Essays on role of banks on economic gaps between borrowers and savers

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Abstract

This thesis presents three chapters that empirically investigate role of banks on wage and wealth gaps between heterogeneous households using DSGE models.

The first chapter analyses the role of an endogenous human capital accumulation channel and solvent banks (demand side of the credit market only) on wage gap. We find that the TFP shock reduces wage and income gaps, whereas preferences or financial shocks increase wage and welfare gaps. The welfare gap will be reduced under a human capital shock, whereas wage gap will only be reduced in the short-run. Our findings also suggest that the presence of lending facilities mitigates the propagation mechanism of the shocks to wage and welfare gaps, whereas the human capital accumulation channel mainly improves the welfare of borrowers and significantly affects wage and wealth gaps. Robustness checks show that our key results remain valid.

The second chapter examines wage and wealth gaps under the presence of an imperfect banking competition, as it allows to study both supply and demand sides of the credit market. The findings show that under the TFP shock, wage gap declines, while it widens under housing preference and human capital transformation shocks. Moreover, under the LTV shock, wage gap shrinks in the short-run only. Robustness checks provide consistent results with the base model. However, testing different modelling assumptions show that there is a lower deviation in wage gap under a higher bank capital adjustment cost.

Finally, the third chapter studies the effects of an insolvent banking sector on skill premium with the presence of a skill accumulation channel. We find that under the TFP shock, a shock to skill transformation and a shock to diversion of assets skill premium reduces. We also find that under the TFP shock, the bank capital declines, but it increases under the other three shocks. The shock to the probability of the number of exiting banks increases skill premium due to higher supply of unskilled labour and lower wages for these workers. Robustness checks are consistent with the base model.
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Dedicated to my son, Daniel, and my parents with love.
Declaration

In accordance with the Regulations for Higher Degrees by Research, I hereby declare that the whole thesis now submitted for the candidature of Doctor of Philosophy is a result of my own research and independent work, except where reference is made to published literature. I also hereby certify that the work embodied in this thesis has not already been submitted in any substance for any other degree and is not being concurrently submitted in candidature for any degree from any other institute of higher learning. I am responsible for any errors and omissions present in the thesis.

Candidate:

Aliya Kenjegalieva
Introduction

This thesis investigates wage and income gaps between heterogeneous households (i.e. between household-savers and household-borrowers) using dynamic stochastic general equilibrium (DSGE) models for the U.S. economy. U.S. wage gap has been rising for the past decades. Several factors might have affected this raise in income gap. Firstly, workers’ skills can be obtained through education (long-term learning process) and job training (short-term accumulation of specific skills). Both these learning processes provide workers with relevant skills in order to be productive at their workplace and earn higher wages. However, 34% of the U.S. population aged 25 and over obtained a bachelors or higher degree while only 10.4% has a college degree. The rest of the population has a high school degree or lower.\(^1\) Moreover, the popularity of zero-hour contracts by employers show that unskilled workers are more likely to have such contracts. Secondly, new technologies have replaced unskilled workers while high-skilled workers have been rewarded with higher wages. Another explanation for the change in wage gap can be globalisation, which creates greater competition in labour markets negatively affecting low-skilled workers.

This study does not exclude the importance of the mentioned factors on wage gap; however, this research only investigates skills and households’ time preference that are assumed as the main aspects of wage gap. Due to the quarterly basis of various financial and productivity shocks, only on- and off-job training are assumed as a main source of accumulating new skills (see, for example, Dadgar and Trimble, 2015). The main assumption is that there are skilled and unskilled workers who accumulate new skills through job training. The 2015 findings of the Organisation for Economic Co-operation and Development (OECD, 2015) show that to reduce income gap lower skilled workers need to invest time and money in their human capital. However, as they have to earn wages for their everyday consumption, they devote most of their time to work instead of human capital accumulation.

\(^1\)See Table 3 Detailed Years of School Completed by People 25 Years and Over by Sex, Age Groups, Race and Hispanic Origin: 2017 in the U.S. Census Educational Attainment in the United States: 2017
There are studies using DSGE models that investigate wealth redistribution across households under changing monetary policies (see Alpanda and Zubairy, 2016, Sommer and Sullivan, 2018), however, these studies do not analyse what happens to wage gap with the presence of financial shocks. Therefore, the main contribution of this thesis is to research the effects of financial and productivity shocks on wage and wealth gaps and how findings can change under different banking sectors.

Starting with a brief description of the thesis, each chapter contributes to the analyses of the role of different types of banking sectors on wage and income gaps by using DSGE models. The first chapter investigates the effects of a solvent banking sector on wage and wealth gaps. The banks operate in a perfect competition. The solvent banking and heterogeneous households are built on Iacoviello (2015). Heterogeneity is presented by two households: household-savers and household-borrowers. The human capital accumulation channel is introduced to examine the skill premium (wage gap) following Jones et al. (1993). The second chapter, which extends Chapter 1, introduces banks that operate under an imperfect competition. A solvent banking sector allows studying only the demand side of the credit market, while imperfect competition allows analysing the effects of both demand and supply sides of the credit market on wage gap. The third chapter accounts for the bank runs and bankruptcies to analyse their effects on wage gap. Another contribution of Chapter 3 is that skilled and unskilled workers come from one household. Therefore, workers can now switch between the skill groups from being unskilled to becoming high skilled, and vice versa. This aspect was missing in Chapter 1 and Chapter 2 due to the modelling assumptions that allowed household-borrowers to borrow. With the new assumptions presented in Chapter 3, the impact on wage gap can be analysed under a stable banking sector or bank runs.

More analytically, the first chapter presents two types of households, who differ in terms of their time preference and, therefore, wealth and skills that create heterogeneity between these two households. First households are called ‘household-savers’ or ‘patient’ households whose discount factor is higher. Thus, these households save part of their income, purchase hous-
ing, invest in their skills and, as a result, earn higher wages. The other group of households are ‘household-borrowers’ who are ‘impatient’ households. Household-borrowers’ wages are lower, even though they also invest in their skills, but their skills are lower than those of household-savers. These households’ wealth is housing, however, their housing wealth is less than those of savers. As household-borrowers’ wages are lower, they can invest in new skills only by borrowing from banks. Moreover, these households can borrow from banks to purchase new housing or to increase the expenditure on consumption of goods and services.

Therefore, having banks in the model is essential for the household-borrowers to borrow and invest in human capital accumulation. Banks represent financial intermediaries who allocate funds from household-savers to borrowers. Banks do not go bankrupt, thus, they are solvent in this model. There are two borrowers in this economy. In addition to household-borrowers, there are also entrepreneurs, who purchase new real estate, which can be used in the production of final goods. Entrepreneurs also use labour force supplied by households in their production. Both borrowers are subject to a borrowing constraint that restricts them to borrow more than the value of a collateral they provide, i.e. housing/real estate.

The findings show that, under a total factor productivity (TFP) shock, skill premium decreases due to higher increase in household-borrowers’ wages relative to household-savers. Similar to Asimakopoulos and Asimakopoulos (2017), as household-borrowers enjoy higher supply of loanable funds, they fund their housing purchasing but also investment in human capital, which increases the human capital stock of these agents. The preferences shock shows that when household-savers decide to consume most of their income and decrease their savings, the supply of loanable funds will drop. As this shock affects both households, household-borrowers cannot enjoy higher consumption or housing purchasing due to lower loan supply. However, they will consume most of their income without investing significant amount of funds and time in their human capital. As a result, skill premium widens and it will be further enlarged due to household-savers increasing human capital. Similar to the preferences shock, the human capital transformation shock shows
the increase in skill premium as household-savers start saving less but investing more in their human capital accumulation, similar to the findings by Dadgar and Trimble (2015). Without enough funds available for household-borrowers to borrow, their human capital will deteriorate due to depreciation of human capital over time. The household-borrowers’ loan-to-value (HH-B LTV) ratio shock or household-borrowers’ borrowing constraint shock leads to lower wage gap as this shock allows household-borrowers to obtain more loans to invest in human capital and consumption expenditures. Because of lower investment in human capital by household-borrowers, the wage gap widens under the shock to entrepreneurs financial constraint or entrepreneurs’ LTV ratio shock. It happens because, under this shock, household-borrowers are not able to enjoy higher loans as entrepreneurs.

The welfare analysis show that household-borrowers are generally better off under the presence of a human capital accumulation channel. However, with the TFP shock household-borrowers gain significantly more than household-savers because of their higher wages. Furthermore, under human capital transformation shock, household-borrowers’ consumption will be higher, although, the wage gap will widen. The household-borrowers’ LTV ratio shock shows that household-borrowers’ welfare improves, although there is a reduction in consumption gap in the short-run only. Under the entrepreneurs’ LTV ratio shock, both wage and consumption gap increase in the short and long-run.

Overall, the findings of the first chapter demonstrate the importance of a human capital accumulation channel, as it can help policymakers control the welfare gap. Moreover, the tighter regulations of banks can significantly mitigate the effects of shocks on wage and wealth gaps.

The second chapter presents a DSGE model similar to the one in Chapter 1, by assuming heterogeneous households (household-borrowers and household-savers) following Gerali et al. (2010). Households can endogenously accumulate human capital by investing in human capital, similar to Jones et al. (1993). However, the main difference between the two chapters is that Chapter 2 investigates the changes in wage and wealth gaps between these two households under the imperfect banking competition, while Chapter 1
studies the perfect competition market.

An imperfect banking sector allows for both the demand and supply sides of credit markets to be examined, while studies of solvent banking neglect the supply side. The imperfect banking competition or market power is determined by the value of bank capital and interest rate setting on both deposits and loans. Tighter regulations of bank capital signal the stronger financial markets and its ability to resist various financial shocks. On the other hand, loose constraints can help household-borrowers to borrow, even though, that will be spent on consumption rather than on investment in human capital. As Piketty and Saez (2014) state, higher human capital leads to higher productivity, which generates greater economic development.

The findings of Chapter 2 demonstrate that under housing preference and human capital transformation shocks, wage gap widens, while short-run wage gap drops under the TFP and household-borrowers’ LTV ratio shocks. With the TFP shock, wages of both households increase allowing households to boost their consumption and investment in human capital. Consequently, wage gap decreases. Housing preference shock leads to higher wage gap due to larger labour supply by household-borrowers and less investment in human capital by household-savers. Household-borrowers’ LTV ratio shock produces lower wage gap in the short-run, but creates larger wage gap in the long-run. It is a result of higher investment in human capital by household-borrowers, and lower investment by household-savers, who increase their deposits instead. Human capital transformation shock results in greater wage gap between the two households. This happens due to higher investment in human capital by household-savers, who are already more skilled than household-borrowers. As a result, household-borrowers will also invest in human capital, but their investment will not allow them to catch up with their savers counterparts.

Different modelling assumptions show that household-borrowers are generally worse off than the household-savers. Moreover, wage gap is lower under the higher capital adjustment parameter across all four shocks compared to other modelling assumptions.

The third chapter investigates the skill premium, the ratio between skilled
and unskilled workers’ wages to analyse the changes in wage and wealth gaps between skilled and unskilled workers. This chapter extends the ideas of Chapter 1 and Chapter 2 by assuming one single household that consists of bankers and workers, as in Gertler and Karadi (2011). However, the main contribution is that workers in this household are both skilled and unskilled, following He and Liu (2008). The skill transformation channel is introduced in order for workers to be able to switch between the groups that contain skilled or unskilled workers. That means that the workers can become more or less skilled and move from an unskilled group to a skilled group. Moreover, this allows for the avoidance of the division of households between household-borrowers and household-savers as each of these groups may contain both skilled and unskilled workers. Furthermore, this helps investigate the supply of labour by two workers and, consequently their wages and skill premium.

Chapter 3 also extends the banking sector, while introducing an unstable financial market that can bankrupt and experience bank runs, which is built on Gertler and Karadi (2011). This helps to analyse how the instability of banking sectors can affect the supply of skilled and unskilled labour and skill premium under various financial and productivity shocks. However, due to the AR(I) processes used in this chapter, the positive shock will be exactly mirroring the negative shock of the same type. Therefore, this chapter presented a strong and stable banking sector.

The findings show that overall the skill premium decreases under the most shocks. For instance, under the TFP shock skill premium decreases due to lower supply of unskilled labour that increases their wages. Shock to skill transformation also produces lower skill premium because of declined wages of skilled workers. It is interesting that wage gap increases under the similar shock in Chapter 1 and Chapter 2. Shock to diversion of funds leads to a significant drop in the skill premium due to the rise in wages of unskilled workers. However, the findings also show that the bank capital decreases under these three shocks, which means that there are less funds available for firms to invest in the production. Finally, the shock to the probability of the number of exiting banks creates an increase in the skill premium, as unskilled workers’ wages go down relative to skilled workers’ wages. Moreover, under
this shock, the bank capital increases, which signals a strong and stable banking sector.

Robustness checks show that the findings stay robust under different modelling assumptions.
Chapter 1: Solvent banks and economic gaps

1.1 Introduction

Income redistribution and wage gap have been examined extensively over the past years (see i.e. Goldin and Katz, 2008 and Hornstein et al., 2005 for reviews). In particular, there has been an upward trend on wage gap since 1980, reaching its highest level, since 1915, in the recent years (see Goldin and Katz, 2008).

However, the aforementioned literature has not systematically examined the effect on income and wage gap with the inclusion of a banking sector and the associated financial frictions. This is despite the fact that previous studies have shown the significant spill-over effects to the economy when a banking sector is included (i.e. Goodhart et al., 2006, Dib, 2010 and Iacoviello, 2015). Moreover, it is crucial to assess the role of the various financial frictions in the economy when housing is being used as collateral, since it is the key asset for about 2/3 of the U.S. population. The value of housing is higher than that of gross domestic product and it has a significant impact on the financially-constrained agents (Alpanda and Zubairy, 2016 and Asimakopoulos and Asimakopoulos, 2017), which becomes more pronounced in the presence of banks (Iacoviello, 2015).

Therefore, the aim of this study is to combine these research streams to examine the impact of various productivity and financial shocks to the economy, and in particular consumption and wage gaps between two households under the presence of endogenous human capital accumulation, banks and financial frictions. Matsuyama (2006) argues that less research has been done using "endogenous formation of class structure" assumptions. Therefore, in this chapter we would like to fill this gap and allow for households to endogenously accumulate their human capital.

In particular, in order to deal with the frequency difference in the occur-

\footnote{See Table 14 of the U.S. Census Bureau Homeownership Rates for the U.S. and Regions for the period 1965-2015.}

\footnote{Galor and Moav (2003) also state that the main driver for economic growth is no longer the physical capital but human capital.}
rence of human capital investment changes, productivity and credit shocks, we use short-time certificates and on- and off-job training as a way of accumulating human capital (see Dadgar and Trimble, 2015 for short-time education). This way we are able to assess the effects of financial shocks on wage and consumption gaps on a quarterly basis taking into account the role of human capital accumulation. To that end, we develop a modified dynamic stochastic general equilibrium (DSGE) model with a banking sector and financial frictions, following Iacoviello (2015), and we introduce a human capital accumulation channel as in Jones et al. (1993).

It is implicitly assumed that households can change their productivity when they are allowed to affect their human capital by educating and/or training themselves. Therefore, households can invest time and goods to become more productive. Agents with higher productivity are more significant in firms’ production process. This leads to higher profits for firms and higher wage rates. The increased wages can be invested in human capital to further boost productivity or accumulate housing, and used as collateral to draw more loans from the banks. Therefore, the human capital accumulation channel can play a crucial role on wage and consumption gaps under the presence of banks.

With respect to the banking sector, it has been shown that banks can amplify and propagate shocks to the real economy (see, for example, Bernanke et al., 1999). This is driven from the decision or inability of the borrowers to pay back their loans causing destructions in bank activities. This way banks cannot repay deposits to savers and receive the part of their own capital they used for loans, leading to creation of shocks with spill-over effects to the entire economy. Therefore, credit markets are a source of propagation and amplification of financial shocks. However, Cooley et al. (2004), focusing solely on the demand side of the credit market, show the limited role of banks as financial intermediaries and that bank capital does not have any influence on bank’s ability to lend.

On the one hand, another stream of literature argues that bank capital substantially distresses investment and amplifies financial shocks. Gerali et al. (2010), for example, show that changing a bank capital to assets ratio
can decrease the impact of technology shocks to investment. Christiano et al. (2008) also give controversial results for banks’ role prior to the crisis. They claim that banks intermediation plays a minor role in a shock creation process and further amplification to other industries.

On the other hand, Meh and Moran (2010) empirically demonstrate that financial and technological shocks are closely related to bank capital. Banks with lower or decreasing capital have to reduce their lending to firms causing a decline in investment and output levels. Therefore, the banking sector is an independent source for creating negative shocks along with propagating and amplifying them.

Similar to Meh and Moran (2010), Dib (2010) shows how financial shocks come from the banking sector and how banks can further propagate the shocks to economies. deWalque et al. (2010) also illustrate that capital requirements can mitigate financial instability in an economy, but it can also negatively affect output in the long-run.

Kamber et al. (2015) empirically demonstrate the negative effects of financial frictions on decision to consume and invest. Under financial shocks borrowers are constrained by the value of their collateral. This restrains them from borrowing while decreasing investment in capital and consumption.

As we can see from the aforementioned studies, the banking sector is an important component in these models. It is proven that the banking sector not only amplifies and propagates financial shocks but also generates them within the sector. However, these studies mainly focus on the effects of the banking sector on total output and investment, but not on wage gap. Although there are studies (see Alpanda and Zubairy, 2016, Sommer and Sullivan, 2018) who incorporate heterogeneous agents in DSGE models with housing market, these papers focus mainly on the implications of changes in interest rates on wealth redistribution across households.

This chapter, therefore, extends this stream of literature in two dimensions: i) we assess the impact of the banking sector on income and wage gaps; and ii) we utilise relatively stylised real business cycle (RBC) model with heterogeneous agents that can endogenously choose their human capital.

Income gap has drawn a lot of attention in the literature. Douglas (1930)
was amongst the first to observe wage distribution, stating that clerical workers are substituted by new equipment. Acemoglu (1998) also shows that there is a direct connection between workers’ level of income and human capital. For example, technological change increases the demand for skilled workers.

In addition, Mankiw (2000) states that households with low wealth are more likely to face binding borrowing constraints since they consume most of their disposal income while making small or no savings. Agents with high level of wealth are able to smooth their consumption and have access to financial markets. Under an increase in interest rates, borrowing becomes more expensive, which means spenders’ debts rise. However, savers (households with high level of wealth) are better off since the interest rate for savings rises too. This leads to a bigger income gap between savers and borrowers.

Lemieux (2006) empirically tests how post-secondary education can increase the level of wage rate and its distribution. He shows that the demand for highly-educated workers is always higher compared to lower educated workers, resulting in a wage gap between the two. Goldin and Katz (2008) also investigate wage gap and they find that wage rates mainly depend on the level of education or skills.

When thinking of gap one might consider gap in wage and/or income. Piketty and Saez (2014) discuss that gap doesn’t only include wage or income. They state that it can consist of two parts: income gap and capital inequality. This also follows the gap decomposition provided by the U.S. Council of Economic Advisers. In this chapter we want to investigate both sources of wage gap. Thus, in our model households differ in both wage and wealth.\(^4\)

OECD (2015) research on wage gap shows the existence of a higher income gap for less educated people. The reason for this is not only due to lower investment in education, but also due to shorter time spent in education compared to skilled people. In addition, when workers invest as much as their skilled counterparts, they will still earn less than the latter. Low-skilled people spend more time working to earn wages to invest in human capital, while devoting less time on the skill accumulation process itself, which results

\(^4\)In our model, we assume housing as a stock that households can purchase depending on their level of income.
in less skills accumulated. This increases income and wealth gaps between the two groups.

Dadgar and Trimble (2015) also assess the effects of increasing human capital on wages using quarterly data for short-term and long-term certificates. They find that increasing human capital has a positive and substantial impact on workers’ earnings. Murphy and Topel (2016) find that larger wage gap, or sluggish rise in skilled workers, leads to a decline in economic growth rate. They also find that workers have incentives to invest more in their human capital as the returns are higher, leading to higher quality of skills. This results in wage gap in workers’ earnings, which creates a wage power for those who are highly skilled. Therefore, to gain new skills and be competitive in the market less skilled workers need to continuously invest in human capital.

The set-up of our model merges the key features of the rich literature in endogenous human capital accumulation and financial frictions. Following Iacoviello (2015), who find that redistribution and other financial shocks are responsible for about 2/3 of output collapse during the recent crisis, we have two types of households, savers and borrowers. Both households provide labour to entrepreneurs and they both invest a fraction of their income in house purchasing. It is widely known, though, that households differ in their level of productivity, which directly affects the level of income they can potentially earn. Therefore, agents can accumulate skills to improve their productivity. Thus, the main departure from Iacoviello’s model is the inclusion of an endogenous human capital accumulation channel, which effectively allows households to improve their productivity by investing in goods and time in human capital.

In addition, following Iacoviello (2015), we assume that the two types of households exhibit different time preferences that lead to different asset holdings and wealth. Patient agents are wealthier and represent savers in the current model. Savers do not need to borrow and are the indirect loan

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5 They use quarterly data because credentials increase every quarter, showing the changes of earnings and wages each quarter. This helps understand the relationship between obtained certificates and inequality.
providers to the economy via their bank deposits. Moreover, since they are wealthier they are able to invest more in human capital compared to household-borrowers, which leads to a wage gap between the two agents.

Household-borrowers represent the impatient households. They invest less in their human capital due to lower income level compared to savers. In order for borrowers to improve their human capital they need to take out loans from banks. The amount of loans they can draw depends on their collateral, which is housing in this model. This is also known as a credit constraint. Banks manage savers deposit accounts and issue loans to household-borrowers and entrepreneurs who produce the final goods and maximise their profit. Banks play essentially the role of intermediaries between savers and borrowers. In other words, banks transfer financial resources between agents over time.

The results of the model show that under productivity shocks, wage and income gaps can be reduced significantly having positive long-run effects. We further find that under preferences or financial shocks, wage gap will rise both in terms of welfare and wage rates. Finally, we find that under a human capital productivity shock the welfare gap will be reduced both in the short-run and in the long-run. However, wage gap will only be reduced in the short-run, leading to higher wage gap in the long-run.

Performing robustness checks on the calibration of the financial frictions we find that the key results remain robust. It is worth noting though that a reduction in capital-asset requirement ratio for banks may lead to higher wage and consumption gap in the long-run compared to our benchmark model.

Finally, we assess the effect of the key modelling assumptions via providing three alternative models. In the first model we eliminate the human capital accumulation channel. In the second model we eliminate banks and in the last model we eliminate both human capital accumulation channel and banks. Our findings suggest that lending facilities in general mitigate the propagation of the shocks to wage and consumption gaps. In addition, the human capital accumulation channel mainly improves the welfare of household-borrowers and it significantly affects wage gap. Finally, savers’ welfare remains at a similar level with the benchmark model under all the
shocks and the different models.

We also find that as household-borrowers are better off with the presence of lending facilities, the role of banks is not important in this model as long as household-savers can play a role of lenders and there are available funds for borrowers to borrow. However, it is convenient to have a banking sector as it helps us to extend it for the rest two chapters.

Therefore, from a policy perspective, human capital accumulation needs to be enhanced via easier access to training and/or education for the welfare gap to be reduced. In addition, the results indicate that the bank capital-asset ratio needs to be efficiently controlled and monitored because it is crucial for the propagation of the shocks to wage and consumption gaps. Moreover, the lending facilities available to borrowers make them better off and be able to invest in human capital.\footnote{The role of banks for the propagation of the shocks to overall output and investment are well documented in the literature and our results do not differ significantly. Therefore, we mainly focus on wage and income gaps.}

In the next section, we extensively discuss the literature review on both DSGE models with banks and human capital. Next, we present the dynamic stochastic general equilibrium model of this chapter. Section 1.4 outlines the decentralised competitive equilibrium. Section 1.5 presents the calibration of our model. Section 1.6 includes the analysis of the results. Section 1.7 provides the various robustness checks, and Section 1.8 concludes the chapter.

1.2 Related literature

In this section we present two streams of literature. Firstly, we discuss the importance of a banking sector and its development in DSGE models. In the second part of the section we discuss the literature that covers human capital and wage gaps.

As we have already mentioned in the introduction of this chapter, there are studies that include heterogeneous agents in DSGE models. Lindquist (2005) implements a DSGE model on Swedish data to understand the factors that have caused changes in equality in Sweden. He and Liu (2008) and He (2012) also use an RBC model to analyse wage gap in the U.S. and the
important role of a human capital accumulation channel in decreasing the wage gap. There are also papers that look at the redistribution of wealth among different types of households (see Alpanda and Zubairy, 2016, Sommer and Sullivan, 2018). However, the papers above do not study the role of a banking sector in wage gap using DSGE models, which we cover in this chapter.

On the other hand, there is a stream of literature that uses causal relationship. For example, Amountzias (2018) studies the causal relationship between income redistribution and financial instability by conducting non-Granger causality test. This income redistribution is caused by a high demand for loans by low-income households who accumulate debt, which creates destructions and uncertainty in financial markets. They found that income gap enlarges instability in financial markets with greater accumulation of debt, caused by excess loan supply.

Another paper by Berisha and Meszaros (2018) examines the economic growth and income gap by conducting VAR informational sufficiency test. Their findings suggest that aggregate household debt creates higher income gap, which leads to a decline in output growth and economic weakness in the long-run. Moreover, economic growth generates a wider gap between top and low-income households.

1.2.1 The importance of banks

Dynamic stochastic general equilibrium models have been developing for many decades (see, for instance, Lucas, 1977). However, the most followed one is presented by Kydland and Prescott (1982) as a theory of real business cycles, which explains the stylised facts through technological change. Classic dynamic stochastic general equilibrium (DSGE) models consist of two agents, i.e. households and firms. However, the importance of a banking sector has led to its inclusion in DSGE models as intermediaries between savers and borrowers.\(^7\)

The banking sector can amplify and propagate shocks to the real economy

\(^7\)See Rebelo (2005) and Christiano et al. (2018) for further details on the history of real business cycle (RBC) models.
(i.e. Kiyotaki and Moore, 1997 and Bernanke et al., 1999). For instance, Kiyotaki and Moore (1997) study the effects of credit constraints on agents and the economy. A temporary productivity shock can negatively affect the ability of agents to borrow and, thus, to invest in production and increase their total expenditure by reducing the value of a collateral. The reduction of a collateral will drop over time while the effects of the shock are propagated further to the real economy.

Therefore, credit markets are a vital source of propagation and amplification of various shocks. This can lead to greater bankruptcies, declines in asset prices and bank failures that play a main role in increasing depression in the economy. These credit-market imperfections are included in DSGE models to analyse their effects. This sheds light on shocks that significantly influence economies while taking into account credit frictions. However, Cooley et al. (2004) use models that focus only on the demand side of credit markets. They claim that banks have a limited role of intermediaries between borrowers and lenders. They also state that bank capital does not significantly influence the ability of banks to lend. In other words, lenders are suppliers of their savings, who lend these funds to borrowers without any need in intermediaries’ interaction, such as banks.

Unlike Cooley et al. (2004), Meh and Moran (2004) show that banks have to rely on their capital when considering risky loans. Their model presents banks who face credit constraints. They also demonstrate the importance of firms’ balance sheets. Their model illustrate two moral hazard problems. Firstly, as a standard, they assume entrepreneurs produce goods. Entrepreneurs undertake risky investments and activities in order to make higher profit. However, the higher the risk the smaller the chance of getting loans from banks, which pushes firms to invest their own money. Secondly, some banks might still issue loans to such firms, while tolerating the higher risk of a borrower to default on loans. Moreover, such banks might not monitor borrowers’ activities and use of loans as screening procedures are expensive and time consuming. Depositors realise the risk, which they do not want to take. Thus, savers require banks to invest their own capital in those risky projects making banks face financial constraints.
By introducing financial intermediaries in DSGE models, researchers seek to forecast and prevent economic crunches before they have already appeared. The role of monetary policies and central banks’ control over commercial banks help to understand the process of transmission of shocks. For instance, Angeloni and Faia (2009) confirm a banks’ central role in shock transmissions. They also prove the significance of bank capital and leverage ratios. Banks can redeploy assets with the aim of liquidating defaults. When banks issue loans they rely on firms’ cash flows. However, cash flows can be volatile and unreliable, which creates uncertainty in a bank’s balance sheet. Moreover, this raises a ratio of bank loans to deposits that can lead to bank runs as savers lose their confidence in bank liquidity. Therefore, banks should mostly rely on their own capital.

Bank runs are dangerous as they can weaken banks’ liquidity and stability in credit markets. If, under some news shocks, depositors assume a bank might collapse then they will start withdrawing their monies. Even strong and stable banks can bankrupt under those conditions. The main reason for this is banks’ lending activities. Banks can demand the earlier issued loans back, but they will lose their borrowers. This will further damage confidence in banks creating discredit, mistrust and instability in financial markets. With constraints in credit markets, production will immediately decline, leading to the slowdown of economic growth.

On the other hand, there is a literature that provides opposite evidence of a role of bank capital. They agree that bank capital has a considerable impact on investment, but argue about its role in amplification of financial shocks. Gerali et al. (2010), for example, show that bank capital doesn’t have significant effect following monetary policy shocks. However, while considering a ratio of capital to assets, bank capital reduces the impact of technology shocks on investment. Similar to previous literature, Gerali et al. (2010) show how banks can decrease output levels during crises.

Following previous studies, deWalque et al. (2010) paper develops a model that shows the relationship between banks and entrepreneurs. The model also presents lending injections into a credit market. If these injections are not financed through taxes then inflation rates can go up. As in Dib (2010),
they illustrate that capital requirements can mitigate financial instability in the economy, but it can also lead to fluctuations in output in the long-run. Moreover, the difference of maturity dates for deposits and for loans can also play an important role in creating shocks in a banking sector.

As discussed above, banking capital is important as it indicates banks’ stability and liquidity. Iacoviello (2015) introduces a model, which can reasonably fit the U.S. data. He produces a model with banks and heterogeneous households. Borrowers pledge their houses as a collateral to get loans from financial intermediaries. As the recent crisis shows, banks have to sell houses at lower prices, which don’t cover the loan value and additional expenses associated with it. Thus, borrowers pay back less than they agreed by credit contracts. It is important to show how the banking sector might transfer and spillover financial shocks causing persistent and substantial effects on the real economy.

Gertler et al. (2017) present a model where they show how bank runs can negatively affect consumption, investment and output. A strong bank with a good balance sheet can be resistant to various financial shocks, which positively affects borrowers and eliminates bank runs. However, banks with weaker balance sheet can experience instabilities even under small shocks. Therefore, it makes the research in banking crucial in understanding the behaviour of agents under the presence of shocks.

1.2.2 Human capital and wage gap

As it can be seen from the previous studies of DSGE models, a banking sector is an important component in these models. It plays a crucial role of shock creator and propagator to an economy. The main aim of this chapter is to examine the effects of financial shocks on wage and wealth gaps between heterogeneous households. Therefore, in this chapter we investigate how household-borrowers and household-savers are affected by financial shocks when there are both a banking sector and an endogenous human capital accumulation channel present in the model.

We assume that households differ in terms of their time preference, which
creates other differences, such as wealth. Therefore, household-savers are patient while household-borrowers are impatient. Households also differ in their level of income and expenditures.

We assume that households can invest goods and time in the skill accumulation process - their human capital. Savers will invest more and spend more time, meaning the return on their human capital will be higher while borrowers will face lower returns. The paper by Ben-Porath (1967) also supports the fact that to accumulate human capital, households have to constantly improve their skills over time. Therefore, the higher the human capital the greater the workers’ earnings will be. Thus, human capital can be assumed as human wealth. We also assume that agents have some initial endowment. As time passes, this endowment increases if they invest in human capital. Furthermore, human capital depreciates over time, which makes households continuously invest in their human capital. Both savers and borrowers are subject to the same depreciation rate. However, since savers have higher wage rates they invest more in their human capital accumulation, while household-borrowers have to borrow from banks in order to invest. Therefore, this increases wage gap between the two households.

Research in income gap has always been a hot topic for discussions. For example, Douglas (1930) is one of the first economists, who observed wage distribution in the U.S. for the period of 1890 to 1926. As he states in his work, if clerical workers are substituted by new equipment then this leads to the decrease of their wages. The wage decrease is also caused by the increasing number of white-collar workers as there was mass access to education. Moreover the wages of uneducated and low-skilled workers were higher, which he assumes was caused by the decrease in numbers of immigrants. These two changes in wages of skilled and unskilled workers caused a growth in wage gap.

Katz and Autor (1999) argue that higher skills lead to a greater wage gap. However, they also analyse factors that affect wage gap, such as demand and supply (including quality) of labour, which play a crucial role in wage gap. When there is a rise in supply of labour in a labour market, the competition will be larger leading to lower wage rates. On the other hand, if there is a
greater demand for skilled labour, then workers’ wages will increase too. This has led to a rise in wage gap that can be seen from the U.S. wage statistics from 1950 onwards, excluding a decline in the 1970s. Caselli (1999) also argues about the role of technological changes in the U.S., which has led to the demand for high-skilled labour. He empirically finds that skilled labour holds more capital because of their higher wages, which also allows them to purchase various assets. Moreover, with realisation of higher returns on capital and skills, workers will move towards human capital and skill accumulation. On one hand, this reduces the wage gap. On the other hand, higher returns on capital stock will divide highly-skilled and low-skilled workers, while leading to a wider wage gap.

Similar to Caselli (1999), Krusell et al. (2000) explain the unfavourable position of unskilled labour. They claim that unskilled labour is more easily replaced by new technologies, while the increase of such technologies improve marginal product of high-skilled workers, making skilled workers more complementary to equipment. Therefore, the optimal policy to improve well-being and wages of unskilled workers is to propose solutions focused on training and skill accumulation for low-skilled labour. With higher skills, workers can improve their productivity by using new machinery and equipment instead of being substituted by new technologies.

Using the Krusell et al. (2000) model, Lindquist (2005) analyses the Swedish wage gap, which has been rising over the past decades. He finds that the main driver for the wage gap between skilled and unskilled workers is the demand for labour. With new technological changes, the demand for skilled workers increased, while the growth of skilled labour supply declined. This has affected the relative wages earned in Sweden, increasing skill premium.

In our model we assume that households are able to attend on- and off-job training to improve their skills and human capital. Similar, Hornstein et al. (2005) explain the importance of investment in training and how it can positively affect the wage gap. They discuss that low-skilled workers with no college education gain skills through on-the-job learning. However, they also

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8See March Current Population Survey, Public Use Micro Samples or Panel Study of Income Dynamics for further data on wage inequality and wage gaps.
stress the fact that under various shocks the wage gap always increases in the long-run, while unskilled workers are being worse off. Moreover, this labour are also more likely to lose their jobs than their skilled counterparts.\footnote{See World Bank Development Indicators for U.S. unemployment by education levels.}

He and Liu (2008) also present a model that shows the importance of human capital and workers’ incentives in investing in skill accumulation because of equipment-skill complementarity effect. As in Krusell et al. (2000), they show that skill premium will increase due to lower marginal productivity of unskilled labour under technological changes. But technological change is not the only factor that affects wage gap. The return on education and training also plays a substantial role in explaining changes in wage gap. Generally, higher supply of skilled workers suppresses wage gap in wages as well as in consumption and wealth.

Similar to the literature above, He (2012) finds that wage gap increases when there is a higher demand for skilled labour. However, this demand motivates low-skilled workers to invest in new skills, which will lead in a greater supply of skilled workers. Additionally, Acemoglu and Autor (2012) state that with increasing changes in technologies it becomes more important for policymakers to improve workers’ skills. This includes the skills of unskilled workers who perform tasks that do not require higher skills. This might change the direction of technological change, which will eventually adjust to a demand for skilled labour.

Autor (2014) then argues that demand for labour depends on their productivity, which also dictates the wages workers will earn. Workers’ productivity depends on skills they accumulate. As he analyses, the wage gap has been rising in the U.S. as workers stopped accumulating necessary skills for various reasons. However, Autor (2014) also states that the wage gap presented in an economy is necessary to encourage workers to accumulate new skills continuously. Wage gap or inequality cannot be fully avoided but it can be decreased through long-term policies, which can stimulate workers productivity through skills and reasonably-paid jobs.

Murphy and Topel (2016) and Gomes and Kuehn (2017) findings are consistent with the literature in human capital. Murphy and Topel (2016)
state that insufficient supply of skilled labour increases gap, and with lower growth in skilled labour the economic growth will slow down too. Gomes and Kuehn (2017) find that highly-skilled workers increase productivity of firms and economy. With higher human capital, individuals can earn greater wages, which can lower wage gap. This incentivises workers to accumulate human capital through acquiring new skills.

1.3 Model outline

The proposed model is a closed economy with four agents: savers, borrowers, entrepreneurs and banks. Households consume final goods and purchase houses. Households consist of savers and borrowers similar to patient and impatient households as in Iacoviello (2015). Both types of households own houses and accumulate their human capital by investing their money and time in training/education, similar to Jones et al. (1993). Entrepreneurs produce the final goods and maximise their profit. Banks, in this model, are the intermediaries between savers and borrowers. They accumulate savings from household-savers in deposit accounts and with their own capital issue loans to entrepreneurs and household-borrowers.

1.3.1 Households

Households, in the model, are represented by a continuum of infinitely living households of a unit mass. There are two types of households in our economy. The first type represents savers, who have access to asset markets. The second type consists of households who are borrowers. Both households own houses. The lifetime utility function of a representative household is given by

$$U_i = \sum_{t=0}^{\infty} \beta_t^i u(C_{i,t}, H_{i,t}, N_{i,t}, N_{i,t}^{HC})$$

where $\beta_t^i$ is the discount factor for an $i$ agent at period $t$, and $0 < \beta_t^i < 1$, $C_t$ is household’s consumption at period $t$; $H_{i,t}$ is housing; and $N_{i,t}^{HC}$ and $N_{i,t}$ represent time spent in human capital accumulation and work respectively. $u(\cdot)$ is strictly increasing, strictly concave and twice continuously differentiable.
**Household-savers**  Each period household-savers choose consumption $C_{H,t}$, housing $H_{H,t}$ and the time they spend in human capital accumulation $N^{HC}_{H,t}$ and at work $N_{H,t}$.\(^{10}\) Therefore, they maximise the following objective function:

$$\max E_0 \sum_{t=0}^{\infty} \beta_H^t \left[ A_{c,t} \log C_{H,t} + j A_{c,t} \log H_{H,t} + \tau \log \left( 1 - N_{H,t} - N^{HC}_{H,t} \right) \right]$$

(1)

where $\beta_H^t$ is discount factor for savers ($0 < \beta_H^t < 1$). $j$ shows the share in housing preference, $A_{c,t}$ is a preferences shock and $\tau$ is the share of leisure.

Savers are subject to the following budget constraint:

$$C_{H,t} + I^{HC}_{H,t} + D_t + q_t (H_{H,t} - H_{H,t-1}) = R_{H,t-1} D_{t-1} + W_{H,t} H_{C,t-1} N_{H,t}$$

(2)

where at period $t$ choose housing $H_{H,t}$ and the level of investment in human capital $I^{HC}_{H,t}$. They also have deposit accounts, $D_{S,t}$, in banks and purchase houses at price $q_t$.\(^{11}\) They receive interest payments, $R_{H,t}$, on savings and get $W_{H,t}$ wage rate for $N_{H,t}$ worked hours. Finally, $HC_{H,t}$ is the level of productivity defined by the human capital accumulation.

**Human capital accumulation channel**  We assume that households are able to accumulate new skills by attending on- and off-job training and obtaining further short-term education. This improves their productivity and allows them to receive higher returns from labour as they earn higher wages. The human capital accumulation channel is set-up as in Jones *et al.* (1993):

$$HC_{H,t} = (1 - \delta_{SK}) HC_{H,t-1} + B_t \left[ (I^{HC}_{H,t})^\theta (HC_{H,t-1} N^{HC}_{H,t})^{(1-\theta)} \right]^\chi$$

(3)

where $B_t$ is the shock to the human capital transformation, $\theta$ shows the importance of goods input in the transformation of skills and $\chi$ is the parameter that shows the returns to scale. Human capital is also subject to depreciation

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\(^{10}\)Throughout the paper we normalise time to unity. As a result, leisure plus time at work and time at human capital accumulation add up to one.

\(^{11}\)The house/real estate prices are the same for all agents in the model.
over time, which is given by $\delta_{SK}$.

**Household-borrowers** Borrowers own houses and borrow, so that they invest enough to improve productivity via increasing their human capital level. At period $t$ household-borrowers maximise their lifetime welfare by choosing $C_{S,t}$ consumption, $H_{S,t}$ housing, time at work $N_{S,t}$ and time in human capital accumulation $N_{HC}^{S,t}$:

$$
\max E_0 \sum_{t=0}^{\infty} \beta^t_S \left[ A_{c,t} \log C_{S,t} + j A_{c,t} \log H_{S,t} + \tau \log (1 - N_{S,t} - N_{HC}^{S,t}) \right]
$$

(4)

where $\beta^t_S$ is household-borrowers’ discount factor and $\beta^t_H > \beta^t_S$.

They are subject to the budget constraint:

$$
C_{S,t} + I_{HC}^{S,t} + q_t (H_{S,t} - H_{S,t-1}) + R_{S,t-1} L_{S,t-1} = L_{S,t} + W_{S,t} HC_{S,t-1} N_{S,t}
$$

(5)

where $I_{HC}^{S,t}$ is the investment in human capital. $L_{S,t}$ determines the amount of borrowing from banks at $R_{S,t}$ interest rate. $W_{S,t}$ is the wages rate and $HC_{S,t}$ is the human capital accumulation in terms of productivity.

Household-borrowers are also subject to the following borrowing constraint:

$$
L_{S,t} \leq \rho_S L_{S,t-1} + (1 - \rho_S) m_S A_{MS,t} \left( \frac{q_{t+1}}{R_{S,t}} H_{S,t} \right)
$$

(6)

where $\rho_S$ measures the slow adjustment of the borrowing constraint over time and $m_S$ indicates the constraint of the households on the amount they are able to borrow by the value of their collateral (loan-to-value ratio). Finally, $A_{MS,t}$ is the exogenous shock which affects the households borrowing ability. Following Iacoviello (2015), we assume that Equation 6 binds in a neighborhood of the steady states if $\beta^t_S$ is lower than the weighted average of the discount factors of household-savers and banks. This assumption is also used in the following chapters.

**Human capital accumulation channel** Similarly to household-savers, the household-borrowers accumulate human capital which improves their pro-
ductivity and helps them compete in the labour market:

\[ HC_{S,t} = (1 - \delta_{SK}) HC_{S,t-1} + B_t \left[ (I_{S,t}^{HC})^\theta \left( HC_{S,t-1} N_{S,t}^{HC} \right)^{(1-\theta)} \right]^\lambda \]  (7)

1.3.2 Banks

Banks in this model are intermediaries between savers and borrowers. They play a crucial role since the banking sector can create shocks and then propagate them to other sectors. Banks maximise the following objective function:

\[
\max E_0 \sum_{t=0}^{\infty} \beta_B^t \left[ \log C_{B,t} \right] \]

where \( \beta_B \) is banks’ discount factor and \( C_{B,t} \) is banks’ consumption at period \( t \).

In addition, banks are subject to the following budget constraint:

\[
C_{B,t} + R_{H,t-1} D_{t-1} + L_t = D_t + R_{S,t} L_{S,t} + L_{E,t-1} \]

where \( L_t = L_{S,t} + L_{E,t} \) measures total loans issued by banks. Banks cannot issue loans more than the capital they have for liquidity and stability reasons.\(^{12}\)

Banks are also subject to the following capital adequacy constraint:

\[
L_t - D_t \geq \rho_D (L_{t-1} - D_{t-1}) + (1 - \gamma)(1 - \rho_D) (L_t) \]

where \( \rho_D \) is the parameter which shows the partial adjustment in bank capital and \( \gamma \) shows the long-term target of capital-asset ratio. Following Iacoviello (2015), we assume that this constraint binds in a neighborhood of the steady states as banks’ discount factor is lower than the discount factor of household-savers, implying a relative impatience assumption. We also use this assumption in Chapter 2 and 3.

Entrepreneurs produce a final good by using housing and labour provided

\(^{12}\)Assuming that banks are solvent simplifies our analysis. Extending the model to include non-solvent banks is very interesting but beyond the scope of this chapter.
by households, and they maximise the following lifetime welfare function:

$$\max E_0 \sum_{t=0}^{\infty} \beta_E^t \left[ \log C_{E,t} \right]$$  \hspace{1cm} (11)

where $\beta_E^t$ is their discount factor and $C_{E,t}$ denotes consumption at period $t$.

They also maximise their profit given by:

$$\Pi = Y_t - W_{H,t} HC_{H,t-1} N_{H,t} - W_{S,t} HC_{S,t-1} N_{S,t} - R_{V,t} q_t H_{E,t-1} - R_{E,t} L_{E,t-1}$$  \hspace{1cm} (12)

where $H_{E,t}$ is entrepreneurs’ commercial real estate. We assume a commercial real estate as a collateral for the loans that entrepreneurs’ obtain from banks. There are two reasons for using commercial real estate here. Housing collateral is practical and crucial. Most borrowings are secured by real estate. Moreover, housing plays crucial role in business fluctuations.

$Y_t$ is output given by a standard Cobb-Douglas production function:

$$Y_t = Z_t H_{E,t-1} [HC_{H,t-1} N_{H,t}]^{(1-\nu)(1-\sigma)} [HC_{S,t-1} N_{S,t}]^{(1-\nu)\sigma}$$  \hspace{1cm} (13)

where $\nu$ is the share of entrepreneur’s real estate in the production function and $\sigma$ is the share of labour input of household-borrowers in the production process. $Z_t$ represents total factor productivity.

Entrepreneurs are also subject to the budget constraint:

$$C_{E,t} + q_t (H_{E,t} - H_{E,t-1}) + R_{E,t} L_{E,t-1}$$
$$+ W_{H,t} HC_{H,t-1} N_{H,t} + W_{S,t} HC_{S,t-1} N_{S,t} = Y_t + L_{E,t}$$  \hspace{1cm} (14)

where $L_{E,t}$ denotes loans from banks at $R_{E,t}$ interest rate.

In addition, entrepreneurs’ borrowing is constrained by their total income after all payments have taken place. Therefore, they need to satisfy the following financial constraint:
\[ L_{E,t} \leq \rho_E L_{E,t-1} + (1 - \rho_E) A_{ME,t} \left[ m_H E_t \left( \frac{q_{t+1}}{q_{R, t+1}} H_{E,t} \right) - m_N \left( W_{H,t} H C_{H,t-1} N_{H,t} + W_{S,t} H C_{S,t-1} N_{S,t} \right) \right] \] (15)

where \( \rho_E \) allows for slow adjustment over time, \( m_H \) is the real estate loan-to-value ratio, \( A_{ME,t} \) is an exogenous shock to entrepreneurs borrowing ability and the term \( m_N \) shows the wage bill paid in advance. As entrepreneurs’ discount factor satisfies the following restriction \( \beta^t_E = \frac{1}{\gamma (1 + \gamma) \alpha} \), then the constraint will be binding in the neighborhood of the steady state. We follow this assumption in Chapter 2 and 3.

1.3.3 Aggregate resource constraint and market clearing conditions

The aggregate resource constraint of our economy is given by:

\[ Y_t = C_t + I_t \] (16)

where total consumption and total investment are given by:

\[ C_t = C_{H,t} + C_{S,t} + C_{E,t} + C_{B,t} \]
\[ I_t = I_{HC}^{H,t} + I_{HC}^{S,t} \]

In addition, we have the following market clearing condition for housing:

\[ H_t = H_{H,t} + H_{S,t} + H_{E,t} = 1 \] (17)

where, we normalise the overall supply of housing to unity, as in Iacoviello (2015).\textsuperscript{13}

\textsuperscript{13}This does not affect at all the key results of the chapter and it greatly simplifies our analysis.
1.3.4 Shocks

There are five shocks in the model: a TFP shock, a shock to entrepreneurs’ borrowing constraint, a shock to household-borrowers’ borrowing constraint, a shock to preferences, and a shock to human capital transformation. These exogenous shocks follow AR(I) processes:

\[
\log(Z_t) = \rho_Z \log(Z_{t-1}) + u_Z
\]

\[
\log(A_{ME,t}) = \rho_{AME} \log(A_{ME,t-1}) + u_{ME}
\]

\[
\log(A_{MS,t}) = \rho_{AMS} \log(A_{MS,t-1}) + u_{MS}
\]

\[
\log(A_{C,t}) = \rho_C \log(A_{C,t-1}) + u_C
\]

\[
\log(B_t) = \rho_B \log(B_{t-1}) + u_B
\]

where \(u_{jj}\) is independently and identically distributed Gaussian random variables with zero mean and \(\sigma_{jj}\) standard deviation, for \(jj = \{Z, ME, MS, C, B\}\).

1.4 Decentralised competitive equilibrium

The non-stochastic decentralised competitive equilibrium (DCE) is summarised by a sequence of allocations \(\{C_{H,t}, C_{S,t}, C_{E,t}, C_{B,t}, H_{H,t}, H_{S,t}, H_{E,t}, N_{H,t}, N_{S,t}, N_{HC}^{H}, N_{HC}^{S}, \bar{I}_{H,t}^{HC}, \bar{I}_{S,t}^{HC}, H_{CH,t}, H_{CS,t}, D_t, L_{S,t}, L_{E,t}, Y_t\}_{t=0}^{\infty}\) and prices \(\{W_{H,t}, W_{S,t}, R_{H,t}, R_{S,t}, R_{E,t}, R_{V,t}, q_t\}_{t=0}^{\infty}\) such that the two types of households solve their optimisation problem, and firms and banks maximise their profits, taking prices and initial conditions for housing as given; and all the markets clear.\(^{14}\)

1.5 Calibration and steady state

In this section we calibrate our model using quarterly U.S. data for the period 1965-2015 and then solve to match the key stylised facts and properties of the U.S. economy.

\(^{14}\)The DCE system is presented in appendix A.
Starting with households’ utility functions we set the weight of leisure $\tau$ equal to 2.25 so that the households on average spend around one third of their time at work. In addition, the share of the housing preference $j$ is set equal to 0.084 so that the ratio of loans for household-borrowers-to-output is about 0.67 as in the quarterly data for loans for the U.S. for the period of 1965Q1 to 2015Q4 taken from the Flow of Funds Accounts. Moreover, following Iacoviello (2015), the discount factors for household-savers $\beta_H$ and household-borrowers $\beta_S$ are set equal to 0.9925 and 0.94 respectively.

Next we proceed with the parameters of the human capital accumulation channel. The parameter $\chi$ is set equal to 0.34 to match the consumption over output ratio of 0.70.\textsuperscript{15} In addition, $\theta$ is set equal to 0.8 to get a steady state skill premium equal to 1.60, consistent with the related literature (see Acemoglu and Autor, 2011 and Angelopoulos et al., 2015). We use skill premium as an indicator of wage gap.

Human capital depreciation rate, $\delta_{SK}$, is usually set to 10%. Heckman (1976) uses values from 4% to 9%, however, these values are sensitive to the model’s specification, while He and Liu (2008) use the value of 8%, as in Stokey (1991). Rosen (1976) gives different values for skill depreciation rate for high school graduates (5%) and university graduates (19%). In this chapter we use the depreciation rate value as in Jones et al. (1993).

The loan-to-value ratios are set to 0.9 as in Iacoviello (2015), except for the $m_N$ which equals to 1 to ensure that workers receive their wages in advance as in Neumeyer and Perri (2005). Also, the capital-asset ratio, $\gamma$, and the partial adjustment parameters in the financial constraints ($\rho_D$, $\rho_E$ and $\rho_S$) are set as in Iacoviello (2015).\textsuperscript{16}

The real estate elasticity $\nu$ and labour share $\sigma$ in the production function are set equal to 0.0331 and 0.28 respectively. These parameters help us pin down the steady state values for the loans to entrepreneurs over output at

\textsuperscript{15}For this target we use hp-filtered and log-transformed U.S. quarterly data for the period 1965-2015 from NIPA tables.

\textsuperscript{16}We perform a robustness analysis later on so as to assess the importance of those parameters in our model.
and total wages over output at about 0.44. We also normalise the steady state values of all the AR(1) processes to unity (i.e., $Z = A_{ME} = A_{MS} = A_C = B = 1$)

Having calibrated the model, as shown in Table 1.1, we solve the non-stochastic DCE system of equations (A1)-(A26), as presented in the appendix. Table 1.2 presents the steady state results of our model together with the U.S. data averages for the period 1965-2015. The steady state of our model is very close to that of the data. This confirms our calibration and the good fit of the model.

### Table 1.1: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_B$</td>
<td>Banks discount factor</td>
<td>0.945</td>
<td>Iacoviello (2015)</td>
</tr>
<tr>
<td>$\beta_E$</td>
<td>Entrepreneurs discount factor</td>
<td>0.94</td>
<td>Iacoviello (2015)</td>
</tr>
<tr>
<td>$\beta_H$</td>
<td>HH-S discount factor</td>
<td>0.9925</td>
<td>Iacoviello (2015)</td>
</tr>
<tr>
<td>$\beta_S$</td>
<td>HH-B discount factor</td>
<td>0.94</td>
<td>Iacoviello (2015)</td>
</tr>
<tr>
<td>$j$</td>
<td>Housing preference share</td>
<td>0.084</td>
<td>Data</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Elasticity of labour supply</td>
<td>2.25</td>
<td>Data</td>
</tr>
<tr>
<td>$m_N$</td>
<td>Wage bill paid in advance</td>
<td>1</td>
<td>Neumeyer &amp; Perri (2005)</td>
</tr>
<tr>
<td>$m_H$</td>
<td>Real estate loan-to-value ratio</td>
<td>0.9</td>
<td>Iacoviello (2015)</td>
</tr>
<tr>
<td>$m_S$</td>
<td>Housing loan-to-value ratio</td>
<td>0.9</td>
<td>Iacoviello (2015)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Capital-asset ratio target</td>
<td>0.9</td>
<td>Iacoviello (2015)</td>
</tr>
<tr>
<td>$\rho_D$</td>
<td>Partial adj. in bank capital</td>
<td>0.233</td>
<td>Iacoviello (2015)</td>
</tr>
<tr>
<td>$\rho_E$</td>
<td>E. borr. constraint adj.</td>
<td>0.63</td>
<td>Iacoviello (2015)</td>
</tr>
<tr>
<td>$\rho_S$</td>
<td>HH-B borr. constraint adj.</td>
<td>0.71</td>
<td>Iacoviello (2015)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Real estate share in prod.</td>
<td>0.0331</td>
<td>Data</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>HH labour share in prod.</td>
<td>0.8</td>
<td>Data</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Goods share in the HCA</td>
<td>0.8</td>
<td>Data</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Returns to scale in HCA</td>
<td>0.34</td>
<td>Data</td>
</tr>
<tr>
<td>$\delta_{SK}$</td>
<td>HC depreciation rate</td>
<td>0.1</td>
<td>Jones et al. (1993)</td>
</tr>
</tbody>
</table>

---

17 Using again quarterly data for loans for the U.S. for the period of 1965Q1 to 2015Q4 taken from the Flow of Funds Accounts as in Iacoviello (2015).
18 For total wages we used data from NIPA Table 2.1 and for output we used data from WDI.
19 Note that throughout the paper we use the abbreviation HH to denote households, mainly in our tables. Therefore, HH-S stands for household-savers and HH-B for household-borrowers.

42
Table 1.2: Steady states

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_Y$</td>
<td>0.70</td>
<td>0.66</td>
</tr>
<tr>
<td>$L_Y$</td>
<td>0.38</td>
<td>0.38</td>
</tr>
<tr>
<td>$L_N$</td>
<td>0.67</td>
<td>0.67</td>
</tr>
<tr>
<td>$W_Y$</td>
<td>0.44</td>
<td>0.44</td>
</tr>
<tr>
<td>$SP$</td>
<td>1.60</td>
<td>1.60</td>
</tr>
<tr>
<td>$N_N$</td>
<td>0.36</td>
<td>0.33</td>
</tr>
<tr>
<td>$N_S$</td>
<td>0.35</td>
<td>0.33</td>
</tr>
</tbody>
</table>

1.5.1 Stochastic processes

We also calibrate the autocorrelation and standard deviation parameters for the TFP AR(1) process to match the autocorrelation and standard deviation of output in our model with the data. Specifically, we set $\rho_Z$ to 0.9 and $\sigma_Z$ to 0.007 to obtain an autocorrelation of output equal to 0.82 and standard deviation equal to 1.3.20

Then we set the remaining $\rho$ and $\sigma$ parameters for the entrepreneurs’ borrowing constraint, households’ borrowing constraint, preferences and human capital transformation processes, to match the relative correlation and standard deviation to output of loans to entrepreneurs, loans to households, consumption and wage series respectively. Table 1.3 shows the relevant calibrated parameters, and Table 1.4 the results from the simulated model in comparison with the data.

In order to obtain the business cycle moments for our model, we perform a second-order approximation of the equilibrium conditions around the deterministic steady state and we simulate time paths under all the shocks presented and calibrated in Table 1.3. Then we conduct 500,000 simulations of 254 periods, where we drop the initial 50, to match the number of observations in the data we used. Table 1.4 shows that the moments of the estimated model are close to the moments of the data, as discussed above.

---

20Quarterly data for output for the period 1965-2015 from WDI were used.
Table 1.3: Stochastic processes

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_Z$</td>
<td>AR(1) coef. of TFP</td>
<td>0.9</td>
<td>Data</td>
</tr>
<tr>
<td>$\sigma_Z$</td>
<td>Std. dev. of TFP</td>
<td>0.007</td>
<td>Data</td>
</tr>
<tr>
<td>$\rho_{AME}$</td>
<td>AR(1) coef. of Entr. borr. constraint</td>
<td>0.7</td>
<td>Data</td>
</tr>
<tr>
<td>$\sigma_{AME}$</td>
<td>Std. dev. of Entr. borr. constraint</td>
<td>0.004</td>
<td>Data</td>
</tr>
<tr>
<td>$\rho_{AMS}$</td>
<td>AR(1) coef. of HH. borr. constraint</td>
<td>0.8</td>
<td>Data</td>
</tr>
<tr>
<td>$\sigma_{AMS}$</td>
<td>Std. dev. of HH borr. constraint</td>
<td>0.005</td>
<td>Data</td>
</tr>
<tr>
<td>$\rho_C$</td>
<td>AR(1) coef. of preference</td>
<td>0.7</td>
<td>Data</td>
</tr>
<tr>
<td>$\sigma_C$</td>
<td>Std. dev. of preference</td>
<td>0.0015</td>
<td>Data</td>
</tr>
<tr>
<td>$\rho_B$</td>
<td>AR(1) coef. of HC transformation</td>
<td>0.6</td>
<td>Data</td>
</tr>
<tr>
<td>$\sigma_B$</td>
<td>Std. dev. of HC transformation</td>
<td>0.007</td>
<td>Data</td>
</tr>
</tbody>
</table>

Table 1.4: Business cycle statistics of the key ratios

<table>
<thead>
<tr>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_i$</td>
<td>$\sigma(X_i)/\sigma(Y)$</td>
</tr>
<tr>
<td>$C$</td>
<td>0.94</td>
</tr>
<tr>
<td>$L_E$</td>
<td>2.71</td>
</tr>
<tr>
<td>$L_S$</td>
<td>1.64</td>
</tr>
<tr>
<td>$W$</td>
<td>0.84</td>
</tr>
</tbody>
</table>

1.6 Impulse response and welfare analysis

In this section, we analyse the effects of various shocks to the economy, and wage gap in particular, under the presence of banks, financial frictions and endogenous human capital accumulation. Later on we are going to illustrate how the existence of banks and/or the endogenous human capital accumulation channel affect the results.

Figures 1.1-1.5 about here

1.6.1 TFP shock

We start with the analysis of a positive one standard deviation temporary shock to the economy. Figure 1.1 shows that immediately after the shock output increases, leading to higher demand for inputs in the production process. This drives both of the wage rates to increase, leading to
higher overall consumption. Moreover, deposits for savers increase, leading to higher supply of loans that are mainly channeled to household-borrowers, similar to Asimakopoulos and Asimakopoulos (2017). Thus, housing demand for household-borrowers increases as a result of the higher loans, as well as the investment in human capital, leading to higher stock of human capital. Specifically, the increase in the stock of human capital for borrowers is higher than that of the savers. This leads to higher increase in their wage rate relative to savers and a lower skill premium as a consequence.

1.6.2 Shock to preference

We now turn to the case of a positive one standard deviation temporary shock to households’ preferences. Figure 1.2 shows that the shock to preferences has asymmetric effects to the two types of households in the economy.\(^{21}\) Savers will decrease their deposits so as to consume more and increase their stock of housing. The decrease in deposits has a knock-on effect on loans supply to borrowers, leading to lower stock of housing for household-borrowers and entrepreneurs. Therefore, even though borrowers have a stronger preference towards consumption and housing they are not able to satisfy it due to the lower availability of funds. As a result, they will tend to increase their labour supply so as to compensate for this loss. This leads to lower wage rate for borrowers which is further pushed downwards from the lower investment and time spent in human capital accumulation. Skill premium will widen under this scenario and it will be further pushed upwards from the increased human capital accumulation from the savers.

1.6.3 Shock to human capital transformation

In this subsection, we analyse the case of a positive one standard deviation temporary shock to human capital transformation efficiency. Figure 1.3 shows that the positive human capital transformation shock leads to a symmetric increase in human capital accumulation via investment in goods and

\(^{21}\)In line with our results, Isore and Szczerbowicz (2017) show that preferences shock creates more impatient households with respect to their consumption expenditures.
time for both types of households. The initial financing of human capital is driven from the lower consumption for both households. However, savers decrease their deposits to further enhance their human capital accumulation and housing stock. This leads to lower loans for borrowers, decreasing their housing stock and driving up their labour supply. Therefore, borrowers’ wage rate decreases resulting in an increase of the skill premium.\footnote{This result is similar to Dadgar and Trimble (2015) and Murphy and Topel (2016), who show a higher wage inequality driven by the assumed endogenous human capital accumulation, albeit at a different set-up.}

### 1.6.4 Shock to household-borrowers financial constraint

Next we examine the case of a positive one standard deviation temporary shock to loan-to-value (LTV) ratio for household-borrowers. In particular, Figure 1.4 shows the effects of a relaxation of the financial friction for household-borrowers. Under this shock they borrow more from banks to invest in housing and human capital, similar to Liu et al. (2013) and Ravn (2016). In our set-up they substitute away from labour to time in human capital accumulation, due to the income effect, which will increase their wage rate. The higher wage rate is also supported by an increase in their productivity. Deposits from savers initially increase to satisfy the higher supply of funds from the banks but as the shock fades away deposits drop and so do the loans. The reduction in the loans and the increase in housing prices drive an increase in labour supply from borrowers which overturns the reduction in wage and consumption inequalities observed in the short-run.

### 1.6.5 Shock to entrepreneurs financial constraint

In Figure 1.5 we examine a positive one standard deviation temporary shock to loan-to-value ratio for entrepreneurs. We observe that this shock will lead to higher loans for entrepreneurs which will be invested in real estate.\footnote{Gambacorta and Signoretti (2014) also find that under a positive borrowing constraint shock borrowers become better off while the demand for housing and goods increases.} Therefore, labour demand will decrease (due to the assumed standard Cobb-Douglas production function) driving wage rates to decrease in the short-
run. Following the initial reaction, the wage rates for savers will tend to increase faster compared to that of the borrowers, due to higher returns from deposits that the savers partially invest in human capital. Household-borrowers increase their time spent in human capital substituting away from labour. However, this is not sufficient to compensate the lower investment of goods in human capital, driving their overall human capital to decrease. As a result this shock will lead to a higher skill premium, and wage gap will widen in the economy.

1.6.6 Welfare effects

In this section, we report the numerical solutions of aggregate welfare from the various shocks presented above using the consumption equivalence approach.

Rotemberg and Woodford (1997) present a loss function that could transform households’ objective function into a quadratic function by using first-order approximation of the constraints, which delivers accurate results for social planners. Benigno and Woodford (2012) developed this idea further by assuming the general setting without the presence of subsidiaries. They also show that households’ objective function can be transformed into a quadratic form of:

\[
\sum_{t=0}^{\infty} E_0 \beta_t^t U(X_t) \simeq \text{constant} - \sum_{t=0}^{\infty} E_0 \beta_t^t X_t^i W_t^i X_t^i
\]

where \(X_t\) is an \(N \times 1\) vector with the variables used in a model with their deviation from the steady state. \(X_t^i W_t^i X_t^i\) is the quadratic approximation of the households’ utility function of \(U(X_t)\).

There is also a literature that describes the analytical approach of the welfare aspects (see, for example, Ferrero et al., 2018 and Rubio and Yao, 2019), who derive the second-order approximation of the welfare function to find consumption gap, output gap and housing gap.

Assuming that the welfare of each agent after the shock is given by \(W_t^a\) and before \(W_t^0\), then the consumption equivalent gain/loss of each agent from
that shock is calculated as:

\[
W^0_i \left( \lambda_i, C^0_i, H_i^0, N_i^0, N_i^{HC,0} \right) = W^{as}_i \left( C^{as}_i, H_i^{as}, N_i^{as}, N_i^{HC,as} \right)
\]  

(23)

\[
\sum_{t=0}^{\infty} \beta^t U \left( (1 + \lambda_i) C^0_i, H_i^0, N_i^0, N_i^{HC,0} \right) = \sum_{t=0}^{\infty} \beta^t U \left( C^{as}_i, H_i^{as}, N_i^{as}, N_i^{HC,as} \right)
\]

(24)

where \( \lambda_i \) is the consumption equivalent gain/loss from the shock.

Using the logarithmic utility function applied in our analysis we get the following expression for the consumption equivalent gain/loss:

\[
\lambda_i = \left[ \exp \left( (1 - \beta_i) \left( W^{as}_i - W^0_i \right) \right) - 1 \right] \times 100
\]

(25)

As a result, Table 1.5 presents the values of the consumption equivalent in percentage terms for each agent (in columns) and for each shock (in rows). Positive values indicate that the agent is better off under the shocks and vice versa. Moreover, the values reported in Table 1.5 are for \( t \to \infty \). Note that Table 1.5 also includes the discounted percent deviation of the skill premium from the steady state\(^ {24} \), expressed as the ratio of the savers over the borrowers wage rate, in the last column. This gives a quantitative indication regarding the wage gap effects of each shock we discussed in the impulse responses earlier.

Starting with the TFP shock, we observe that borrowers gain significantly more, relative to the savers, due to their higher wage rates. This is mainly driven by the increased loans to borrowers that are invested in both housing and human capital accumulation. Moving to the preferences shock we observe that savers are the only ones that marginally increase their welfare. This is mainly due to the lower availability of funds for the borrowers, which also has a negative effect on wage gap that increases at about 2%.

Interestingly, under the human capital efficiency transformation shock, household-borrowers will increase their consumption more relative to savers.

\(^{24}\) As a discount factor, we have used the households-borrowers time discount factor for the calculations shown in the tables. Even if we use the savers’ discount factor the results are qualitatively similar.
However, the wage gap will widen at about 0.6% due to the shift of investment from deposits to human capital accumulation from savers, leading to lower available funds for the borrowers.

Under the positive financial shock to households’ borrowing constraint we observe that borrowers are better off compared to savers in terms of welfare, even though there is a reduction in consumption gap only in the short-run, as we discussed earlier in the impulse responses. Regarding wage gap, we observe that skill premium increases at about 0.2% indicating that the short-run reduction of wage gap after the shock is not sufficient to reduce wage gap in the long-run.

Finally, under the positive shock to the entrepreneurs’ borrowing constraint we observe an increase in wage and consumption gaps both in the short-run (see Figure 1.5) and in the long-run. Specifically, Table 1.5 indicates that savers will be marginally better off in terms of welfare and wage gaps will increase at about 1.2%. As it is expected, entrepreneurs will also be better off, as well as banks, due to the higher supply of loans to entrepreneurs.

<table>
<thead>
<tr>
<th>Table 1.5: Welfare and skill premium effects</th>
</tr>
</thead>
<tbody>
<tr>
<td>HH-savers</td>
</tr>
<tr>
<td>$\text{TFFP}$</td>
</tr>
<tr>
<td>$\text{A}_C$</td>
</tr>
<tr>
<td>$B$</td>
</tr>
<tr>
<td>$A_{MS}$</td>
</tr>
<tr>
<td>$A_{ME}$</td>
</tr>
</tbody>
</table>

1.7 Robustness check

1.7.1 Assessing the calibration of financial frictions

As a robustness check we change the parameters that enter the financial frictions. These parameters affect the financial frictions of the borrowers and the banks. In particular, in our experiments we decrease the capital-asset ratio requirement for the banks, $\gamma$, the adjustment of the borrowing constraints for the households and entrepreneurs, $\rho_S$ and $\rho_E$, and the partial
adjustment in bank capital, $\rho_D$.

Table 1.6 below compares the welfare results from these cases to the base results presented earlier, as well as the effects on wage gap. We can see that the adjustment parameters of the borrowing constraints do not affect the results significantly. However, the capital-asset ratio requirement for the banks, $\gamma$, appears to have a strong impact on wage and consumption gaps. Specifically, under a TFP shock there is an increase of about 2% in wage gap compared to the base case, if we relax the capital-asset ratio requirement. In addition, consumption gap increases compared to the base case since borrowers’ consumption will not increase as much and savers’ consumption marginally increases relative to the base case. Finally, under preferences and human capital efficiency shocks the capital-asset ratio requirement seems to marginally reduce wage gap at about 0.5%.

Therefore, the decrease in capital-asset requirement will increase consumption and wage gap under a TFP shock. This is due to the fact that under a TFP shock the lower capital-asset ratio requirement will lead to a reduction in the demand for deposits from the savers increasing their disposable income that they could spend in human capital accumulation and/or consumption. The reduction of wage gap under a positive preferences and human capital transformation shocks is mainly driven from the fact that with a lower $\gamma$ the bank can increase its borrowing to household-borrowers that can be spent in human capital accumulation.

1.7.2 Different modelling assumptions

In this section we assess the importance of the endogenous human capital accumulation channel and the existence of banks. To that end we construct three different models. In the first model we eliminate the endogenous human capital channel (model 1). In the second model we re-introduce the endogenous human capital channel but we eliminate the banks (model 2). In this model household-savers will provide the loans to borrowers and there will be no capital adequacy constraint, leading to a faster supply of funds. Finally, in the third model we eliminate both the endogenous human capital
Table 1.6: Robustness checks for the welfare and skill premium effects

<table>
<thead>
<tr>
<th></th>
<th>HH-savers</th>
<th>HH-borrowers</th>
<th>Entrepr.</th>
<th>Bank</th>
<th>$W_H/W_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$TP$</td>
<td>0.0210</td>
<td>0.2965</td>
<td>0.1977</td>
<td>0.3519</td>
<td>-13.2112</td>
</tr>
<tr>
<td>$AC$</td>
<td>0.0014</td>
<td>-0.0364</td>
<td>-0.0054</td>
<td>-0.0941</td>
<td>2.0524</td>
</tr>
<tr>
<td>$B$</td>
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<td>0.0556</td>
<td>0.0900</td>
<td>-0.0822</td>
<td>0.6347</td>
</tr>
<tr>
<td>$AMS$</td>
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<td>0.0028</td>
<td>0.0000</td>
<td>0.0340</td>
<td>0.2068</td>
</tr>
<tr>
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<td>0.0166</td>
<td>0.2171</td>
<td>1.1856</td>
</tr>
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<tr>
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</tr>
<tr>
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</tr>
<tr>
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</tr>
<tr>
<td>$B$</td>
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</tr>
<tr>
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</tr>
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<td>0.5519</td>
</tr>
<tr>
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<td>0.2261</td>
</tr>
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<td>0.0156</td>
<td>0.1448</td>
<td>1.1549</td>
</tr>
</tbody>
</table>

channel and banks (model 3).

**Steady state analysis of the different models** Before we show the impulse responses of these models and perform an analysis on the wage gap effects, we would like to mention that these models lead to a different steady state (Table 1.7), similar to Iacoviello (2015) and Asimakopoulos and Asimakopoulos (2017) when they examine models with and without banks.

Specifically, Model 1 (without human capital) leads to a significantly lower level of consumption, at about 80% lower than the benchmark case.
Moreover, deposits and loans drop significantly. Hours worked decrease as well at about 10%, leading to a decrease of about 84% in output compared to the base case. Wage gap increases significantly, to about 66%, compared to the base case and assets wage gap increases since housing is being reallocated to savers. Therefore, cutting down the endogenous human capital channel has significant steady-state effects in our model.

Moving to Model 2 (without banks), there is an increase in overall consumption of about 5%, and housing is being re-allocated to borrowers due to higher availability of funds from the elimination of the bank’s capital adequacy constraint. In addition, output decreases, in contrast to Iacoviello (2015), due to the decrease in human capital accumulation compared to the base case.

Finally, Model 3 provides a combined outcome of Model 1 and 2 discussed above.

<table>
<thead>
<tr>
<th>Table 1.7: Steady state deviations from the benchmark model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variables</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>Cons.</td>
</tr>
<tr>
<td>Banks</td>
</tr>
<tr>
<td>Entrepreneurs</td>
</tr>
<tr>
<td>HH-Savers</td>
</tr>
<tr>
<td>HH-Borrowers</td>
</tr>
<tr>
<td>Deposits</td>
</tr>
<tr>
<td>Housing</td>
</tr>
<tr>
<td>Entrepreneurs</td>
</tr>
<tr>
<td>HH-Savers</td>
</tr>
<tr>
<td>HH-Borrowers</td>
</tr>
<tr>
<td>Hours worked</td>
</tr>
<tr>
<td>HH-Savers</td>
</tr>
<tr>
<td>HH-Borrowers</td>
</tr>
<tr>
<td>Wages</td>
</tr>
<tr>
<td>HH-Savers</td>
</tr>
<tr>
<td>HH-Borrowers</td>
</tr>
<tr>
<td>HC</td>
</tr>
<tr>
<td>HH-Savers</td>
</tr>
<tr>
<td>HH-Borrowers</td>
</tr>
<tr>
<td>Output</td>
</tr>
<tr>
<td>Loans</td>
</tr>
<tr>
<td>HH-Borrowers</td>
</tr>
<tr>
<td>Entrepreneurs</td>
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<tr>
<td>Skill premium</td>
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</tbody>
</table>

52
Impulse response and welfare analysis of the various models  

We perform again the impulse response analysis for the three models under different assumptions and we plot them on the same graph. Figures 1.6 - 1.10 show the IRs of all the models and for every shock. The solid line is for the benchmark model, the dashed line is for Model 1, the dotted line is for Model 2, and the dashed-dotted line is for Model 3. We also provide Table 1.8 that presents the welfare and skill premium effects for each model under each shock.

Under the TFP shock we can see that the existence of human capital and banks mitigates the reduction in wage gap mainly via two channels. On the one hand, human capital allows both agents to improve productivity but the savers can do so without the need to increase borrowing (see Figure 1.6). On the other hand, the existence of banks and the relevant capital adequacy constraint provide a friction in the availability of funds for the borrowers. Therefore, those two features keep skill premium closer to the steady state. Regarding consumption gap, we can see that consumption of both households is mostly affected by the existence of the human capital channel.

Moving on to the preferences shock, Figure 1.7, we observe that both wage and consumption gaps increase as we move from the base model to Models 1-3. Under this shock, savers can keep the same level of consumption, as in the base case, by decreasing their level of deposit. Household-borrowers increase their labour supply to be able to support their level of consumption at the expense of receiving lower wage rates since there is no human capital channel to invest in skills. Under the case without banks, savers benefit the most since they now become the provider of loans, and are able to sustain the same level of consumption, human capital accumulation and housing as in the base case. Household-borrowers, however, reduce their human capital investment so as to minimise the deviations of consumption and housing from the base case, leading to higher wage gap.

Figure 1.8 compares the three models under the human capital transformation efficiency shock. Here we can only compare our base model with
Model 2 that has human capital and no banks. We can see that the existence of banks under a human capital shock is significant for the economy. First of all, wage gap will rise considerably without banks since both agents will increase their human capital at a similar rate but given the initial difference in human capital, savers will keep and extend their comparative advantage. Since borrowers’ wage rate drops they try to increase their labour supply so as to sustain their income level and be able to invest more in human capital. In addition, consumption gap increases mainly due to the lower wage rate for borrowers.

The positive shock to the financial constraint of household-borrowers, Figure 1.9, indicates that the existence of human capital and banks leads to a smoother reaction of skill premium (lower reduction compared to the base case) and lower consumption gap. This is due to the fact that under this shock household-borrowers increase their housing level, leading to lower available stock of housing for the savers, given the assumed bounded availability of housing stock. Their wealth increases substituting away from labour supply and increasing their consumption. We should also note here that there is a more pronounced effect on consumption gap under no banks because of the frictionless transfer of funds from savers to borrowers.

Finally, under the financial shock to entrepreneurs, Figure 1.10, we observe that under no banks, entrepreneurs can increase their stock of real estate considerably, due to the frictionless transfer of funds from savers (as in the previous case), leading to higher future output level, similar to Asimakopoulos and Asimakopoulos (2017). This also results in higher consumption and wage gaps because of the increased demand for the more productive labour supply. The existence of human capital channel in this case does not create significant deviations from the base model.

In terms of welfare we can see from Table 1.8 that household-savers will always be marginally worse off or at a similar level with the base model. However, household-borrowers will benefit the most when the human capital accumulation channel is present and there is a frictionless flow of funds (no banks). This is fairly intuitive since the endogenous human capital accumulation channel can be more effective when agents can more easily draw more
funds to invest on. Even though under no human capital but with frictionless flow of funds (no banks) they can still benefit more, in terms of welfare, compared to the base case. Thus, banks tend to mitigate the positive spillover effects due to the assumed capital adequacy constraint. Entrepreneurs also benefit the most under no banks but with human capital, even though they will also benefit under the case without human capital and without banks.

Table 1.8: Welfare and skill premium effects of alternative model specifications

<table>
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<tr>
<th></th>
<th>HH-savers</th>
<th>HH-borrowers</th>
<th>Entrepr.</th>
<th>Bank</th>
<th>$W_H/W_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>TFP</strong></td>
<td>0.0210</td>
<td>0.2965</td>
<td>0.1977</td>
<td>0.3519</td>
<td>-13.2112</td>
</tr>
<tr>
<td>$A_C$</td>
<td>0.0014</td>
<td>-0.0364</td>
<td>-0.0054</td>
<td>-0.0941</td>
<td>2.0524</td>
</tr>
<tr>
<td>$B$ Base</td>
<td>0.0199</td>
<td>0.0556</td>
<td>0.0900</td>
<td>-0.0822</td>
<td>0.6347</td>
</tr>
<tr>
<td>$A_{MS}$</td>
<td>0.0001</td>
<td>0.0028</td>
<td>0.0000</td>
<td>0.0340</td>
<td>0.2068</td>
</tr>
<tr>
<td>$A_{ME}$</td>
<td>0.0008</td>
<td>-0.0086</td>
<td>0.0166</td>
<td>0.2171</td>
<td>1.1856</td>
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<td><strong>TFP</strong></td>
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<td>0.2791</td>
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<td>$B$ Model 1</td>
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<td>-</td>
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<td>-</td>
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1.8 Conclusion

This chapter presents an empirical analysis of wage and welfare gaps in a DSGE model with heterogeneous agents, financial frictions and endogenous human capital accumulation. We initially calibrated our model for the U.S. economy using quarterly data for the period 1965-2015 to match first and
second moments of the key variables.

We found that under productivity shocks, wage and income gaps are reduced significantly, whereas under preferences or financial shocks, wage gap increased both in terms of welfare and wage rates. Finally, under a human capital productivity shock we find that the welfare gap was reduced both in the short-run and in the long-run. However, wage gap was reduced only in the short-run, leading to higher wage gap in the long-run.

The robustness checks regarding the calibration of the financial frictions indicated that the key results remained unaffected. However, the reduction in capital-asset requirement for banks could lead to higher wage and consumption gap in the long-run, compared to the benchmark model.

Finally, we assessed the effect of the key modelling assumptions, and our findings suggested that the existence of solvent banks and their associated capital adequacy constraint can mitigate the propagation of the shocks to wage and consumption gaps. However, we also find that any lending facilities are beneficial for household-borrowers, even though the presence of a banking sector as long as household-savers can represent lenders. In addition, the human capital accumulation channel mainly improved the welfare of household-borrowers and it significantly affected wage gap. Moreover, savers’ welfare remained at a similar level as the benchmark model under all the shocks and the different models that we analysed. Therefore, our results indicated that policymakers need to foster human capital accumulation to efficiently control welfare gap, but also need to tightly regulate banks’ capital-asset ratio to control for the propagation of the shocks to wage and consumption gaps.
Appendix A

A Chapter 1

This appendix shows the DCE system of equations for Chapter 1 model for each agent.

A.1 Households

A.1.1 Household-savers

\[
0 = \lambda_t^H - E_t \beta_H \lambda_{t+1}^H R_{H,t} \quad (A1)
\]

\[
0 = -\frac{j A_{C,t}}{H_{H,t}} + \lambda_t^H q_t - E_t \beta_H \lambda_{t+1} q_{t+1} \quad (A2)
\]

\[
0 = -\frac{\tau}{1 - N_{H,t} - N_{HC,t}} - \lambda_t^H W_{H,t} H C_{H,t-1} \quad (A3)
\]

\[
0 = SK_{H,t} - \lambda_t^H W_{H,t+1} N_{H,t+1} - E_t \beta_H SK_{H,t+1} (1 - \delta_{SK}) - \\
E_t \beta_H S K_{H,t+1} B_{H,t+1} \left[ (H C_{H,t} N_{H,t+1})^{(1-\theta)} \left( I_{H,t+1}^{HC} \right)^{\theta} \right]^{\chi} \quad (A4)
\]

\[
0 = -\frac{\tau}{1 - N_{H,t} - N_{HC,t}} - SK_{H,t+1} \frac{B_t (1 - \theta) \chi}{N_{HC,t}} \left[ (H C_{H,t+1} N_{HC,t})^{(1-\theta)} \left( I_{H,t+1}^{HC} \right)^{\theta} \right]^{\chi} \quad (A5)
\]

\[
\lambda_t^H - SK_{H,t} \frac{B_t \theta \chi}{I_{HC,t}} \left[ (I_{HC,t}^{\theta}) \left( H C_{H,t-1} N_{HC,t} \right)^{(1-\theta)} \right]^{\chi} = 0 \quad (A6)
\]

\[
H C_{H,t} = (1 - \delta_{SK}) H C_{H,t-1} + B_t \left[ (I_{HC,t}^{\theta}) \left( H C_{H,t-1} N_{HC,t} \right)^{(1-\theta)} \right]^{\chi} \quad (A7)
\]

where \( \lambda_t^H = -\frac{A_{C,t}}{C_{H,t}} \)

A.1.2 Household-borrowers

\[
0 = \frac{A_{C,t}}{C_{S,t}} + \lambda_t^S \quad (A8)
\]

\[
0 = -\frac{j A_{C,t}}{H_{S,t}} + \lambda_t^S q_t - E_t \beta_S \lambda_{t+1}^S q_{t+1} - \\
\mu_t^S (1 - \rho_S) m_S A_{MS,t} \left( \frac{q_{t+1}}{R_{S,t}} \right) \quad (A9)
\]
\[ 0 = -\lambda^S_t + \mu^S_t + E_t \beta_S \lambda^S_{t+1} R_{S,t} - E_t \beta_S \mu^S_{t+1} \rho_S \]  
\[ 0 = -\frac{\tau}{1 - N_{S,t} - N_{HC,t}^S} - \lambda^S_t W_{S,t} HC_{S,t} \]  
\[ 0 = SK_{S,t} - \lambda^S_t W_{S,t+1} N_{S,t+1} - E_t \beta_S SK_{S,t+1} (1 - \delta_{SK}) - E_t \beta_S SK_{S,t+1} \frac{B_h (1 - \theta)_X}{HC_{S,t}} \left[ \left( I_{S,t+1}^{HC} \right)^\theta \left( HC_{S,t} N_{HC,t}^S \right)^{(1 - \theta)} \right]^X \]  
\[ 0 = -\frac{\tau}{1 - N_{S,t} - N_{HC,t}^S} SK_{S,t} \frac{B_h (1 - \theta)_X}{HC_{S,t}} \left[ \left( I_{S,t}^{HC} \right)^\theta \left( HC_{S,t-1} N_{HC,t}^S \right)^{(1 - \theta)} \right]^X \]  
\[ 0 = \lambda^S_t - SK_{S,t} \frac{B_h (1 - \theta)_X}{HC_{S,t}} \left[ \left( I_{S,t}^{HC} \right)^\theta \left( HC_{S,t-1} N_{HC,t}^S \right)^{(1 - \theta)} \right]^X \]  
\[ HC_{S,t} = (1 - \delta_{SK}) HC_{S,t-1} + B_t \left[ \left( I_{S,t}^{HC} \right)^\theta \left( HC_{S,t-1} N_{HC,t}^S \right)^{(1 - \theta)} \right]^X \]

where \( \lambda^S_t = -\frac{\lambda_{AC,t}^S}{\lambda_{HC,t}^S} \).

**A.2 Banks**

\[ 0 = \frac{1}{C_{B,t}^t} + \lambda^B_t \]  
\[ 0 = -\lambda^B_t - \mu^B_t + E_t \beta_B \lambda^B_{t+1} R_{H,t} - E_t \beta_B \mu^B_{t+1} \rho_D \]  
\[ 0 = \lambda^B_t - \mu^B_t (1 - \gamma)(1 - \rho_D) - E_t \beta_B \lambda^B_{t+1} R_{E,t+1} - E_t \beta_B \mu^B_{t+1} \rho_D \]  
\[ 0 = \lambda^B_t - \mu^B_t (1 - \gamma)(1 - \rho_D) - E_t \beta_B \lambda^B_{t+1} R_{S,t+1} - E_t \beta_B \mu^B_{t+1} \rho_D \]

**A.3 Entrepreneurs**

\[ 0 = \frac{1}{C_{E,t}^t} + \lambda^E_t \]  
\[ 0 = -\lambda^E_t + \mu^E_t + E_t \beta_E \lambda^E_{t+1} R_{E,t+1} - E_t \beta_E \lambda^E_{t+1} \rho_E \]  
\[ 0 = \lambda^E_t q_t - E_t \beta_E \lambda^E_{t+1} q_{t+1} - \mu^E_t (1 - \rho_E) A_{ME,t} m_H E_t \left( \frac{q_{t+1}}{R_{E,t+1}} \right) \]  
\[ 0 = v \frac{Y_t}{H_{E,t-1}} - q_t R_{V,t} \]
0 = \frac{(1 - v)(1 - \sigma)Y_t}{N_{H,t}} - W_{H,t}HC_{H,t-1}(1 + \mu_t^E(1 - \rho_E)A_{ME,t}m_N) \quad (A24)

0 = \frac{(1 - v)\sigma Y_t}{N_{S,t}} - W_{S,t}HC_{S,t-1}(1 + \mu_t^E(1 - \rho_E)A_{ME,t}m_N) \quad (A25)

### A.4 Market clearing condition for housing

\[ H_{H,t} + H_{S,t} + H_{E,t} = 1 \quad (A26) \]

### A.5 Shocks

\[ \log(Z_t) = \rho_Z \log(Z_{t-1}) + u_Z \quad (A27) \]
\[ \log(A_{ME,t}) = \rho_{AME} \log(A_{ME,t-1}) + u_{ME} \quad (A28) \]
\[ \log(A_{MS,t}) = \rho_{AMS} \log(A_{MS,t-1}) + u_{MS} \quad (A29) \]
\[ \log(A_{C,t}) = \rho_C \log(A_{C,t-1}) + u_C \quad (A30) \]
\[ \log(B_t) = \rho_B \log(B_{t-1}) + u_B \quad (A31) \]
Figure 1.1: Impulse responses to a one standard deviation increase in TFP. All variables are in percentage deviations from their steady state. X-axis is in quarters.
Figure 1.2: Impulse responses to a one standard deviation increase in preferences. All variables are in percentage deviations from their steady state. X-axis is in quarters.
Figure 1.3: Impulse responses to a one standard deviation increase in human capital transformation efficiency. All variables are in percentage deviations from their steady state. X-axis is in quarters.
Figure 1.4: Impulse responses to a one standard deviation increase in LTV ratio of HH-B. All variables are in percentage deviations from their steady state. X-axis is in quarters.
Figure 1.5: Impulse responses to a one standard deviation increase in LTV ratio of entrepreneurs. All variables are in percentage deviations from their steady state. X-axis is in quarters.
Figure 1.6: Comparing the impulse responses to a one standard deviation increase in TFP across the various models. Solid lines are for the benchmark model. Dashed lines represent Model 1 without HC accumulation. Dotted lines represent Model 2 without Banks and dotted-dashed lines represent Model 3 without the HC channel and without Banks. All variables are in percentage deviations from their steady state. X-axis is in quarters.
Figure 1.7: Comparing the impulse responses to a one standard deviation increase in preferences across the various models. Solid lines are for the benchmark model. Dashed lines represent Model 1 without HC accumulation. Dotted lines represent Model 2 without Banks and dotted-dashed lines represent Model 3 without the HC channel and without Banks. All variables are in percentage deviations from their steady state. X-axis is in quarters.
Figure 1.8: Comparing the impulse responses to a one standard deviation increase in HC transformation efficiency across the various models. Solid lines are for the benchmark model. Dashed lines represent Model 1 without HC accumulation. Dotted lines represent Model 2 without Banks and dotted-dashed lines represent Model 3 without the HC channel and without Banks. All variables are in percentage deviations from their steady state. X-axis is in quarters.
Figure 1.9: Comparing the impulse responses to a one standard deviation increase in LTV of HH-B across the various models. Solid lines are for the benchmark model. Dashed lines represent Model 1 without HC accumulation. Dotted lines represent Model 2 without Banks and dotted-dashed lines represent Model 3 without the HC channel and without Banks. All variables are in percentage deviations from their steady state. X-axis is in quarters.
Figure 1.10: Comparing the impulse responses to a one standard deviation increase in LTV of entrepreneurs across the various models. Solid lines are for the benchmark model. Dashed lines represent Model 1 without HC accumulation. Dotted lines represent Model 2 without Banks and dotted-dashed lines represent Model 3 without the HC channel and without Banks. All variables are in percentage deviations from their steady state. X-axis is in quarters.
Chapter 2: Economic gaps under banking competition

2.1 Introduction

The past financial crisis shows the important role of the banking sector in creating and propagating financial shocks. Researchers investigate this important issue by simulating dynamic stochastic general equilibrium (DSGE) models (Dib, 2010; Gerali et al., 2010; Iacoviello, 2015). Some DSGE models include a banking sector, banks are presented in a perfectly competitive market rather than in a monopolistic market. The World Bank Report states that banking competition is vital for economic efficiency and for the highest social welfare\textsuperscript{25}, while the Bank of England’s views are that the greater the banking competition the better the service banks provide to customers.

This chapter aims to fill this gap by presenting a DSGE model with imperfectly competitive banks as in Gerali et al. (2010). As in the first chapter, we introduce a human capital accumulation channel built on Jones et al. (1993), which allows households to endogenously accumulate human capital through investment and time devoted, and, thus boost productivity and wages. The contribution of this study is to analyse how wage and wealth gaps between heterogeneous workers is affected by imperfect banking competition and a human capital accumulation channel. Existence of imperfect banking competition allows households to choose between banks for higher gains in terms of consumption, housing and investment in human capital. In Chapter 1, we have found that banks mitigate the effects of financial shocks on wage gap. However, does this remain true when an imperfect banking sector is considered instead of perfect competition?

Literature on banking presents models with imperfect competition in financial markets, where banks offer different interest rates on loans and deposits to attract clients. Moreover, banking competition helps investigate economic volatility under imperfections in the banking sector (see Claessen,

In this research we analyse both the demand and supply sides of the credit market. Perfect banking competition adjusts to credit demand, while neglecting the supply side. Market power, on the other hand, is determined by bank capital and setting interest rates on loans and deposits. Bank capital regulations are important as they are the main drivers for continuous loan supply and changing costs of loans (i.e. de Walque et al., 2010). Therefore, banks use their profit to increase their capital and loan supply. Stronger bank capital regulations also support the economy’s resistance to various shocks.

With greater loan supply, borrowers can boost consumption and, thus, output. However, loose constraints negatively affect human capital, since household-borrowers borrow and spend on their consumption rather than on human capital.

Inclusion of a human capital channel is vital, as it allows households to endogenously accumulate their human capital through attending short-term training. This gives impatient households a chance to compete in the labour market, as they are less skilled (see, for example, Goldin and Katz, 2008; Piketty and Saez, 2014). Thus, higher productivity results in higher economic development and explains the wage distribution among households.

OECD (2015) study shows that lower the skills workers have, the greater the income gap, which results in less investment in human capital and less time spent training. Low-skilled workers have to devote most of their time to work instead of spending time in the human capital accumulation process. Therefore, they will always be worse off than their high-skilled peers. Thus, it is important to analyse the roles of both inputs in accumulating human capital, as it is presented in this chapter.

We use quarterly data to analyse both financial shocks and the human capital accumulation, following Dadgar and Trimble (2015). They investigate short-term and long-term credentials, which increase every quarter. Their findings suggest that quarterly skill improvements lead to positive changes in workers’ income. It is vital to study these changes on a smaller basis than yearly, to understand how accumulated skills affect deviations in wage gap between heterogeneous households. This also helps to study the effects of
quarterly basis shocks on human capital.

Therefore, this work aims to answer the following questions: i) can human capital reduce the wage and wealth gaps under various shocks; ii) how does imperfect competition affect households’ preferences for consumption, housing and human capital; iii) how do borrowing and saving decisions change under the presence of monopolistic banking competition; iv) does the finding of the first chapter stay consistent under new banking assumptions.

The findings show that with the presence of an endogenous human capital accumulation channel, wage gap increases under a housing preference and human capital shocks. Interestingly, a human capital transformation shock provides only a temporary decline in wage gap, while it widens in the long-run. Furthermore, wage gap decreases under a total factor productivity (TFP) and a loan-to-value (LTV) ratio shocks. These results are consistent with the outcomes of the first chapter.

Robustness checks show that with the different modelling assumptions the results of the model stay consistent. However, lower loans elasticity of substitutions provide greater steady state deviations and the largest wage gap among all modelling assumptions.

The rest of the chapter is as follows: Section 2.2 revises the literature review in banking, while the literature on human capital accumulation was extensively discussed in Chapter 1. Section 2.3 describes the model set-up of the current chapter. Section 2.4 presents the decentralised competitive equilibrium. The calibration, steady states and stochastic processes are discussed in Section 2.5. Impulse response analysis and welfare effects are given in Section 2.6, whereas the robustness check is presented in Section 2.7. Finally, conclusion is discussed in Section 2.8.

2.2 Related literature

As intermediaries between savers and borrowers, banks propagate and amplify financial shocks to the real economy. The literature dedicated to DSGE models has included banks decades ago, however, the significant realisation of the importance of including banks in models came after the financial cri-
sis. This inclusion allows us to analyse financial frictions and destructions in shock propagation.

One of the earliest papers in this area is by Bernanke et al. (1999), which indicates the vital role of a banking system and its inclusion in DSGE models. They show that agents' decisions of whether to repay loans back or not can create considerable destructions in various banking activities. This also leads to bank defaults on contract obligations to depositors, which can cause further damages, and defaults in financial markets. While these situations are grounds for runs and shortages in loan supply, they also generate instability and insolvency of banks.

Following Bernanke et al. (1999), papers such as Meh and Moran (2004), Curdia and Woodford (2009) and Iacoviello (2015) also empirically prove the importance of including a banking sector in DSGE models. However, these papers investigate the demand side of the credit market, while the supply side of it is neglected. Thus, these papers present banks that operate under a perfect competition.

The models of banking in DSGE models' literature assume a representative bank that is solvent and homogeneous. This bank does not bankrupt while performing in the market with a perfect competition. Unlike these studies, our assumption follows the other side of the banking literature where banks differ in terms of their main two operations (loan and deposit contracts) and setting interest rates. These heterogeneous banks can default on their obligations and bankrupt. Therefore, this chapter focuses on the both sides of a credit market, where financial market can be imperfect in terms of loan supply.

Salop (1979) is among the firsts to show a monopolistic competition within industries, and whose location model is widely used in banking. The Smith (1998) paper demonstrates a heterogeneous loan market whereas this market is not centralised. The economic activity improves when there is a healthy competition between banks. However, an imperfect competition might lead to severe macroeconomic consequences by affecting output and creating larger cyclical fluctuations. Smith's (1998) findings show that macroeconomic costs are mainly associated with banking regulations. Moreover,
these regulations depress the competition among banks while reducing the number of operating banks.

Goodhart et al. (2005) state that bank operations are risky by definition. Questioning the interactions between banks in an interbank market helps to measure the risks these banks can take to be potentially able to disturb an economy. For these reasons, an interbank market and heterogeneous banks shall be taken into consideration in models. Additionally, according to Goodhart et al. (2006) identical or homogenous banks cannot default because they hedge against bankruptcies. This constrains researchers in finding out how shocks are created within a credit supply side.

Therefore, loan market imperfections and financial institutions’ activities can create the environment for financial shocks, as the recent financial crisis has shown. These imperfections in credit markets explain the existence of heavy banking regulation policies by authorities as banks are loan generators in an economy. It also sheds light on central banks’ immediate reaction to any volatility in a financial sector.

de Walque et al. (2010) extend the Goodhart et al. (2006) model by including heterogeneous banks that are regulated by authorities and enjoy liquidity injections. Their paper includes regulations for a bank capital and a supervisory authority in an interbank market for liquidity injection purposes. They provide the evidence for the importance of a bank capital. However, they state that this capital is rather endogenous than exogenous. Thus, banks do not consume their profit fully, instead they partially invest it into their capital in order to increase the volume of issuing loans. Moreover, bank capital can also prevent a bank from any bank failure.

Therefore, bank capital regulations help financial institutions and economies to be more resistant to shocks, although it suppresses output growth in the long-term. Injections, which are used to stabilise and improve the liquidity, can positively influence an economy while stimulating the progress and financial stability. However, the outcomes of these stimulations fade away quickly, and only appear to be successful in the short-run.

The importance of various bank capital levels across different banks (i.e. size) and the competition among them are also presented by the Gerali et al.
Bank capital affects the quality and the cost of loans banks can potentially issue to borrowers. Therefore, the authors look at the supply side of a loan market, claiming that there are also shocks to interest rates of both deposits and loans, and to bank capital. With the presence of a monopolistic competition across banks, the banking sector can significantly attenuate the output under TFP and monetary shocks. This relaxes the effects of these shocks on both consumption and investment.

Gerali et al. (2010) have also found that in the Euro area, costs of loans have been the main driver of financial shocks during the last financial crisis. Under the presence of a financial crunch, bank capital shrinks, which causes further loan supply instability. Without sufficient loan availability, firms and entrepreneurs will struggle to enlarge their production and invest in current and new projects. The decline in the production will affect the employment rate, thus, reducing the working hours and wages of households. This explains why financial shocks negatively affect real variables, such as output, consumption and investment.

Andres and Arce (2012) also demonstrate an imperfect competition in a credit market, whereas there is a perfect competition in a deposit market. They also assume an endogenous spread in interest rates. They stress the idea of a stronger banking competition, where there will be a greater output in the long-run. To attract more borrowers, banks might charge lower interests on loans, which increases the volume of loan supply and demand. Consequently, housing, that impatient households can afford to purchase on a mortgage, will go up. Moreover, loan availability improves aggregate investment by firms, which increases output and consumption. However, real variables are more volatile in the short-run.

Schliephake and Kirstein (2013) research capital regulations in the banking sector. Their findings suggest that with higher capital requirements banking lending will drop. However, their empirical test shows that these requirements lead to higher loan costs, which increases bank profit. This can stabilise the economy by eliminating risky activities by banks, however, a reduction of the competition in the market will lead to declined loan supply. Moreover, Egan et al. (2017) also investigate banking competition, the
supply side of a credit market and regulations in deposit markets. They find that stricter capital and deposit regulations eliminate severe outcomes of financial destructions. On the other hand, harsher capital regulations and requirements can lead to stability in the market, but remove the welfare.

Rubio and Carrasco-Gallego (2017) also show the importance of capital regulations, which can decrease excessive credit growth by raising capital requirements for banks. They discuss the channels that can reduce welfare, and that there is a welfare trade-off that appears between savers and borrowers. Similar to our analyses, borrowers are generally worse off, and savers better off due to their savings. Banks are affected negatively in this scenario as with higher capital requirements they produce less credits that reduce their profit initially. However, these requirements stabilise financial sectors that make borrowers and banks better off eventually.

The purpose of this chapter is to analyse wage gap under the presence of both human capital accumulation channel and imperfect banking competition. Thus, in this current research we combine two streams in one single model. It is known that savers and borrowers are affected by different shocks including financial. But, as we mentioned previously, these studies do not analyse the effects of both imperfect banking sector and human capital accumulation.

There are several reasons that explain the importance of human capital. Firstly, it allows households to accumulate new skills by attending on- and off-job training. Secondly, this gives households a chance to affect their own productivity through endogenously accumulating human capital. Lastly, with greater human capital, households can be more competitive in a labour market, and their wages will be higher.

In our model we introduce two types of households: household-savers and household-borrowers. They differ in preferences in their current and future consumption and savings. These households also differ in terms of wage rates, which comes from the differences in skills and productivity. This widens wage gap between two households. Moreover, with human capital accumulation channel, households can decide how much time and final goods they are willing to invest in their skills in order to improve their human capital and
productivity. Along with this, households need to decide on working hours, expenditure on consumption and housing. Household-savers are more skilled whereas household-borrowers are less skilled. Moreover, household-borrowers are more impatient, which explains why these agents tend to borrow. Human capital in this modelling set-up can also be assumed as human wealth.

2.3 Model outline

Here we introduce a model of a closed economy presented by two different types of households who have different human capital levels, entrepreneurs who produce wholesale goods, capital and final goods producers, and banks who compete in an imperfect market. The initial model is based on the model of the Iacoviello (2015) and Gerali et al. (2010) papers. Moreover, the current model is the extension of the model in Chapter 1. We extend it by replacing a solvent banking sector with an imperfect banking competition. Therefore, each \( j \) bank holds a degree of monopolistic power in terms of deposits and loans in these markets. Furthermore, we include capital and final goods producers in the model. These producers, as well as imperfect competition in the banking sector, are built on the Gerali et al. (2010) paper.

We assume that there are two types of households: household-savers and household-borrowers. They differ in terms of their time preference, which leads to differences in their wealth, wages and human capital. Thus, they have different levels of productivity, which creates a wage gap. A human capital accumulation channel is introduced following the Jones et al. (1993) paper. Entrepreneurs in the model produce wholesale goods by using capital stock and labour provided by households. Both entrepreneurs and household-borrowers borrow from banks to increase their expenses on consumption, housing or capital stock. Household-borrowers also invest in human capital in order to be more competitive in a labour market and earn higher wages. As stated earlier, there are also capital and final goods producers. Producers of capital operate in a perfectly competitive market, while goods retailers operate in a monopolistic market.
2.3.1 Households

There are two types of representative households: household-savers and household-borrowers, similar to patient and impatient households in Gerali et al. (2010). Both households own houses, but differ in wealth. Households also hold human capital, which is higher for savers as they are more patient and productive than household-borrowers. This difference in time preference causes wage gap between these two agents. Both households maximise their utility and are subject to budget constraints. In the utility, both agents are subject to a consumption habit coefficient \( a_H \). Both households are able to affect their productivity and enjoy a human capital transformation shock.

**Household-savers** Each representative household-saver decides between consumption \( C_{H,t} \), housing \( H_{H,t} \), time they devote for work \( N_{H,t} \) and for human capital \( N_{H,t}^{ED} \) at period \( t \). The agent has the following maximisation problem:

\[
\max E_0 \sum_{t=0}^{\infty} \beta_H^t \left[ (1 - a_H) \log \left( C_{H,t} - a_H C_{H,t-1} \right) + j \varepsilon_{H,t} \log H_{H,t} - \right] \frac{\left( N_{H,t} + N_{H,t}^{ED} \right)^{1+\phi}}{1+\phi}
\]

where \( \beta_H^t \) is a household-saver’s discount factor \( (0 < \beta_H^t < 1) \).\(^{26}\) Saver’s consumption is presented in the utility as a current and lagged aggregate consumption. \( j \) shows the share of the housing preference. Labour is presented by a disutility of worked hours with an elasticity of labour supply \( \phi \). \( \varepsilon_{H,t} \) is a housing preference shock.

A household-saver is subject to the following budget constraint:

\[
C_{H,t} + I_{H,t}^{ED} + d_{H,t} + q_{H,t} (H_{H,t} - H_{H,t-1}) \\
= (1 + R_{D,t-1}) d_{H,t-1} + W_{H,t} HC_{H,t-1} N_{H,t} + \frac{J_{at}}{7_H}
\]

At a period \( t \) a saver makes spending on consumption, new housing and investment in human capital \( I_{H,t}^{ED} \). She also holds deposit contracts of \( d_{H,t} \)

\(^{26}\)See Samuelson (1937) and Laibson (1997) for further reference to the households’ time preference.
in retail banks. Housing price equals to \(q_{H,t}\), which is the same across all households. \((1 + R_{D,t-1}) d_{H,t-1}\) are the gross payments the saver is paid by a bank for holding deposit contracts, where \(R_{D,t}\) are the net interest payments. The agent is also paid \(W_{H,t}\) wages for \(N_{H,t}\) worked hours and according to the current human capital \(H_{C,H,t}\) she has already obtained. As an owner of banks and firms, a saver is paid \(\frac{J_{\gamma H}}{\gamma H}\) dividends by firms and banks.

**Human capital accumulation channel**  We assume that a household-saver is able to endogenously accumulate new skills by attending extra training. This allows the agent to increase the returns on investment in human capital as s/he becomes more productive and earns higher wages. The human capital accumulation channel follows the Jones et al. (1993) paper:

\[
H_{C,H,t} = (1 - \delta_{SK}) H_{C,H,t-1} + B_t \left[ (I_{E,H,t}^{ED})^{\theta} \left( H_{C,H,t-1} N_{E,H,t}^{ED} \right)^{(1-\theta)} \right]^\chi \tag{28}
\]

A household-saver invests \(I_{E,H,t}^{ED}\) in human capital whereas \(\theta\) shows the importance of goods input in the human capital accumulation process, while \(\chi\) shows the returns to scale. \(B_t\) is a shock to the production of new skills or a human capital transformation shock. Human capital is also subject to depreciation \(\delta_{SK}\). Thus, household-savers need to continuously improve their human capital to increase their productivity level over time.

**Household-borrowers**  Each representative household-borrower owns housing and borrows to increase consumption, to purchase new housing and to invest in human capital. Thus, household-borrower is willing to increase human capital if returns from it are higher. At period \(t\) a household-borrower maximises her utility, which is similar to the one for savers, by choosing between consumption \(C_{S,t}\), housing \(H_{S,t}\) and time at work \(N_{S,t}\) and in human capital accumulation \(N_{E,D,S,t}^{ED}\):
The objective function is given by:

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t \left[ \left(1 - a_H \right) \log \left(C_{S,t} - a_H C_{S,t-1}\right) + j \varepsilon_H \log \left(H_{S,t}\right) \frac{(N_{S,t} + N_{ED,t})^{1+\phi}}{1+\phi} \right] \tag{29}
\]

where \( \beta^t \) is a household-borrower’s discount factor and \( \beta_H^t > \beta_S^t \) as household-savers are more patient than household-borrowers.

A household-borrower is subject to the budget constraint:

\[
C_{S,t} + I_{S,t}^{ED} + q_{H,t} (H_{S,t} - H_{S,t-1}) + (1 + R_{S,t-1}) l_{S,t-1} = l_{S,t} + W_{S,t} H_{C_{S,t-1} N_{S,t}} \tag{30}
\]

A household-borrower invests \( I_{S,t}^{ED} \) in human capital and borrows \( l_{S,t} \) amount of funds from banks for the cost of net interest \( R_{S,t} \). \( W_{S,t} \) is the wage, which is paid for \( N_{S,t} \) worked hours and \( HC_{S,t} \) human capital the agent has obtained.

As household-borrowers borrow from banks they are subject to the borrowing constraint. Therefore, they cannot borrow more than the value of their collateral, i.e. houses, to cover both the loan amount and the interests on it:

\[
(1 + R_{S,t}) l_{S,t} \leq m_{S,t} q_{H,t+1} H_{S,t} \tag{31}
\]

where \( m_{S,t} \) is an exogenous shock, which affects households’ borrowing abilities or a loan-to-value ratio shock. If this shock is positive, then household-borrowers can borrow more for the same value of their collateral.

**Human capital accumulation channel** Similar to household-savers, household-borrowers accumulate human capital, which improves their wage rates as it affects their productivity, and helps them compete with savers in a labour market:

\[
HC_{S,t} = (1 - \delta_{SK}) HC_{S,t-1} + B_t \left[ \left(I_{S,t}^{ED}\right)^{\theta} \right] \left(HC_{S,t-1} N_{S,t}^{ED}\right)^{(1-\theta)} \right] \tag{32}
\]
2.3.2 Entrepreneurs

Entrepreneurs produce wholesale goods by using their capital stock and labour force provided by households. They maximise the following utility:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t E (1 - a_E) \log (C_{E,t} - a_E C_{E,t-1})$$

(33)

where \( \beta^t E \) is the entrepreneurs’ discount factor and \( C_{E,t} \) is their consumption at period \( t \) with a lagged consumption parameterised by a habit coefficient \( a_E \).

Their production function \( Y^W_t \) is the Cobb-Douglas type, which is given by:

$$Y^W_t = Z_t (u_t K_{E,t-1}^{1-\alpha} [L^u H^v L^1 S^u S^v])^{1-\alpha}$$

(34)

where, for simplicity, \( L_{HD} = HC_{H,t-1} N_{H,t} \) and \( L_{SD} = HC_{S,t-1} N_{S,t} \). \( Z_t \) is a total factor productivity shock. \( u_t \) is a utilisation rate. \( \alpha \) is the share of capital stock in the production function and \( v \) is the share of household-savers.

Entrepreneurs are subject to the budget constraint:

$$C_{E,t} + q K_{E,t} (K_{E,t} - (1 - \delta_K) K_{E,t-1}) + (1 + R_{E,t-1}) l_{E,t-1}$$
$$+ W_{HD} L^u H^v + W_{SD} L^1 S^u S^v + \Lambda (u_t) K_{E,t-1} = \frac{Y^W_t}{X_t} + l_{E,t}$$

(35)

They borrow \( l_{E,t} \) loans from banks and pay \( R_E \) net interest payments on them. Entrepreneurs own capital presented by \( K_{E,t} \), which depreciates at \( \delta_K \) rate. Utilisation rate \( u_t \) is associated with a setting of a real cost \( \Lambda (u_t) = \xi_1 (u_t - 1) + \frac{\xi_2}{2} (u_t - 1)^2 \), where \( \xi_1 > 0 \) and \( \xi_2 > 0 \).\(^{27}\) This means that we allow entrepreneurs to be able to control a utilisation rate of their capital stock. Moreover, with a utilisation cost a depreciation rate of capital stock should take higher value. \( \frac{1}{X_t} \) is a relative competitive price level of wholesale.

\(^{27}\)\( \Lambda (u_t) \) is presented as in the Schmitt-Grohé and Uribe (2006) paper.
goods. Therefore, the $\Lambda(u_t)K_{E,t}$ is a cost of utilising capital stock. To be able to borrow from banks, the fraction of entrepreneurs’ capital should be more than a value of a loan. Therefore, entrepreneurs are restricted by the value of their collateral:

$$(1 + R_{E,t}) l_{E,t} \leq q_{K,t+1}(1 - \delta K)K_{E,t}$$

(36)

2.3.3 Demand for loans and deposits

Demand functions of borrowers’ loans and savers’ deposits are given using the Dixit and Stiglitz (1977) demand functions.

**Demand for household-borrowers and entrepreneurs loans** There are continuum of $j$-s banks in an economy as well as $i$-s number of households and entrepreneurs. The aggregate demand for loans by household-borrowers from a $j$ bank is as follow:

$$l_{S,t}(j) = \overline{l_{S,t}} \left( \frac{R_{S,t}(j)}{R_{S,t}} \right)^{-\varepsilon_{BS,t}}$$

(37)

where $R_{S,t}(j)$ is an interest rate for loans charged by a $j$ bank. $\varepsilon_{BS,t}$ is the elasticity of substitution and $\varepsilon_{BS,t} > 1$. $\overline{l_{S,t}}$ are aggregate loans to household-borrowers.

We take the same approach for the loan demand by a continuum of entrepreneurs:

$$l_{E,t}(j) = \overline{l_{E,t}} \left( \frac{R_{E,t}(j)}{R_{E,t}} \right)^{-\varepsilon_{BE,t}}$$

(38)

where $\varepsilon_{BE,t}$ is the elasticity of substitution and $\varepsilon_{BE,t} > 1$. $\overline{l_{E,t}}$ are aggregate loans to entrepreneurs.

**Demand for household-savers deposits** The aggregate demand for household-saver’s deposits at each $j$ bank is:

$$d_{H,t}(j) = \overline{d_t} \left( \frac{R_{H,t}(j)}{R_{D,t}} \right)^{-\varepsilon_{BD,t}}$$

(39)
where $\varepsilon_{BD,t}$ is the elasticity of substitution and $\varepsilon_{BD,t} < -1$. $d_t$ represents aggregate deposits in an economy.

The elasticities of substitution presented above play a crucial role in determining the interest rate spreads, which are derived in the next section. Here we present the elasticities of substitution for loans and deposits. The values are given to differentiate two different types of financial products: loan and deposit contracts. We assume that these elasticities are stochastic permanent shocks. Their effects on interest rate spreads are separate from those of the monetary policy.

These elasticities show the market power. The higher the market power for setting interest rates the lower the elasticities.

### 2.3.4 Banks

As mentioned earlier, the banking sector of this model closely follows the paper by Gerali et al. (2010). There are three types of branches of each $j$ bank: a wholesale branch and two retail banks. A wholesale bank operates under a perfect competition. It is also in charge of bank capital among all three branches. Thus, it accumulates aggregate deposits from one retail bank and, while combining it with bank capital, issues aggregate loans to another retail bank. Retail banks, one of which issues loans to borrowers while the other one collects deposits from savers, operate under a monopoly.

**Wholesale branch** A wholesale bank accumulates bank capital as follows:

$$K_{B,t} = (1 - \delta_B) K_{B,t-1} + j_{B,t-1}$$  \hspace{1cm} (40)

where $K_{B,t}$ is the total bank capital, $\delta_B$ represents all resources used to manage bank capital and $j_{B,t}$ shows total profits, which are made by three branches.

Wholesale banks maximise the following objective function:
\[ \max_{\{L_t, D_t\}} \sum_{t=0}^{\infty} \beta^t \left[ (1 + R^B_{L,t}) L_{B,t} - L_{B,t+1} + D_{B,t+1} - (1 + R^B_{D,t}) D_{B,t+1} + \right. \\
\left. (K_{B,t+1} - K_{B,t}) - \frac{\kappa_{KB}}{2} \left( \frac{K_{B,t}}{L_{B,t}} - \nu_B \right)^2 \right] \]

where \( \beta^t \) is a banks’ discount factor, \( L_{B,t} \) are wholesale loans, \( D_{B,t} \) are wholesale deposits, \( R^B_{L,t} \) is a net wholesale loan rate, \( R^B_{D,t} \) is a net wholesale deposit rate, which equals to the policy rate \( R_t \) by arbitrage. \( \kappa_{KB} \) is an adjustment parameter and \( \frac{K_{B,t}}{L_{B,t}} \) is a capital-to-asset ratio whereas \( \nu_B \) is its target level.

Wholesale banks are subject to a balance-sheet constraint:

\[ L_{B,t} = D_{B,t} + K_{B,t} \]

To see the spread on interest rates in the wholesale side we find a first order condition, which is given by the following:

\[ R^B_{L,t} - R^B_{D,t} + \kappa_{KB} \left( \frac{K_{B,t}}{L_{B,t}} - \nu_B \right) \left( \frac{K_{B,t}}{L_{B,t}} \right)^2 = 0 \]

thus, taking into account this FOC and \( R^B_{D,t} = R_t \) (as banks have unlimited access to funds from central banks) we get a spread for wholesale banks:

\[ S^W_t = R^B_{L,t} - R_t = -\kappa_{KB} \left( \frac{K_{B,t}}{L_{B,t}} - \nu_B \right) \left( \frac{K_{B,t}}{L_{B,t}} \right)^2 \]

From the given spread condition, it is seen that if wholesale banks decide to increase interest rates to get higher interest payments, then marginal costs have to change too.

**Retail branch** There are two retail banks in the model. Both retail branches operate under monopolistic competition in loans and deposits markets. The first bank issues loans to household-borrowers and entrepreneurs while the second one offers deposit contracts to household-savers.
**Loans branch** Loan branches issue loans to borrowers. They obtain wholesale loans from wholesale banks at a cost of \( R_{L,t}^{B} \), differentiate and reissue these loans to agents. The objective function of a \( j \) retail loan bank is:

\[
\max_{\{R_{S,t}(j), R_{E,t}(j)\}} \sum_{t=0}^{\infty} \beta^{t} E_{0} \left[ R_{S,t}(j) L_{S,t}(j) + R_{E,t}(j) L_{E,t}(j) - R_{L,t}^{B} L_{t}(j) \right]
\]

(45)

The maximisation problem is subject to the loan demand functions of two borrowers (Equation 37 and 38). It is also subject to \( L_{t}(j) = L_{S,t}(j) + L_{E,t}(j) \) by definition.

A first order condition with respect to interest rates on loans to household-borrowers and entrepreneurs are given as follows:

\[
1 - \varepsilon_{SS,t} + \varepsilon_{SS,t} \frac{R_{L,t}^{B}}{R_{SS,t}} = 0
\]

(46)

Log-linearising and rearranging this FOC gives us the condition for the interest rate on loans, which depends on the expected mark-up shocks’ values and expected values of wholesale interest rates in the next periods as well as the marginal costs. When assuming the flexible interest rates we get the following spread between a loan interest rate and policy rate:

\[
S_{t}^{L} = R_{SS,t} - R_{t} = \frac{\varepsilon_{SS,t}}{\varepsilon_{SS,t} - 1} S_{t}^{W} + \frac{1}{\varepsilon_{SS,t} - 1} R_{t}
\]

(47)

where the flexible rate is \( R_{SS,t} = \frac{\varepsilon_{SS,t}}{\varepsilon_{SS,t} - 1} R_{L,t}^{B}. \) The spread of the retail loans shows that it increases with the rise in the policy rate and is also proportional to the wholesale spread. Moreover, the greater the monopolistic power the wider the retail spread for loans.

**Deposits branch** A deposit branch accumulates deposits of savers by offering them deposit contracts, which further transferred to a wholesale branch for the interest rate of \( R_{D,t}. \) A deposit branch maximises the following objective function:
\[
\max_{\{R_{D,t(j)}\}} \sum_{t=0}^{\infty} \beta^t_B \left[ R_t D_t(j) - R_{H,t}(j) D_{H,t}(j) \right] \tag{48}
\]

This function is subject to \( D_t(j) = d_{H,t}(j) \) and to the deposit demand function (Equation 39). We find a first order condition with respect to deposit interest rate as follows:

\[-1 + \varepsilon_{BD,t} - \varepsilon_{BD,t} \frac{R_t}{R_{H,t}} = 0 \tag{49}\]

**Bank profit** The total bank profit consists of the profits made by a wholesale branch and two retail banks, giving the following earnings equation:

\[ j_{B,t} = R_{S,t} l_{S,t} + R_{E,t} l_{E,t} - R_{D,t} d_{H,t} - \frac{\kappa_{KB}}{2} \left( \frac{K_{R,B,t}}{L_{B,t} - \nu_B} \right)^2 K_{B,t} \tag{50}\]

### 2.3.5 Capital producers

There are firms, owned by entrepreneurs, which operate under a perfect competition. These firms purchase capital from their owners at a price of \( Q_{K,t} \). This capital stock has been undepreciated in the last period. They also buy final goods from retail firms, \( I_t \) units of which at \( P_t \) retail price. Therefore, taking into account inputs, these firms’ flow is the following:

\[ \Delta \overline{K}_t = K_t - (1 - \delta_K) K_{t-1} \tag{51}\]

Equation 51 improves the effective capital stock \( \overline{K}_t \), which these firms sell back to entrepreneurs at price of \( Q_{K,t} \), where \( q_{K,t} = \frac{Q_{K,t}}{P_t} \) is its real price. \( K_t \) is an aggregate capital stock. We assume that in steady state capital price is unity.

Capital producers decide on the level of \( \overline{K}_t \) and \( I_t \) to maximise the function:

\[ E_0 \sum_{t=0}^{\infty} \beta^t_K [q_{K,t} \Delta \overline{K}_t - I_t] \tag{52}\]
subject to

\[ K_t = K_{t-1} + I_t \]  \hspace{1cm} (53)

### 2.3.6 Retailers

Following Bernanke et al. (1999) retailers of final goods operate in a monopolistic market. The prices for the final goods are subject to price stickiness following Calvo (1983). Therefore, retailers choose price \( \{P_t(j)\} \) to maximise the following objective function:

\[ E_0 \sum_{t=0}^{\infty} \beta_R^t \left[ P_t(j)Y_t(j) - P_t^WY_t(j) \right] \]  \hspace{1cm} (54)

where \( \beta_R^t \) is the retailers’ discount factor.\(^{28}\)

The objective function is subject to the demand

\[ Y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\varepsilon_{Y,t}} Y_t \]  \hspace{1cm} (55)

where \( \varepsilon_{Y,t} \) is a price elasticity of demand.

### 2.3.7 Aggregate resource constraint

As it is assumed all goods produced in the economy should be consumed, thus, consumption, investment in human capital, bank capital and adjustment costs cannot exceed current production. This is shown in the standard aggregate resource constraint:

\[ Y_t = C_t + q_{K,t}(K_t - (1 - \delta_K)K_{t-1}) + \Lambda(u_t)K_{E,t-1} + I_t^{ED} + \delta_BK_{B,t-1} \]  \hspace{1cm} (56)

where the total consumption:

\(^{28}\) As households-savers own banks and retail firms, there is a \( \beta_H^t \) discount factor to assess savers’ future profit. Therefore, a discount factor is the same across these agents.

As entrepreneurs are owners of the capital producers, the latter will have the same discount factor as entrepreneurs.
\[ C_t = C_{H,t} + C_{S,t} + C_{E,t} \]  \hspace{1cm} (57)

and the total investment in human capital:

\[ I_{t}^{ED} = I_{H,t}^{ED} + I_{S,t}^{ED} \]  \hspace{1cm} (58)

For simplicity the housing in the model is normalised to unity as in Iacoviello (2015):

\[ H_t = H_{S,t} + H_{H,t} = 1 \]  \hspace{1cm} (59)

2.3.8 Shocks

There are four shocks in the model: a TFP shock, a shock to household-borrowers’ borrowing constraint, a shock to human capital transformation and a shock to housing preference. These exogenous shocks follow AR(I) process:

\[
\log \varepsilon_{H,t} = (1 - \rho_{\varepsilon_H})\varepsilon_{H}^{ss} + \rho_{\varepsilon_H} \log \varepsilon_{H,t-1} + u_H
\]  \hspace{1cm} (60)

\[
\log Z_t = (1 - \rho_Z)Z^{ss} + \rho_Z \log Z_{t-1} + u_Z
\]  \hspace{1cm} (61)

\[
\log B_t = (1 - \rho_B)B^{ss} + \rho_B \log B_{t-1} + u_B
\]  \hspace{1cm} (62)

\[
\log m_{S,t} = (1 - \rho_{MS})m_S^{ss} + \rho_{MS} \log m_{S,t-1} + u_{MS}
\]  \hspace{1cm} (63)

There are also shocks in the banking side of the model, which do not follow an AR(I) process. We present these shocks as permanent shocks, which are explained in the following sections.
2.4 Decentralised competitive equilibrium

The decentralised competitive equilibrium is presented by a series of allocations \( \{C_{H,t}, C_{S,t}, C_{E,t}, H_{H,t}, H_{S,t}, K_{B,t}, N_{H,t}, N_{S,t}, N^E_{H,t}, N^E_{S,t}, I^E_{H,t}, I^E_{S,t}, HC_{H,t}, HC_{S,t}, d_{H,t}, I_{E,t}, Y_t^{W}, Y_t\}_{t=0}^{\infty} \) and prices \( \{W_{H,t}, W_{S,t}, R_{D,t}, R_{S,t}, R_{E,t}, R_{V,t}, q_{H,t}, q_{K,t}\}_{t=0}^{\infty} \) so that agents will maximise their profits.

The DCE equations are given in Appendix B.

2.5 Calibration and steady state

This section provides the relevant calibration of the model. For the calibration we have used the U.S. quarterly data for the period of 1965Q1-2016Q4\(^{29}\) to match the steady states with the great ratios.

On the households’ side, we set a consumption habit coefficient \( a_H \) to be 0.867 following Gerali et al. (2010). Housing preference share \( j \) is set to equal 0.1 to match household-borrowers’ loans-to-output ratio of 0.68.\(^{30}\) The elasticity of labour supply \( \phi \) in the labour disutility equals to 1, following the Gerali et al. (2010) paper. Discount factors of household-savers \( \beta_H \) and household-borrowers \( \beta_S \) are set to 0.9943 and 0.94 respectively, as in Iacoviello (2015).

There is a share of the firms’ and banks’ profit \( \gamma_H \), that household-savers receive as dividends. This parameter \( \gamma_H \) equals to 1 as in the Gerali et al. (2010) paper.

On both households’ human capital accumulation side we have three parameters: human capital depreciation \( \delta_{SK} \), goods’ inputs importance \( \theta \) in accumulating human capital, and returns to scale \( \chi \). \( \theta \) is set to equal 0.7 to match the investment in human capital over output ratio of 2.11 that is close to the data of the private investment in skills-to-output.\(^{31}\) \( \chi \) is set to be 0.35

\(^{29}\)U.S. quarterly data for 1965Q1-2016Q4 has been hp-filtered and log-transformed.
\(^{30}\)The quarterly U.S. data for the output for the period of 1965Q1-2016Q4 is obtained from the NIPA tables. The quarterly U.S. data for the households’ loans for the 1965Q1-2016Q4 is taken from the Flow of Funds Accounts, as it is given in Iacoviello (2015).
to match the consumption-to-output ratio of 0.66. Following Jones et al. (1993), we set human capital depreciation rate $\delta_{SK}$ at 0.1 to match the skill premium of 1.79.

On the entrepreneurs’ side, we start with the production function. The share of entrepreneurs’ capital stock $\alpha$ in the production is set to be 0.33. Household-savers’ labour share $\nu$ in the production process equals to 0.68. These two parameters are set in order to match wages-to-output ratio to be equal to 0.45 and the entrepreneurs’ loans-to-output ratio of 0.38.

The depreciation rate of capital stock $\delta_K$ equals to 0.03. The reason to have a higher value than the standard one is stated by Schmitt-Grohé and Uribe (2006). Higher $\delta_K$ is associated with the inclusion of the utilisation cost in the production function. Capital depreciation rate ensures the investment-to-output ratio of 0.16.

Parameters of capacity utilisation costs $\xi_1$ and $\xi_2$ are given following Gerali et al. (2010).

On the banking sector side, we have calibrated the parameter for the cost of managing bank capital $\delta_B$ to be 0.1659 for the capital-to-asset ratio $\nu_B$ to be 0.09, similar to Gerali et al. (2010). Adjustment parameter $\kappa_{KB}$ in the wholesale banks’ utility equals to 5. This parameter gives the deposits-to-output ratio of 0.40.

The steady state values of the stochastic processes are normalised to unity $\varepsilon_H = Z = B = m_S = 1$.

The calibration of the parameters is presented in Table 2.1. In Table 2.2 we compare the averages of the U.S. quarterly data for the period of 1965Q1-2016Q4 with the steady states obtained from the model.

---

32 Households consumption is a quarterly U.S. data for 1965Q1-2016Q4, which is taken from the NIPA tables.
33 The quarterly U.S. data for households’ wages and entrepreneurs loans for the period of 1965Q1-2016Q4 is obtained from the NIPA tables and Flow of Funds Accounts, respectively.
34 The quarterly data for the private fixed non-residential investment for the period of 1999Q1-2016Q4 is obtained from the NIPA tables.
35 Following Gerali et al. (2010) $\xi_1 = (1 - \delta_K) - \frac{m_S}{\beta_E} (1 - \delta_K) \left( \frac{1}{1 + \kappa_{KB}} - \beta_E \right) + \frac{1}{\beta_E}$ and $\xi_2 = 0.1 \xi_1$.
36 Households’ deposits are a quarterly U.S. data for 1965Q1-2016Q4, which is obtained from the FRED.
Table 2.1: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_B$</td>
<td>Banks discount factor</td>
<td>0.9925</td>
<td>Iacoviello (2015)</td>
</tr>
<tr>
<td>$\beta_E$</td>
<td>Entrepreneurs discount factor</td>
<td>0.94</td>
<td>Iacoviello (2015)</td>
</tr>
<tr>
<td>$\beta_H$</td>
<td>HH-S discount factor</td>
<td>0.9925</td>
<td>Iacoviello (2015)</td>
</tr>
<tr>
<td>$\beta_S$</td>
<td>HH-B discount factor</td>
<td>0.94</td>
<td>Iacoviello (2015)</td>
</tr>
<tr>
<td>$\beta_K$</td>
<td>Capital producers discount factor</td>
<td>0.94</td>
<td>Iacoviello (2015)</td>
</tr>
<tr>
<td>$\beta_R$</td>
<td>Retailers discount factor</td>
<td>0.9925</td>
<td>Iacoviello (2015)</td>
</tr>
<tr>
<td>$j$</td>
<td>Housing preference share</td>
<td>0.1</td>
<td>Data</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Elasticity of labour supply</td>
<td>1</td>
<td>Gerali et al. (2010)</td>
</tr>
<tr>
<td>$a_H$</td>
<td>Consumption habit</td>
<td>0.867</td>
<td>Gerali et al. (2010)</td>
</tr>
<tr>
<td>$\gamma_H$</td>
<td>Firms’ and banks’ profit share</td>
<td>1</td>
<td>Gerali et al. (2010)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Share of capital stock in prod.</td>
<td>0.3</td>
<td>Data</td>
</tr>
<tr>
<td>$\psi$</td>
<td>HH-S labour share in prod.</td>
<td>0.647</td>
<td>Data</td>
</tr>
<tr>
<td>$\delta_K$</td>
<td>Capital depreciation</td>
<td>0.035</td>
<td>Schmitt-Grohé and Uribe (2006)</td>
</tr>
<tr>
<td>$\xi_1$</td>
<td>Capacity utilisation costs</td>
<td>0.0912</td>
<td>Gerali et al. (2010)</td>
</tr>
<tr>
<td>$\xi_2$</td>
<td>Capacity utilisation costs</td>
<td>0.0091</td>
<td>Gerali et al. (2010)</td>
</tr>
<tr>
<td>$\delta_B$</td>
<td>Managing a bank capital cost</td>
<td>0.1049</td>
<td>Gerali et al. (2010)</td>
</tr>
<tr>
<td>$\kappa_{KB}$</td>
<td>Adjustment parameter</td>
<td>5</td>
<td>Data</td>
</tr>
<tr>
<td>$\nu_B$</td>
<td>Target of capital-to-asset ratio</td>
<td>0.09</td>
<td>Data</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Goods share in the HCA</td>
<td>0.7</td>
<td>Data</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Returns to scale in HCA</td>
<td>0.35</td>
<td>Data</td>
</tr>
<tr>
<td>$\delta_{SK}$</td>
<td>HC depreciation rate</td>
<td>0.1</td>
<td>Jones et al. (1993)</td>
</tr>
</tbody>
</table>

2.5.1 Stochastic processes of the model

We calibrate the parameters of autocorrelations and standard deviations of the AR(I) processes to match autocorrelations and standard deviations of the data moments. Table 2.3 shows the values for these parameters, whereas Table 2.4 shows the moments for the data and the model.

We set the autocorrelation $\rho_Z$ and standard deviation $\sigma_Z$ of the TFP shock 0.86 and 0.004, respectively, to find the standard deviation and autocorrelation of output in the model to be 0.014 and 0.76, respectively.

Autocorrelation $\rho_{ms}$ and standard deviation $\sigma_{ms}$ parameters for the LTV ratio shock of household-borrowers are 0.9 and 0.0003, respectively. The parameters of this shock ensure the standard deviation and autocorrelation

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of household-borrowers’ loans-to-output to be closer to the data moments.

Autocorrelation $\rho_B$ and standard deviation $\sigma_B$ for the human capital transformation shock equal to 0.65 and 0.008, respectively. The parameters of this shock give the autocorrelation and standard deviation of the wages-to-output ratio similar to the data moments.

We set autocorrelation $\rho_{\varepsilon_H}$ and standard deviation $\sigma_{\varepsilon_H}$ for housing preference shock to be 0.8 and 0.004, respectively. The parameters are set for the standard deviation and autocorrelation of consumption-to-output ratio to be a closer match of the data moments.

### Table 2.3: Stochastic processes

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_Z$</td>
<td>AR(1) coef. of TFP</td>
<td>0.86</td>
<td>Data</td>
</tr>
<tr>
<td>$\sigma_Z$</td>
<td>Std. dev. of TFP</td>
<td>0.004</td>
<td>Data</td>
</tr>
<tr>
<td>$\rho_{mS}$</td>
<td>AR(1) coef. of HH borr. LTV ratio</td>
<td>0.9</td>
<td>Data</td>
</tr>
<tr>
<td>$\sigma_{mS}$</td>
<td>Std. dev. of HH borr. LTV ratio</td>
<td>0.0003</td>
<td>Data</td>
</tr>
<tr>
<td>$\rho_B$</td>
<td>AR(1) coef. of HC transformation</td>
<td>0.65</td>
<td>Data</td>
</tr>
<tr>
<td>$\sigma_B$</td>
<td>Std. dev. of HC transformation</td>
<td>0.008</td>
<td>Data</td>
</tr>
<tr>
<td>$\rho_{\varepsilon_H}$</td>
<td>AR(1) coef. of housing preference</td>
<td>0.8</td>
<td>Data</td>
</tr>
<tr>
<td>$\sigma_{\varepsilon_H}$</td>
<td>Std. dev. of housing preference</td>
<td>0.004</td>
<td>Data</td>
</tr>
</tbody>
</table>

### 2.6 Impulse response analysis and welfare effect

This section provides the impulse responses of the model following the four shocks: a TFP shock, a shock to households’ housing preference, a shock to
Table 2.4: Business cycle statistics of the key ratios

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_i$</td>
<td>$\tilde{\sigma}(X_i)$</td>
<td>$\hat{\sigma}(X_i)/\tilde{\sigma}(Y)$</td>
</tr>
<tr>
<td>$C$</td>
<td>0.01</td>
<td>0.83</td>
</tr>
<tr>
<td>$L_S$</td>
<td>0.06</td>
<td>4.78</td>
</tr>
<tr>
<td>$W$</td>
<td>0.01</td>
<td>0.77</td>
</tr>
</tbody>
</table>

Household-borrowers LTV ratio and a human capital transformation shock. All shocks are positive and temporary. The graphs present the responses of the main variables under the presence of a banking competition and human capital accumulation channel to a one standard deviation shock (see Appendix B for Figures 2.1-2.4).

Figures 2.1-2.4 about here

2.6.1 TFP shock

When a TFP shock hits the economy, production inputs become more effective (see Figure 2.1). Following the shock, increasing output requires more labour and capital inputs. This leads to a raise in investment and households’ wages, allowing households to boost their consumption and investment in human capital. As a result wages will go up further. As a result, households spend more time in human capital accumulation than at work. This growth is also supported by credit availability, leading to enlarged demand for savers’ deposits. Therefore, household-borrowers can purchase more housing and invest more in human capital. Consequently, the growth of household-borrowers’ wages is greater than it is for household-savers, resulting in tighter wage gap in the long-run.

2.6.2 Housing preference shock

A housing preference shock leads to an increased housing demand, predominantly by household-borrowers (see Figure 2.2). As per substitution effect, this lowers their consumption and investment in human capital. Higher demand for real estate results in greater house prices. To satisfy this de-
mand, household-borrowers can either supply more labour or borrow more from banks. Larger labour forces generate higher wages in the short-run. Household-borrowers also start demanding more loans, increasing the demand for deposits. Savers invest less in long-term human capital, which decreases their earnings in the long-run. Therefore, this shock creates a larger wage gap in wage and wealth gaps in the long-run.

2.6.3 Household-borrowers loan-to-value ratio shock

Under a borrowing constraint shock, household-borrowers are able to borrow more loans (see Figure 2.3). With these loans they bring up their consumption, housing and human capital. Raised demand for loans leads to a greater credit supply, which increases household-savers’ deposits and bank capital.

This shock negatively affects entrepreneurs who are not able to get loans affecting investment and capital stock in the short-run.

Household-borrowers increase their human capital expenditures, which rapidly goes up, allowing them to earn greater wage. As savers do not invest in human capital, wage gap decreases.

2.6.4 Human capital transformation shock

With the presence of this shock, human capital of both households immediately improves in the long-run (see Figure 2.4). Households realise that the returns on human capital are higher and will invest more, while spending less on housing and savings. They spend more time in human capital as a substitute to working hours. As human capital goes up, they become more productive, leading to higher wages and increased overall consumption and output.

On the other hand banks face deposit runs. To keep their liquidity and solvency, banks increase their capital. Even though the demand for loans initially drops, it increases in the long-run. This will require more deposits, which increase as a result of higher interest payments.

Although household-borrowers accumulate higher levels of human capital, wage gap rises in the long-run, as household-savers always invest more in their
2.6.5 Shocks under the presence of banking competition

Here we present the set of shocks, which are simulated under the presence of imperfect competition in the banking sector. As we have mentioned in the previous sections, these shocks are permanent and do not follow AR(I) process. We increase the value of a banks’ adjustment parameter $\kappa_{KB}$ from 5 to 8. We change the steady state value of the deposit elasticity of substitution $\varepsilon_{BD}^{SS}$ from $-2.2$ to $-5$, and for the steady state values of the two borrowers loans’ elasticity of substitutions from 3.5 to 2.5.

Table 2.5: Steady state deviations of various models

<table>
<thead>
<tr>
<th>Model - Base</th>
<th>$\kappa_{KB}= 8$</th>
<th>$\varepsilon_{BD}^{SS}=-5$</th>
<th>$\varepsilon_{BE}^{SS} = \varepsilon_{BS}^{SS}= 2.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{Y}$</td>
<td>0.77</td>
<td>0.76</td>
<td>0.77</td>
</tr>
<tr>
<td>$\frac{L_F}{Y}$</td>
<td>0.42</td>
<td>0.42</td>
<td>0.42</td>
</tr>
<tr>
<td>$\frac{L_S}{Y}$</td>
<td>0.46</td>
<td>0.46</td>
<td>0.50</td>
</tr>
<tr>
<td>$\frac{K}{Y}$</td>
<td>0.62</td>
<td>0.62</td>
<td>0.62</td>
</tr>
<tr>
<td>$\frac{F_BD}{Y}$</td>
<td>0.12</td>
<td>0.12</td>
<td>0.12</td>
</tr>
<tr>
<td>$SP$</td>
<td>1.60</td>
<td>1.60</td>
<td>1.60</td>
</tr>
<tr>
<td>$I$</td>
<td>0.09</td>
<td>0.09</td>
<td>0.09</td>
</tr>
</tbody>
</table>

From Table 2.5 we see that when banks’ adjustment parameter and deposit elasticity of substitution permanently changes, the steady states of the main ratios stay constant. However, the ratio of wages-to-output decreases, which means, holding output constant, households wages decline under these shocks. Interestingly, when two borrowers loans’ elasticity of substitutions become lower, wages-to-output ratio stays as low as under previous two shocks. However, consumption-to-output ratio, investment-over-output ratio and entrepreneurs’ loans-over-output ratio increase. On the other hand, the ratio of household-borrowers’ loans-to-output ratio decreases. This shock makes entrepreneurs better off, while the two households are worse off. It is worth mentioning that investment-in-training-to-output
ratio does not change under these three shocks, which means households do not invest more in their skills, which can also explain the drop in wages.

2.6.6 Welfare analysis

This section describes the welfare analysis of gains/losses in terms of the consumption equivalence under the shocks described in the previous section. We follow the same approach as given in Chapter 1. We assume that each agent’s welfare is presented by following:

\[ W_0^i \left( \lambda_i, C_0^i, H_0^i, N_0^i, N_{ED,0}^i \right) = W_1^i \left( C_1^i, H_1^i, N_1^i, N_{ED,1}^i \right) \]  \hspace{1cm} (64)

where \( W_0^i \) is the welfare before the shock and \( W_1^i \) after the shock, and \( \lambda_i \) is the gain/loss in terms of the consumption. Substituting this with the logarithmic utility function given in the model, we get the following expression for the gain/loss in terms of the consumption:

\[ \lambda_i = \left[ \exp \left( (1 - \beta_i) (1 - a_i) \left( W_1^i - W_0^i \right) \right) - 1 \right] \times 100 \]  \hspace{1cm} (66)

Table 2.6 presents the consumption gain/loss for each agent under the four shocks simulated in the model. The values are expressed in percentage change. The positive values demonstrate an agent’s gain, while negative values reveal the agent’s consumption equivalent loss. The table also includes the values for the skill premium, the ratio of savers’ wages over the borrowers’ wages. The discounted percent deviation of the skill premium sheds the light on the households’ wage gap and the rise/decline of the wage gap under each shock.

Starting with the TFP shock we can see that all agents are better off under this shock. The wage gap decreases too. This is because more loans are available to household-borrowers to invest in their human capital. As
Table 2.6: Welfare and skill premium effects

<table>
<thead>
<tr>
<th></th>
<th>HH-savers</th>
<th>HH-borrowers</th>
<th>Entrepr.</th>
<th>$W_H/W_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP</td>
<td>0.0069</td>
<td>1.4098</td>
<td>0.1632</td>
<td>-0.0599</td>
</tr>
<tr>
<td>B</td>
<td>0.0157</td>
<td>0.0027</td>
<td>-0.0316</td>
<td>0.0068</td>
</tr>
<tr>
<td>$m_S$</td>
<td>0.0000</td>
<td>0.0069</td>
<td>-0.0010</td>
<td>-0.0008</td>
</tr>
<tr>
<td>$\varepsilon_H$</td>
<td>-0.0019</td>
<td>-0.1299</td>
<td>0.0036</td>
<td>0.5298</td>
</tr>
</tbody>
</table>

Households are more productive, the production process is more efficient as well.

The consumption responses are positive for both households, indicating the gain under human capital transformation shock. On the other hand, the wage gap widens (0.0068%) as a result of savers’ transactions of deposits to human capital, leading to a decrease in loan supply.

The household-borrowers loan-to-value ratio shock makes both households better off in terms of consumption, while entrepreneurs are worse off. As a response to this shock the wage gap shrinks at 0.0008% as it allows household-borrowers’ invest more in human capital and housing.

Under the housing preference shock, both households try to purchase more houses, which negatively affects their wealth in terms of consumption. Entrepreneurs gain in this case, as the labour supply rises, allowing entrepreneurs to increase production. Due to higher labour supply, which is provided largely by household-borrowers, the wage gap increases at 0.529%.

### 2.7 Robustness check

#### 2.7.1 Impulse responses and welfare analysis of the various models

In this section we present impulse responses to three shocks in banking side as a comparison to a base model, which are given in Figures 2.5-2.8 (see Appendix B). The solid lines in graphs present the base model. Dashed lines are given for the model with $\varepsilon_{SS}^{BD} = -5$, dotted lines are for $\varepsilon_{SS}^{BE} = \varepsilon_{SS}^{BS} = 2.5$ and finally dotted-dashed lines are for $\kappa_{KB} = 8$.

Table 2.7 presents the comparison of the results of the base model and the models with the changed parameters under the four simulated shocks.
Table 2.7: Robustness checks for the welfare and skill premium effects

<table>
<thead>
<tr>
<th>Response</th>
<th>HH-savers</th>
<th>HH-borrowers</th>
<th>Entrepr.</th>
<th>( W_H / W_S )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>TFP</strong></td>
<td>0.0069</td>
<td>1.4098</td>
<td>0.1632</td>
<td>-0.0599</td>
</tr>
<tr>
<td>( \varepsilon_H )</td>
<td>-0.0019</td>
<td>-0.1299</td>
<td>0.0036</td>
<td>0.5298</td>
</tr>
<tr>
<td>( B )</td>
<td>0.0157</td>
<td>0.0027</td>
<td>-0.0316</td>
<td>0.0068</td>
</tr>
<tr>
<td>( m_S )</td>
<td>0.0000</td>
<td>0.0069</td>
<td>-0.0010</td>
<td>-0.0008</td>
</tr>
<tr>
<td><strong>TFP</strong></td>
<td>0.0077</td>
<td>1.3358</td>
<td>0.1529</td>
<td>-0.0613</td>
</tr>
<tr>
<td>( \varepsilon_H )</td>
<td>-0.0019</td>
<td>-0.1322</td>
<td>0.0032</td>
<td>0.5121</td>
</tr>
<tr>
<td>( B )</td>
<td>0.0158</td>
<td>0.0254</td>
<td>-0.0279</td>
<td>0.0067</td>
</tr>
<tr>
<td>( m_S )</td>
<td>0.0000</td>
<td>0.0072</td>
<td>-0.0009</td>
<td>-0.0008</td>
</tr>
<tr>
<td><strong>TFP</strong></td>
<td>0.0034</td>
<td>1.4482</td>
<td>0.1591</td>
<td>-0.0569</td>
</tr>
<tr>
<td>( \varepsilon_H )</td>
<td>-0.0021</td>
<td>-0.1250</td>
<td>0.0034</td>
<td>0.6211</td>
</tr>
<tr>
<td>( B )</td>
<td>0.0172</td>
<td>-0.0420</td>
<td>-0.0358</td>
<td>0.0053</td>
</tr>
<tr>
<td>( m_S )</td>
<td>0.0000</td>
<td>0.0089</td>
<td>-0.0009</td>
<td>-0.0008</td>
</tr>
<tr>
<td><strong>TFP</strong></td>
<td>0.0092</td>
<td>1.2131</td>
<td>0.1451</td>
<td>-0.0637</td>
</tr>
<tr>
<td>( \varepsilon_H )</td>
<td>-0.0017</td>
<td>-0.1454</td>
<td>0.0030</td>
<td>0.4237</td>
</tr>
<tr>
<td>( B )</td>
<td>0.0126</td>
<td>0.1518</td>
<td>-0.0148</td>
<td>0.0078</td>
</tr>
<tr>
<td>( m_S )</td>
<td>-0.0000</td>
<td>0.0060</td>
<td>-0.0008</td>
<td>-0.0007</td>
</tr>
</tbody>
</table>

Generally, we can see that household-savers are always better off, just like in the base model. Moreover, the different modelling assumptions present that household-borrowers are always worse off than their savers counterparts. We see a similar pattern in entrepreneurs as they are also worse off. Household-borrowers will gain mostly under lower loan elasticity of substitution, while they will lose more under the lower deposits elasticity of substitution. Furthermore, with the latter, and under human capital transformation shock, household-borrowers will lose in terms of consumption while they gain under the same shock with other modelling assumptions.

The wage gap will always decrease under the TFP and LTV shocks with three different modelling set-ups. The values are similar across the different modelling assumptions. However, the wage gap widens under housing preference and human capital transformation shocks. Overall, the higher capital adjustment parameter produces lower wage gap across all four shocks.
2.8 Conclusion

In this chapter we present a model with heterogenous banks that operate under imperfect competition. This model also includes a human capital accumulation channel, which allows heterogenous households to endogenously accumulate human capital and be more productive and competitive in a labour market. In this research we aim to see how the wealth and wage gaps changes under the current model set-up and how borrowers and savers change their decisions under the presence of an imperfect banking competition.

The calibration part of this model is based on a U.S. quarterly data for the 1965Q1-2016Q4. We match moments produced by the model with the steady states and the moments of the given dataset.

We find that under the TFP shock households are able to accumulate human capital, which reduces the wage gap in the short-run. More importantly this wage gap also decreases in the long-run. Under the housing preference shock. The output increases in the short-run, but this shock causes larger wage gap between the two households’ wages in the long-run.

Under households LTV ratio shock wage gap decreases in the short-run only, while it widens in the long-run.

Human capital transformation shock in this model shows that both households are better off and their wages improve in the long-run. However, it results in a higher wage gap. It is because of household-savers’ higher skills and, even though household-borrowers invest in human capital, they will not catch up with the skills of savers.

Permanent change of the parameters on the supply side of the credit market show that with the imperfect banking competition, wage and wealth gaps between households increases in the long-run under housing preference and human capital transformation shocks. These modelling assumptions also show that household-borrowers are generally worse off than the savers. Furthermore, under the capital adjustment cost, wage gap reduces across all four shocks.

Moreover, the robustness check shows that the results of the model stay robust. However, with higher capital managing costs, household-borrowers
gain less than other agents, but it also leads to lower deviations of wage gap compared to other modelling assumptions.
Appendix B

B Chapter 2

Here we present the decentralised competitive equilibrium for each agent in Chapter 2 model.

B.1 Households

B.1.1 Household-savers

\[
\lambda^H_t = \frac{(1 - a_H)}{C_{H,t} - a_HC_{H,t-1}} \quad (B1)
\]

\[
\lambda^H_t = E_t\beta_H \lambda^H_{t+1} (1 + R_{D,t}) \quad (B2)
\]

\[
j \frac{\varepsilon_{H,t}}{H_{H,t}} - \lambda^H_t q_{H,t} + E_t\beta_H \lambda^H_{t+1} q_{H,t+1} = 0 \quad (B3)
\]

\[
(N_{H,t} + N_{H,t}^{ED})^\phi = \lambda^H_t W_{H,t} HC_{H,t-1} \quad (B4)
\]

\[
SK_{H,t} - E_t\beta_H \lambda^H_{t+1} W_{H,t+1} N_{H,t+1} - E_t\beta_H SK_{H,t+1}(1 - \delta_{SK}) - E_t\beta_H SK_{H,t+1} \frac{B_{t+1}(1-\theta)\chi}{HC_{H,t}} \left[ (HC_{H,t}N_{H,t+1}^{ED})^{(1-\theta)} (I_{H,t+1}^{ED})^\theta \right]^\chi = 0 \quad (B5)
\]

\[
(N_{H,t} + N_{H,t}^{ED})^\phi = SK_{H,t} \frac{B_t(1-\theta)\chi}{N_{H,t}^{ED}} \left[ (HC_{H,t-1}N_{H,t}^{ED})^{(1-\theta)} (I_{H,t}^{ED})^\theta \right]^\chi \quad (B6)
\]

\[
\lambda^H_t = SK_{H,t} \frac{B_t \theta \chi}{I_{H,t}^{ED}} \left[ (I_{H,t}^{ED})^\theta (HC_{H,t-1}N_{H,t}^{ED})^{(1-\theta)} \right]^\chi \quad (B7)
\]

\[
HC_{H,t} = (1 - \delta_{SK}) HC_{H,t-1} + B_t \left[ (I_{H,t}^{ED})^\theta (HC_{H,t-1}N_{H,t}^{ED})^{(1-\theta)} \right]^\chi \quad (B8)
\]
\[ C_{H,t} + I_{H,t}^{ED} + d_{H,t} + q_{H,t} (H_{H,t} - H_{H,t-1}) = (1 + R_{D,t-1}) d_{H,t-1} + W_{H,t} HC_{H,t-1} N_{H,t} + \frac{J_n}{\gamma H} \]  

(B9)

\section*{B.1.2 Household-borrowers}

\[ \lambda_t^S = \frac{(1 - a_S)}{C_{S,t} - a_S C_{S,t-1}} \]  

(B10)

\[ j \frac{q_{H,t}}{H_{S,t}} - \lambda_t^S q_{H,t} + E_t^S \beta_t \lambda_t^{S+1} q_{H,t+1} + \mu_t^S m_{S,t} q_{H,t+1} = 0 \]  

(B11)

\[ \lambda_t^S = \mu_t^S (1 + R_{S,t}) + E_t^S \beta_t \lambda_t^{S+1} (1 + R_{S,t}) \]  

(B12)

\[ (N_{S,t} + N_{S,t}^{ED})^\phi = \lambda_t^S W_{S,t} H C_{S,t-1} \]  

(B13)

\[ S K_{S,t} - E_t^S \beta_t \lambda_{t+1}^S W_{S,t+1} N_{S,t+1} - E_t^S \beta_t S K_{S,t+1} (1 - \delta SK) - E_t^S \beta_t S K_{S,t+1} B_{t+1} (1-\theta) \chi \left[ (I_{S,t+1}^{ED})^\theta (HC_{S,t} N_{S,t}^{ED})^{(1-\theta)} \right]^\chi = 0 \]  

(B14)

\[ (N_{S,t} + N_{S,t}^{ED})^\phi = SK_{S,t} \frac{B_t (1 - \theta) \chi}{N_{S,t}^{ED}} \left[ (I_{S,t}^{ED})^\theta (HC_{S,t-1} N_{S,t}^{ED})^{(1-\theta)} \right]^\chi \]  

(B15)

\[ \lambda_t^S = SK_{S,t} \frac{B_t \theta \chi}{I_{S,t}^{ED}} \left[ (I_{S,t}^{ED})^\theta (HC_{S,t-1} N_{S,t}^{ED})^{(1-\theta)} \right]^\chi \]  

(B16)

\[ (1 + R_{S,t}) l_{S,t} = m_{S,t} q_{H,t+1} H_{S,t} \]  

(B17)

\[ HC_{S,t} = (1 - \delta SK) HC_{S,t-1} + B_t \left[ (I_{S,t}^{ED})^\theta (HC_{S,t-1} N_{S,t}^{ED})^{(1-\theta)} \right]^\chi \]  

(B18)

\[ C_{S,t} + I_{S,t}^{ED} + q_{H,t} (H_{S,t} - H_{S,t-1}) + (1 + R_{S,t-1}) l_{S,t-1} = l_{S,t} + W_{S,t} HC_{S,t-1} N_{S,t} \]  

(B19)
B.2 Entrepreneurs

\[ \lambda_t^E = \frac{(1 - a_E)}{C_{E,t} - a_E C_{E,t-1}} \]  
(B20)

\[ \lambda_t^E = +\mu_t^E (1 + R_{E,t}) + E_t \beta E \lambda_{t+1}^E (1 + R_{E,t}) \]  
(B21)

\[ \lambda_t^E q_{K,t} - E_t \beta E \lambda_{t+1}^E q_{K,t+1} (1 - \delta_K) - \mu_t^E q_{K,t+1} (1 - \delta_K) + E_t \beta E \lambda_{t+1}^E \left( \xi_1 (u_{t+1} - 1) + \xi_2 (u_{t+1} - 1)^2 \right) = 0 \]  
(B22)

\[ \frac{\alpha Y_t^W}{K_{E,t-1}} = q_{K,t} R_{V,t} \]  
(B23)

\[ \frac{v (1 - \sigma)}{N_{H,t}} Y_t^W = W_{H,t} H C_{H,t-1} \]  
(B24)

\[ \frac{(1 - v) (1 - \sigma)}{N_{S,t}} Y_t^W = W_{S,t} H C_{S,t-1} \]  
(B25)

\[ C_{E,t} + q_{K,t} (K_{E,t} - (1 - \delta_K) K_{E,t-1}) + (1 + R_{E,t-1}) l_{E,t-1} + W_{H,t} L_{HD} + W_{S,t} L_{SD} + \Lambda(u_t) K_{E,t-1} = \frac{Y_t^W}{X_t} + l_{E,t} \]  
(B26)

\[ Y_t^W = Z_t (u_t K_{E,t-1})^\alpha \left[ L_{H,t}^{1-v} L_{SD} \right]^{(1-\alpha)} \]  
(B27)

\[ (1 + R_{E,t}) l_{E,t} = q_{K,t+1} (1 - \delta_K) K_{E,t} \]  
(B28)

B.3 Capital producers

\[ \Delta K_t = K_t - (1 - \delta_K) K_{t-1} \]  
(B29)

B.4 Retailers

\[ A_p = Y_t \left[ 1 - \frac{1}{X_t} \right] \]  
(B30)
$\varepsilon_{Y,t} = X_t (\varepsilon_{Y,t} - 1)$ \hspace{1cm} (B31)

**B.5 Banks**

**B.5.1 Wholesale branch**

\[ R^B_{L,t} - R^B_{D,t} + \kappa_B \left( \frac{K_{B,t}}{L_t} - \nu_B \right) \left( \frac{K_{B,t}}{L_t} \right)^2 = 0 \] \hspace{1cm} (B32)

\[ K_{B,t} = (1 - \delta_B) K_{B,t-1} + j_{B,t-1} \] \hspace{1cm} (B33)

\[ L_t = D_t + K_{B,t} \] \hspace{1cm} (B34)

\[ R^B_{D,t} = R_t \] \hspace{1cm} (B35)

**B.5.2 Loan branch**

\[ 1 - \varepsilon_{BS,t} + \varepsilon_{BS,t} \frac{R^B_{L,t}}{R_{S,t}} = 0 \] \hspace{1cm} (B36)

\[ 1 - \varepsilon_{BE,t} + \varepsilon_{BE,t} \frac{R^B_{L,t}}{R_{E,t}} = 0 \] \hspace{1cm} (B37)

\[ R_{S,t} - R_t = \frac{\varepsilon_{BS,t}}{\varepsilon_{BS,t} - 1} \left( R^B_{L,t} - R_t \right) + \frac{1}{\varepsilon_{BS,t} - 1} R_t \] \hspace{1cm} (B38)

\[ R_{E,t} - R_t = \frac{\varepsilon_{BE,t}}{\varepsilon_{BE,t} - 1} \left( R^B_{L,t} - R_t \right) + \frac{1}{\varepsilon_{BE,t} - 1} R_t \] \hspace{1cm} (B39)

\[ L_t(j) = L_{S,t}(j) + L_{E,t}(j) \] \hspace{1cm} (B40)

**B.5.3 Deposit branch**

\[ -1 + \varepsilon_{BD,t} - \varepsilon_{BD,t} \frac{R_t}{R_{H,t}} = 0 \] \hspace{1cm} (B41)
\[ D_t(j) = D_{H,t}(j) \]  

(B42)

**B.5.4 Banks profit**

\[ j_{B,t} = R_{S,t}L_{S,t} + R_{E,t}L_{E,t} - R_{H,t}D_{H,t} - \frac{K_{KB}}{2} \left( \frac{K_{R,t}}{L_t} - \nu_{B,t} \right)^2 K_{B,t} \]  

(B43)

**B.6 Aggregate resource constraint**

\[ Y_t = C_t + q_{K,t}(K_t - (1 - \delta_K)K_{t-1}) + \Lambda(u_t)K_{E,t-1} + I_{t}^{ED} + \delta_B K_{B,t-1} \]  

(B44)

\[ C_t = C_{H,t} + C_{S,t} + C_{E,t} \]  

(B45)

\[ I_{t}^{ED} = I_{H,t}^{ED} + I_{S,t}^{ED} \]  

(B46)

\[ H_t = H_{S,t} + H_{H,t} = 1 \]  

(B47)

**B.7 Shocks**

\[ \log \varepsilon_{H,t} = (1 - \rho_{\varepsilon_H})\varepsilon_{H,t}^{ss} + \rho_{\varepsilon_H} \log \varepsilon_{H,t-1} + u_H \]  

(B48)

\[ \log Z_t = (1 - \rho_Z)Z_{t}^{ss} + \rho_Z \log Z_{t-1} + u_Z \]  

(B49)

\[ \log B_t = (1 - \rho_B)B_{t}^{ss} + \rho_B \log B_{t-1} + u_B \]  

(B50)

\[ \log m_{S,t} = (1 - \rho_{MS})m_{S,t}^{ss} + \rho_{MS} \log m_{S,t-1} + u_{MS} \]  

(B51)
Figure 2.1: Impulse responses to a one percentage point deviation increase in TFP. All variables are in percentage deviations from their steady states. X-axis is in quarters.
Figure 2.2: Impulse responses to a one percentage point increase in housing preferences. All variables are in percentage deviations from their steady states. X-axis is in quarters.
Figure 2.3: Impulse responses to a one percentage point increase in HH-B LTV ratio. All variables are in percentage deviations from their steady states. X-axis is in quarters.
Figure 2.4: Impulse responses to a one percentage point increase in HC transformation. All variables are in percentage deviations from their steady states. X-axis is in quarters.
Figure 2.5: Comparison of the impulse responses to a one percentage point increase in TFP shock across various models. Solid lines are for base model. Dashed lines are for $\varepsilon^{SS}_{BD} = -5$. Dotted lines are for $\varepsilon^{SS}_{BE} = \varepsilon^{SS}_{BS} = 2.5$. Dotted-dashed lines are for $\kappa_{KB} = 8$. 
Figure 2.6: Comparison of the impulse responses to a one percentage point increase in housing preferences shock across various models. Solid lines are for base model. Dashed lines are for $\varepsilon_{BD}^{SS} = -5$. Dotted lines are for $\varepsilon_{BE}^{SS} = \varepsilon_{BS}^{SS} = 2.5$. Dotted-dashed lines are for $\kappa_{KB} = 8$. 
Figure 2.7: Comparison of the impulse responses to a one percentage point increase in HH-B LTV ratio shock across various models. Solid lines are for base model. Dashed lines are for $\kappa_{BD} = -5$. Dotted lines are for $\kappa_{BE} = \kappa_{BS} = 2.5$. Dotted-dashed lines are for $\kappa_{KB} = 8$. 
Figure 2.8: Comparison of the impulse responses to a one percentage point increase in HC Transformation shock across various models. Solid lines are for base model. Dashed lines are for $\varepsilon_{BD} = -5$. Dotted lines are for $\varepsilon_{BE} = \varepsilon_{BS} = 2.5$. Dotted-dashed lines are for $\kappa_{KB} = 8$. 
Chapter 3: Role of bank runs on economic gaps

3.1 Introduction

Wage gap has been increasing over the past years. On average, even high-skilled workers earn less than they would have before 2007 due to a higher competition between skilled workers that suppresses their wages. Furthermore, there is a constant decrease in lower-skilled workers’ wages, leading to a rise in skill premium due to demand for more productive workers (see Acemoglu and Autor, 2012 and Autor, 2014). Higher skills ensure greater productivity, efficiency and higher wages for workers. In order to obtain new skills, workers need to invest and spend time in a skill accumulation process. Additionally, the accumulation of new skills and, thus, supply of skilled labour will decrease the skill premium (He, 2012).

The aim of this study is to research how skill premium and supply of skilled workers are affected by bank destructions, such as bank runs. For instance, the collapse of U.S. financial institutions during the recent financial crisis has caused increasing panic, loss of consumer and business confidence, as well as caused vulnerable financial sectors to steer fast bank runs and the rough global crisis. From the CPS data, we see that the recent financial crisis has led to an increased skill premium due to a higher unemployment rate among low-skilled labour.\(^{37}\)

Although there is a rich literature that analyses a banking sector as a creator and propagator of financial shocks (including bank runs) to the rest of economy (see, for example, Iacoviello, 2015 and Gertler and Kiyotaki, 2015), there are limited studies on wage gap and skill premium under bank runs and possibility of bank shutdowns. This chapter fills this gap by investigating the impacts of bank failures on the labour market in terms of demand and supply of skilled and unskilled workers.

Taking into account this limitation of the literature, we present a single

\(^{37}\)For further details see The Current Population Survey (CPS) Earnings by Educational Attainment Table for the period of 1975 to 2016
DSGE model with bank runs and skill premium that analyses the ability of workers to choose between different skill levels. It is also unclear how transformation from unskilled to skilled workers and its supply can be affected by the presence of various financial shocks. This chapter fills this gap by introducing a new approach in analysing skill premium: i) a banking sector with possibility of runs or bankruptcies and ii) a skill accumulation channel to analyse skilled labour supply.

The importance of including both banking sector and skill accumulation is explained by three main reasons. Firstly, from the recent financial crisis we see that banks face runs and bankruptcies (see for example, Lehman Brothers or Northern Rock). The probability of bank bankruptcies will shed light on severe destructions in deposit and loan markets affecting demand and supply of skilled and unskilled workers. OECD statistics for the U.S. show that, in 2010, there was less increase in unemployment among high-skilled workers (at around 5 percentage points), whereas for low-skilled workers it increased up to 17 percentage points.\(^{38}\)

Thus, this leads us to the second point of workers heterogeneity. In this chapter we present one household, which consists of a banker and two workers, one skilled and one unskilled. We assume that low-skilled workers have incentives to accumulate skills as a guarantee of greater wages and higher demand for this labour force. Therefore, the inclusion of a skill accumulation channel permits workers to invest in their skills to improve their productivity, which positively affects their wages. Furthermore, this inclusion allows workers to switch between skilled and unskilled groups, which has been impossible to implement in the previous two chapters. The presence of banks and workers heterogeneity will explain what happens to the supply of skilled and unskilled workers, to their productivity and, most importantly, to their wages during various shocks. Hence, thirdly, these two channels help us to study the changes in skill premium under the presence of various shocks.

The skill transformation channel is built on He and Liu (2008). For the banking sector and possibility of bank runs, we follow the model by Gertler

---

\(^{38}\)Source: OECD World Development Indicators of Skills for Employment in the U.S. for the period of 1981-2017
and Karadi (2011) who also replicate shadow banking. These banks do not have any regulations to follow, they issue short-run debts while holding long-term securities. This creates a positive environment for the possibility of bank runs.

Therefore, the set-up of the model in this chapter combines two streams of literature: skill accumulation and banking bankruptcies. OECD (2015) research highlights the importance of both investment and time spent in gaining new skills. Thus, workers in the model are allowed to accumulate new skills through investing and spending time in a skill gaining process.

Our model shows that under a total factor productivity (TFP) shock, a shock to skill transformation and a shock to diversion of assets, skill premium between skilled and unskilled workers’ wages reduces. Therefore, we show that both workers can be better off under the provided assumptions. We also find that under these shocks the bank capital decreases. Bank capital is important to investigate under these modelling assumptions as it provides stability and shock resistance for financial markets. However, higher business confidence can lead to lower bank capital to increase bank assets.

The shock to the probability of the number of exiting banks increases skill premium due to higher labour supply by unskilled workers, decreasing their wages. The bank capital increases under this shock, which indicates the resistance for instabilities.

Conducted robustness checks are consistent with the benchmark model. As robustness checks, we compare the benchmark model with three different models. Model 1 assumes that $A^S = A^U = 1$ where workers are indifferent in accumulating new skills. Model 2 presents a model where both workers have the same weight in the production function by setting this parameter to be equal to 0.335. Model 3 presents a model where there is no investment adjustment cost.

We find that unskilled workers’ supply of labour does not drop as much as it does in the benchmark model, which explains smaller deviation of the skill premium under these modelling assumptions. However, with $v = 0.335$, the skill premium increases under the skill transformation shock as unskilled workers’ labour supply increases significantly.
There are two policy implications presented in this chapter. First is the importance of a skill accumulation channel. Low-skilled workers are better off when they are able to endogenously accumulate new skills, which reduces wage gap in the long-run and provides financial wellness among workers. Secondly, under the model provided in this study, the bank capital decreases under most shocks. Therefore, with a greater supply of high-skilled workers in the economy, the bank capital is lower, which presents financial confidence.

The rest of the chapter is as follows: Section 3.2 provides the literature review in banking and skill premium. Section 3.3 presents the DSGE model with a banking sector and the presence of a skill accumulation channel. Section 3.4 provides decentralised competitive equilibrium (DCE). The calibration of our model is given in Section 3.5. Section 3.6 provides the discussion of empirical results, and Section 3.7 describes the robustness checks. Finally, the conclusion can be found in Section 3.8.

3.2 Related literature

There is a vast amount of literature on financial instability, bank runs and bankruptcies that can severely damage the economic situation of a country. Even strong banks with stable profits and good management are unsafe against bank runs if agents believe these banks will collapse. However, there is limited literature investigating the effects of instability in a banking sector on skill premium and supply and demand for workers with different skill levels.

There is also literature on skill premium and skill accumulation, which is rich and fruitful. Accumulation of new skills is important as it affects supply and demand for workers. High-skilled workers are in greater demand by employers compared to their lower-skilled counterparts (see, for example, Acemoglu, 1998, 2002, Mankiw, 2000, and Goldin and Katz, 2008).

These papers also suggest that workers who are able to advance their skills and develop additional productivity also improve their financial wellbeing by earning higher wages. The recent papers find that a greater labour force with higher skills leads to economic prosperity. For example, Acemoglu et al.
(2014) propose that better skills and productivity jointly create a positive environment for long-term economic growth and development. Higher wages allow workers to increase consumption expenditures, thus, increasing the demand for goods and services, which also supports development in financial sectors and economic growth.

Murphy and Topel (2016) state that a decline in skill investment can cause higher wage gap growth, which leads to losses in the banking sector and negative economic outcomes. They indicate that to affect wage gap, implementing policies should support larger investment in skill accumulation to increase the supply of skilled workers. Furthermore, bank runs do not significantly affect skilled workers, while unskilled workers are at a higher risk of losing their jobs or working less hours and earning lower wages.

Thus, we examine the other stream of literature that looks at bank runs and instability in financial sectors.

The literature covering bank runs before the global financial crisis state that bank runs might occur even with a low probability of default (see Zhu, 2005). It can arise when agents genuinely need their deposits back. Requirements, such as a reserve rate and deposit insurance, are efficient in preventing runs. However, these requirements might force banks to invest inefficiently and create asymmetric information in credit markets. Ennis and Keister (2006) show that when banks invest more, economic growth also increases, however, with less investment, banks have more resources in case of runs. These two outcomes depend on the probability or expectation of runs.

Asymmetric information can arise when deposit holders are not aware of banks’ risky activities, which might cause illiquidity of assets and bank runs. For example, before the recent financial crisis, banks used asset-backed securities or created CDOs (see Calvo, 2009). Chen and Hasan (2008) demonstrate that runs happen when depositors expect information that reveals instability or signals banks’ risky activities. Furthermore, they demonstrate that with full and clear information agents might panic, causing runs even when agents are rational.

Bernanke (2010) explains this as follows: withdrawing is an easier way to deal with the information individuals hold rather than analysing the safety
of their funds in bank deposits. The uncertainty also comes from the shadow banking, which, prior to the recent financial crisis, was not regulated, leaving investment under high risk. Moreover, most financial institutions have relied on short-term funding when they started selling assets at a fire price. This made short-term funding costly and hard to obtain, causing lower credit supply, more runs and further instability in the financial sector. As Bernanke (2010) states, defaulting subprime mortgages wouldn’t damage the U.S. economy as much as bank runs did.

For instance, Gertler and Kiyotaki (2015) also find that depositors might assume a bank is planning to default if the bank has stopped issuing loans to the public, which usually happens during recessions. Thus, most recent papers look at both micro- and macroeconomic levels to understand the behaviour of individuals and firms on the aggregate economy. Combining two levels helps to link consequences of financial accelerator and bank liquidity at individual and aggregate levels.

Gertler et al. (2016, 2017) further research bank runs. In particular, they emphasise their attention on wholesale banks and their role in the financial crisis as wholesale banks were severely affected during the crisis. Wholesale banks financed themselves with short-term debts through interbank markets by lending and borrowing from/to other banks. Retail banks, on the other hand, relied on funds from households. Authors also find that wholesale banks generally lead to higher productivity and economic growth, however, expansion of this sector results in a weaker economic resistance to changes.

3.3 Model

3.3.1 Households

Households are presented as persistent supply of workers with a unit measure. Households save, consume and supply labour force. Each household consists of workers and bankers. Bankers are intermediaries who pay any profit they earn back into households they come from. Thus, banks are owned by households, however, savings that households hold are not in the same bank they own. At each period of time, workers are either skilled or unskilled. Unskilled
workers are able to become skilled through the skill transformation channel, which is built on He and Liu (2008).

Following Gertler and Karadi (2011), we assume that the fraction of households are workers \( (1 - f) \) with the rest \( f \) being bankers. The probability for bankers to stay in a banking sector is \( \sigma \) while the survival time for any banker is \( \frac{1}{1 - \sigma} \) on average. Therefore, \((1 - \sigma)f\) of bankers leave the banking sector and become workers, while a similar number of workers become bankers, keeping the fraction of each group constant. Each household funds the start-up of a banker from that household. The representative household’s maximisation problem is as follows:

\[
\max \sum_{i=0}^{\infty} \beta_i \left[ \log (C_t - aC_{t-1}) - \frac{\gamma (S_t + U_t)^{1+\varphi}}{1 + \varphi} \right]
\]

where \( \beta_i \in (0, 1) \) is the household’s discount factor. \( C_t \) represents the household’s consumption and \( a \) is a consumption habit coefficient. Disutility of labour is given by fraction of \( S_t \) skilled and \( U_t \) unskilled workers with a \( \varphi \) labour supply elasticity and \( \gamma \) weight given to the disutility of labour.

The representative household is subject to the following budget constraint:

\[
C_t + I_{H,t} + B_{t+1} = \Pi_t + R_t B_t + W_{S,t} S_t + W_{U,t} (1 - e_t) U_t
\]

where \( I_{H,t} \) is an investment in skill accumulation. Households purchase \( B_{t+1} \) short-term debt, which pays gross real return of \( R_t \). Short-term debt can be deposits or government bonds, which are perfectly substitutable. Skilled households are paid \( W_{S,t} \), whereas unskilled households \( W_{U,t} \). \( e_t \) shows the time unskilled workers invest in the skill accumulation process. \( \Pi_t \) is a net start-up fund provided to a banker by a household.

Households are subject to the skill accumulation channel, which allows unskilled worker to become skilled by investing goods and time:

\[
S_{t+1} = (1 - \eta) S_t - Skill_t \left[ (I_{H,t})^{\alpha} (e_t U_t)^{(1 - \alpha)} \right]^x
\]

where \( \eta \) is a depreciation rate of current skills, \( Skill_t \) is a shock to efficient
skill transformation process, \( \alpha \) shows the importance of goods input relative to time inputs, \( \chi \) is the return to scale.

### 3.3.2 Firms

Production side of the model is presented by firms who produce final goods. Their production process follows Cobb-Douglas production function:

\[
Y_t = A_t \left( u_t K_t \right)^\theta \left( A^S S_t \right)^\upsilon \left( A^U U_t (1 - \epsilon_t) \right)^{(1-\theta-\upsilon)}
\]

where \( Y_t \) is the output produced by firms and \( A_t \) is a TFP shock. Firms hire workers according to their current skills. \( \upsilon \) shows the weight of skilled workers’ input in the production process. As skilled and unskilled workers’ weight in the production process are supplements, households are indifferent in terms of their skill accumulation. Therefore, to get exogenous productivity and exogenous skill premium, we introduce \( A^S \) and \( A^U \). \( K_t \) is capital stock whereas \( \theta \) shows the importance of capital stock input in the production process. To buy capital stock firms use intermediaries service. Firms issue \( A^S_t \) claims to buy \( K_{t+1} \) capital stock at a price of \( Q_t \). Therefore,

\[
Q_t A^S_t = Q_t K_{t+1}
\]

### 3.3.3 Capital producers

Capital producers buy capital stock from firms at the end of the time \( t \), renovate it and form new capital. They sell this new repaired capital at price of \( Q_t \). We also assume that capital producers face adjustment costs while producing new capital. We assume that households, owners of capital producers, receive profit from this production. The discounted return for capital producers is given by

\[
\max \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta_{T-t} A_{t,\tau} \left\{ (Q_\tau - 1)I_t - f (I_\tau - I_{ss}) I_\tau \right\}
\]

where
\[ K_{t+1} = (1 - \delta(u_t))K_t - I_t \]  

(73)

and \( \delta(u_t) \) is given as in Christiano et al. (2005):

\[ \delta(u_t) = \delta_K + \frac{b^U}{1 + \zeta^U} (u_t)^{1+\zeta^U} \]  

(74)

where \( \delta(u_t)K_t \) presents the quantity of capital that has been renovated.

### 3.3.4 Banks

Here we present a banking sector, which is built on Gertler and Karadi (2011). Banks are the financial intermediaries that transfer funds from households to firms. Their assets are long-term, while liabilities are short-term.

The balance sheet of a \( j \) bank takes the form of:

\[ Q_tA_s_{jt} = N_{jt} + B_{jt+1} \]  

(75)

where \( A_s_{jt} \) are financial claims on firms in the production side. \( N_{jt} \) is the net worth a \( j \) bank has. Alternatively, we can assume that \( N_{jt} \) is the bank’s equity capital. \( B_{jt+1} \) are households deposits in banks, for which banks pay \( R_{t+1} \) real gross return at time \( t + 1 \). \( A_s_{jt} \) are assets that are paid \( R_{K,t+1} \) return.

The bank’s equity capital changes over time according to the following process, which comes from the difference between returns on assets and interest payments on liabilities:

\[ N_{jt+1} = R_{K,t+1}Q_tA_s_{jt} - R_{t+1}B_{jt+1} \]  

(76)

\[ = (R_{K,t+1} - R_{t+1})Q_tA_s_{jt} + R_{t+1}N_{jt} \]  

(77)

Higher returns on assets depend on interest rate premium \( (R_{K,t+1} - R_{t+1}) \) and the total amount of assets \( Q_tA_s_{jt} \).

Financial intermediaries are subject to inequality constraint:
\[ E_t \beta_i \Lambda_{i,t+1+i} (R_{K,t+1+i} - R_{t+1+i}) \geq 0 \]  

(78)

where \( i \geq 0 \). The inequality constraint shows that assets with discounted return are not funded if their return is less than the discounted cost of borrowing. Moreover, in perfect markets the risk premium equals to zero, making the above equation hold with equality. The premium, however, is positive under imperfect markets.

Therefore, as long as banks can at least earn zero return from their activity they will be funded by households. Banks’ wealth maximisation problem is then:

\[
V_{jt} = \max E_t \sum_{i=0}^{\infty} (1 - \sigma * Ext_{t+1}) \sigma^i \beta_{i+1} \Lambda_{i,t+1+i} N_{jt+1+i} = \\
\max E_t \sum_{i=0}^{\infty} (1 - \sigma * Ext_{t+1}) \sigma^i \beta_{i+1} \Lambda_{i,t+1+i} \left[ (R_{K,t+1+i} - R_{t+1+i}) Q_{i+1} A_s_{j,t+i} + \frac{R_{t+1+i} N_{jt+i}}{R_{t+1+i}} \right] 
\]

(79)

As \( \sigma \) shows the probability of bankers staying bankers and not becoming workers, we introduce a shock to this probability. Thus, this shock affects the fraction of surviving banks in the market.

We assume that as long as \( \beta_{i+1} \Lambda_{i,t+1+i} (R_{K,t+1+i} - R_{t+1+i}) \) is positive, banks will demand more funds from households to enlarge their assets. However, this operation is costly for banks and it raises the moral hazard problem. For instance, banks can choose to transfer \( \lambda \) portion of project funds back to the household they come from. In this case, depositors can enforce banks to bankrupt and recover \( (1 - \lambda) \) of the project funds, but it is too costly to recover \( \lambda \) funds.

Therefore, for depositors to keep providing funds to banks, the following constraint should satisfy:

\[
V_{jt} \geq (\lambda * Dvr_t) Q_t A_s_{jt} 
\]

(80)

where \( V_{jt} \) shows the loss if bankers transfer a fraction of assets whereas
\( \lambda Q_t As_{jt} \) is the gain from diverting funds. We also introduce here a shock \( Dvrt_t \) that increases the possibility of a larger diversion, leading to bank runs and bank bankruptcies.

Alternatively, we can present \( V_{jt} \) as:

\[
V_{jt} = \nu_t Q_t As_{jt} + \eta_t N_{jt}
\]

where

\[
\nu_t = E_t \left[ (1 - \sigma * Ext_{t+1}) \beta \Lambda_{t,t+1} (R_{K,t+1} - R_{t+1}) + \beta \Lambda_{t,t+1} (\sigma * Ext_{t+1}) X_{t+1} \nu_{t+1} \right] 
\]

\[
\eta_t = E_t \left[ (1 - \sigma * Ext_t) + \beta \Lambda_{t,t+1} (\sigma * Ext_{t+1}) Z_{t+1} \eta_{t+1} \right] 
\]

where \( X_{t+1+i} = \frac{Q_{t+i} As_{jt+i}}{Q_t As_{jt}} \) is the assets’ gross growth rate. \( Z_{t+1+i} = \frac{N_{t+i}}{N_{jt}} \) is the net worth’s gross growth rate. \( \nu_t \) is an expected discounted marginal gain if a bank increases its asset holdings by one unit while its net worth is constant. The larger \( \nu_t \), the greater is the opportunity cost to the banker from being forced into bankruptcy. \( \eta_t \) is an expected discounted value of additional net worth while assets are fixed. Under the presence of competition in a capital market with no presence of frictions, banks will be borrowing until rates of return change so that \( \nu_t \) equals to zero. To avoid this, banks are also subject to the incentive constraint:

\[
\eta_t N_{jt} + \nu_t Q_t As_{jt} \geq (\lambda * Dvrt_t)Q_t As_{jt}
\]

Whenever this constraint is binding then the amount of assets banks can have will depend on banks equity:

\[
Q_t As_{jt} = \Phi_t N_{jt} = \frac{\eta_t}{(\lambda * Dvrt_t) - \nu_t} N_{jt}
\]

where \( \Phi_t \) is a leverage ratio. Higher \( As_{jt} \) will lead banks to be willing to divert the fraction of \( \lambda \) while holding \( N_{jt} \) constant.

If \( \nu_t > 0 \) then banks will increase their assets as it is more profitable.

Banks’ net worth is then given by the following equation:
\[ N_{jt+1} = [(R_{K,t+1} - R_{t+1})\Phi_t + R_{t+1}] N_{jt} \]  
(86)

Also,
\[ Z_{t+1} = \frac{N_{jt+1}}{N_{jt}} = (R_{K,t+1} - R_{t+1})\Phi_t + R_{t+1} \]  
(87)

\[ X_{t+1} = \frac{Q_{t+1}As_{jt+2}}{Q_tAs_{jt+1}} = \frac{\Phi_{t+1} N_{jt+1}}{N_{jt}} = \frac{\Phi_{t+1}}{\Phi_t} Z_{t+1} \]  
(88)

Thus, we can derive total demand for assets as:
\[ Q_tAs_t = \Phi_t N_t \]  
(89)

where \( N_t \) is the bank’s aggregate capital. The net worth of banks comes from the net worth of existing banks \( N_{et} \) and of those who enter industry \( N_{nt} \).
\[ N_t = N_{et} + N_{nt} \]  
(90)

where
\[ N_{et} = (\sigma \times Ext_t) [(R_{K,t} - R_t)\Phi_{t-1} + R_t] N_{t-1} \]  
(91)

\[ N_{nt} = \omega Q_tAs_{t-1} \]  
(92)

\( \omega \) is the fraction of \((1 - \sigma)Q_tAs_{t-1} \) transferred from households to new banks.

Replacing Equations 91 and 92 in 90 gives the following:
\[ N_t = N_{et} + N_{nt} = (\sigma \times Ext_t) [(R_{K,t} - R_t)\Phi_{t-1} + R_t] N_{t-1} + \omega Q_tAs_{t-1} \]  
(93)

### 3.3.5 Aggregate resource constraint
\[ Y_t = C_t + I_t + I_{H,t} + f (I_{\tau} - I_{ss}) I_{\tau} \]  
(94)
3.3.6 Shocks

This section presents four shocks in the model: a TFP shock, a shock to skill accumulation channel, a shock to diversion of funds, and a shock to the probability of exiting banks.

\[
\log TFP_t = \rho_{TFP} \log TFP_{t-1} + u_{TFP}
\]

(95)

\[
\log \text{Skill}_t = \rho_{SK} \log \text{Skill}_{t-1} + u_{SK}
\]

(96)

\[
\log \text{Dvrt}_t = \rho_D \log \text{Dvrt}_{t-1} + u_D
\]

(97)

\[
\log \text{Ext}_t = \rho_E \log \text{Ext}_{t-1} + u_E
\]

(98)

3.4 Decentralised competitive equilibrium

The non-stochastic decentralised competitive equilibrium (DCE) is summarised by a sequence of allocations \(\{C_t, K_t, S_t, U_t, e_t, I_{H,t}, u_t, B_t, A_{st}, I_t, Y_t, \nu_t, \Lambda_{t+1}, X_t, \eta_t, Z_t, \Phi_t, N_t\}_{t=0}^{\infty}\) and prices \(\{W_{S,t}, W_{U,t}, R_{K,t}, R_t, Q_t\}_{t=0}^{\infty}\) such that the two types of workers solve their optimisation problem, and firms, capital producers and banks maximise their profits, taking prices and initial conditions for capital stock as given; and all the markets clear (see Appendix C).

3.5 Calibration

We calibrate the parameters of the model by following Gertler and Karadi (2011), He and Liu (2008), Jones et al. (1993) and Christiano et al. (2005), which is presented in Table 3.1.

We also set \(A^S\) and \(A^U\) to 1.45 and 1 to get a skill premium of 1.6.

Stochastic processes We set parameters for autocorrelation and standard deviations for TFP and skill transformation shocks following the values given
Table 3.1: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>HH discount factor</td>
<td>0.99</td>
<td>Gertler &amp; Karadi (2011)</td>
</tr>
<tr>
<td>$a$</td>
<td>HH consumption habit coefficient</td>
<td>0.815</td>
<td>Gertler &amp; Karadi (2011)</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Elasticity of labour supply</td>
<td>0.276</td>
<td>Gertler &amp; Karadi (2011)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Weight of labour disutility</td>
<td>1</td>
<td>Gertler &amp; Karadi (2011)</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Skill depreciation rate</td>
<td>0.1</td>
<td>Jones et al. (1993)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Goods share in the skill process</td>
<td>0.4</td>
<td>He and Liu (2008)</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Returns to scale in skill process</td>
<td>0.75</td>
<td>He and Liu (2008)</td>
</tr>
<tr>
<td>$\delta_K$</td>
<td>Capital depreciation rate</td>
<td>0.025</td>
<td>Gertler &amp; Karadi (2011)</td>
</tr>
<tr>
<td>$\xi^u$</td>
<td>Marginal depreciation elasticity wrt $u_t$</td>
<td>7.2</td>
<td>Christiano et al. (2005)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Weight of capital stock input</td>
<td>0.33</td>
<td>Gertler &amp; Karadi (2011)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Weight of skilled workers’ input</td>
<td>0.4</td>
<td>Gertler &amp; Karadi (2011)</td>
</tr>
<tr>
<td>$\psi^u$</td>
<td>Weight of utilisation rate</td>
<td>0.0376</td>
<td>Gertler &amp; Karadi (2011)</td>
</tr>
<tr>
<td>$I^{ss}$</td>
<td>Steady state investment</td>
<td>0.3387</td>
<td>Gertler &amp; Karadi (2011)</td>
</tr>
<tr>
<td>$\phi_i$</td>
<td>Inverse elasticity of investment</td>
<td>1.728</td>
<td>Gertler &amp; Karadi (2011)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Bank’s survival rate</td>
<td>0.972</td>
<td>Gertler &amp; Karadi (2011)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Fraction of funds diverted by a banker</td>
<td>0.381</td>
<td>Gertler &amp; Karadi (2011)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Proportional transfer to entering bank</td>
<td>0.02</td>
<td>Gertler &amp; Karadi (2011)</td>
</tr>
</tbody>
</table>

in the Gertler and Karadi (2011) and He and Liu (2008) papers, respectively. Unlike in Gertler and Karadi (2011), we present a positive TFP shock. We use standard values for the parameters of the autocorrelation and the standard deviation of the two financial shocks as there is no literature to base them on. Therefore, we look at the main variables, their reactions, and the directions of the changes while considering these two shocks. The parameters of the autocorrelation and the standard deviation of the AR(I) processes are given in Table 3.2:

### 3.6 Impulse response analysis

This section describes the impulse response functions after the four shocks: a TFP shock, a shock to skill transformation, a shock to diversion of funds, and a shock to the probability of the number of exiting banks (see Appendix C).
Table 3.2: Stochastic processes

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{TFP}$</td>
<td>AR(1) coef. of TFP</td>
<td>0.95</td>
</tr>
<tr>
<td>$\sigma_{TFP}$</td>
<td>Std. dev. of TFP</td>
<td>0.01</td>
</tr>
<tr>
<td>$\rho_{SK}$</td>
<td>AR(1) coef. of HH skill transformation</td>
<td>0.85</td>
</tr>
<tr>
<td>$\sigma_{SK}$</td>
<td>Std. dev. of HH skill transformation</td>
<td>0.01</td>
</tr>
<tr>
<td>$\rho_{D}$</td>
<td>AR(1) coef. of diversion of funds</td>
<td>0.85</td>
</tr>
<tr>
<td>$\sigma_{D}$</td>
<td>Std. dev. of diversion of funds</td>
<td>0.01</td>
</tr>
<tr>
<td>$\rho_{E}$</td>
<td>AR(1) coef. of probability of number of exiting banks</td>
<td>0.85</td>
</tr>
<tr>
<td>$\sigma_{E}$</td>
<td>Std. dev. of probability of number of exiting banks</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Figures 3.1-3.4 about here

3.6.1 TFP shock

We start analysing IRs with a positive TFP shock presented in Figure 3.1. Under this shock, output increases immediately, while production inputs become more efficient. Capital producers using the increased productivity will start investing more in capital, leading to a higher supply of capital. However, the demand for the capital starts rising as capital prices decline, leading to an increased demand for loans. Moreover, we see that risk premium drops as a result of higher deposits and initial lower demand for loans. Skilled workers’ wages increase slightly. Unlike skilled workers, unskilled workers supply less labour that raises their wages, which decreases skill premium. Moreover, workers deposit more, which explains a decline in the bank capital.

3.6.2 Shock to skill transformation

Here we present a shock to a skill transformation channel and its effects on real variables (see Figure 3.2). This shock improves the productivity of all workers, which leads to a larger supply of skilled workers. Due to this increase in supply of skilled workers, their wages drop significantly compared to the wages of unskilled workers. As workers do not generally earn much, the deposits drop too. To ensure the increase in deposits, banks will increase
the interest rates on deposits, which explains the drop in risk premium. However, we also see a drop in the bank capital due to a significant decline in risk premium, which also negatively affects the production in the economy. However, as the productivity of workers increase, firms start producing more output and claim more loans to purchase capital, which explains the rise in the bank capital the following quarters.

Consequently, we see that skill premium drops as skilled workers’ wages drop. This is the opposite result to those under solvent banks, where skill premium increases under a similar shock to the human capital transformation shock.

3.6.3 Shock to diversion of funds

In this subsection, we present IRs after a positive shock that affects the fraction of total assets which a banker decides to divert back to the household they originally came from (see Figure 3.3). The diversion of a fraction of assets can lead to bank runs, and depositors might force bankers to bankrupt. We see that the positive diversion shock leads to lower bank capital and its net worth through decreased risk premium. The optimal leverage ratio has to decline after periods, following this shock, to eliminate any incentives for a banker to divert funds in the future and balance the cost of doing so. Lower bank capital explains the decrease of loans to firms, which drops production. To increase the production, firms require more capital and labour. Moreover, households’ consumption decreases overall due to fewer bonuses and dividends paid by banks in terms of λ.

However, the wages increase only for unskilled workers, as their labour supply drops, leading to a significant decrease in skill premium.

3.6.4 Shock to the probability of the number of exiting banks

Finally, there is a positive shock to the probability of the number of bankers exiting the market in the next period (see Figure 3.4). Under this shock, we assume that σ becomes lower, because the probability of bankers to stay in the financial market is higher, leading to stability and confidence in this
market. Banking capital increases as deposits decrease due to lower wages. Risk premium increases as bankers don’t require deposits, but have high demand for loans. As a result, output goes up, which induces a greater supply of workers, mainly low-skilled labour. Therefore, as the supply of unskilled workers increases their wages go down, while it is the opposite for skilled labour, though their wages do not increase significantly. Consequently, we see that skill premium increases.

3.7 Robustness check

This section presents robustness checks, which analyse different modelling assumptions: i) we assume that workers are indifferent in accumulating new skills by assuming that $A^S = A^U = 1$. We use $A^S$ and $A^U$ to get the skill premium value of 1.6. However, when we assume that $A^S = A^U = 1$ then skill premium is calculated based on skills workers already have, and will equal to 1.1 in the steady state. Workers are still able to change their skills but they will be indifferent in doing so. However, as $A^S$ is lower, we expect output and investment to be lower than in the benchmark model; ii) next we introduce a model where both workers have the same weight in the production function by setting $v = 0.335$. In the benchmark model we assumed that skilled workers have a higher weight and importance in the production process than unskilled workers. Here, as in the previous model, we assume that output will be affected in this model as unskilled workers, who are less productive, are as important as skilled workers; iii) and finally, we analyse the model where investment adjustment parameter $\phi_i = 0$. Here we are simplifying the model to have a standard law of motion equation. As there is no adjustment cost affecting investment, then investment and capital are higher than they are in the benchmark model.

Figures 3.5-3.8 present the IRs under each shock for the three modelling assumptions. The solid lines are for the benchmark model. The dashed lines present Model 1 where $A^S = A^U = 1$. The dotted lines are for Model 2 where $v = 0.335$. Lastly, the dashed-dotted lines are for Model 3, which is without investment adjustment costs ($\phi_i = 0$).
We start our interpretation of IRs with a TFP shock (see Figure 3.5). Under $A^S = A^U = 1$, skilled and unskilled workers earn similar wages, which is slightly higher for skilled workers. As workers do not change their skills, the demand for capital is higher under this model. While there is a higher demand for loans and a greater supply of deposits, the optimal leverage ratio will decrease. Model 3 presents the highest raise in investment and output due to exclusion of the investment adjustment costs. With the higher wages of unskilled labour in each model, the skill premium will be decreasing. However, in Model 1, both workers’ wages are similar, which explains that there is no change in skill premium under $A^S = A^U = 1$.

Figure 3.6 presents IRs after a skill accumulation shock. When workers have the same weight in the production function ($v = 0.335$), the supply of labour increases, which decreases in other modelling assumptions, leading to a rise in output. That makes firms demand more loans to purchase capital. Risk premium increases as there is no great need in attracting loanable funds from depositors. As the supply of unskilled workers increase their wages drop, which results in the skill premium going up. It is interesting that under this shock the bank capital decreases in the base model and under $\phi_i = 0$, however, it increases under $v = 0.335$ and $A^S = A^U = 1$. Overall, skill premium declines under this shock across the rest of the models, which increased in the first two chapters under a similar shock to skills.

Next, we present IRs after the shock to the diversion of assets by banks, shown in Figure 3.7. As mentioned previously, this shock leads to a lower fraction of diverted funds by banks to the households they came from. It is interesting that under $A^S = A^U = 1$ real variables deviate less than in the benchmark model, with investment and output increasing slightly due to the lower $A^S$, and workers productivity staying almost unchanged. We see similar results in Model 2 with $v = 0.335$ for each type of workers. Investment and output are lower than in the benchmark model because unskilled workers are as important as skilled workers in the production. Even though their
productivity is lower than that of skilled workers, leading to lower output. In Model 3, the output and investment increase due to $\phi_i = 0$. Moreover, in all three models, unskilled workers supply less labour, which leads to higher wages for these agents and, thus, a fall in skill premium.

Finally, we discuss IRs after a shock to the probability of the number of exiting banks under the three modelling assumptions (see Figure 3.8). This shock increases the possibility of individuals to stay as bankers in the next period. Model 1 shows that due to the nature of the shock, loan demand decreases as lower $\sigma$ affects bank capital and the availability of loans under $A^S = A^U = 1$. Moreover, under this assumption, workers will be less productive but provide more labour, which drops wages, thus, creating less deposits. Taking this into account, we see that output doesn’t deviate much from its steady state under Model 1. Model 2 shows similar results because less productive workers have the same weight in the production, which paired with a lower demand for loans, leads to slight increase in output, compared to Model 3 and the base model. The skill premium is increasing under these two models, as skilled workers earn slightly more wages, while unskilled labour’s wages fall. Model 3, on the other hand, shows that under this shock, and with $\phi_i = 0$, output increases due to higher investment and a greater demand for loans. However, skill premium in this model increases, similar to the previous two models, due to lower wages of unskilled workers.

Overall, the three models and the benchmark model show that skill premium generally decreases under all shocks, apart from the shock to the probability of the number of exiting banks, which leads to a rise in skill premium under different modelling assumptions, including the base model.

3.8 Conclusion

In this chapter we present a DSGE model with skilled and unskilled workers in one household. These workers can switch between the two groups, thus, becoming more or less skilled. The banking sector represents the sector in which banks can bankrupt and experience bank runs. However, in this chapter we assume two financial shocks that show stability in the sector,
which is at the opposite to the financial instability caused by bank runs. The latter allows us to investigate the changes in skill premium under various shocks, and workers’ decision on skill accumulation.

Our findings show that under the TFP shock, unskilled workers supply less labour that increases their wages. Skilled workers, on the other hand, supply more labour, leading to lower wages. This, in turn, shrinks the skill premium. This result is consistent with the findings of the first two chapters under the same shock. Under the shock to the transformation of skills, the skill premium decreases following the drop in skilled workers’ wages. It is interesting as under solvent banks and a competition in the banking sector, that was presented in the previous chapters, the skill premium increases under the similar shock. We find that under the shock to diversion funds by bankers, skill premium drops as the wages rise only for unskilled workers following the decline in their labour supply. The skill premium increases under the shock to the probability of the number of exiting banks. This shock represents the stability of the financial market, leading to higher output production. This rise induces higher labour supply by unskilled workers, leading to lower wages for these agents.

Furthermore, the robustness check shows that the findings stay robust under different modelling assumptions. However, as we assume \( \phi_i = 0 \), investment and capital will always be higher than in the benchmark model.
Appendix C

C Chapter 3

Here we present the decentralised competitive equilibrium for each agent in Chapter 3 model.

C.1 Households

\[
\lambda_t = \frac{1}{C_t - aC_{t-1}} - \beta \frac{a}{C_{t+1} - aC_t} \quad (C1)
\]

\[
\lambda_t = E_t \beta \lambda_{t+1} R_t \quad (C2)
\]

\[-\beta \gamma (S_t + U_{t+1})^\alpha + E_t \beta \lambda_{t+1} W_{S,t+1} + SK_t - E_t \beta SK_{t+1} (1 - \eta) \quad (C3)\]

\[-\gamma (S_{t-1} + U_t)^\alpha + \lambda_t W_{U,t}(1 - e_t) - SK_t \frac{Skill_t (1 - \alpha) \chi}{U_t} \left[ (I_{H,t})^\alpha (e_t U_t)^{(1-\alpha)} \right]^{\chi} = 0 \quad (C4)\]

\[-\lambda_t - SK_t \frac{Skill_t \alpha \chi}{I_{H,t}} \left[ (I_{H,t})^\alpha (e_t U_t)^{(1-\alpha)} \right]^{\chi} = 0 \quad (C5)\]

\[-\lambda_t W_{U,t} U_t - SK_t \frac{Skill_t (1 - \alpha) \chi}{e_t} \left[ (I_{H,t})^\alpha (e_t U_t)^{(1-\alpha)} \right]^{\chi} = 0 \quad (C6)\]

\[S_t = (1 - \eta) S_{t-1} + Skill_t \left[ (I_{H,t})^\alpha (e_t U_t)^{(1-\alpha)} \right]^{\chi} \quad (C7)\]

C.2 Firms

\[\frac{\theta Y_t}{u_t} = b^U a_t \xi u_t K_{t-1} \quad (C8)\]

\[W_{S,t} = \frac{vY_t}{A_t S_{t-1}} \quad (C9)\]

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\[ W_{U,t} = \frac{(1 - \theta - \nu)Y_t}{A^U_t U_t(1 - e_t)} \]  
(C10)

\[ R_{K,t} = \frac{\frac{\delta Y_v}{K_{t-1}} + Q_t - \delta}{Q_{t-1}} \]  
(C11)

\[ \delta = \delta_K + \frac{b^U}{(1 + \zeta^U) u_t(1 + \zeta^U)} \]  
(C12)

\[ Y_t = A_t (u_t K_t)\theta (A^S S_t)^\nu (A^U U_t(1 - e_t))^{(1 - \theta - \nu)} \]  
(C13)

\[ Q_t A_s = Q_t K_t \]  
(C14)

### C.3 Capital producers

\[ Q_t = 1 + \frac{\phi_i}{2} (I_t - I^{SS})^2 + \phi_i (I_t - I^{SS}) I_t \]  
(C15)

\[ K_t = (1 - \delta) K_{t-1} + I_t \]  
(C16)

### C.4 Banks

\[ \nu_t = E_t \left[ (1 - \sigma * Ext_{t+1}) \beta \Lambda_{t,t+1} (R_{K,t+1} - R_t) + \beta \Lambda_{t,t+1} (\sigma * Ext_{t+1}) X_{t+1} \nu_{t+1} \right] \]  
(C17)

\[ \eta_t = E_t \left[ (1 - \sigma * Ext_t) + \beta \Lambda_{t,t+1} (\sigma * Ext_{t+1}) Z_{t+1} \eta_{t+1} \right] \]  
(C18)

\[ \Phi_t N_t = \frac{\eta_t}{(\lambda * Dwrt_t) - \nu_t} N_t \]  
(C19)

\[ Z_t = (R_{K,t} - R_t) \Phi_{t-1} + R_t \]  
(C20)
\[ X_t = \frac{\Phi_t}{\Phi_{t-1}} Z_t \]  
\[ (C21) \]

\[ Q_t A_{st} = \Phi_t N_t \]  
\[ (C22) \]

\[ N_t = (\sigma \cdot Ext_t) [(R_{K,t} - R_{t-1})\Phi_{t-1} + R_{t-1}] N_{t-1} + \omega Q_{t} A_{st-1} \]  
\[ (C23) \]

C.5 \textbf{Aggregate resource constraint}

\[ Y_t = C_t + I_t + I_{H,t} + \frac{\phi_i}{2} I_t (I_t - I^{SS})^2 \]  
\[ (C24) \]

C.6 \textbf{Shocks}

\[ \log TFP_t = \rho_{TFP} \log TFP_{t-1} + w_{TFP} \]  
\[ (C25) \]

\[ \log Skill_t = \rho_{SK} \log Skill_{t-1} + u_{SK} \]  
\[ (C26) \]

\[ \log Dvrt_t = \rho_{D} \log Dvrt_{t-1} + u_{D} \]  
\[ (C27) \]

\[ \log Ext_t = \rho_{E} \log Ext_{t-1} + u_{E} \]  
\[ (C28) \]
Figure 3.1: Impulse responses to a one standard deviation increase in TFP. All variables are in percentage deviations from their steady state. X-axis is in quarters.
Figure 3.2: Impulse responses to a one standard deviation increase in skill accumulation. All variables are in percentage deviations from their steady state. X-axis is in quarters.
Figure 3.3: Impulse responses to a one standard deviation increase in diversion of assets. All variables are in percentage deviations from their steady state. X-axis is in quarters.
Figure 3.4: Impulse responses to a one standard deviation increase in probability of the number of exiting banks. All variables are in percentage deviations from their steady state. X-axis is in quarters.
Figure 3.5: Comparing the impulse responses to a one standard deviation increase in TFP across the various models. Solid lines are for the benchmark model. Dashed lines represent Model 1 with $A^S = A^U = 1$. Dotted lines represent Model 2 where $v = 0.335$, and dotted-dashed lines represent Model 3 with $\phi_i = 0$. All variables are in percentage deviations from their steady state. X-axis is in quarters.
Figure 3.6: Comparing the impulse responses to a one standard deviation increase in skill accumulation across the various models. Solid lines are for the benchmark model. Dashed lines represent Model 1 with $A^S = A^U = 1$. Dotted lines represent Model 2 where $\nu = 0.335$, and dotted-dashed lines represent Model 3 with $\phi_i = 0$. All variables are in percentage deviations from their steady state. X-axis is in quarters.
Figure 3.7: Comparing the impulse responses to a one standard deviation increase in diversion of assets across the various models. Solid lines are for the benchmark model. Dashed lines represent Model 1 with $A^S = A^U = 1$. Dotted lines represent Model 2 where $v = 0.335$, and dotted-dashed lines represent Model 3 with $\phi_i = 0$. All variables are in percentage deviations from their steady state. X-axis is in quarters.
Figure 3.8: Comparing the impulse responses to a one standard deviation increase in probability of exiting banks across the various models. Solid lines are for the benchmark model. Dashed lines represent Model 1 with $A^S = A^U = 1$. Dotted lines represent Model 2 where $v = 0.335$, and dotted-dashed lines represent Model 3 with $\phi_i = 0$. All variables are in percentage deviations from their steady state. X-axis is in quarters.
Concluding remarks

This thesis fills the gap in the literature by analysing the effects of various banking sectors and financial shocks they generate on wage and wealth gaps and the supply of skilled labour. There are studies that have attempted to investigate the wealth redistribution across households under the policies that change interest rates in the economy (see, for example, Alpanda and Zubairy, 2016). However, these studies do not empirically analyse the effects of financial and productivity shocks on wage and wealth gaps between heterogeneous households with the presence of different types of banking sectors.

Each chapter introduces a DSGE model with a banking sector and a channel that allows households to invest in their skills in order to become more productive and earn higher wages. The significance of these studies to policymakers is that it shows the vital role of skill accumulation in decreasing wage gap. As the reviewed literature in human capital show, wage gap is unavoidable. However, it is possible to create an environment in which unskilled workers are able to accumulate new skills that will lower the wage gap. Therefore, it is important to introduce policies that will allow workers to attend various on- and off-job training to gain essential skills.

For example, Chapter 1 shows that with the presence of the solvent banking sector, wage gap can be significantly reduced under the TFP shock, which has positive long-run effects. The preferences and financial shocks create higher wage gap both in terms of welfare and wage rates. Human capital productivity shock leads to lower welfare gap, which will be reduced both in the short-run and in the long-run. Furthermore, a lower capital-asset requirement ratio leads to higher wage and consumption gaps in the long-run. We also find that the presence of banks considerably mitigates the effects of financial shocks on wage gap.

Chapter 2 finds that with the imperfect banking competition the TFP shock shrinks the wage and wealth gaps, whereas the housing preference and financial shocks lead to higher wage gap in terms of households’ welfare and wage rates in the long-run. Similar to Chapter 1, under the shock to human capital accumulation, welfare is reduced both in the long and short-
run, however, wage gap widens in the long-run. Moreover, higher capital managing costs lead to lower deviations of skill premium.

Chapter 3 presents a model with a skill accumulation channel and insolvent banking sector. Findings show that under the TFP shock the wage gap shrinks, as it is the case in Chapter 1 and Chapter 2. However, under the skill accumulation shock the wage gap decreases, which contrasts to the findings in Chapter 1 and Chapter 2. We also find that under the diversion funds shock the wage gap decreases, while it widens under the probability of the number of exiting banks. The latter shock produces interesting results as it show the stability in the banking sector, which leads to lower wages for unskilled workers creating higher wage gap.

For further research, it would be interesting to study wage gap under the presence of monetary and fiscal policies. For example, Gertler and Karadi (2011) present a model with a credit policy, where a central bank can inject funds into the economy during crises or provide funds as a "lender of last resort". This injection could replicate the quantitative easing policies that have been implemented during the recent crisis. Fiscal policies can also help to analyse wage gap as it can, for instance, affect the disposable or "take home" income, which will reduce the funds to invest in human capital. Moreover, it would be worthwhile conducting a crisis experiment in a similar way to Gertler and Karadi (2011). It is also worth assuming the lower bound interest rate and how this will affect wage gap. Furthermore, the type of production function proposed by Krusell et al. (2000) may also be considered. It is assumed that this production function matches skill premium data better than the Cobb-Douglas production function we have used in this thesis. This would allow researches to compare the results under the two production functions to also match other variables in the models. In addition, it could be worthwhile to conduct a historical decomposition of the three models, similar to the one in the Iacoviello (2015) paper.
References


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