Production and joint emission reduction decisions based on
two-way cost-sharing contract under cap-and-trade regulation

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Abstract

Cap-and-trade regulation is widely applied as a carbon policy in low-carbon supply chain management. This study investigates production and carbon emission reduction strategies that are based on such regulation in a two-echelon supply chain, which comprises one manufacturer and one retailer. In this supply chain, the manufacturer directly participates in carbon emission reduction while the retailer is indirectly involved in low-carbon promotion. On this basis, we establish single and joint emission reduction models, in which supply chain members may adopt the one-way or two-way cost-sharing contracts. We then analyze the optimal strategy design for supply chain and the appropriate sharing rate contract. We find that the implementation of contracts can increase carbon emission abatement level, product quantity and supply chain profit. The one-way cost-sharing contract is beneficial for supply chain, whereas the two-way cost-sharing contract is also beneficial for supply chain when the sharing rate is in a small range. Under certain conditions, joint emission reduction model is optimal choice for supply chain. Meanwhile, the sharing rate can affect supply chain choice between decentralized and centralized decisions. Then we propose the extended multiple retailers model and find that this model offers better performance. In addition, carbon emission abatement level increases with carbon trading price by numerical study. The government can stimulate supply chain to reduce carbon emission by regulating carbon trading price, and should also pay attention to the impact on supply chain production and profits.

Keywords: cap-and-trade, carbon emission reduction, low-carbon supply chain, cost-sharing contract.

1 Introduction

With the rapid development of the global economy and technology, the environmental issues (such as global warming) faced by governments and enterprises are increasingly important. Massive carbon emissions arising from individual activities and industrial production are a major cause of global warming (Xu et al., 2016). Many environmental policies and measures have been proposed by national governments to control carbon emissions. One common measure is the carbon tax mechanism, in which the government imposes a certain tax on enterprises for their carbon emissions. According to the Center for Climate and Solutions (2013), Finland,
the Netherlands, Norway, Sweden, the UK, and Australia implement the carbon tax mechanism (Aysegül and Bilgesu, 2017). Another measure is cap-and-trade regulation, which is generally considered an effective way to reduce carbon emissions (Wang and Wang, 2015). A few territories, such as the EU, Australia, New Zealand, northeastern United States, and Tokyo, are currently implementing cap-and-trade regulation (Aysegül and Bilgesu, 2017). Under this policy, firms are first provided carbon credits by their national governments and then purchase additional carbon quotas when their carbon emissions exceed the initial credits; otherwise, these companies can sell excess carbon quotas to the carbon emission trading market (Du et al., 2014). Therefore, a firm has two options when its carbon credits are insufficient, namely, reducing their carbon emission level or buying added carbon quotas. Therefore, enterprises must urgently consider the impact of carbon emissions on production operation.

In December 2015, 195 nations participating in the UN Climate Change Conference negotiated and unanimously passed the Paris agreement, which is to be implemented after 2020. According to the agreement, average global temperature rise must be limited to less than 2 °C above pre-industrial levels and ideally limited to within 1.5 °C. Several enterprises have responded to this agreement and actively undertaken the corresponding social responsibility. For example, Coca-Cola aims to reduce its carbon emissions by a quarter by 2020, while Unilever seeks to halve theirs (Lei and Qin, 2017).

Consumers are becoming increasingly aware of environmental protection and prefer low-carbon products. Hence, several enterprises in such countries as the US, Europe, Japan, and other developed countries have begun displaying their carbon information on their commodity labels, thereby making carbon footprint labeling on food packaging increasingly common (Lei and Qin, 2017). For example, Walmart plans to display carbon information on product labels along with price information. To gather environmental information and thus accomplish this goal, Walmart requires its suppliers to answer a few questions regarding energy costs, waste emissions, material efficiency, and natural resource utilization (Bustillo, 2009). On the other hand, an increasing number of firms, such as H&M, Marks & Spencer, and Levi’s, is now producing low-carbon products using new techniques to curtail the carbon emissions in their production processes (Li and Li, 2014).

The government regulations and consumer preferences increasingly motivate enterprises to not only maximize profits but also consider environmental impact, thereby making the study of low-carbon supply chains important and meaningful. For instance, Chen (2001) developed a quality-based model which considers consumer preferences, product decisions and government environmental protection policies to analyze the impact of green product development on consumers, industries and society. In a low-carbon supply chain, the manufacturer directly participates in carbon emission reduction while the retailer is involved in low-carbon promotion. Considering the pressure regarding carbon emission reduction, the two enterprises
establish a two-way cost-sharing contract, which may allow them to share reduction costs. The supply chain must make a trade-off between profit and carbon emission abatement level. Therefore, the supply chain should rationally decide commodity production, carbon emission abatement level, sharing rate, and other operational strategies.

This research considers a two-stage supply chain, which includes a single manufacturer and a single retailer, working under cap-and-trade regulation. The manufacturer and the retailer are directly and indirectly committed to reducing carbon emissions, respectively. This study attempts to illustrate whether the supply chain’s carbon emission abatement level or profit is improved by the implementation of a one-way or two-way cost-sharing contract and attempts to determine the optimal sharing rate for supply chain. This study also aims to analyze the difference in emission abatement level, product quantity, and supply chain profit under various research models and study the specific impact of sharing rate on supply chain profit and the adjusting effect of carbon trading price on supply chain.

The general structure of this paper is as follows. Section 1 introduces the research background of this study. Sections 2 provides the relevant literature review. Problem description, model assumptions, and related parameters are presented in Section 3. Section 4 explains and analyzes the research models, namely, the single and joint emission reduction models, then proposes the extended model on the basis of the single emission reduction model. Section 5 shows the numerical analysis of the sharing rate and carbon trading price. Managerial implications and conclusions are detailed in Section 6.

2 Literature review

Low-carbon supply chain management has become a research hot topic due to government control of carbon and the change in public attitude to environmental protection. Most studies investigate the carbon policies in supply chains at the macro levels (Liu et al., 2015; Cheng et al., 2016). Studies related to this work can be divided into two aspects. The first aspect studies supply chain decisions under cap-and-trade regulation, and the second investigates sharing contracts between firms in supply chains.

2.1 Supply chain decisions under cap-and-trade regulation

Research on the decision making in supply chains under cap-and-trade regulation is currently mainly at the macro and micro levels. The macro level pertains to the study of low-carbon supply chain design, and the micro level investigates the specific operation decisions of supply chains under cap-and-trade regulation.

In view of low-carbon supply chain design, Ramudhin et al. (2008) proposed a mixed-integer programming method to solve the problem of green supply chain network design on the basis of an integrated logistics model under cap-and-trade regulation and then obtained the relationship between supply chain cost and carbon footprint. Sundarakani et al. (2010) studied the carbon footprint across supply chains
and found that it influences supply chain design. Considering the life cycle perspective, Agrawal et al. (2012) concluded that promotion is more environmentally friendly than leasing because the direct control of products is superior to indirect leasing. Chaabane et al. (2012) introduced mixed-integer linear programming into low-carbon supply chains on the basis of life cycle assessment and found that the current regulation should be optimized in long-term environmental strategies.

From the perspective of operation decisions, Plambeck (2012) summarized methods of carbon emission reduction and quantified the impact of supply chain performance on the environment. Drake et al. (2010) considered a two-stage technology choice and capacity investment under cap-and-trade or tax regulation. Zhang et al. (2011) studied the problem of optimal production strategy of manufacturers under stochastic demand in cap-and-trade regulation. Similarly, Gong and Zhou (2013) proposed an optimal production strategy that is based on a dynamic model. Du et al. (2013) investigated optimal production decisions and found an interval of cap that can be simultaneously utilized by the manufacturer and the retailer for coordination to earn additional profits. Different from Du et al., Cao and Yu (2018) considered the capital-constrained supply chain under the cap-and-trade regulation policy, and found that the product quantity is not affected by the carbon quotas. Yu and Han (2017) investigated the impact of carbon emission taxation on the product pricing and carbon emission reduction under the centralized and decentralized circumstances. Similarly, Yu et al. (2020) studied the impact of carbon emission tax on the choice of supply chain channel in the vertical supply chain. Benjaafar et al. (2013) discussed the impact of four low-carbon policies on production and inventory decisions. Yang et al. (2014) also analyzed the effects of four kinds of policies but on supply chain channel coordination.

Different from the analysis of the impact of carbon emission policies, Tseng and Hung (2014) presented a decision-making model that includes operation and the social cost of carbon emission and found that forcing enterprises to accept this social cost can effectively reduce carbon emissions. Xu et al. (2015) investigated the production and pricing problem of two products in the MTO (make to order) supply chain under cap-and-trade regulation and concluded that the optimal product quantity decreases with an increase in cap. Similarly, Li et al. (2016) investigated the pricing problem and greening strategy of supply chains. Wang et al. (2016) studied the decisions of manufacturers regarding carbon emission reduction that consider the low-carbon preference of consumers on the basis of the game model. However, most of the current research only focuses on the manufacturer’s carbon emission reduction, but a few do analyze the case that manufacturer and retailer are involved in reducing carbon emission at the same time. This study considers that the manufacturer and retailer will participate in carbon emission reduction simultaneously, and the two sides may share costs with each other. Furthermore, the sharing rate will ultimately affect the emission abatement level and relevant operational decisions of supply chain.
2.2 Sharing contract in supply chains

According to the research on supply chain contracts, many contracts are available, such as buy-back, wholesale price, and cost- or revenue-sharing contracts (Sluis and Giovanni, 2016). Several previous works covered revenue-sharing contracts, which are widely implemented in supply chains. Most studies achieved supply chain coordination by establishing revenue-sharing contracts. Giannoccaro and Pontrandolfo (2004) investigated the decision and coordination condition deviation in two- and three-echelon supply chains with revenue-sharing contracts. Yao et al. (2008) studied the coordination effect of revenue-sharing contracts on supply chains with a manufacturer and two competing retailers and found that revenue-sharing contracts are more beneficial than price contracts. Veen and Venugopal (2005) found that revenue-sharing contracts can effectively coordinate profit distribution to improve performance. In addition to only analyzing revenue-sharing contracts, Ai X et al. (2012) compared the two kinds of contracts under competitions among chains. Similarly, Bellantuono et al. (2009) considered two contracts capable of coordination to increase sales and found that each contract can improve the expected profit of the supply chain. Pan et al. (2010) analyzed different contracts under various situations in the supply chain and concluded that at least one member becomes profitable after the implementation of a revenue-sharing contract.

Distinct from the above, Hsueh (2014) established a new revenue-sharing contract by adding corporate social responsibility to supply chain coordination in a two-echelon supply chain. Hsueh found that this contract can increase corporate social responsibility performance and the profits of entire supply chains, thereby benefiting all members.

Different from revenue sharing contracts, cost sharing contracts pay more attention to the sharing of R&D, investment, operation and other costs between the cooperating firms. Chao et al. (2009) studied the cost sharing contract in the traditional supply chain and concluded that it can clearly improve product quality. Ghosh and Shah (2015) explored two models of cost sharing contract in the global green supply chain, and analyzed the impact of contract on the product green level, price and profit, then found that the contract resulted in reducing carbon emissions and higher profits of the supply chain members. Leng and Parlar (2010) also considered the cost sharing contract in which the retailer shares part of total cost of the manufacturer. Wang et al. (2016) established the game model considering the wholesale price premium contract and the cost sharing contract simultaneously under the low-carbon preference market, then analyzed the impact of different contracts on the supply chain decisions and profits. Similarly Xu et al. (2017) also considered the wholesale price contract and cost sharing contract in the supply chain. Their results showed that both contracts could coordinate supply chain. Yang and Chen (2018) investigated the impact of both revenue-sharing and cost-sharing contracts in the retailer driven supply chain. They found that both contracts are beneficial to the improvement of supply chain performance and promote the manufacturer to reduce
carbon emissions. Wang and Shin (2015) compared three different contracts and concluded that the optimal contract that supply chain members tend to choose are inconsistent. In addition, Dai. et al. (2017) combined external environmental factors with internal technology capability differences, then investigated the influence of the cost sharing contract and cartel on supply chain decision-making. The results indicated that both modes can achieve Pareto-improving in contrast to the non-cooperative case. Zhou et al. (2018) studied the pricing strategies of products in the supply chain with the service-cost sharing contract, and concluded that this contract is beneficial to supply chain performance. Similarly, He et al. (2020) investigated three types cost-sharing contracts under different situations in the two-echelon supply chain which considered the low-carbon advertising and corporate social responsibility, then found that the consumer’s low-carbon preference, marginal profit and corporate social responsibility can influence the optimal decisions.

In the present paper, we combine the two-way cost-sharing contract with the cap-and-trade regulation to consider carbon emission reduction in the two-echelon supply chain, in which the manufacturer implements carbon emission reduction and the retailer conducts low-carbon promotion.

There are also some practical examples of cost sharing. Plambeck (2007) indicated that Walmart actively participated in the green product production and process reformation of suppliers through the contractual mechanism. This included sharing the cost of farmers’ planting organic cotton to promote the transformation of the land from the traditional cultivation to the organic cultivation. Bhaskaran and Krishnan (2009) investigated that Mega Pharmaceuticals company cooperated with Alpha Labs to equally share the cost of drug development investment, so that the lab can reduce the cost burden of drug development and research. The pharmaceutical company can then better commercialize and promote the product.

This study makes the following contributions, distinct from the existing literature. Firstly, the study combines the two-way cost-sharing contract with the cap-and-trade regulation in two-echelon supply chain which is not usually involved in the previous studies. Secondly, the study establishes the joint emission reduction model in which considers the process of joint emission reduction between the manufacturer and retailer, then extends the existing model to the case of multiple retailers in the market. Thirdly, the study considers the two-way cost-sharing contract in which the manufacturer and retailer can share the carbon emission abatement cost of each other. In addition, analyzing the impact of the changes in sharing rate on emission abatement level and supply chain decisions, as well as the influence of carbon trading price on supply chain. The study concludes that the two-way cost-sharing contract is beneficial to carbon emission reduction, then obtains the optimal sharing rate portfolio under different supply chain situations. Meanwhile the study finds that the government can control carbon emission through regulating carbon emission trading price. This study also discuss the impact of cap-and-trade regulation policy on supply chain decisions under different cost-sharing contracts, which provides insights for government to
control carbon emission, as well as the managerial inspiration for supply chain.

3 Problem description and assumptions

In this paper, similar to Xu. et al. (2017), the model is based on the assumption of an MTO supply chain, which is widely used in supply chain research and practical operation. The upstream manufacturer produces the finished products, which are ordered by the downstream retailer. Then the retailer packages and sells the products to the final consumers. Since the entire process is produced on order, the manufacturer’s inventory cost is not taken into account. The product quantity of manufacturer derives from market demand, so the market demand is assumed to be always equal to product quantity. For instance, an electric car is sold by car retailer, who orders the products from the car manufacturer. Therefore, we consider a two-echelon supply chain which consists of one manufacturer and one retailer based on the above description.

Government emphasis on environmental issues has resulted in a series of policies and regulations regarding sustainable development. These rules compel supply chain enterprises to contemplate on their impact during regular operation on the environment. Meanwhile, the increasing environmental awareness of consumers also encourages firms to actively undertake additional social responsibility related to environmental problems. Therefore, we consider the carbon emissions of supply chains under conventional operations. In this model, the manufacturer and the retailer may be involved in carbon emission reduction. First, the manufacturer obtains a specific initial carbon quota (also called “cap”) from the government; the amount is derived from the manufacturer’s prior experience (Sadegheih, 2011). When the actual carbon emissions produced fall short of or exceed the initial carbon quota, the enterprise can sell or purchase (also called “trade”) quotas in the carbon emission trading market. Due to the environmental protection advertisement can positively influence the purchase behavior of consumers (Cherian and Jacob, 2012), so the retailer will consider low-carbon advertising to better sell the products. In this way, the retailer participates in the emission reduction indirectly. Hence, we investigate the joint reduction decisions between the manufacturer and retailer under cap-and-trade regulation.

The emission reduction of supply chains comprises three stages. In the first stage, supply chain enterprises are aware of the importance of reducing carbon emissions. The manufacturer takes part in the carbon emission while the retailer maintains normal operations. In the second stage, the retailer considers the manufacturer’s pressure to decrease emissions or increase the sales of low-carbon products in the market. The retailer may share a portion of the manufacturer’s carbon abatement costs, pursue low-carbon advertising, or both, because low-carbon products tend to have higher prices than ordinary commodities. In the last stage, the manufacturer may consider sharing part of the retailer’s promotion costs. Therefore, both enterprises share partial costs to optimize their own profit or supply chain profits. These processes are described in Fig. 1.
Before the establishment of the mathematical model, several assumptions are provided as follows.

**Assumption 1.** Product demand is linear with the selling price set by the retailer and the emission abatement level of the manufacturer.

The setting of price to demand function is common in existing literature. In addition, carbon emissions affect product demand. The higher the abatement level of the manufacturer, the lower the unit carbon emission of its products and the higher the market demand. Customers also increasingly prefer low-carbon products as their environmental awareness increases (Liu et al., 2012). Hence, market demand should be linear with the firm’s emission abatement level.

**Assumption 2.** The initial carbon emission per unit product is constant, and total carbon emissions linearly increase with quantity.

The carbon emission of the manufacturer is mainly derived from production. For model simplicity, we ignore carbon emissions produced during transportation and other processes. The carbon emissions per unit product remain constant when the manufacturer does not implement any emission reduction measure. Therefore, carbon emissions are linearly dependent on production quantity (Du et al., 2013; Yang et al., 2014).

**Assumption 3.** We consider the manufacturer’s production cost only and ignore inventory or transportation cost.

**Assumption 4.** The manufacturer can always trade quotas in the carbon emission trading market when its initial carbon quota is insufficient or excessive.

First, the manufacturer is allocated an initial carbon quota by the government. The specific amount of this quota depends on previous experience and the anticipated market demand. The manufacturer then operates according to the received carbon quota. When the actual carbon emissions are larger than the initial quota, the manufacturer must purchase additional quotas to address the excess carbon emissions, whereas any surplus can be sold for profit. Furthermore, the sale price is less than the purchase price; otherwise, the initially allocated carbon quotas will be meaningless. The carbon trading price is an exogenous variable that is determined by the carbon emission trading market.

**Assumption 5.** The manufacturer’s cost of carbon emission reduction is related to the abatement level, and the retailer’s low-carbon promotion cost is associated with the promotion degree.

For the manufacturer, the abatement level directly determines the cost of carbon emission reduction. Similarly for the retailer, the low-carbon promotion degree...
determines the promotion cost, thus determining the cost for retailer to participate in carbon emission reduction. Other activities may also generate corresponding costs. Since the research mainly analyzes the retailer’s decisions to take part in carbon emission reduction, we only consider the cost of emission reduction generated by retailer’s promotion. The final optimization results are not affected by this. The same setting is seen in Ji. et al. (2017).

The cost of carbon emission reduction for the manufacturer is assumed to be a one-time investment that is a quadratic function (similar to the retailer’s low-carbon product promotion cost), which has been widely used in prior literature (Bansal and Gangopadhyay, 2003; Yao and Liu, 2005; Liu et al., 2012; Xu. et al., 2017; Wei et al., 2015; Yang et al., 2017). First, the quadratic function makes the profit function concave for carbon emission abatement level. If the impact of carbon emission abatement level on profit is monotonous, then the optimal abatement level is always the bigger the better or as small as possible, thus it cannot be used as decision variable. The research core of this paper is to study carbon emission reduction decision of supply chain, hence this form can highlight the impact of abatement level on supply chain decision. Second, in practice, the carbon emission reduction cost is invested in increasing abatement level. As the abatement level increases, the investment cost is bound to increase. And when the abatement level increases in the later period, this cost increases sharply, the manufacturer should pay more costs in carbon emission reduction. Therefore the process of carbon emission reduction is from easy to difficult in the market, the quadratic form can better fit this process. Thus we use the function \( c(e) = \frac{1}{2}k_1e^2 \) to define the manufacturer’s emission abatement cost, where \( e \) is the emission abatement level and considered as the manufacturer’s decision variable. For retailers, those with a high promotion degree must pay more in additional advertising cost and displaying low-carbon products, thereby increasing promotion cost. Therefore, we can use the function \( c(r) = \frac{1}{2}k_2r^2 \) to represent the retailer’s low-carbon product promotion cost, where \( r \) denotes the promotion degree and is regarded as the retailer’s decision variable. To ensure that the supply chain profit function is concave to the decision variables, as well as the carbon emission abatement level \( e \), product quantity \( q \) and other decision variables are non-negative, so the parameters \( k_1 \) and \( k_2 \) are assumed to be very large (Wang et al., 2016; Ji et al., 2017).

The relevant model parameters are shown in Table 1.

<table>
<thead>
<tr>
<th>Model parameters</th>
<th>Unit</th>
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<tbody>
<tr>
<td>( w ) ($/unit)</td>
<td>unit wholesale price of manufacturer</td>
</tr>
<tr>
<td>( p ) ($/unit)</td>
<td>unit retail price of retailer</td>
</tr>
<tr>
<td>( c ) ($/unit)</td>
<td>unit production cost of manufacturer</td>
</tr>
</tbody>
</table>
\( q \) (unit) production quantity of manufacturer, also the product market demand
\( a \) (unit) market potential demand
\( e_0 \) (kg/unit) initial unit product carbon emission of manufacturer
\( e \) (kg/unit) emission abatement level of unit product
\( r \) (/unit) promotion degree of retailer
\( E \) (kg) initial unit product carbon emission of manufacturer
\( p \) (unit$/kg) unit carbon trading price, determined by carbon emission trading market
\( k_1 \) (unit$^2$/kg) coefficient of manufacturer’s emission reduction cost
\( k_2 \) (unit$^2$/kg) coefficient of retailer’s promotion cost
\( b_1 \) (unit$^2$/kg) sensitivity coefficient of emission abatement level to market demand
\( b_2 \) (unit$^2$/kg) sensitivity coefficient of promotion degree to market demand
\( \varphi \) retailer share rate of manufacturer’s emission reduction cost
\( \theta \) manufacturer share rate of retailer’s promotion cost

4 Model and analysis

The three phases of emission reduction for single manufacturer–single retailer supply chain are divided into two categories for discussion and analysis. In the beginning, the manufacturer participates in carbon emission reduction, whereas the retailer only shares a portion of emission abatement cost (one-way cost-sharing contract). This arrangement is called the single emission reduction mode. The second category is the joint emission reduction mode, in which the retailer undertakes advertising and promotion for low-carbon products in supply chain and both firms simultaneously share part of each other’s cost (two-way cost-sharing contract). When the manufacturer and the retailer both join in carbon emission reduction, we can define it as the joint emission reduction (Ji. et al., 2017). Under the above mentioned conditions, decentralized or centralized decisions are considered in the supply chain in all cases. Thus, a trade-off should be established among product quantity, emission abatement level, promotion degree, and actual profit, and then corresponding operational decisions are made.

Before exploring joint emission reduction, we analyze the single emission reduction situation.

4.1 Single emission reduction model

In this model, the manufacturer participates in carbon emission reduction and the retailer shares part of the manufacturer’s emission abatement cost (as indicated by the fine dashed line in Fig. 1). According to Assumption 1, market demand is related to retail price and emission abatement level; that is,

\[ q = a - p + b_1 e. \]  

A similar demand function setup is seen in Liu et al. (2012) and Xu. et al. (2017). We ignore the price elasticity of market demand that does not affect analysis results, for model simplicity, and to highlight the carbon emission reduction. We now discuss equilibrium solutions for the supply chain in decentralized and centralized cases.
4.1.1 Decentralized situation

The manufacturer and the retailer competitively strategize to maximize their own profits in the situation where a Stackelberg game exists between them. As the Stackelberg leader, the manufacturer first determines the wholesale price \( w \) and emission abatement level \( e \) simultaneously on the basis of its maximum profit. The retailer then sets the retail price \( p \) according to the received decisions portfolio to maximize its profit. On the basis of the preceding assumptions, the profits of the manufacturer and the retailer are respectively set as follows:

\[
\pi_m = (w - c)q - p_c \left[ e_0 - e \right]q - E - \frac{1}{2} (1 - \varphi) \eta_k e^2, \tag{2}
\]

\[
\pi_r = (p - w)q - \frac{1}{2} \varphi \eta_k e^2. \tag{3}
\]

The notations \( \pi_m, \pi_r \) represent the profit functions of the manufacturer and the retailer, respectively, and superscripts \( sd, sc \) denote the decentralized and centralized cases, respectively, under the single emission reduction model. We use a similar marking method for the rest of the paper. By applying the backward induction procedure to find the optimal solutions for the model, we obtain the following propositions about the optimal prices and emission abatement level, product quantity, and firms’ profits.

**Proposition 1.** In the decentralized single emission reduction model, the optimal equilibrium solutions are as follows:

\[
w_{sd} = \frac{2 k_1 (1 - \varphi) b_c \left[ a + c + p_c e_0 \right] - p_c^2 a - b_1^2 (c + p_c e_0) \right] 4 k_1 (1 - \varphi) - (b_1 + p_c)^2, \]

\[
p_{sd} = \frac{3 k_1 (1 - \varphi) b_c \left[ a + c + p_c e_0 \right] - b_c^2 a - b_1 \left[ c + p_c e_0 \right] - b_1 \left[ 1 - \varphi \right] b_c \left[ c + p_c e_0 \right] \right] 4 k_1 (1 - \varphi) - (b_1 + p_c)^2, \]

\[
e_{sd} = \frac{b_1 + p_c \left[ a - c - p_c e_0 \right] \right] 4 k_1 (1 - \varphi) - (b_1 + p_c)^2. \]

The proofs of all propositions are provided in Appendix I to enhance the readability of this paper. The superscript * denotes the optimal solutions under this model. The preceding solutions are substituted into Eqs. (1)–(3) to derive the optimal product quantity and profits of the supply chain members.

\[
q_{sd} = \frac{k_1 (1 - \varphi) \left[ a - c - p_c e_0 \right] \right] 4 k_1 (1 - \varphi) - (b_1 + p_c)^2 \]

\[
\pi_m^{sd} = \frac{2 k_1^2 (1 - \varphi)^2 - \frac{1}{2} k_1 (1 - \varphi) (b_1 + p_c)^2 \left[ a - c - p_c e_0 \right]^2 \right] 4 k_1 (1 - \varphi) - (b_1 + p_c)^2 + p_c E
\]
The following corollary is obtained according to the optimal solutions in Proposition 1.

**Corollary 1.** (1) \( \frac{\partial e^{sd\varphi}}{\partial \varphi} > 0, \frac{\partial q^{sd\varphi}}{\partial \varphi} > 0 \), (2) \( e^{sd\varphi} > e^{sd\varphi, q^{sd\varphi} > q^{sd\varphi}} \).

The proofs of all corollaries are also shown in Appendix I.

Corollary 1(1) shows the influence of the retailer’s share proportion on emission abatement level and product quantity. Such an effect is consistent, and any fluctuation in the share rate \( \varphi \) eventually affects abatement level and quantity. The manufacturer’s emission abatement level and product quantity increase with the retailer’s share rate. This finding indicates that when the retailer willingly shares the manufacturer’s emission reduction cost, the manufacturer also willingly participates in the carbon emission reduction, thereby producing an increased quantity of environmentally friendly products. This result is consistent with the situation in the market, in which manufacturers can easily reduce carbon emissions when retailers’ sharing reduction cost rises, thereby resulting in an increase in emission abatement level. Thus, supply chain products become increasingly environmentally friendly and popular.

Corollary 1(2) analyzes whether emission abatement level and production change when the retailer shares manufacturer’s emission reduction cost. The emission abatement level and production increase after the retailer has shared manufacturer’s emission reduction cost. Hence, this corollary suggests that the retailer’s one-way cost-sharing contract is beneficial for carbon emission reduction and production.

### 4.1.2 Centralized situation

In this situation, the manufacturer and the retailer make operational decisions together to maximize supply chain profits under the decentralized case. Hence, the profit function is redefined as follows:

\[
\pi = \pi_m + \pi_r = (p - c)q - p_r [e_0 - (e + E)] - \frac{1}{2} k_1 e^2. \tag{4}
\]

We then obtain the following proposition about the optimal price, emission abatement level, product quantity, and supply chain profit.

**Proposition 2.** In the centralized single emission reduction model, the optimal equilibrium solutions are as follows:

\[
p^{ucr*} = \frac{(k_1 - b_r p_r)(a + c + p_r e_0) - p_r^2 a - b_r^2 (c + p_r e_0)}{2k_1 - (b_r + p_r)^2},
\]

\[
e^{ucr*} = \frac{(b_r + p_r)(a - c - p_r e_0)}{2k_1 - (b_r + p_r)^2}.
\]
By substituting the above solutions into Eqs. (1) and (4), we obtain the following optimal product quantity and profit of the supply chain.

\[
q^* = \frac{k_1(a-c-p_e^0)}{2k_1 - (b_1 + p_c)^2},
\]

\[
\pi^* = \frac{k_1^2 - \frac{1}{2}k_1(b_1 + p_c)^2}{2k_1 - (b_1 + p_c)^2}(a-c-p_e^0)^2 + p_cE
\]

The following corollary can be derived from optimal solutions in Proposition 2.

**Corollary 2.**

(1) If \( \frac{1}{2} < \varphi < 1 \), then \( e^{da} > e^{ce} \); if \( 0 \leq \varphi < \frac{1}{2} \), then \( e^{da} < e^{ce} \).

(2) If \( \frac{2k_1}{2k_1 + (b_1 + p_c)^2} < \varphi < 1 \), then \( q^{da} > q^{ce} \); if \( 0 \leq \varphi < \frac{2k_1}{2k_1 + (b_1 + p_c)^2} \), then \( q^{da} < q^{ce} \).

According to Corollary 2, which shows an analysis of the difference in the manufacturer’s emission abatement level and product quantity between the decentralized and centralized situations, derives that the centralized decision is not always optimal for abatement level and product quantity in all cases. From Corollary 2(1), when \( \frac{1}{2} < \varphi < 1 \), the emission abatement level in the decentralized situation is higher than that in the centralized case; whereas when \( 0 < \varphi < \frac{1}{2} \), the centralized situation is superior to the decentralized one. This finding indicates that when the retailer is willing to share more of the manufacturer’s emission reduction cost, the decentralized decision is more conducive to carbon emission reduction, conversely the centralized decision should be selected. When the retailer does not share the emission reduction cost, the centralized decision is undoubtedly superior to the decentralized decision.

For Corollary 2(2), when \( \frac{2k_1}{2k_1 + (b_1 + p_c)^2} < \varphi < 1 \), the product quantity in the decentralized situation is larger than that in the centralized case. Meanwhile, the centralized situation produces more products when \( 0 < \varphi < \frac{2k_1}{2k_1 + (b_1 + p_c)^2} \). This finding suggests that when the retailer shares the reduction cost at a high rate, the products under the decentralized decision are more popular in the consumer market. Conversely, the centralized decision is optimal for product quantity, especially when
the retailer does not share the emission reduction cost, the centralized situation is better for product production than the decentralized one.

4.2 Joint emission reduction model

Considering the manufacturer’s pressure to reduce carbon emissions and increase the sales of low-carbon products, the retailer opts for the promotion of low-carbon products under this model. Thus, the manufacturer is still directly involved in reducing carbon emissions, and the retailer is indirectly involved via low-carbon product promotion. In addition, both firms share partial costs (as indicated by the thick dashed line in Fig. 1). From the assumption 1, the market demand changes to the following:

\[ q = a - p + b_1 e + b_2 r. \] (5)

A similar demand function setup is seen in Ji. et al. (2017). We then discuss the equilibrium solutions in the decentralized and centralized cases in the following subsection.

4.2.1 Decentralized situation

A two-stage game is established between the manufacturer and the retailer to maximize profits in this situation. First, as the Stackelberg leader, the manufacturer simultaneously provides the wholesale price \( w \) and emission abatement level \( e \). The retailer then simultaneously sets its retail price \( p \) and promotion degree \( r \) according to the received decisions portfolio. According to the preceding assumptions, the profit function of the supply chain members is as follows:

\[ \pi_m = (w - c)q - p_e[(e_0 - e)q - E] - \frac{1}{2}(1 - \varphi)k_1 e^2 - \frac{1}{2} \varphi k_2 r^2, \] (6)

\[ \pi_r = (p - w)q - \frac{1}{2} \varphi k_1 e^2 - \frac{1}{2}(1 - \theta)k_2 r^2. \] (7)

The following proposition is then obtained about the optimal prices, emission abatement level, promotion degree, product quantity, and firms’ profits. The superscripts \( jd, jc \) denote the decentralized and centralized situations, respectively, under the joint emission reduction model.

**Proposition 3.** In the decentralized joint emission reduction model, the optimal equilibrium solutions are as follows:

\[ e^{\text{jd*}} = \frac{k_2^2 (1 - \theta)^2 (b_1 + p_c)(a - c - p_c e_0)}{k_1 k_2 (1 - \varphi) \left[ 4k_2^2 (1 - \theta)^2 - 2b_2^2 + 3\beta b_2^2 \right] \left[ k_2 (1 - \theta)(b_1 + p_c) \right]^2}, \]

\[ p^{\text{jd*}} = \frac{k_1 (1 - \varphi) k_2 (1 - \theta) b_2 (a - c - p_c e_0)}{k_1 k_2 (1 - \varphi) \left[ 4k_2^2 (1 - \theta)^2 - 2b_2^2 + 3\beta b_2^2 \right] \left[ k_2 (1 - \theta)(b_1 + p_c) \right]^2}, \]

\[ w^{\text{jd*}} = a + b_1 e^{\text{jd*}} - \frac{k_1 (1 - \varphi) e^{\text{jd*}} [2k_2 (1 - \theta) - b_2^2]}{k_2 (1 - \theta)(b_1 + p_c)}, \]
\[ p^{\text{id}e} = \frac{k_2(1-\theta)(a+b_1e^{\text{id}e}+w^{\text{id}e})-b_2^2w^{\text{id}e}}{2k_2(1-\theta)-b_2^2}. \]

Part of the formula is complex and thus partially replaced by the emission abatement level \( e^{\text{id}e} \). By substituting the above optimal solutions into Eqs. (5)–(7), we obtain the optimal product quantity and profits of the manufacturer and the retailer, respectively.

\[ q^{\text{id}e} = \frac{k_1(1-\varphi)k_2^2(1-\theta)^2(a-c-p_e\theta)}{k_2(1-\varphi)[4k_2(1-\theta)^2-2b_2^2+3\theta b_2^2]-[k_2(1-\theta)(b_1+p_e)]^2} \]

\[ \pi^{\text{m}d} = \left\{ \frac{k_2^2(1-\theta)^2}{k_2^2(1-\theta)^2} \left[ \frac{2k_2^2(1-\theta)^2-k_2^2b_2^2+\frac{3}{2}\theta k_2b_2^2}{k_2^2(1-\theta)^2} \right] - \frac{1}{2} k_1(1-\varphi) \right\} e^{\text{id}e^2} + p_e \]

\[ \pi^{\text{r}d} = \left\{ \frac{k_2^2(1-\theta)^2}{k_2^2(1-\theta)^2} \left[ \frac{\frac{1}{2}(1-\theta)k_2b_2^2}{k_2^2(1-\theta)^2} \right] - \frac{1}{2} k_2(1-\varphi) \right\} e^{\text{id}e^2} \]

The following corollary is obtained according to the preceding optimal solutions in Proposition 3.

**Corollary 3.**

(1) If \( \frac{1}{3} < \theta < 1 \), then \( \frac{\partial e^{\text{id}e}}{\partial \theta} < 0, \frac{\partial q^{\text{id}e}}{\partial \theta} < 0 \); if \( 0 < \theta < \frac{1}{3} \), then \( \frac{\partial e^{\text{id}e}}{\partial \theta} > 0, \frac{\partial q^{\text{id}e}}{\partial \theta} > 0 \); if \( \frac{1}{3} < \theta < 1 \), then \( e^{\text{id}e} > e^{\text{id}e}_{\theta=0}, q^{\text{id}e} < q^{\text{id}e}_{\theta=0} \); if \( 0 < \theta < \frac{1}{2} \), then \( e^{\text{id}e} > e^{\text{id}e}_{\theta=0}, q^{\text{id}e} > q^{\text{id}e}_{\theta=0} \).

Corollary 3(1) reveals the influence of the manufacturer’s sharing rate on the emission abatement level and product quantity. Such an effect is consistent and non-monotonous, and any fluctuation in the sharing rate \( \theta \) eventually affects the abatement level and quantity. When \( \frac{1}{3} < \theta < 1 \), the emission abatement level and product quantity decrease with an increase in the manufacturer’s share of the retailer’s promotion cost; when \( 0 < \theta < \frac{1}{3} \), the emission abatement level and product quantity increase with an increase in the manufacturer’s sharing rate. This finding indicates that the manufacturer taking a small share of the retailer’s promotion cost is beneficial to reducing carbon emissions and increasing commodity production. Conversely, the emission abatement level and quantity are reduced when the manufacturer’s sharing rate rises. If the manufacturer shares the retailer’s promotion cost, then the retailer is
encouraged to conduct additional low-carbon promotion. This condition increases product sales and drives manufacturers to produce more environmentally friendly products, which results in an increase in emission abatement level and product quantity. Otherwise, a high sharing proportion influences the manufacturer’s normal operation, thereby producing the decreasing effect on emissions reduction and product production.

Corollary 3(2) analyzes the changes in emission abatement level and production that occur after the implementation of one-way and two-way cost-sharing contract between the manufacturer and retailer. When $\frac{1}{2} < \theta < 1$, the abatement level and production after the manufacturer has shared promotion cost are less than that in the case where the manufacturer does not share promotion cost. When $0 < \theta < \frac{1}{2}$, the situation in which the manufacturer shares promotion cost is superior to that where the manufacturer does not practice sharing. The abatement level and production after the retailer has shared emission reduction cost are higher than that in the situation where the retailer does not share emission reduction cost. This is consistent with Corollary 1(2). Hence, this corollary suggests that the retailer’s or manufacturer’s one-way cost-sharing contract, or two-way cost-sharing contract can increase carbon emission abatement level and production. However, the manufacturer’s sharing rate of promotion cost should be in the interval $(0, \frac{1}{2})$.

4.2.2 Centralized situation

Based on the decentralized situation, the manufacturer and the retailer make operational decisions together in this case, and the supply chain profit is as follows.

$$\pi = \pi_{m} + \pi_{r} = (p - c)q - p_{e}[e_{0} - c]q - E\left[-\frac{1}{2}k_{1}e^{2} - \frac{1}{2}k_{2}r^{2}\right].$$  

(8)

We then obtain the following proposition about the optimal price, emission abatement level, promotion degree, product quantity, and supply chain profit.

**Proposition 4.** In the centralized joint emission reduction model, the optimal equilibrium solutions are as follows:

$$e^{ice} = \frac{k_{2}(b_{1} + p_{e})(a - c - p_{e}e_{0})}{k_{1}(2k_{2} - b_{2}^{2}) - k_{2}(b_{1} + p_{e})^{2}},$$

$$r^{ice} = \frac{k_{2}b_{2}(a - c - p_{e}e_{0})}{k_{1}(2k_{2} - b_{2}^{2}) - k_{2}(b_{1} + p_{e})^{2}},$$

$$p^{ice} = \frac{k_{2}[k_{1} - (b_{1} + p_{e})p_{e}][a - c - p_{e}e_{0})]}{k_{1}(2k_{2} - b_{2}^{2}) - k_{2}(b_{1} + p_{e})^{2}} + c + p_{e}e_{0}.$$  

By substituting the above solutions into Eqs. (5) and (8), we obtain the following optimal product quantity and profit of the supply chain.
\[ q^{ic} = \frac{k_1 k_2 (a - c - p_e e_0)}{k_1 (2k_2 - b^2_2) - k_1 (b_1 + p_e)^2} \]

\[ \pi^{ic} = \frac{k_1 k_2 [k_1 k_2 - \frac{1}{2} k_1 b_2^2 - \frac{1}{2} k_2 (b_1 + p_e)^2 (a - c - p_e e_0)^2 + p_e E}{[k_1 (2k_2 - b^2_2) - k_1 (b_1 + p_e)^2]} \]

The following corollary is derived on the basis of the preceding optimal solutions in Proposition 4.

**Corollary 4.**

1. If \[ 2k_2 (1 - 2\varphi) (1 - \theta)^2 > \left[ (3\theta - 2) \varphi - \theta^2 - \theta + 1 \right] b^2_2 \], then \( e^{idw} < e^{ic} \); if \[ 2k_2 (1 - 2\varphi) (1 - \theta)^2 < \left[ (3\theta - 2) \varphi - \theta^2 - \theta + 1 \right] b^2_2 \], then \( e^{idw} > e^{ic} \);

2. If \[ \frac{\varphi (b_1 + p_e)^2}{k_1 (1 - \varphi)} + \frac{\left( 1 - \theta^2 - \theta \right) b^2_2}{k_2 (1 - \theta^2)} < 2 \], then \( q^{idw} < q^{ic} \); if \[ \frac{\varphi (b_1 + p_e)^2}{k_1 (1 - \varphi)} + \frac{\left( 1 - \theta^2 - \theta \right) b^2_2}{k_2 (1 - \theta^2)} > 2 \], then \( q^{idw} > q^{ic} \).

According to Corollary 4, we compare the manufacturer’s emission abatement level and product quantity under the decentralized and centralized situations in the joint emission reduction model. The comparison results indicate that the centralized decision is always optimal under certain conditions. From Corollary 4(1), when \[ 2k_2 (1 - 2\varphi) (1 - \theta)^2 > \left[ (3\theta - 2) \varphi - \theta^2 - \theta + 1 \right] b^2_2 \], the emission abatement level in the decentralized situation is lower than that in the centralized case. In Corollary 4(2), when \[ \frac{\varphi (b_1 + p_e)^2}{k_1 (1 - \varphi)} + \frac{\left( 1 - \theta^2 - \theta \right) b^2_2}{k_2 (1 - \theta^2)} < 2 \], products under the centralized situation are widely popular in the consumer market. Otherwise, product quantity in the decentralized situation is higher than that in the centralized situation. Hence, the sharing rate of two-way cost-sharing contract can affect the decision choice of supply chain. The centralized situation is generally the optimal choice. Given that the optimal carbon emission abatement level and production, there is a threshold from centralized to decentralized situation. This corollary indicates that when the manufacturer’s and retailer’s sharing rates meet certain conditions, the centralized decision is beneficial for carbon emission reduction and production, and conversely the decentralized decision is optimal.

**4.3 Comparisons between supply chain models**

The following discussion presents a comparison of the optimal equilibrium solutions of the two kinds of supply chains, in order to explore the impact of supply chain structure on the optimal decisions. We then determine the best structure under
specific conditions on the basis of different criteria.

**Corollary 5.** According to a comparison of the single and joint emission reduction models in the decentralized situation, if $0 < \theta < \frac{2}{3}$, then $e^{id} > e^{sd}; q^{id} > q^{sd}$; if $\frac{2}{3} < \theta < 1$, then $e^{id} < e^{sd}; q^{id} < q^{sd}$.

Corollary 5 compares the emission abatement levels and product quantities in the two models under the decentralized situation. When $0 < \theta < \frac{2}{3}$, the abatement level and product quantity in the joint emission reduction model are higher than those under the single emission reduction model. When $\frac{2}{3} < \theta < 1$, the abatement level and quantity in the single emission reduction model are higher than those in the joint emission reduction model. This finding indicates that when the manufacturer shares a small proportion of the retailer’s promotion cost, the joint emission reduction model is superior to the single emission reduction model for carbon emission reduction and commodity production. However, if the sharing rate is excessively high, then the single emission reduction model should be selected. The manufacturer’s sharing of the cost will encourage the retailer to increasingly promote low-carbon products, thereby increasing the emission abatement level and product quantity. Once the sharing proportion exceeds a certain value, it will influence the manufacturer’s normal operation and the entire supply chain.

**Corollary 6.** A comparison of the single and joint emission reduction models in the centralized situation reveals that $e^{jca} < e^{ica}; q^{jca} < q^{ica}$.

Corollary 6 shows the difference in the emission abatement levels and product quantities between the single and joint emission reduction models in the centralized situation. The results indicate that the abatement level and product quantity in the joint emission reduction model are larger than those in the single emission reduction model. Moreover, the supply chain can produce more environmentally friendly products in the consumer market in the joint emission reduction model. This is because the sharing of each other’s partial cost can reduce the pressure of the manufacturer to reduce carbon emissions and that of the retailer to promote low-carbon products. Consequently, the emission abatement level and product quantity increase.

We now analyze the profits of the supply chain and its members in the different cases to obtain the following corollary. The individual profits of the manufacturer and the retailer are incomparable. Detailed explanations are provided in the subsequent numerical analysis.

**Corollary 7.** A comparison of the single and joint emission reduction models in the centralized situation indicates that $\pi^{ic} < \pi^{jc}$.
According to Corollary 7, the profit in the joint emission reduction model is higher than that in the single emission reduction model. This finding suggests that when the retailer promotes low-carbon products and both firms share partial costs simultaneously, the supply chain profit increases. Combined with Corollary 6, Corollary 7 states that the joint emission reduction model is the optimal one in the centralized situation.

4.4 The extended model

This subsection analyzes the case where there is more than one retailer which participates in carbon emission reduction in the supply chain. In essence, we add the competition in the retail portion to the single emission reduction model introduced in section 4.1. The purpose of competition is to raise manufacturer’s emission abatement level and reduce carbon emission, as well as both the retail and wholesale prices. Based on the model in section 4.1, assume that there are \( n \) retailers, and the subscript \( R_i \) represents a retailer in the consumer market, \( i = 1, 2, 3, \ldots, n \). The \( n \) retailers simultaneously and independently make their own decisions, and competition from the retailers may impact the manufacturer’s decision-making process.

In this model, the manufacturer and \( n \) identical retailers make the competitive strategies to maximize their profits respectively. Similar to the decentralized situation where there is only one single retailer in the subsection 4.1.1, this situation extends to include \( n \) retailers. Assume that the \( n \) retailers are exactly the same, because if each retailer faces a common manufacturer and consumers then the sale volume is equal, that is \( \sum \limits_{i=1}^{n} q_i = nq, i = 1, 2, 3, \ldots, n \) (9). Hence the manufacturer, as the stackelberg game leader, firstly determines the wholesale price \( w \) and emission abatement level \( e \) simultaneously to maximize its profit, then the followers make their own decisions respectively to respond to manufacturer. Based on the above description, the profits of manufacturer and retailer can be rewritten as follows, respectively,

\[
\pi_\text{m} = (w - c)q - p \left[ (e_0 - e)q - E \right] - \frac{1}{2} (1 - \phi)k_i e^2
\]

(10)

\[
\pi_i = (p - w)q_i - \frac{1}{2} \phi k_i e^2
\]

(11)

The subscript \( i \) denotes the related information of the \( i \)th retailer, the superscript \( E \) in the next proposition represents the multiple retailers situation under the extended model. By employing the backward induction procedure, then we get the following proposition about the optimal retail price and the emission abatement level as well as the product quantity and profits of firms.

**Proposition 5.** In the multiple retailers extended model, the optimal equilibrium solutions are shown as follows,
$$w_{E^*} = \frac{(n+1)k_i(1-\varphi) - nb_i p_c}{2(n+1)k_i(1-\varphi) - n(b_i + p_c)^2} (a + c + p_c e_0) - np_i^2 a - nb_i^2 (c + p_c e_0)$$

$$e_{E^*} = \frac{n(b_i + p_c)(a - c - p_c e_0)}{2(n+1)k_i(1-\varphi) - n(b_i + p_c)^2}$$

$$q_i^{E^*} = \frac{k_i(1-\varphi)(a - c - p_c e_0)}{2(n+1)k_i(1-\varphi) - n(b_i + p_c)^2}$$

$$q^E = \frac{nk_i(1-\varphi)(a - c - p_c e_0)}{2(n+1)k_i(1-\varphi) - n(b_i + p_c)^2}$$

Substituting the above solutions into Eqs. (9)-(11), deriving the optimal quantity and profits of supply chain members.

$$p_i^{E^*} = \frac{(n+2)k_i(1-\varphi) - nb_i p_c + p_c}{2(n+1)k_i(1-\varphi) - n(b_i + p_c)^2} [a + nk_i(1-\varphi) - n(b_i + p_c)] (c + p_c e_0)$$

$$n_{E^*} = \frac{n(n+1)k_i^2(1-\varphi)^2 - \frac{1}{2} k_i (1-\varphi)n^2 (b_i + p_c)^2 (a - c - p_c e_0)^2}{2(n+1)k_i(1-\varphi) - n(b_i + p_c)^2} + p_c E$$

$$\pi r_{E^*} = \frac{k_i^2(1-\varphi)^2 - 1}{2(n+1)k_i(1-\varphi)} [a - c - p_c e_0]$$

From the optimal solutions in the proposition 5, getting the following corollaries.

**Corollary 8.**

1. \( \frac{\partial e^{E^*}}{\partial n} > 0; \frac{\partial q_i^{E^*}}{\partial n} < 0; \frac{\partial q^{E^*}}{\partial n} > 0 \),

2. \( e^{id^*} \leq e^{E^*}; q_i^{E^*} \leq q^{id^*} \leq q^{E^*} \),

3. \( \lim_{n \to \infty} e^{E^*} = \frac{(b_i + p_c)(a - c - p_c e_0)}{2k_i(1-\varphi) - (b_i + p_c)^2} \); \( \lim_{n \to \infty} q^{E^*} = \frac{k_i(1-\varphi)(a - c - p_c e_0)}{2k_i(1-\varphi) - (b_i + p_c)^2} \).

Following corollary 8, we analyze the impact of retailer numbers on the emission abatement level and product quantity under the multiple retailers situation. From the corollary 8(1), when the retailers’ number increases, so does the emission abatement level. Meanwhile, the amount of product sold by a single retailer decreases with the increase of parameter \( n \), while the total quantity of product from the manufacturer increases gradually. It indicates that as retailer numbers increases, the manufacturer will produce more environmentally and friendly products.

Corollary 8(2) compares the emission abatement level and product quantity in the decentralized single emission reduction model and the extended model. It shows
that the emission abatement level in the extended model is higher than that in the decentralized situation. The single retailer’s sales volume in the extended model is lower than that in the decentralized situation, but the total product quantity in the extended model is higher. Combined with the corollary 8(1) and (2), it can be concluded that the existence of multiple retailers is conducive to the carbon emission reduction of manufacturer, and the products are popular with the consumer market. In addition, as the number of retailers increases, so does the emission abatement level and product quantity produced by the manufacturer.

Corollary 8(3) indicates the best results (that is the highest emission abatement level and the highest product quantity) in the extended model. Although the emission abatement level and product quantity will increase with the retailers’ number, there is an upper limit for them, as shown in the corollary 8(3) which represents the best conditions that can be achieved.

Since it is a multi-retailer situation, we only compare the profit of manufacturer in the decentralized single emission reduction model and the extended model, getting the following corollary.

**Corollary 9.** Compared the profit of manufacturer in the decentralized single emission reduction model and the extended model, we can get that\( \pi^{de} \leq \pi^{ex} \).

From the corollary, it can be inferred that the manufacturer’s profit in the extended model is larger than that in the decentralized situation. It indicates that when multiple retailers participate in the supply chain, the manufacturer will gain more profit. Combined with corollary 8, it can be derived that when there are multiple retailers, the manufacturer will produce more environmentally friendly products and obtain more profit. Therefore, the extended model is more beneficial to carbon emission reduction than single retailer mode, and the extended model is better for the supply chain.

This subsection analyzed the situation that the multiple retailers are identical. However, in future research, it is necessary to explore the case that the multiple retailers are not identical and analyze the impact of different retailers decisions on supply chain simultaneously.

### 5 Numerical analysis

To present evident comparisons, we provide the following series of numerical studies to discuss the impacts of parameter that are inconvenient to analyze directly. The values of the parameters are assumed as \( a = 40, \ c = 5, \ b_1 = 2, \ b_2 = 1, \ e_0 = 2, \ p_c = 2, \ E = 5, \ k_1 = 100, \ k_2 = 20, \ \varphi = 0.2, \) and \( \theta = 0.2 \). The numerical study comprises a two-part analysis. In the first part, we present the impact of the manufacturer’s sharing rate of retailer’s promotion cost on supply chain members’ profits. The next part shows the influence of carbon emission trading price on emission abatement level, product quantity, and the profits of supply chain members.
5.1 Impact of manufacturer’s sharing rate

In this section, we set the manufacturer’s sharing rate to vary in the interval of [0, 0.8]. Completely sharing the retailer’s low-carbon product promotion cost (that is, \( \theta = 1 \)) is not reasonable for the manufacturer. Fig. 2 shows the impact of the manufacturer’s sharing rate on the profits of supply chain members.

In the single emission reduction model, Fig. 2 shows that after the retailer has shared emission reduction cost, the profits of supply chain members become larger than those in the case where the retailer does not share the cost (as shown in Fig. 2, \( \pi^{\text{sd}}, \varphi = 0 \) and \( \pi^{\text{sr}}, \varphi = 0 \)). This finding indicates that when the retailer shares this cost, the profits of the manufacturer and the retailer simultaneously increase, thereby producing a mutually beneficial situation. Hence, the retailer’s one-way cost-sharing contract is beneficial for supply chain. In the joint emission reduction model, the profit of the manufacturer first increases slowly and then decreases as the manufacturer’s sharing rate increases. From Fig. 2, we can get that when the sharing rate \( \theta = \frac{1}{3} \), the manufacturer’s profit is optimal (the maximum is shown in Fig. 2). If the manufacturer shares retailer’s promotion cost within a small rate interval (\( \theta < \frac{1}{3} \)), the manufacturer’s profit will increase with this sharing rate, while the retailer’s profit will decrease accordingly. Meanwhile, the higher the sharing rate, the faster the retailer’s profit decreases. Hence, the manufacturer’s one-way cost-sharing contract is beneficial for the manufacturer when the sharing rate is in the interval of \((0, \frac{1}{3})\), but is detrimental for the retailer.

According to a comparison of the two models in Fig. 2, when the sharing rate satisfies \( \theta < \frac{2}{3} \), the manufacturer’s profit in the joint emission reduction model exceeds that in the single emission reduction model, on the contrary the single emission reduction model is more profitable. Furthermore, if this sharing rate approximately meets \( \theta < \frac{1}{2} \), the retailer’s profit in the joint emission reduction model is higher than that in the single emission reduction model. When \( \theta > \frac{1}{2} \), the retailer’s profit in the joint emission reduction model is low. The joint emission reduction model is optimal for supply chain members’ profits when this sharing rate is in the interval of \((0, \frac{1}{2})\). Hence, combining with corollary 5 we derive that joint emission reduction model is optimal choice for supply chain when the sharing rate is small.
5.2 Impact of carbon emission trading price

Carbon trading price is determined by the carbon emission trading market. We assume that this price vary in the interval of $[0,8]$. Fig. 3 describes the impact of carbon trading price on emission abatement level in four scenarios. As shown in this figure, emission abatement level increases with carbon trading price in all situations. Meanwhile, the larger the price, the faster the level increases in the centralized situation. This finding indicates that carbon trading price can effectively promote carbon emission reduction of the manufacturer. Under the cap-and-trade regulation policy, after the manufacturer has reduced carbon emission, if the carbon emission level is still higher than initial carbon quotas at the moment, then the manufacturer still needs to buy extra quotas from the carbon trading market to remain regular operation. The increase of carbon trading price increases the purchasing cost. However, if the carbon emission level is lower than initial carbon quotas at this moment, the manufacturer can sell the remaining quotas to the carbon trading market to gain additional profits. Therefore, the increase of carbon trading price results in the increase of additional profits. Thus the manufacturer will reduce carbon emission under both two situations when the carbon trading price increases. Because of the cap-and-trade policy, the manufacturer’s carbon trading behavior results in the purchasing cost or additional profit which will be affected by carbon trading price. From the perspective of policy-level, in the early stage of carbon emission reduction, the abatement level is always low, the carbon trading market can increase trading price to enforce manufacturer to reduce carbon emission. Whereas in the later stage, the abatement level is high, then the manufacturer may sell remaining quotas to market, the market can decrease this price properly to avoid loss. Hence, carbon emission trading market can stimulate supply chain to reduce carbon emission by regulating carbon trading price.

![Fig. 2. Impact of manufacturer’s sharing rate on profits of supply chain members](image1)

![Fig. 3. Impact of carbon trading price on emission abatement level](image2)

We now analyze the influence of carbon trading price on supply chain production, and Fig. 4 shows the impact in four situations. Product quantity decreases with an
increase of carbon trading price in the decentralized situation, which indicates that a high carbon trading price leads to a small product quantity. By contrast, the influence of carbon trading price is non-monotonous in the centralized situation. When carbon trading price is $p_c \leq 4$, product quantity decreases as trading price increases; when $p_c > 4$, product quantity increases with trading price. Under the cap-and-trade regulation policy, at the beginning of the increase of carbon trading price, the carbon emission abatement level is low. Hence the manufacturer still needs to buy additional quotas from carbon trading market. The increase of purchasing cost cuts down the investment cost in product production, thus product quantity decreases. Later, with the increase of carbon trading price, the carbon emission abatement level increases sharply, which results in the increase of product market demand. At this moment, the manufacturer sells the remaining quotas to obtain additional profits. The additional profits can provide the cost for producing more products, which leads to the increase of production quantity. Meanwhile the manufacturer can produce more products to meet the market demand and expand the market share. From the perspective of policy-level, the carbon trading market can raise carbon trading price to pursue a small number products that are more environmentally friendly in the early stage of carbon emission reduction. In the later stage, the market can consider appropriate decrease this price due to the high carbon emission abatement level in the decentralized decision situation. Hence, the changes in carbon trading price can affect supply chain production.

Fig. 4. Impact of carbon trading price on product quantity

Fig. 5. Impact of carbon trading price on profits of supply chain members

Corollary 7 analyzes the profit of different models under the centralized decision, and Fig. 5 describes the impact of carbon trading price on profits of supply chain members’ in two other circumstances. According to Fig. 5, the impacts of carbon trading price on the profits of supply chain members are consistent in the decentralized situation, and the profit decreases with the increase of carbon trading price. The increase in this price leads to a decline in production, which directly limits
supply chain members’ profits. Meanwhile, supply chain members’ profits in the joint emission reduction model are higher than that in the single emission reduction model under this parameter setting. Hence, the increase of carbon trading price is detrimental for the profits of supply chain members. From the perspective of policy-level, the carbon trading market should set carbon trading price in the medium interval to coordinate carbon emission abatement level and supply chain profit. To ensure that the supply chain can obtain enough profit, and there are more environmentally friendly products in the market simultaneously.

Next we analyze the impact of retailers’ number and carbon trading price on the carbon emission abatement level and product quantity, as shown in Fig. 6 and 7 respectively. In Fig. 6, regardless of the number of retailers, the carbon emission abatement level increases with carbon trading price, which is similar to the situation of single emission reduction model. And the upward trend becomes more obvious when this retailers’ number increases. Meanwhile as the retailers’ number increases, the carbon emission abatement level increases as well, which indicates that the competition between retailers is conducive to carbon emission reduction. The more intense the competition, the more incentive the manufacturer has to produce more environmentally friendly products.

In Fig. 7, the impact of carbon trading price on product quantity is non-monotonic. With the increase of carbon trading price, the product quantity first decreases and then increases, which is similar to the centralized situation under single emission reduction model. Meanwhile, with the increase of retailers’ number, the product quantity of individual retailer gradually decreases, which accords with the market rule. Combining with corollary 8 (1) and (2), we conclude that market competition is conducive to produce more environmentally friendly products, while adjusting carbon trading price can stimulate the manufacturer to reduce carbon emission.

Fig. 6. Impact of retailer’s number and carbon trading price on carbon emission abatement level

Fig. 7. Impact of retailer’s number and carbon trading price on product quantity

From the preceding analysis, because of the cap-and-trade regulation policy, the supply chain should pay (or gain) cost (or profit) during the carbon trading process, thus carbon trading price can affect carbon emission reduction, production and supply
chain profits. The higher carbon emission price is, the higher emission abatement level will be, but production and profit will decrease. In the early stage, the carbon trading market can raise the carbon trading price to get higher carbon emission abatement level regardless of the change in the product quantity and profit. However, in the later stage, the market should balance the emission abatement level and supply chain profit, thus it can decrease carbon trading price properly. Hence, carbon emission trading market can stimulate supply chain to reduce carbon emission by regulating this price, meanwhile should focus on the decrease of supply chain production and profits.

6 Conclusions

This study develops a model comprising one manufacturer and one retailer to investigate supply chain production and carbon emission reduction decisions under the cap-and-trade regulation policy. Single and joint emission reduction modes under cap-and-trade regulation are considered, and one-way and two-way cost-sharing contracts between supply chain members are discussed. A Stackelberg game, where the manufacturer is the leader, exists between the manufacturer and the retailer in the decentralized solution. However, in the centralized situation, supply chain members act together to decide regarding production, cost sharing rate, carbon emission abatement level, and overall profit. This research presents the following main results, which can provide managerial insights for supply chain and the basis for the government’s policy making.

First, we analyze the retailer’s sharing rate of manufacturer’s carbon emission reduction cost in the single emission reduction model. The sharing rate can influence carbon emission abatement level and product quantity, thereby ultimately affecting the profits of supply chain members. After retailer has shared cost, carbon emission abatement level, product quantity, and manufacturer’s and retailer’s profits increase. Hence, the retailer’s one-way cost-sharing contract is beneficial for supply chain. When this sharing rate is low, supply chain selects centralized decision; by contrast, when sharing rate exceeds a certain threshold, the decentralized decision is optimal choice for supply chain.

In the joint emission reduction model, we analyze the two-way cost-sharing contract between supply chain members. The manufacturer’s sharing rate of retailer’s promotion cost also affects abatement level, product quantity, and supply chain profits. When this sharing rate is small, after manufacturer has shared cost, abatement level, product quantity, and manufacturer’s profit improve, and all three variables increase further with sharing rate. Whereas when this rate exceeds a certain value, all three variables decrease after manufacturer has shared cost, and decrease further with an increase in sharing rate. The retailer’s profit decreases when the manufacturer shares cost. Hence, the two-way cost-sharing contract is beneficial for carbon emission reduction, production and manufacturer’s profit, but is detrimental for retailer’s profit. Meanwhile, this sharing rate can affect supply chain choice between decentralized and centralized decisions.
Second, when the sharing rate of manufacturer’s one-way cost-sharing contract is low, a comparison of two models shows that carbon emission abatement level, product quantity, and supply chain profits in the joint emission reduction model are satisfactory in the decentralized situation; whereas when this sharing rate is high, the single emission reduction mode is beneficial for supply chain. In the centralized situation, the joint emission reduction model is superior to the single emission reduction model for carbon emission reduction, production, and supply chain profits. In addition, emission abatement level and product quantity increase with retailer’s number in the extended model. The extended model is more conducive to carbon emission reduction by comparing the single emission reduction model and the extended model.

Third, based on the cap-and-trade regulation policy, there will be transaction cost which is affected by carbon trading price during the carbon trading process, thus carbon trading price can affect carbon emission reduction, production and profits of supply chain members. The specific effects are as follows, carbon emission abatement level increases with this price. With the increase of carbon trading price, product quantity decreases in the decentralized situation, and decreases first then increases in the centralized situation. Supply chain members’ profits decreases with the increase in this price. Taking the policy-level and supply chain performance into consideration, the carbon trading market can increase carbon trading price in the early stage to promote supply chain to reduce carbon emission, then this price can be lowered appropriately in the later stage as the carbon emission abatement level of products increases. Hence, the government can promote supply chain to reduce carbon emission by regulating carbon trading price, but cannot ignore the impact on supply chain production and profits.

Several areas in this study require further extension and improvement. First, this study considers only the manufacturer’s carbon emissions in production process. In practice, carbon emissions may exist in any link of supply chain. Thus, the equilibrium of carbon quotas between supply chain members, and total carbon emissions of entire supply chain are worth studying in the future. Second, this study only discusses the single principal-agent game mode under information symmetry. In practice, there are many different game modes. Such as the large retailer (for example, the Wal-Mart) as decision marker, the manufacturer or supplier as decision follower. The different game modes lead to different optimal solutions, which ultimately affects the supply chain operation decisions. Therefore, the future research can analyze and compare the situations between different game modes. On the other hand, in reality, the information between supply chain members is often asymmetric, hence this study can discuss the carbon emission reduction and product production decisions under the condition of information asymmetry. Third, the multiple retailers in the extended model are identical, while the retailers are not identical in practice. Hence, the future research can analyze the mode with different multiple retailers. Finally, this study assumes a linear market demand. In reality, demand is random and influenced by
Appendix I

Proof of proposition 1.

The manufacturer determines the wholesale price and carbon emission abatement level simultaneously, and then the retailer sets the retail price. Using backward induction procedure, in order to get stackelberg equilibrium solutions, the response of follower in the second stage should be determined at first. The leader’s optimal decision is solved based on the follower’s response.

Therefore, we firstly solve for the retailer’s profit function. The first order derivative of profit \( \pi_r \) to retail price \( p \) is shown as follows, \( \frac{\partial \pi_r}{\partial p} = a - 2p + b_e + w \).

Then the second order derivative condition is \( \frac{\partial^2 \pi_r}{\partial p^2} = -2 < 0 \), so retailer’s profit function is strictly concave to retail price \( p \). Let \( \frac{\partial \pi_r}{\partial p} = 0 \) we can get \( p = \frac{a + b_e + w}{2} \).

As supply chain information is symmetric, substitute the value of price into Eq. (2), then solve the optimal decision of manufacturer, the first order condition is

\[
\frac{\partial \pi_m}{\partial w} = \frac{1}{2} a - w + \frac{1}{2}(c + p\epsilon_0) + \frac{1}{2}(b_1 - p_c)e
\]

\[
\frac{\partial \pi_m}{\partial \epsilon} = \frac{1}{2} p_c a + \frac{1}{2} (b_1 - p_c)w - \frac{1}{2}(c + p\epsilon_0)b_1 + [b_1p_c - k_1(1 - \phi)]e
\]

From the above conditions, we can obtain the second order condition as follows,

\[
\frac{\partial^2 \pi_m}{\partial w^2} = -1 < 0, \quad \frac{\partial^2 \pi_m}{\partial w \partial \epsilon} = \frac{\partial^2 \pi_m}{\partial \epsilon \partial w} = \frac{1}{2} (b_1 - p_c), \quad \frac{\partial^2 \pi_m}{\partial \epsilon^2} = b_1p_c - k_1(1 - \phi)
\]

Thus the Hessian matrix of \( \pi_m \) is

\[
H(\pi_m) = \begin{bmatrix}
-1 & \frac{1}{2}(b_1 - p_c) \\
\frac{1}{2}(b_1 - p_c) & b_1p_c - k_1(1 - \phi)
\end{bmatrix}
\]

\( H \) is a negative definite because \( H_{11} < 0 \), from the assumption that the value of \( k_1 \) is large. To ensure that the profit function in this model has joint concavity for the decision variables and has the maximum value, then making \( \det(H) = k_1(1 - \phi) - \frac{1}{4}(b_1 + p_c)^2 > 0 \), deriving that \( k_1 > \frac{(b_1 + p_c)^2}{4(1 - \phi)} \). Hence manufacturer’s profit \( \pi_m \) is jointly concave in \( w \) and \( e \). Let \( \partial \pi_m / \partial w = 0 \) and \( \partial \pi_m / \partial e = 0 \) we can get \( w^{de}, e^{de}, p^{de} \), then derive corresponding \( q^{de}, \pi_m^{de}, \pi_r^{de} \).

Proof of corollary 1.

(1) Obviously, \( \frac{\partial e}{\partial \phi} = \frac{4k_1(b_1 + p_c)(a - c - p\epsilon_0)}{4k_1(1 - \phi) - (b_1 + p_c)^2} > 0 \), \( \frac{\partial q}{\partial \phi} = \frac{k_1(b_1 + p_c)(a - c - p\epsilon_0)}{4k_1(1 - \phi) - (b_1 + p_c)^2} > 0 \).
(2) From (1) we can derive that the variables $e$ and $q$ increase with the increase of $\varphi$, so when $\varphi = 0$, the $e_{\varphi=0}$ and $q_{\varphi=0}$ are the minimum.

**Proof of proposition 2.**

Similar to proposition 1, for the supply chain profit function in the Eq. (4), getting the first order condition as follows,

$$
\frac{\partial \pi_{sc}}{\partial p} = a - 2p + (b_1 - p_c)e + p_c e_0 + \frac{\partial \pi_{sc}}{\partial e} = p_c + (b_1 - p_c)p + (2b_1 p_c - k_1)e - b_1 (c + p_c e_0).
$$

Then the second order condition is

$$
\frac{\partial^2 \pi_{sc}}{\partial p^2} = -2, \quad \frac{\partial^2 \pi_{sc}}{\partial p \partial e} = \frac{\partial^2 \pi_{sc}}{\partial e \partial p} = b_1 - p_c, \quad \frac{\partial^2 \pi_{sc}}{\partial e^2} = 2b_1 p_c - k_1.
$$

Hence the Hessian matrix is $H(\pi_{sc}) = \begin{bmatrix} -2 & b_1 - p_c \\ b_1 - p_c & 2b_1 p_c - k_1 \end{bmatrix}$. $H$ is a negative definite because $H_{11} < 0$. From the assumption that the value of $k_1$ is large, to ensure that the profit function in this model has joint concavity for the decision variables and has the maximum value, then making $\det(H) = 2k_1 - (b_1 + p_c)^2 > 0$, getting $k_1 > \frac{1}{2}(b_1 + p_c)^2$. So the profit $\pi_{sc}$ is jointly concave in $p$ and $e$, let $\frac{\partial \pi_{sc}}{\partial p} = 0$ and $\frac{\partial \pi_{sc}}{\partial e} = 0$ we can obtain $p_{sc}, e_{sc}$, then get $q_{sc}, \pi_{sc}$.

**Proof of corollary 2.**

The value of $k_1$ as the prerequisite, it should meet $k_1 > \frac{(b_1 + p_c)^2}{4(1 - \varphi)}$ and $k_1 > \frac{1}{2}(b_1 + p_c)^2$. We mainly analyze the impact of the retailer’s sharing rate on the emission abatement level and product quantity.

(1) From the proposition 1 and 2, $e_{sc} - e_{id} = \frac{(2k_1 - 4k_1 \varphi)(b_1 + p_c)(a - c - p_c e_0)}{2k_1 - (b_1 + p_c)^2 4k_1 (1 - \varphi) - (b_1 + p_c)^2}$, thus when $\varphi < \frac{1}{2}$, derive that $e_{sc} - e_{id} > 0$, conversely as $\varphi > \frac{1}{2}$, $e_{sc} - e_{id} < 0$.

(2) Similar to (1), obviously get $q_{sc} - q_{id} = \frac{2k_1 (1 - \varphi) - (b_1 + p_c)^2 \varphi k_1 (a - c - p_c e_0)}{2k_1 - (b_1 + p_c)^2 4k_1 (1 - \varphi) - (b_1 + p_c)^2}$, so when $\varphi < \frac{2k_1}{2k_1 + (b_1 + p_c)^2}$, $q_{sc} > q_{id}$, conversely $q_{id} > q_{sc}$.

**Proof of proposition 3.**
The process of proof is similar to that of the proposition 1, so we first solve for the optimal retail price and promotion degree.

$$\max_{p^{jd}} \pi^{jd}(p^{jd}, r^{jd}, w^{jd}, e^{jd}) \max_{e^{jd}} \pi^{jd}(r^{jd}, p^{jd}, w^{jd}, e^{jd})$$

Second, from above formulas, substituting the value of $p^{jd}$ and $r^{jd}$ into Eq. (6), then deriving the optimal wholesale price and carbon abatement level.

$$\max_{w^{id}} \pi^{id}(w^{id}, e^{jd}) \max_{e^{jd}} \pi^{id}(e^{jd}, w^{id})$$

According to the above process, getting $w^{jd}, e^{jd}, r^{jd}, p^{jd}$, then deriving $q^{jd}, \pi^{jd}, \pi^{jd}$.

**Proof of corollary 3.**

(1) Obviously, from the proposition 3 can get

$$\frac{\partial e}{\partial \theta} = \frac{(1-3\theta)b_2^2k_2(1-\varphi)k_2^2(1-\theta)(a-c-p_e e_0)(b_1 + p_c)}{k_2(1-\varphi)[4k_2(1-\theta)^2 - 2b_2^2 + 3ab_2^2] - [k_2(1-\varphi)(b_1 + p_c)^2]^2}$$

$$\frac{\partial q}{\partial \theta} = \frac{(1-3\theta)b_2^2k_2(1-\varphi)k_2^2(1-\theta)(a-c-p_e e_0)k_2(1-\varphi)}{k_2(1-\varphi)[4k_2(1-\theta)^2 - 2b_2^2 + 3ab_2^2] - [k_2(1-\theta)(b_1 + p_c)^2]^2}$$

Therefore, when $\theta < \frac{1}{2}, \frac{\partial e}{\partial \theta} > 0, \frac{\partial q}{\partial \theta} > 0$, conversely as $\theta > \frac{1}{2}, \frac{\partial e}{\partial \theta} < 0, \frac{\partial q}{\partial \theta} < 0$.

(2) To simplify the abatement level in the proposition 3 can be derived

$$e^{jd} = \frac{k_2^2(a-c-p_e e_0)(b_1 + p_c)}{k_2(1-\varphi)[4k_2 + (3\theta-2)b_2^2]} - [k_2(1-\varphi)(b_1 + p_c)^2]^2$$

Hence $e^{jd} = e^{jd}_{\theta=0}$, so combined with (1) can be obtained, when $0 < \theta < \frac{1}{2}, e^{jd} > e^{jd}_{\theta=0}$, and while $\frac{1}{2} < \theta < 1, e^{jd} < e^{jd}_{\theta=0}$. In addition, the product quantity $q^{jd}$ is proved in the same way.

**Proof of proposition 4.**

The proof is similar to that of proposition 2, so the paper omits it.

**Proof of corollary 4.**

Similar to corollary 2, combining with proposition 3 and 4, we can easily get this corollary.

**Proof of corollary 5.**

According to the proposition 3 and 4, we simplify the abatement level and product quantity.
Comparing the above two abatement levels, it can be easily derived that when \( \theta < \frac{2}{3} \), \( e^{id} > e^{ad} \), conversely while \( \theta > \frac{2}{3} \), \( e^{id} < e^{ad} \). Moreover, the proof of product quantity can be achieved in the same way.

**Proof of corollary 6.**

We can obtain this corollary from the comparison of the optimal solutions in the proposition 2 and 4 directly.

**Proof of corollary 7.**

The proof is similar to that of corollary 6, so the paper omits it.

**Proof of proposition 5.**

Similar to the proof of proposition 1, we firstly solve for each retailer’s profit function. In the multi-retailer situation, the total product quantity is

\[
q = \sum_{i=1}^{n} q_i = q_1 + q_2 + \ldots + q_i + \ldots + q_n,
\]

so the first derivative of single retailer’s profit \( \pi_i \) with respect to product quantity \( q_i \) is

\[
\frac{\partial \pi_i}{\partial q_i} = a - q + b_i e - w - \frac{\partial q}{\partial q_i} q_i
\]

\( = a - q + b_i e - w - q_i \). Since assumed that \( n \) retailers are exactly the same, the product quantity is equivalent for each retailer. Hence the total product quantity \( q \) is numerically equal to \( nq \). Simplified the above derivative expression, we can get that

\[
\frac{\partial \pi_i}{\partial q_i} = a - nq_i + b_i e - w - q_i = a - (n+1)q_i + b_i e - w .
\]

Then the second derivative condition is

\[
\frac{\partial^2 \pi_i}{\partial q_i^2} = -(n+1) < 0 ,
\]

so the retailer’s profit function is strictly concave to product quantity. Let \( \frac{\partial \pi_i}{\partial q_i} = 0 \), getting \( q_i = \frac{a + b_i e - w}{n+1} \). As supply chain information is symmetric, substitute the value of product quantity into Eq. (10), then solve the optimal decision of manufacturer, the first order condition is

\[
\frac{\partial m}{\partial w} = \frac{n[a - 2w + c + p_e e_0 + (b_1 - p_e)]}{n+1}
\]

\[
\frac{\partial m}{\partial e} = \frac{n[p_c a + (b_1 - p_c)w - b_c - b_p e_0 + 2b_p e] - (1 - \varphi)k_y e}{n+1}
\]

From the above conditions, obtaining the second order condition as follows,

\[
\frac{\partial^2 m}{\partial w^2} = -\frac{2n}{n+1}, \quad \frac{\partial^2 m}{\partial w \partial e} = \frac{n(b_1 - p_e)}{n+1}, \quad \frac{\partial^2 m}{\partial e^2} = \frac{2n b_p e}{n+1} - (1 - \varphi)k_y e
\]
Thus the Hessian matrix of $\pi m$ is:

$$H(\pi m) = \begin{bmatrix}
\frac{2n}{n+1} & \frac{n(b_i - p_c)}{n+1} \\
\frac{n(b_i - p_c)}{n+1} & \frac{2nb_i p_c}{n+1} - (1-\phi)k_i 
\end{bmatrix}.$$  

$H$ is a negative definite because $H_{11} < 0$, from the assumption that the value of $k_i$ is large. To ensure that the profit function in this model has joint concavity for the decision variables and has the maximum value, then making $\det(H) > 0$, deriving that $k_1 > \frac{n(b_i + p_c)^2}{2(n+1)(1-\phi)}$. Hence manufacturer’s profit $\pi m$ is jointly concave in $w$ and $e$. Let $\partial\pi m/\partial w = 0$ and $\partial\pi m/\partial e = 0$, getting $w^{E^*}, e^{E^*}, q_i^{E^*}$, then deriving corresponding $p_i^{E^*}, \pi m^{E^*}, \pi r^{E^*}$.

**Proof of corollary 8.**

(1) From the optimal situation in proposition 5, for the emission abatement level $e^{E^*}$ and product quantity $q_i^{E^*}, q^{E^*}$, the derivative with respect to $n$ can lead to this corollary.

(2) We can get this corollary from the comparison of the optimal situations in proposition 1 and 5 directly.

(3) By L’Hospital Law, we can easily get this corollary.

**Proof of corollary 9.**

The proof is similar to that of corollary 8(2), so the paper omits it.

**References**


Yu, W., & Han, R., 2017. Coordinating a two-echelon supply chain under carbon tax. Sustainability, 9(12), 2360.

