year	-0.0001 0.0001 0.0001	0.0006 0.0010 0.0016
7	**France**	2-year 5-year 10-year	0.0001 0.0001 0.0001	0.0004 0.0011 0.0012
8	**Germany**	2-year 5-year 10-year	0.0001 0.0001 0.0001	0.0003 0.0008 0.0015
9	**Ireland**	2-year 5-year 10-year	0.0001 0.0001 0.0001	0.0012 0.0010 0.0013
10	**Italy**	2-year 5-year 10-year	0.0001 0.0001 0.0001	0.0010 0.0010 0.0014
11	**Japan**	2-year 5-year 10-year	-0.0001 0.0001 0.0002	0.0003 0.0007 0.0011
12	**New Zealand**	2-year 5-year 10-year	0.0000 0.0000 0.0001	0.0007 0.0010 0.0015
13	**Netherlands**	2-year 5-year 10-year	-0.0001 0.0001 0.0001	0.0003 0.0007 0.0014
14	**Norway**	5-year 10-year	0.0001 0.0001	0.0011 0.0016

Table 1.A (*Continued*).

	Panel b. Government bonds			
	Country	**Used Gov. bonds data**	**Mean**	**Std. Deviation**
15	**Portugal**	2-year	0.0001	0.0013
		5-year	0.0001	0.0022
		10-year	0.0001	0.0029
16	**Spain**	2-year	0.0001	0.0010
		5-year	0.0001	0.0010
		10-year	0.0001	0.0013
17	**Sweden**	2-year	-0.0001	0.0010
		5-year	0.0001	0.0011
		10-year	0.0001	0.0016
18	**Switzerland**	2-year	-0.0001	0.0007
		5-year	0.0001	0.0008
		10-year	0.0001	0.0010
19	**U.K**	2-year	0.0000	0.0007
		5-year	0.0001	0.0015
		10-year	0.0001	0.0017
20	**U.S**	2-year	0.0001	0.0010
		5-year	0.0001	0.0027
		10-year	0.0001	0.0046

Notes: the time series data are collected from Datastream global equity indexes database. The end data for all the countries in the sample is 30/11/2016. Panel A.2 (B.2) provides the statistics on the 2 year-government bond return (first row), 5 year-government bond return (second row) and 10-year bonds return (third row). The starting dates for the stock market data are match with their counterparts for the bond market data.

Table 1.B Description of the data and Summary Statistics- Emerging Markets.

Notes: see notes on table 1.a.

#	Country	Sample starting date	Mean	Std. Deviation
	Panel (a) Stock markets			
1	**Czech Republic**	05/01/2000	0.0001	0.0060
2	**Greece**	01/10/1999	-0.0003	0.0070
3	**Hungary**	01/02/1999	0.0001	0.0063
4	**Poland**	01/02/2001	0.0001	0.0054
5	**South Africa**	01/09/2000	0.0002	0.0050
	Panel (b) Government bonds			
#	**Country**	**Used Gov. bonds data**	**Mean**	**Std. Deviation**
1	**Czech Republic**	2-year	-0.0001	0.0003
		5-year	0.0001	0.0001
		10-year	0.0001	0.0014
2	**Greece**	2-year	-0.0001	0.0035
		5-year	-0.0002	0.0040
		10-year	-0.0002	0.0052
3	**Hungary**	2-year	0.0001	0.0010
		5-year	0.0001	0.0020
		10-year	0.0001	0.0032
4	**Poland**	2-year	0.0000	0.0010
		5-year	0.0001	0.0012
		10-year	0.0001	0.0021
5	**South Africa**	5-year	0.0000	0.0013
		10-year	0.0001	0.0023

Table 2

The difference in Wavelet-based variance of stock and bond market returns between recessions and expansions periods.

Notes: the analysis is carried out based on MODWT with L8 wavelet filter and up to the sixth time-scale. The estimation used the equations and written Matlab codes in used Percival and Walden (2000). For each country, the variance of the return series during the expansion is subtracted from its counterpart during the recession to examine how the variance is changing over time and across market states. The analyses for the emerging and developed markets are depicted in Panels A and B, respectively. Panels a.1 (b.1), a.2. (b.2), a.3 (b.3) show the variations in stock market return, 2-year and 10 year bonds respectively. Figures in bold denote the lowest value across the time-scales in the same country

Panel a. Developed markets

(a.1) Stock markets

	[2-4] days	[4-8] days	[8-16] days	[16-32] days	[32-64] days	[64-128] days
Australia	0.03402041	**0.00017499**	0.02842040	0.03042508	0.06849111	0.11824488
Austria	0.01580691	0.01220526	0.00299974	-0.05133939	0.15742379	**-0.28718129**
Belgium	0.01541151	0.01956468	0.01544142	**0.00471086**	0.01223005	0.01635040
Canada	0.10494077	0.10194940	0.11270736	0.04088486	0.06422471	**-0.07588195**
Denmark	0.02075658	0.04045847	0.04219801	0.06840647	0.06195627	**0.01339562**
France	0.16251047	0.12067294	0.16468978	**-0.03892049**	0.42190973	0.23884361
Finland	0.00000608	0.00000884	0.00000730	-0.00000074	0.00001235	**-0.00000894**
Germany	0.15060211	0.11158491	0.07588393	0.09185956	0.12402763	**0.02099630**
Ireland	0.20629637	0.22682175	0.19763561	0.09922527	0.28882460	**-0.30718902**
Italy	0.28068826	0.27363956	0.19088941	0.19596258	0.41522345	**-0.05161571**
Japan	0.14343477	0.08043347	0.03876407	0.05975935	0.16142237	**-0.05807799**
New Zealand	0.00886755	0.00461033	0.01989489	**-0.01073607**	0.02393962	-0.06701588
Netherlands	0.15447019	0.11921557	0.08966553	0.08513025	0.15845259	**-0.12237759**
Norway	0.00001920	0.00002113	0.00002932	0.00002212	0.00002877	**0.00001552**
Portugal	0.00000916	0.00001277	0.00000378	-0.00000373	0.00000722	**-0.00001131**
Spain	0.10965405	0.09814243	0.11385619	0.07942721	0.06599061	**-0.09367738**
Sweden	0.34571468	0.25922044	0.16992664	0.12728259	0.23219534	**-0.09556679**
Switzerland	0.17274006	0.17650731	0.14588124	0.08723346	0.16352433	**-0.06891692**
U.K	0.13785039	0.09006316	0.07189948	0.02935636	0.16920494	**-0.07565517**
U.S	0.60715159	0.47931967	0.38586761	0.27550071	0.62872613	**-0.09471362**

(a.2) 2-year bond markets

	[2-4] days	[4-8] days	[8-16] days	[16-32] days	[32-64] days	[64-128] days
Australia	-0.00001634	0.00027482	0.00014755	0.00052613	**-0.00042823**	0.00167873
Austria	0.00102132	0.00108534	0.00105491	**0.00082898**	0.00122788	0.00115443
Belgium	0.00102403	0.00124077	0.00167514	0.00191981	0.00155526	**0.00086387**
Canada	**-0.00050027**	0.00027624	0.00081674	-0.00044626	-0.00032823	0.00071234
Denmark	0.00169499	0.00306285	0.00255386	0.00288956	0.00100259	**0.00026575**
France	0.00181333	0.00175304	0.00220540	0.00091344	0.00100778	**-0.00081868**
Finland	0.00000508	0.00000078	0.00000003	-0.00000004	0.00000235	**-0.00893676**
Germany	0.00045197	0.00040427	0.00025041	0.00019103	0.00052431	**-0.00044863**
Ireland	0.00014835	0.00023425	0.00039625	0.00048262	0.00065743	**0.00026575**
Italy	0.00015898	0.00023028	0.00008897	0.00011950	0.00011059	**0.00009265**
Japan	0.00000073	**-0.00000005**	0.00000148	0.00000027	0.00000210	0.00000004
New Zealand	**0.00002414**	0.00002533	0.00003139	0.00004143	0.00004670	0.00003515
Netherlands	0.00000469	0.00000427	0.00000550	0.00000451	0.00000449	**-0.00000293**
Norway	0.00001880	0.00001983	0.00001939	0.00001882	0.00002774	**0.00000982**
Portugal	0.00000273	0.00000528	0.00000456	0.00000438	0.00000304	**-0.00000108**
Spain	0.00009776	0.00019397	0.00010710	0.00009620	**0.00007552**	0.00008844
Sweden	0.00005942	0.00004800	0.00005098	**0.00002722**	0.00005301	0.00003325
Switzerland	0.00004643	0.00004623	0.00004371	0.00002887	0.00003316	**-0.00000956**
U.K	0.00001423	0.00001861	0.00002560	0.00000492	0.00001531	**0.00000379**
U.S	0.00020765	0.00019436	0.00015039	**0.00008169**	0.00009349	0.00008633

(a.3) 10-year bond markets

	[2-4] days	[4-8] days	[8-16] days	[16-32] days	[32-64] days	[64-128] days
Australia	0.01093479	0.00875752	**0.00435373**	0.00796051	0.01067160	0.01420494
Austria	0.01322399	0.01218215	0.01473175	**0.00186894**	0.00940858	0.00795897
Belgium	**0.00401076**	0.01603212	0.01047759	0.00715247	0.00766476	0.00703983
Canada	0.00032818	0.00007934	0.00374161	**-0.00000024**	0.00230810	0.00942256
Denmark	0.00298347	0.00536689	**-0.00241294**	0.00174839	0.00623887	0.01211602
France	-0.00638110	**-0.00758065**	-0.00508158	-0.00364811	0.00078015	-0.00079891

Table 2, Panel (a.3) (*Continued*).

	[2-4] days	[4-8] days	[8-16] days	[16-32] days	[32-64] days	[64-128] days
Finland	0.00000050	0.00000049	-0.00000013	-0.00000005	0.00000118	**-0.00000155**
Germany	0.00444430	0.00419667	0.00126429	**-0.00137349**	0.00557793	-0.00006701
Ireland	0.00000160	0.00001482	-0.00003132	0.00006433	0.00001539	**-0.00013963**
Italy	0.00003999	-0.00002651	-0.00000229	0.00000898	0.00000802	**-0.00008394**
Japan	-0.00000164	-0.00000702	-0.00001196	-0.00004104	0.00002739	**-0.00013215**
New Zealand	-0.00004610	-0.00007919	-0.00009087	**-0.00009244**	-0.00001434	0.00017641
Netherlands	0.00006332	0.00004140	0.00004725	**0.00001586**	0.00006128	0.00004800
Norway	0.00000031	0.00000031	0.00000002	0.00000021	-0.00000069	**-0.00000086**
Portugal	0.00000884	0.00001677	0.00002805	0.00001929	0.00000463	**0.00000166**
Spain	0.00005780	-0.00000552	0.00008373	-0.00001835	0.00004380	**-0.00014785**
Sweden	0.00003081	-0.00009919	-0.00009629	-0.00001599	0.00000414	**-0.00018812**
Switzerland	0.00000922	0.00001295	0.00000788	0.00000875	0.00001089	**-0.00003637**
U.K	0.00036403	0.00039922	0.00031912	0.00025808	0.00020181	**0.00010111**
U.S	0.00287262	0.00299358	0.00224804	0.00146656	0.00474031	**-0.00123509**

Panel b. Emerging markets.

(b.1) Stock markets

	[2-4] days	[4-8] days	[8-16] days	[16-32] days	[32-64] days	[64-128] days
Czech Republic	0.00000558	0.00000377	0.00000097	0.00000064	**0.00000008**	0.00000028
Greece	0.00004371	0.00003162	0.00003053	**0.00002136**	0.00004232	0.00003498
Hungary	0.00004482	0.00002648	0.00004830	0.00002026	0.00005484	**-0.00000040**
Poland	0.00000884	0.00000589	0.00000997	**0.00000168**	0.00002794	0.00000738
South Africa	0.00001619	0.00001867	0.00001279	0.00000502	0.00003179	**-0.00000500**

(b.2) 2-year bond markets

	[2-4] days	[4-8] days	[8-16] days	[16-32] days	[32-64] days	[64-128] days
Czech Republic	0.00000527	0.00000310	0.00000086	0.00000061	**0.00000006**	0.00000011
Greece	0.00003310	0.00002765	0.00001547	0.00001440	0.00000974	**-0.00002726**
Hungary	0.00000079	0.00000123	0.00000069	0.00000080	0.00000583	**0.00000065**
Poland	0.00882423	0.00825291	0.00373851	0.00196781	0.00289404	**0.00165882**

(b.3) 10-year bond markets

	[2-4] days	[4-8] days	[8-16] days	[16-32] days	[32-64] days	[64-128] days
Czech Republic	0.00551375	0.00203863	0.00151105	0.00079212	0.00025584	**0.00021605**
Greece	0.00006234	0.00004822	0.00003904	0.00004676	0.00003621	**-0.00002913**
Hungary	0.00000714	0.00001138	0.00001404	0.00000865	0.00002358	**-0.00000077**
Poland	0.01833204	0.01979507	**0.01030713**	0.01497052	0.03386584	0.02383410
South Africa	0.00000580	0.00000507	0.00000362	0.00000304	0.00000535	**-0.00000007**

Table 3. VaR ratio (restricted/unrestricted) on time-scales.

Country		Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6	Country		Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
Panel A: Developed countries															
AU	EXP.	1.04180	1.04570	1.02701	1.04020	0.98803	**0.90202**	**AR**	EXP.	0.93776	0.93107	0.91364	0.84903	0.93167	**0.79249**
	REC.	1.02281	1.00474	1.01492	1.03095	**0.97135**	1.07483		REC.	1.10947	**0.94180**	1.09232	1.17908	1.10068	1.11033
BE	EXP.	0.86117	0.83799	0.80810	0.79840	0.82548	**0.77473**	**CA**	EXP.	1.00562	0.98974	1.01169	1.02268	**0.94361**	1.01030
	REC.	0.87969	0.85880	0.83538	0.84834	0.82427	**0.78149**		REC.	1.09405	1.06941	1.07773	1.05926	**0.94702**	1.05778
DE	EXP.	0.84185	0.86025	0.84032	0.88697	**0.83420**	0.97863	**FI**	EXP.	0.99529	0.94822	0.93928	0.93549	**0.91362**	0.95152
	REC.	0.92612	0.90501	0.92499	0.94777	**0.83732**	0.91645		REC.	1.03525	1.09173	1.05407	0.99533	**0.93215**	1.09635
FR	EXP.	0.96217	**0.85370**	1.02083	1.02878	0.98544	0.98581	**GE**	EXP.	1.01020	1.01426	**1.00481**	1.03736	1.04901	1.05566
	REC.	1.01787	1.01770	1.03764	0.99272	**0.99197**	1.05251		REC.	1.12840	1.17619	**1.09724**	1.10498	1	1.16655
IR	EXP.	1	0.99462	0.99473	1	0.99379	**0.99248**	**IT**	EXP.	0.98990	0.99043	0.99009	0.99035	**0.98577**	0.99138
	REC.	0.99606	1	0.99616	1	1	**0.98918**		REC.	1	0.99661	0.99626	0.99631	**0.99084**	0.99583
JP	EXP.	0.99609	0.99599	0.99567	0.99542	0.99578	**0.98484**	**NE**	EXP.	1	1	0.99224	1	**0.98012**	0.99213
	REC.	1.00980	**1.00456**	1.00931	1.01029	1.01739	1.01262		REC.	1.00746	1	1.00708	1.01487	**1.00617**	1.01939
NT	EXP.	1.00546	1.00505	1	1.00505	1.00536	**1.00497**	**NO**	EXP.	1.01086	0.99493	1.02749	0.93520	0.93575	**0.90988**
	REC.	1.01256	1.01682	1.00949	1.00930	**0.99596**	1.02052		REC.	1.07964	1.09885	1.03982	1.12898	**1.02437**	1.17738
PR	EXP.	0.94224	0.93250	0.90179	0.90867	**0.90097**	0.95022	**SP**	EXP.	0.99537	0.99546	0.99493	0.98967	0.99116	**0.97926**
	REC.	0.94094	0.93280	**0.85047**	0.94118	0.89999	0.93313		REC.	1	1.00397	**0.99585**	1	1	1.00474
SW	EXP.	0.99510	0.99561	0.99525	0.99493	0.98981	**0.98751**	**SZ**	EXP.	1.00207	**1.00120**	1.00519	1.00604	1.00430	1.00507
	REC.	**0.99679**	1.00329	1.00375	1	1	1.00848		REC.	1.00913	1.00873	1.00475	1.00535	**0.99834**	0.99940
UK	EXP.	1	1.00578	0.99370	0.99384	**0.98640**	1.00649	**US**	EXP.	1.02381	**1.02272**	1.03205	1.03336	1.02749	1.06548
	REC.	1.01351	1.00953	0.98973	1	0.98653	**0.97517**		REC.	1.03779	1.04088	1.03481	1.04784	**0.94312**	1.10946
Panel B: Emerging countries															
GR	EXP.	1	1.49941	1.50136	1.50335	1.50174	**0.84952**	**HU**	EXP.	1	0.92429	0.93614	0.94957	1.04263	**0.79926**
	REC.	1	0.83320	0.83366	1	0.83328	**0.82960**		REC.	1.04224	0.91481	0.94332	0.93815	**0.75243**	1
PO	EXP.	0.99645	0.99741	0.99518	0.99784	**0.99478**	0.99753	**SO**	EXP.	0.92340	0.89468	0.93524	0.94647	**0.87595**	0.98519
	REC.	0.99688	0.99494	0.99525	0.99342	**0.98705**	0.99639		REC.	0.91893	0.99364	0.90995	0.95831	**0.82399**	1.02092
CZ	EXP.	1.00141	1	0.99949	1.00140	1.00086	**0.99570**								
	REC.	1.00277	1.00053	1	0.99955	**0.99736**	0.99155								

Notes: the estimates in the table are the VaR ratios. The ratio is calculated by dividing the VaR from the unrestricted model over it is counterpart from the restricted model. The latter assumes no co-movement over specific time-scale, while the former estimates the covariance part in the portfolio variance equation and assumes co-movement at the same time-scale. Ratio less than one indicates that the formulated portfolio minimizes the loss of the portfolio. Entries in bold denote for the lowest VaR ratio in across the time-scales during the same market state.

Figure 1. Estimated unbiased wavelet correlation between stock and bond returns for the full-sample period.

Notes: the solid line shows the unbiased wavelet-based correlation, while the upper and lower lines represent the 95% upper and lower confidence intervals, respectively. The inputs for the estimation are as follows: boundary condition: circular, confidence interval method: Gaussian and the wavelet filter is L8. The estimation used the equations and written Matlab codes as exactly used Percival and Walden (2000). Panels A and B concern the analyses for the developed markets and emerging markets, respectively. Panel A.1 (B.1) shows the correlation patterns when the 2-year bond is included in the analysis, panel A.2 (B.2) shows the results with 5-year bond and panel A.3 (B.3) indicate to the results with 10-year bond.

Panel a. Developed Markets

(a.1) Stock and 2-year bond

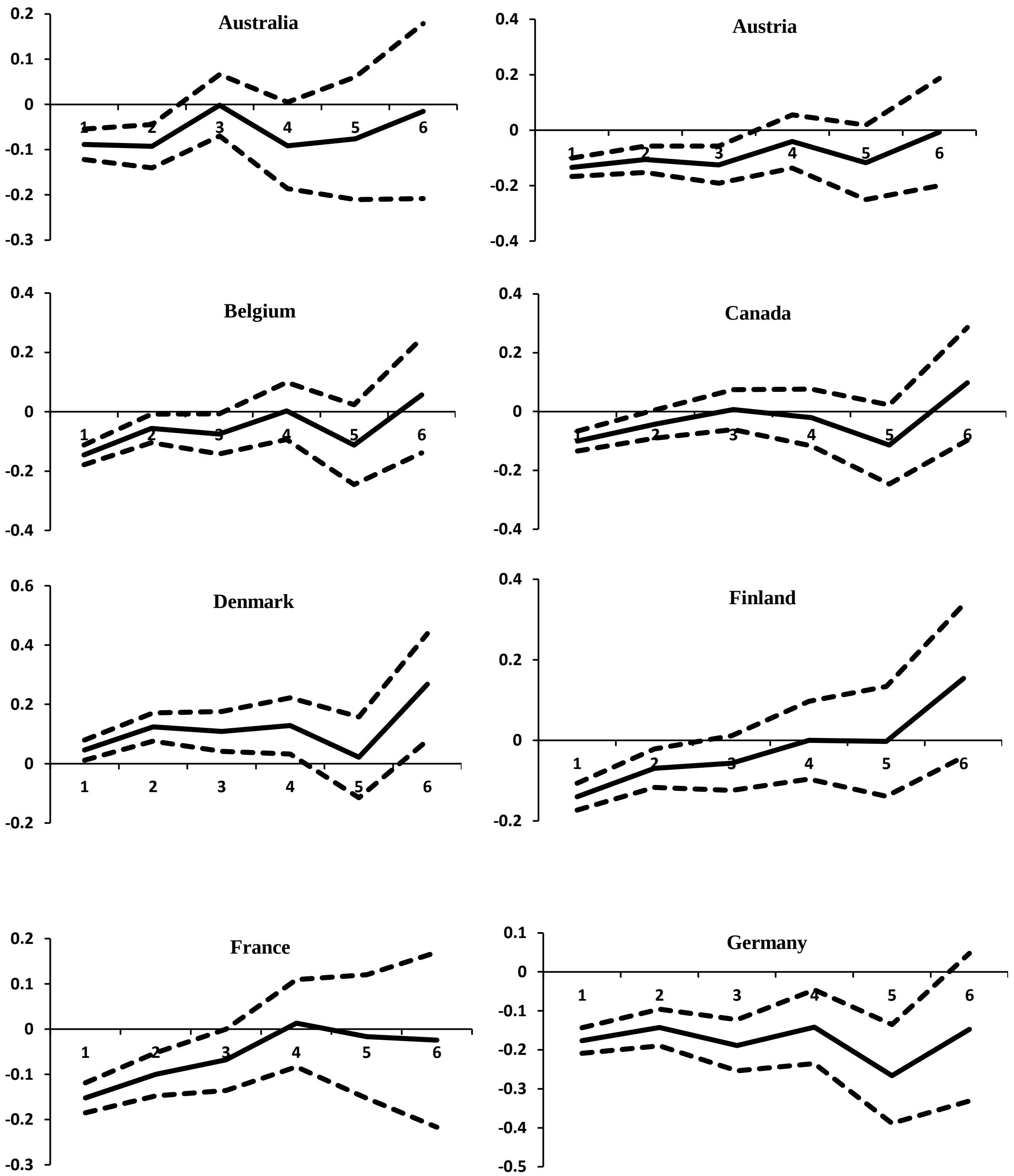


Figure 1. (a.1) (*Continued*).

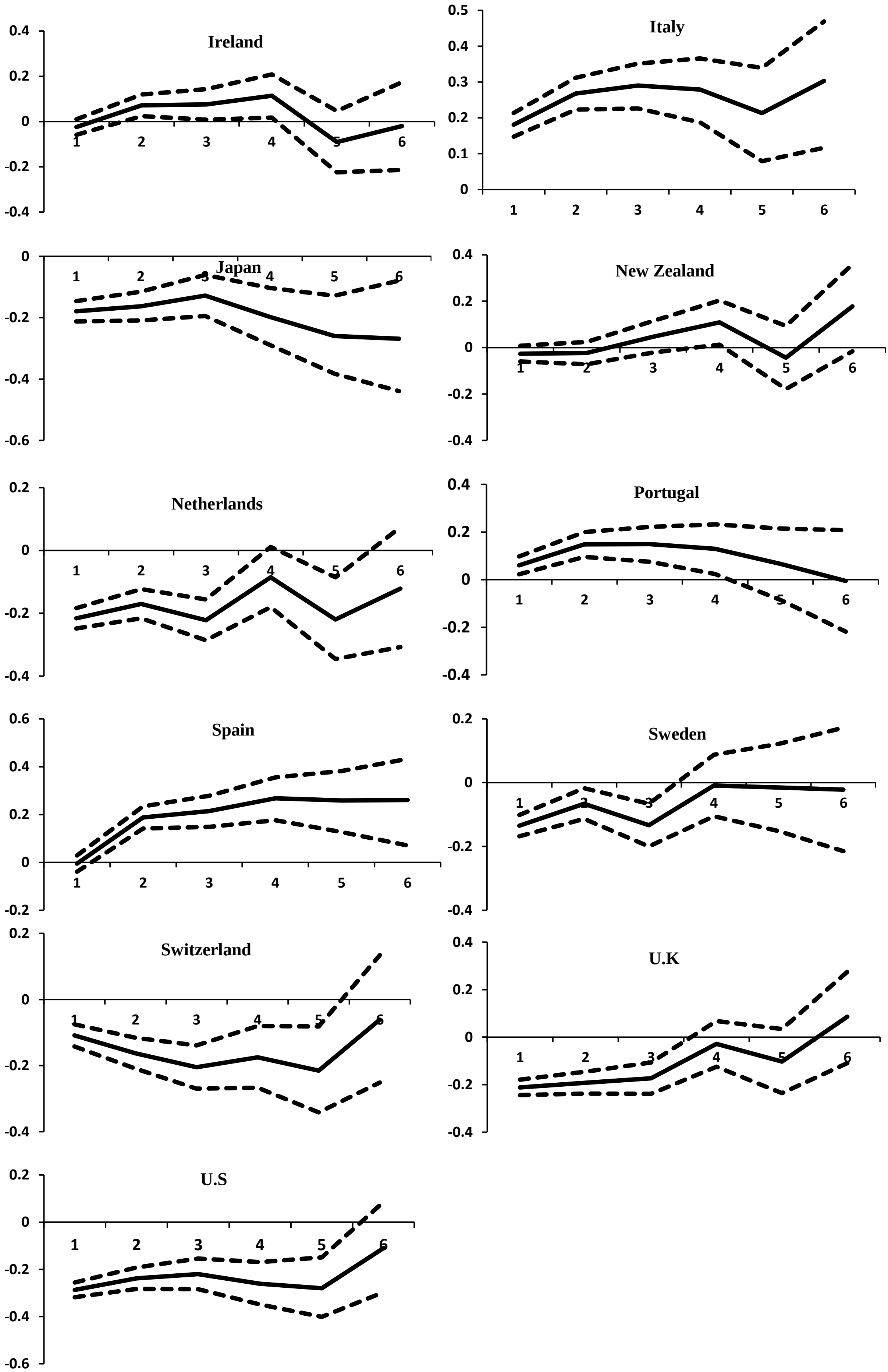


Figure 1. (*Continued*).

(a.2) Stock and 5-year bond

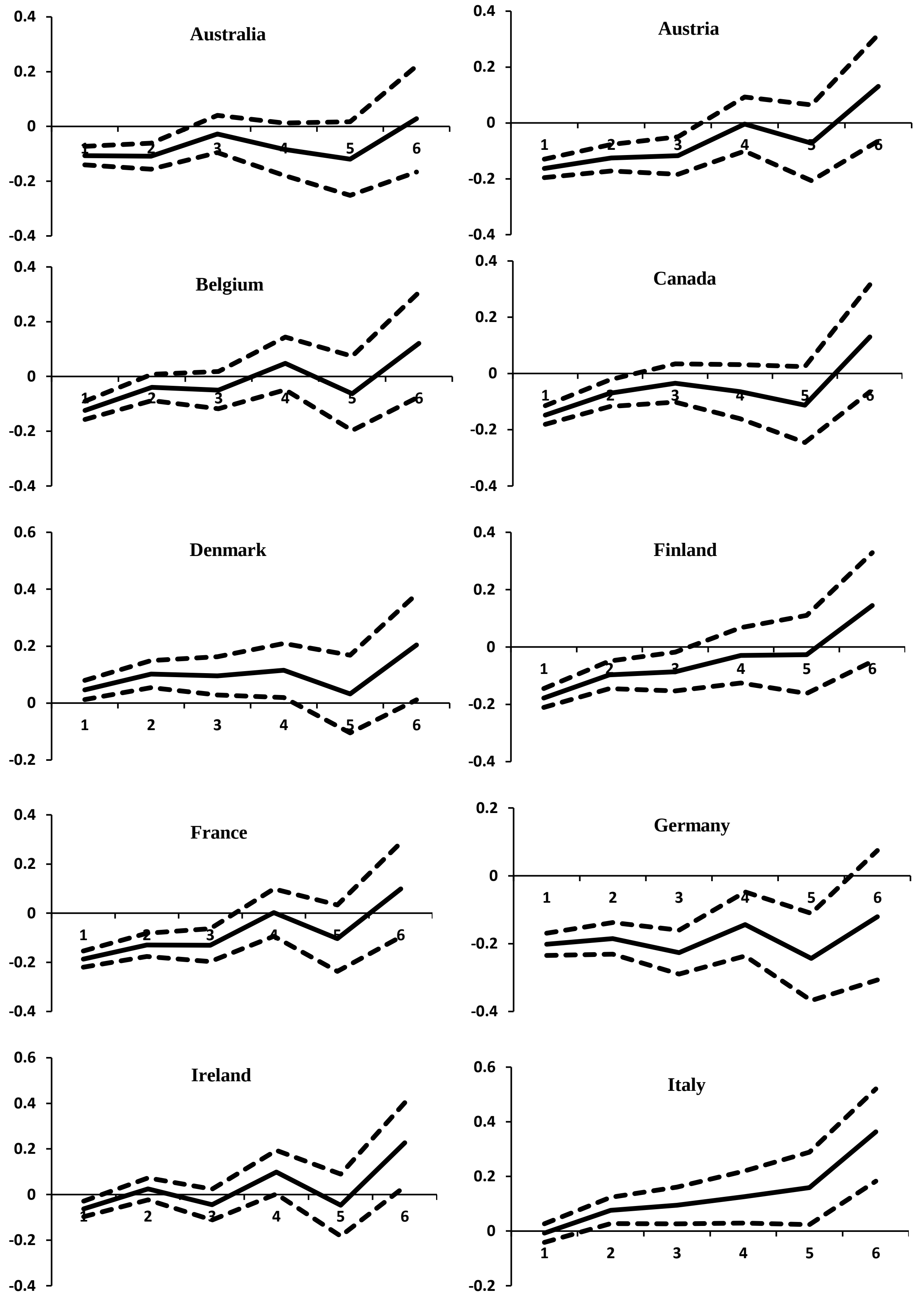


Figure 1. (a.2) (*Continued*).

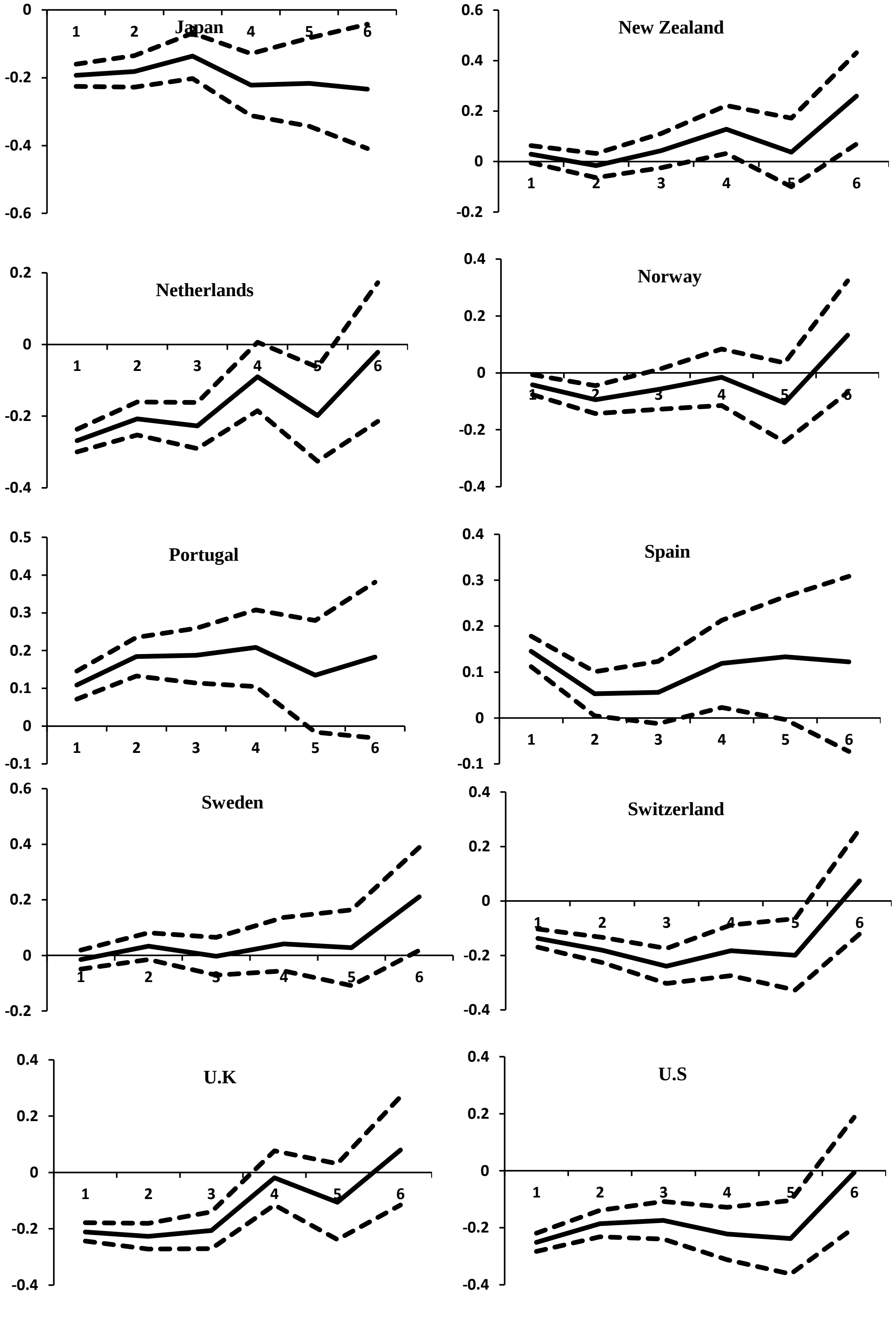


Figure 1. (*Continued*).

(a.3) Stock and 10-year bond

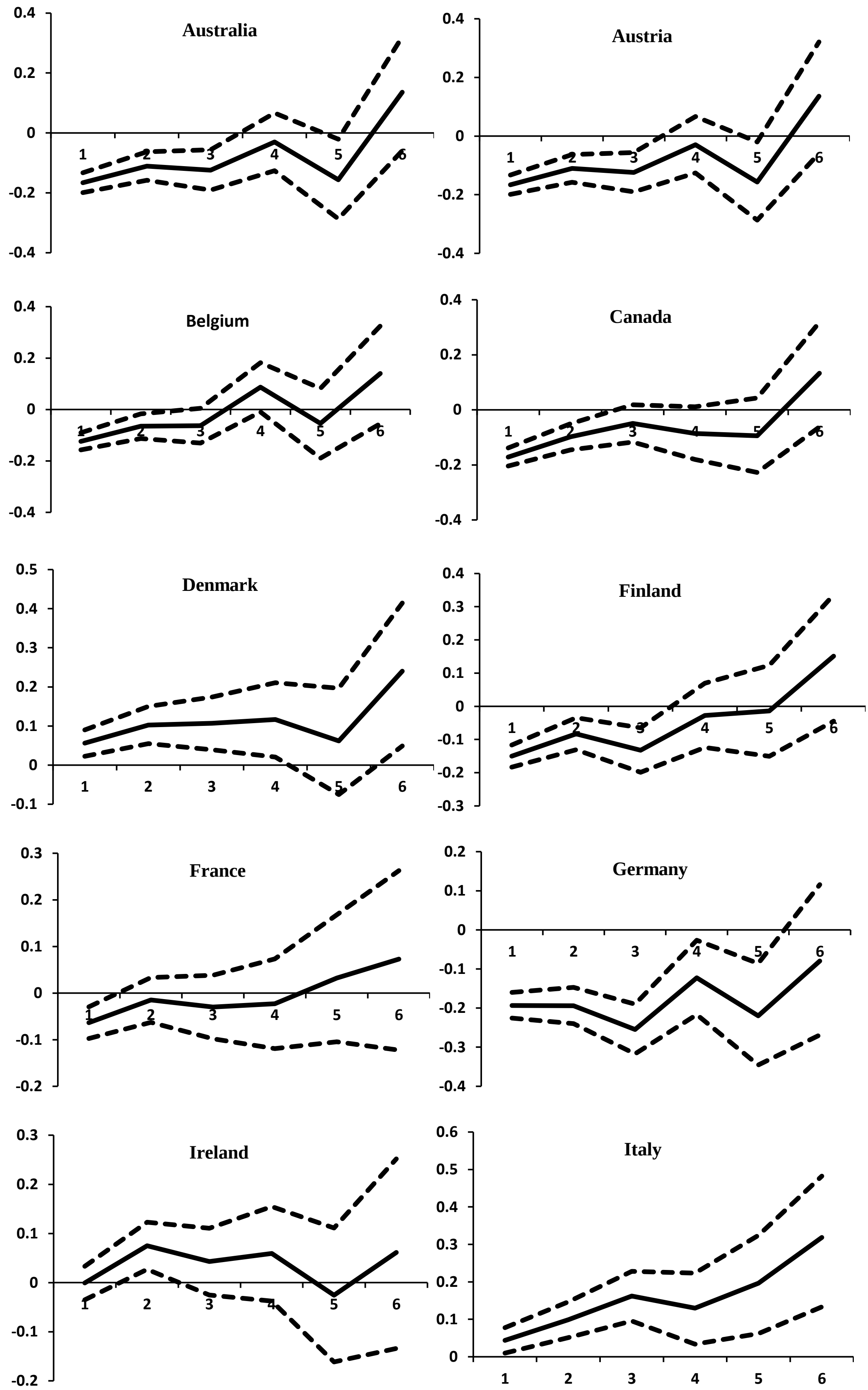


Figure 1. (*Continued*).

(a.3) Stock and 10-year bond

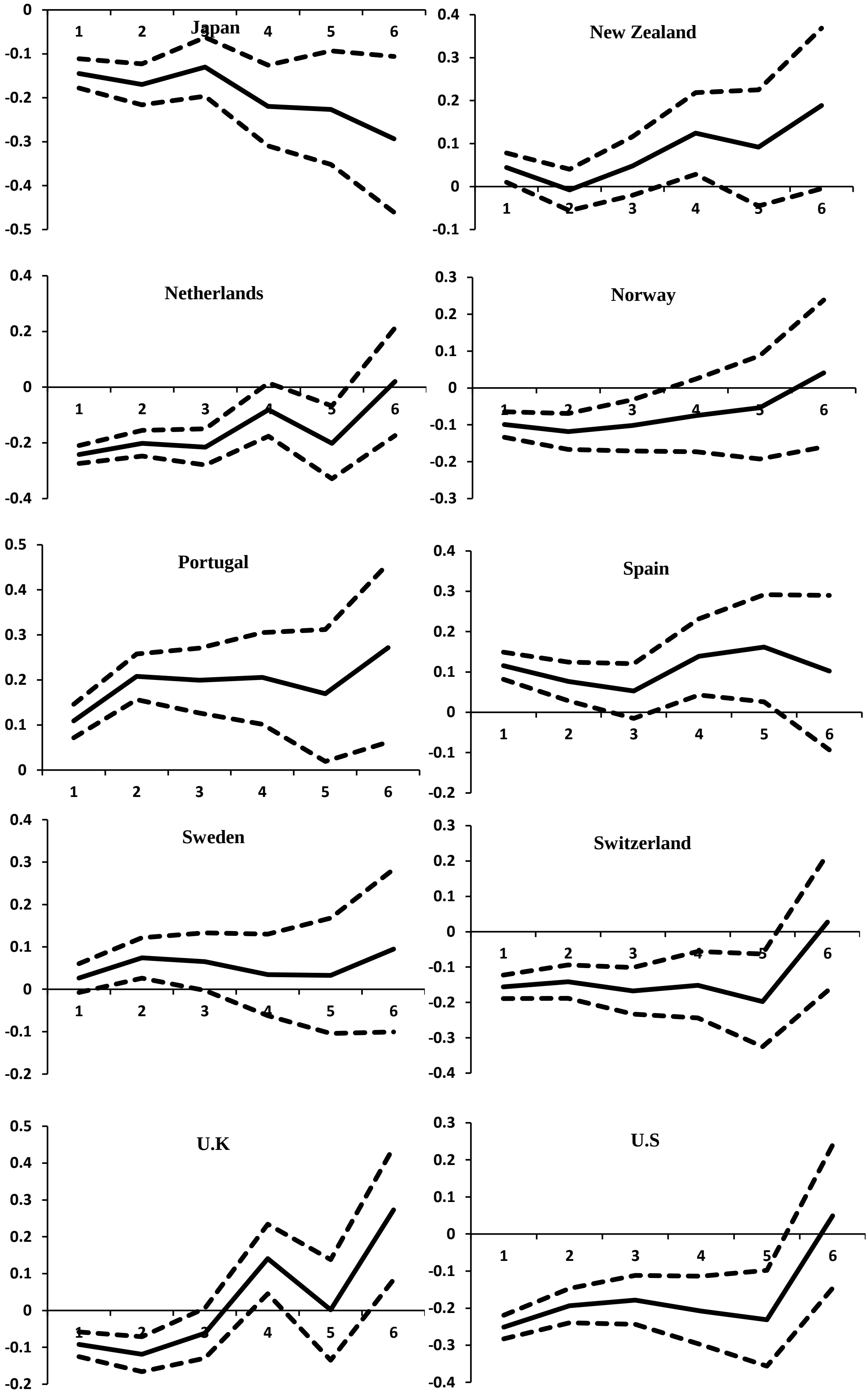
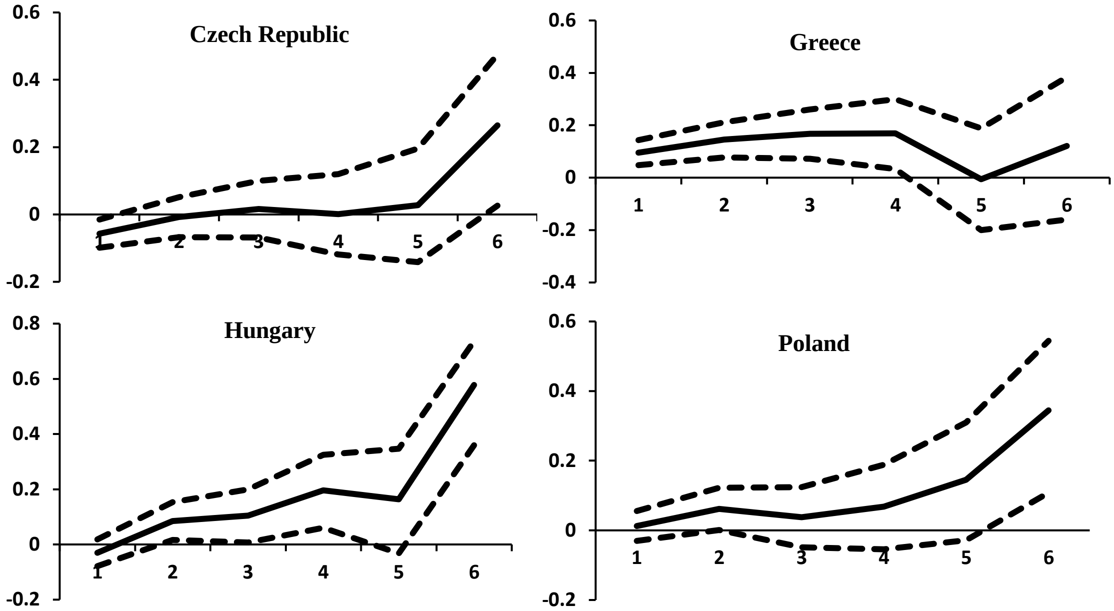


Figure 1. (*Continued*).

Panel b. Emerging Markets

(b.1) Stock and 2-year bond



(b.2) Stock and 5-year bond

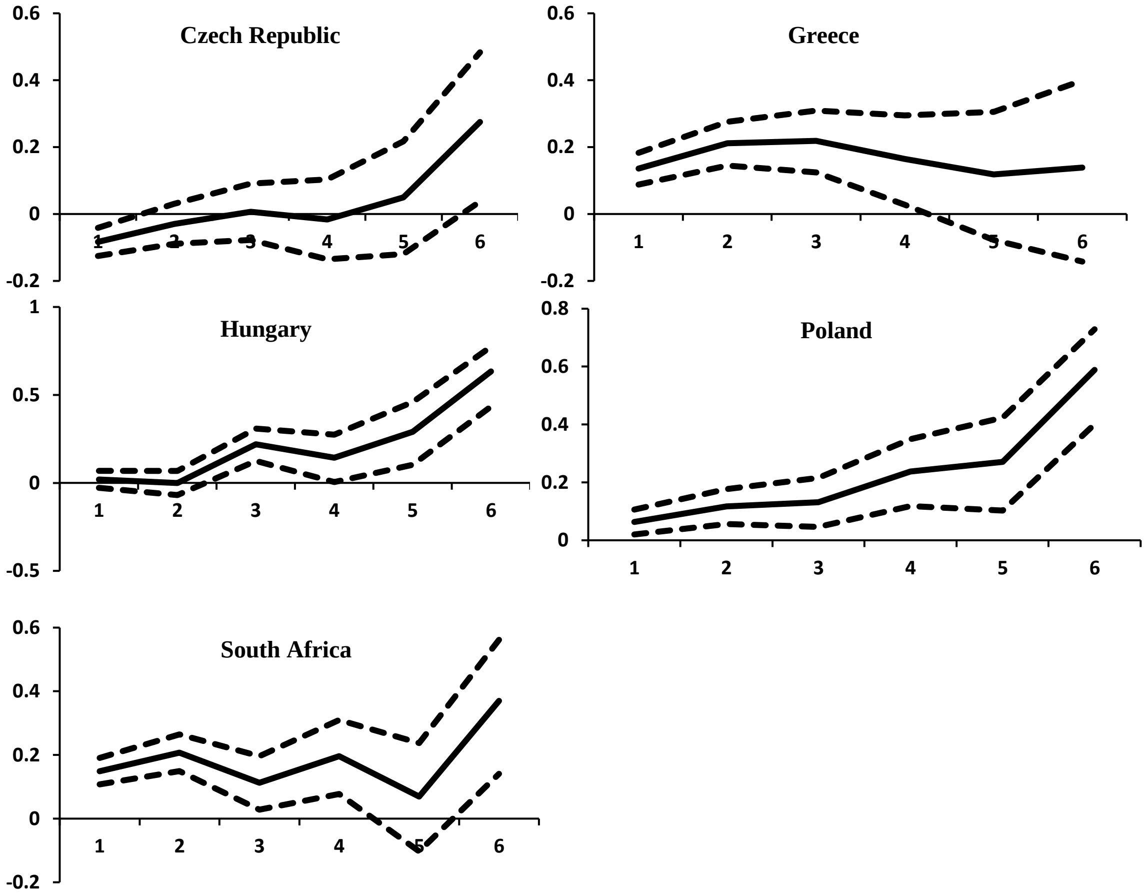


Figure 1 (b.3) (*Continued*).

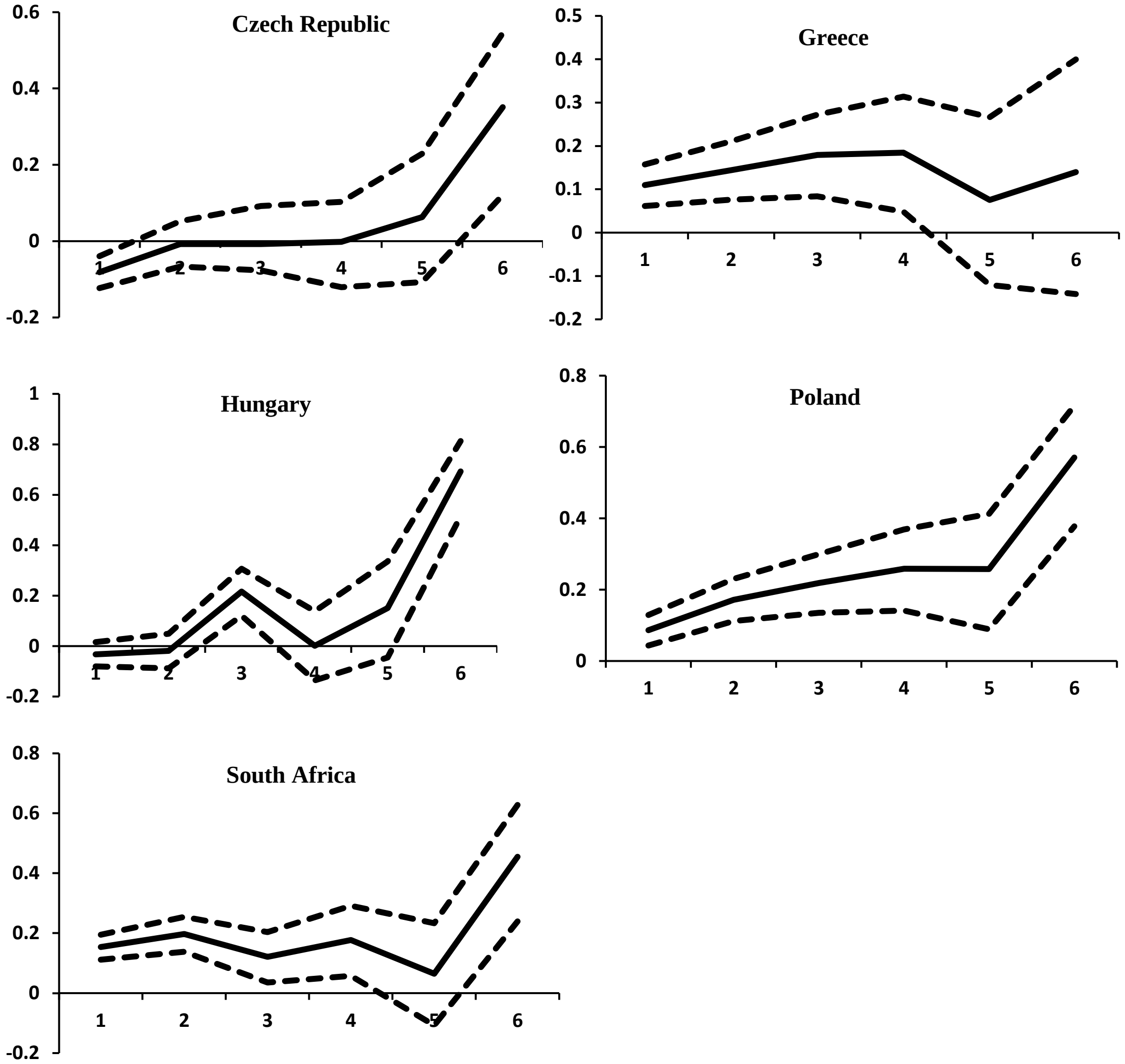
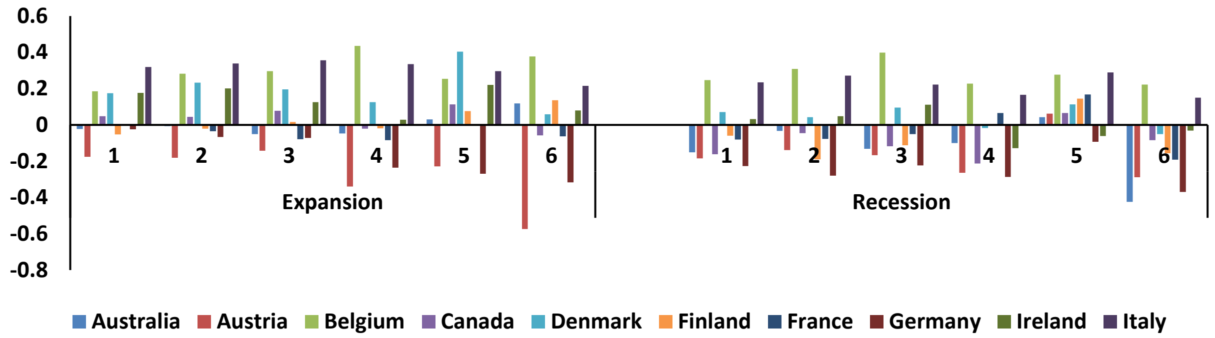


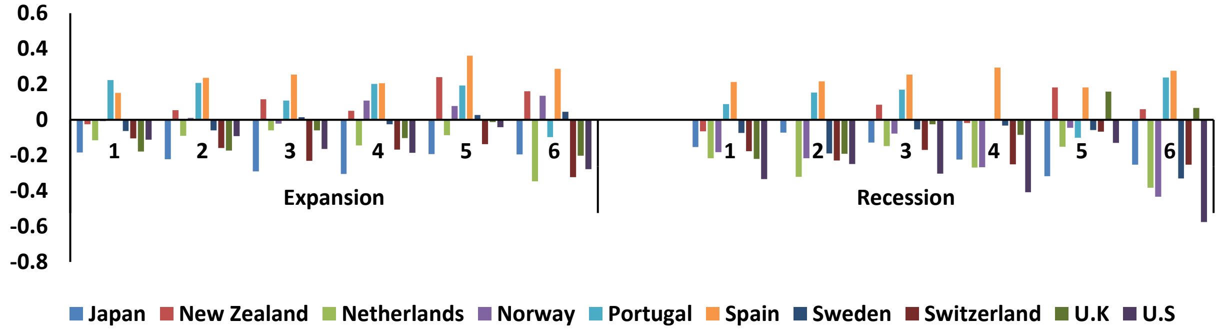
Figure 2. Wavelet-based correlation during expansion Vs. recession market states.

Notes: the analysis is carried out based on MODWT with L8 wavelet filter and up to the sixth time-scale. The estimation used the equations and written Matlab codes as exactly used Percival and Walden (2000). The analyses for the emerging and developed markets are depicted in Panels a. and b, respectively. Panels a.1 (b.1) and a.2. (b.2) show the correlation with 2-year and 10 year bonds respectively.

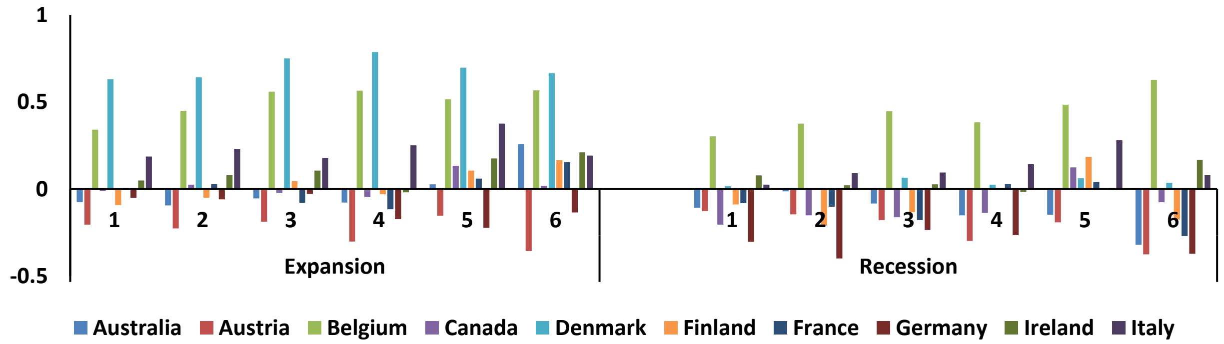
Panel a. Developed Markets

(a.1) Stock and 2-year bond





(a.2) Stock and 10-year bond



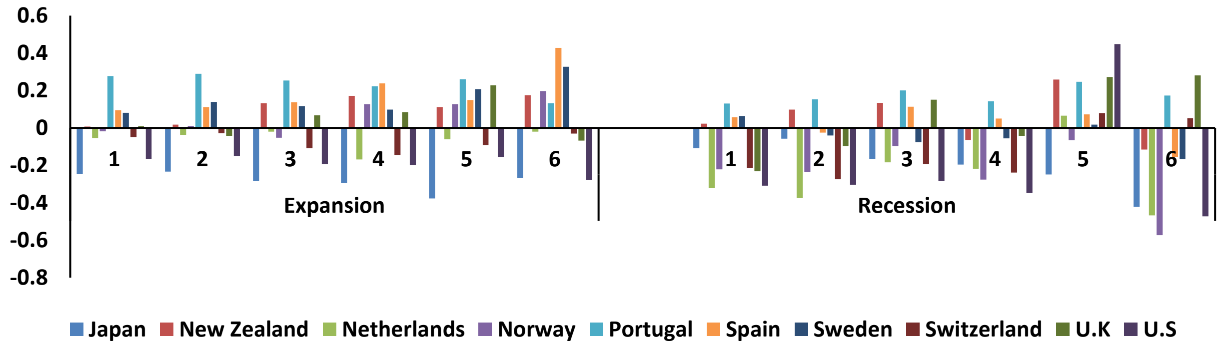
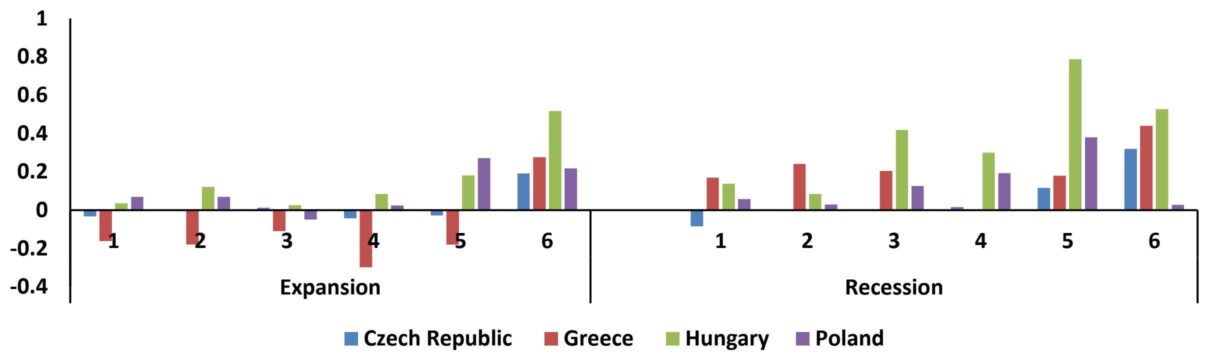


Figure 2 (*Continued*).

Panel b. Emerging Markets

(b.1) Stock and 2-year bond



(b.2) Stock and 10-year bond