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The Production Economics of Red Deer Husbandry for Commercial Venison

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The Production Economics of Red Deer Husbandry for Commercial Venison.


Abstract.

The thesis is concerned with the study of the economics of venison production in Scotland. It examines the various production systems utilised at present both in the wild and on an intensive farm and analyses the factors affecting productivity and their relative importance in each system.

The evolution of the red deer in Scotland and more recent developments in the market for venison are examined in the early chapters. In addition, the production system in the wild is analysed and a production function is developed. In order to study the population dynamics of the red deer, a mathematical model is constructed. This is used in conjunction with the production function developed previously to illustrate the interaction between the ecological and economic factors which govern the productivity of red deer populations. In the light of this analysis, a number of recommendations are made for the improvement of productivity in the wild. The limitations of this production system, however, pose a problem for the practical implementation of such proposals. To some extent, these may be overcome by the adoption of a more intensive production system, although this in turn creates its own problems.

The second part of this thesis is thus concerned with the study of the intensive system. The experimental deer farm at Glensaugh forms the basis for the investigation of this system. The data thus obtained are used in conjunction with a modified version of the mathematical model previously developed. This is incorporated in a linear programming format so that the farming system may be analysed and the operating strategies compared. The objective of the analysis is to determine which factors exert the greatest influence upon the operating strategies in
terms of operating profit. Once these critical areas are identified, research effort may be directed more effectively to improve the system performance.

The above analysis leads to a number of recommendations regarding the operating strategies on a deer farm. The financial aspects of such an operation are examined and provide some basis against which the future viability of the farming operation might be assessed. No attempt is made to define levels of acceptable returns on investment. The intention is rather to determine the effects which changes in the economic and environmental conditions have upon investment potential.

Although this study provides some tentative solutions as to how improvements in productivity may be obtained these should not be regarded as final. As knowledge of the production system improves, the solutions to the problems will change as will the problems themselves.
I am most grateful for all the help and encouragement given to me during the preparation of this thesis by the staff at the Glensaugh Farm from both the Hill Farming Research Organisation and the Rowett Research Institute. In addition, I would like to thank the staff of the Institute of Terrestrial Ecology Research Station at Banchory for all their advice and encouragement. Among other organisations to whom I am indebted are the Highlands and Islands Development Board, the Red Deer Commission and the Scottish Game and Venison Processors Association. My thanks also to May Hunter for typing and Gordon McHugh for photographs and to Professor B.J. Loasby and M. Makower for their guidance throughout this study.

Finally, I owe my biggest debt of gratitude to my wife, without whose support this thesis would not have been possible.
COLOUR PLATES

Plate

1. A view of the farm showing the handling facilities in the centre. The animals may be given access to some of the forestry area to the right of the handling facilities. The light green areas consist of bracken with grassland; the dark areas are predominantly heather.

2. Even in the presence of strangers the animals show no signs of alarm.

3. The tameness is evident, although to a lesser extent, in the newborn calves suckled by their dams. Note the mature heather stands which constitute the majority of the vegetation on the upper reaches of the farm.

4. Antler development, even on stags aged 3 and 4 years, is well advanced. The smaller antlers are typical of those preferred by the Far Eastern buyers.

5. Prior to slaughter in their second year the animals are grazed on the improved re-seeded areas. Note the antler development of the animals even at 13 months of age.
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Introduction</td>
<td>1</td>
</tr>
<tr>
<td>2. A Short History of the Red Deer in Scotland</td>
<td>4</td>
</tr>
<tr>
<td>3. The Production and Marketing of Venison in Scotland</td>
<td>16</td>
</tr>
<tr>
<td>4. An Analysis of the Productivity of Red Deer</td>
<td>34</td>
</tr>
<tr>
<td>5. Management Strategies for Increased Productivity in the Wild</td>
<td>63</td>
</tr>
<tr>
<td>6. The Glensaugh Project and the Development of Red Deer Farming</td>
<td>81</td>
</tr>
<tr>
<td>7. Population Dynamics at Glensaugh</td>
<td>101</td>
</tr>
<tr>
<td>8. A Linear Programming Model of the Farm Operation</td>
<td>112</td>
</tr>
<tr>
<td>9. Model Output Analysis - The Effects of Economics on Operating Strategies</td>
<td>130</td>
</tr>
<tr>
<td>10. Model Output Analysis - The Effects of Population Parameter Changes on Operating Strategies</td>
<td>154</td>
</tr>
<tr>
<td>11. Deer Farming - An Investment Appraisal</td>
<td>170</td>
</tr>
<tr>
<td>12. Summary and Conclusions</td>
<td>192</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>APPENDICES</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Production Inputs in the Wild</td>
<td>201</td>
</tr>
<tr>
<td>B. Rainfall Data</td>
<td>204</td>
</tr>
<tr>
<td>C. Calculation of Dominant Eigenvalues</td>
<td>205</td>
</tr>
<tr>
<td>D. Generation of Linear Programming Input Matrix</td>
<td>207</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

The objective of this thesis is to examine the venison production systems in use in Scotland at present and the prospects for their future development. It is thus concerned with the analysis of the factors which affect productivity in the wild and on the farm and their relative importance to the viability of each system.

The evolution of the red deer and its status in Scotland has been influenced by a number of factors, which are discussed in the second chapter. This provides a background against which the exploitation of the animal and its habitat may be assessed. More recent developments in the venison market are examined in the third chapter, in which a production function, which is used to study the production relationships in the wild system, is also developed. The subsequent analysis leads to the construction of a mathematical model of a red deer population, so that a study may be made of the way in which productivity is affected by changes in population parameters. Although the applicability of the model to the general situation in Scotland is restricted, it permits a comparison of the effects of various measures upon population productivity. The production function developed previously is re-examined in light of the analysis of population productivity and a number of recommendations for the improvement of production from wild deer are made. However, the limitations of venison production in the wild pose problems for the practical implementation of such proposals.

Many of these problems may be overcome by the adoption of a more intensive production system utilising tamed stock.
This in turn, however, creates a further series of problems. In order to investigate some of these the development of the experimental farm at Glensaugh is examined in Chapter 6 and the implications of the results obtained there for the future viability of commercial deer farming are discussed. The data obtained from the Glensaugh project are used in conjunction with a modified version of the population model developed in Chapter 4 to depict the deer farming system. This model is incorporated in a linear programming (L.P.) format so that management strategies for the farming system may be evaluated. The objective of this analysis is not primarily to obtain the "optimal" solution; it is also to determine which factors exert the greatest influence upon the operating strategies. The greater the effect of a parameter on the model output, in terms of operating profit, the more important it is to obtain precise information about it. Research effort may then be directed more effectively to supplying critical data necessary to improve the knowledge of the system performance.

As a result of the model analysis carried out in Chapters 9 and 10 a number of recommendations are made regarding the adoption of operating strategies on a deer farm. These are incorporated in a projected farming operation, the financial aspects of which are examined in Chapter 11. This in turn provides some further feedback in regard to the importance of a number of production inputs and operating conditions.

In this study, the analysis of the wild deer system is restricted to the problems of venison production. Many estates, however, also derive revenue from the leasing of deer stalking rights to sportsmen. For this reason any comparison of venison production
in the wild and on the farm must take into account the different objectives of each system. There is no suggestion that either is the "better"; each must be regarded on its own merits and in its own context.
Chapter 2.

A Short History of the Red Deer in Scotland

The evolution of the red deer throughout its range has been greatly influenced by two factors. These are, firstly, environmental factors such as climate and vegetation and, secondly, economic factors dictated by man's use of the land for a variety of purposes. The latter played a major role only as man himself began to have a greater impact upon his environment. The aim of this chapter is thus to present a perspective against which we may view the development of the Scottish red deer and its exploitation through the years.

Because of the interdependence of the economic and environmental factors, the history of the red deer in Scotland includes a history of the land use. In addition to a discussion of practices in relation to the exploitation of red deer in the past, particular attention is given to the effects of sheep farming. Finally, the recent development of an experimental deer farm is described and its impact on the future development of the red deer is discussed.

Geographical Distribution of the Red Deer

The geographical distribution of the red deer Cervus Elaphus can be divided into three main regions.

a) regional or ecotypes of the red deer living on the European Continent including Great Britain, Sardinia, Corsica, North Africa (the Atlas Mountains), Crimea, the Caucasus and the area north of Iran.

b) sub-species occupying the Asian Continent east of Iran.
c) regional variations of the wapiti inhabiting the North American Continent.

Apart from the above which were the result of natural migrations, red deer have been introduced from Scotland and England to New Zealand (1861, 1862, 1871) and Argentina (1906). It should be noted that in both the above cases the introduced animals improved in size and condition due to the more favourable environment. Red deer are to be found, to a greater or lesser extent, in many European countries with the exception of Iceland, North Finland and Southern Europe. In a few European countries red deer occur only as small isolated groups, notably Spain, Portugal and Italy. In others such as Scandinavia, France, Belgium, Holland and Denmark only small proportions of the country are inhabited by red deer. The main areas which they inhabit are Germany, Poland, Yugoslavia, Hungary and the mountains of Switzerland, Austria and Rumania (Dzięgielewski 1973).

In general, the smallest deer occur in Western Europe improving in size and condition as they move eastwards. This is true of deer living both in the mountains and on the lowlands. Average carcase weights of around 130-150 kgs. in Western Europe compared with 200-250 kgs. in the Danube Basin illustrate this point. The Scottish counterparts, however, average only around 80-100 kgs. (op cit). One of the reasons for such a difference is the climate, which in Western Europe comes under the Atlantic influence and is typified by high humidity and mild winters. The poorer summer conditions also have a derogatory effect upon the growth of the
animals. As the Atlantic influence is reduced moving eastwards, the size of the animals increases. The most favourable conditions for the deer are those of a "Continental" climate in which the winters are considerably more severe than the maritime. In addition, lower humidity and precipitation rates increase the beneficial effects of solar radiation. The relationship between climate and body weight, which is not restricted to the red deer but manifests itself in other species too, has been explained by Bergmann's Rule (op cit), which states that warm blooded animals tend to be heavier in the colder parts of their range in order to minimise the ratio of surface area to body weight and maintain body heat during the cold winters.

Red Deer in Scotland

As we have seen above the difference in size between the red deer in Scotland and its Continental counterpart may be attributed, to a large extent, to the climatic differences. However, during prehistoric times, at the end of the last Ice Age, when the animal was first introduced to Scotland the decline in size occurred very slowly only gaining in pace during historical times (Lowe 1961). By that time man was already beginning to exert an influence upon the environment.

Settlements had already become well established in the area south of the Highland Fault and in the Scottish Lowlands in general. Areas which had been heavily wooded and afforded the red deer protection and fodder had been cleared to make way for man's settlements. As they progressed further and cleared land for
agricultural use the deer were moved further north and west into
the boggy regions, where food was of poor quality. With the
onset of the Iron Age further large quantities of timber were
used for smelting and building construction making further in­
roads into the upland pine and birch woods and the lowland hard­
woods (Lowe 1961).

Despite improving climatic conditions during the period
0 - 1200 AD, conditions which should have resulted in a natural
increase in afforestation, the area of land under forest decreased.
The woodland around the settlements was thinned out to provide a
measure of protection from wolves and thieves and further areas
were cleared for tillage and grazing. Iron smelting too con­
tinued to make considerable inroads into the forest. Even as
man was playing the dominant role in determining the environment
of the red deer, nature again took a hand in the proceedings as
the onset of the Little Ice Age (1550 - 1850) accelerated the
rate of devastation of the forests (Ritchie 1920).

In addition to the damage wreaked by climate, the Lowlanders'
sphere of influence spread further afield and, as their needs for
timber increased, they "discovered" the timber resources of the
Highlands. Thus was started the exploitation of the last
remaining afforested areas of Scotland.

By this time the lowland red deer had been almost totally
ousted from their traditional grazings and the end of the 17th
century saw the virtual extinction of the species in this area.
Coupled with this eradication of deer from lowland areas was the
continued decrease in the size of the animal. It is, however, difficult to ascertain exactly how much of this reduction in size was due to climatic changes and how much can be attributed to the expansion of man's activities (Lowe 1961). What is important to note here is that to some extent these changes are reversible - as was demonstrated when the animals were introduced to more favourable environments (New Zealand, Argentina).

**Sheep and Deer in the Highlands**

The introduction of large-scale sheep farming into the Highlands led to great fluctuations in the Highland farmers' fortunes during the second half of the 19th century and by the end of that century had led to a further transformation of the pattern of land use in the Highlands (Hunter 1973). It is the latter transformation, less revolutionary perhaps than the ones which had preceded it by 100 years, which was of greater importance to the red deer. At the time it was the centre of social and political controversy, in which the issue at stake was the conversion of pasture land to deer forest.

Commercial sheep farming made its first appearance in the Southern Highlands in the 1760s when Thomas Geddes became the first Lowland sheep farmer north of the Great Glen (Gray 1957). Following their introduction the sheep farms steadily extended northwards and westwards until, after the collapse of the kelp industry in the 1820s and 1830s, it became the major source of revenue on all Highland and Hebridean estates. In general, however, it was not the landlords who farmed the sheep, a fact of some subsequent importance.
Thus a small number of tenant farmers, mostly from outside the Highlands, took over the vast tracts of land carrying many thousands of sheep.

The period from the repeal of the Corn Laws in 1846 to the early 1870s was the heyday of apparently limitless expansion in the British economy. The agricultural sector shared in this boom with sheep flourishing particularly well and the Highland sheep farmer shared to the full in this profitability. Both sheep and wool prices increased steadily during this period and during the American Civil War, when cotton imports were greatly restricted, wool prices attained record heights. Thus almost all the forests which had been spared by the lumbering and smelting industries were now cleared to make grazing. The fortunes of the red deer in Scotland had reached their nadir.

The position of the sheep farmers was, however, a precarious one. For as the newly settled lands of Australia and America increased their capacity to undersell the British farmer, even in his home market, he was powerless to influence them. By the end of the 1860s wool prices were retreating from the heights of the mid-1860s and by the mid-1870s the fall in prices reached catastrophic proportions. Records of the Inverness Market showed that the mean prices for the various grades of wool were £2.12, £1.45 and £0.80 in the period 1883-66; in 1881-84 the corresponding prices were £1.40, £0.83, £0.38 (Hunter 1973).

In addition to the price slump due to competition from abroad, production costs at home were increasing due to a number of factors. Among these was the great deterioration in the quality of Highland
pastures. The old agricultural system had given to the sheep farmers a great store of fertility in large areas of low ground, which had been highly cultivated, and in great expanses of hill land, which had only been grazed by small stocks of black cattle for a few months in the year. The sheep farmers had in turn drawn heavily on these resources and the high profits realised in the early years consisted, in effect, of capital extracted from the land. (Hunter 1973). With no reinvestment taking place, partly because the effects of this new agricultural policy did not manifest themselves for as long as 40 years, by the 1870s the capacity of the grazings had declined markedly. As a result more sheep had to be wintered outside the Highlands than had formerly been the case. With both overwintering and wage costs rising sheep farms were becoming less attractive to tenants and by the early 1880s many "especially those of south country origin, were manifesting a very decided desire to escape from the business" (Napier Commission Report 1884).

As leases lapsed, they were not renewed, nor were new tenants coming forward. To make matters worse for the landlord, when the tenant declined to renew a lease, and if no new tenant came forward, under the terms of the lease the landlord himself was obliged to take the stock from the outgoing tenant at valuation. Highland landlords in the 1880s were therefore facing a grave crisis as they sought a more profitable use for their land. Their solution was to convert the land use from sheep farming to deer forest and so by the mid 1880s the area given over to deer forests began to expand.
As the demand for Highland shootings began to increase towards the middle of the 18th century, stalking rentals increased too. Furthermore, during this period, deer stalking, as we know it now, received a tremendous boost in popularity by the publication of Scrope's book in 1838 entitled "The Art of Deer Stalking". According to Fraser Darling and Morton Boyd (1964), this one book "had an influence on the Highlands almost as great as the Rising of 100 years before". Thus was opened up a very welcome source of revenue to the Highland landowners.

The 1870s saw the beginnings of the move to provide the gentleman stalker with the amenities he desired and so the Highland glens became littered with imitation "schloss" and "chateaux". This heralded the period of great expansion in the number of deer forests as the following table (2.1) shows:

Table 2.1 The Growth in Deer Forest Area

<table>
<thead>
<tr>
<th>Year</th>
<th>No. of Forests</th>
<th>Area (000's acres)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1790</td>
<td>9</td>
<td>-</td>
</tr>
<tr>
<td>1838</td>
<td>45</td>
<td>-</td>
</tr>
<tr>
<td>1883</td>
<td>99</td>
<td>1975</td>
</tr>
<tr>
<td>1892</td>
<td>N.A.</td>
<td>3327</td>
</tr>
<tr>
<td>1912</td>
<td>203</td>
<td>3585</td>
</tr>
</tbody>
</table>

Source: O'Dell & Walton (1963)

As the number of graziers of southern origin decreased, so the deer forest became not so much an attractive alternative to sheep farming, but more an essential substitute for it. In the
following 20 years to 1900 the profitability of deer forests continued to improve relative to that of sheep farms. Enormous sums of money were expended on the deer forests in that period and by the early 1890s the annual expenditure of all kinds (including roads, fences, stalkers, ghillies etc.) on the Highland deer forests was estimated by the Napier Commission to be no less than £325,000. Thus it can be seen that although initially the upsurge in the number of deer forests was a consequence of the sheep farming crisis, the sporting boom soon developed its own momentum.

As might have been expected, this led to an encroachment of the deer forests onto the less marginal tracts of land. However, sporting tenants were able to pay the landowner considerably more than any farmer could afford, and since it was considered that the quality of a deer forest increased with its size, there was a move to place more and more land under deer. This tendency continued up to the Great War when the total stood at approximately 3.6 million acres, which according to Fraser Darling (1955) was 1 million acres more than the "optimum" according to agricultural and ecological criteria. As far as the landlords were concerned, however, these agriculturally detrimental effects were far outweighed by the benefits accruing from the sporting boom.

However, as it had begun so the sporting boom, and thus the area of land under deer, declined due to social and economic change. The turning point came mid-way through the 1914-18 war when many deer forests were unlet and when sheep and cattle were introduced
to increase the domestic production of agricultural products to counter the U-boat blockade (Hunter 1973). The status quo was not restored after the war due to higher levels of rating and taxation and somewhat harder times for the entrepreneurial classes who had formed a high proportion of tenants. Even though sheep and cattle values rose to new heights during this period, the Departmental Committee on Deer Forests, 1919; reported that few of the forests were capable of carrying sheep or cattle in sufficient numbers to make them a worthwhile proposition.

During the next 30 years, little change took place in the fortunes of the red deer. Committees came and went without their recommendations being implemented. Following the Second World War, however, while meat rationing was still in force, wholesale poaching was rife. Due to outdated legislation, the penalties for poachers, if caught, were derisory. After repeated efforts had been made to limit the number of deer killed by the introduction of a statutory close season, in 1959 the Deer (Scotland) Act came into force to provide the necessary machinery to control and conserve the red deer in Scotland (Lowe 1964).

**Deer Farming in Scotland**

An upsurge in the price of venison during the late 1960s led to an increased interest being taken in deer by the Hill Farm Research Organisation. One reason for this was that the monetary return per carcase from red deer had become double that of a hill ewe and in terms of land utilisation deer were not far behind sheep (Blaxter et al 1974). It also seemed likely that a native wild
animal, which over many hundreds of years had evolved under poor conditions, might be better adapted to them than the comparatively recently introduced sheep.

The exploitation of native wild animals for commercial meat production was not a new development (Dasman 1964). African game ranching had been initiated in the late 1950s and had proved successful. There was, however, at that time, an indication that the Federal German authorities were soon to introduce more rigorous hygiene requirements regarding the handling of game. (Germany constituted a major market for Scottish venison). This was one of the reasons which prompted the adoption of "farming" as a means of exploitation rather than hunting. In addition, problems of handling, feeding and identification of the animals could best be overcome if the deer were farmed as a domesticated animal. There was also the added advantage, that under the more favourable conditions which would prevail on a farm, the latent growth potential could be realised. (cf. New Zealand).

The establishment of an experimental deer farm in 1970, under the auspices of the Hill Farm Research Organisation and the Rowett Research Institute, thus saw the most recent development in the evolution of the Scottish red deer. What its effects will be in the future will depend, as before, on the various economic and environmental forces.

It has been the aim of this chapter to illustrate how the development of red deer in Scotland has been determined by both natural forces outwith man's control and by the social, political and economic factors directly under his influence. We have seen how
the depletion of the land resource by the sheep farmers during the 19th century left vast tracks of land to be converted into deer forests, and how both climate and man's exploitation of the natural habitat influenced the size of the red deer. It is not unlikely that the recent development in the exploitation of red deer for venison production through deer farming will also be dependent upon both environmental and economic factors.
Chapter 3

The Production and Marketing of Venison in Scotland

The previous chapter described the evolutionary steps undergone by the red deer in Scotland up to the present day. The objective of this chapter is to study, in greater detail, the production and marketing of Scottish venison in recent years.

The latest census data supplied by the Red Deer Commission (R.D.C., 1975) estimate the red deer population of Scotland to be in excess of 0.2m animals grazing over 7.0m acres of hill land. However, much of this is land to which deer only have temporary or seasonal access and on which they will be found at very low densities, sharing the grazing with some 1.4m hill sheep and 0.2m cattle. The "deer forests" are those core areas within this deer occupied ground which are predominantly, or even exclusively, utilised by the red deer. These total approximately 3.0m acres and accommodate some 60-65% of the Scottish red deer population. If we include a further 1.0m acres not exclusively used by red deer, the total 4.0m acres accounts for 80% of the population, leaving only 20% of the population on the remaining 3.0m acres.

Although we refer to deer occupied land as deer forests, these "forests" generally consist of open and mountainous moorland varying in altitude from 1,000-4,000 feet. The name is kept as a reminder of the fact that the land was once wooded. The majority of the deer forests offer little scope for afforestation and the grazing capacity for cattle and sheep is limited by winter conditions. A high degree of correlation exists between deer forests and areas
of high amplitude relief. It has been shown that 84% of forests occur in areas where the amplitude is greater than 2,000 feet and 27% in areas of over 3,000 feet relief amplitude (Mather, 1972). Deer forests appear, therefore, to be associated with the highest and most rugged areas of the Highlands. If some financial contribution is to be obtained from these areas, it is likely that red deer will play an important part.

The following Table 3.1 gives a range of possible annual revenues available to the estates from the sale of venison depending upon the price (£1.33/kg 1976/77).

The annual crop is calculated on the basis that $\frac{1}{6}$ of the total population of 200,000 animals is culled. This is the proportion recommended by the R.D.C. Although additional income is available to the estates in the form of rentals of shooting rights, we shall confine this study to the marketing and production of venison.

Table 3.1

<table>
<thead>
<tr>
<th>Price per kg (£)</th>
<th>Revenue (£m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>1.50</td>
</tr>
<tr>
<td>1.10</td>
<td>1.65</td>
</tr>
<tr>
<td>1.20</td>
<td>1.80</td>
</tr>
<tr>
<td>1.30</td>
<td>1.95</td>
</tr>
<tr>
<td>1.40</td>
<td>2.10</td>
</tr>
</tbody>
</table>
The Venison Market in Scotland

Venison dealing in Scotland is restricted to licensed game dealers. Licenses are, however, generally available on application. The Scottish market is dominated by four main dealers who collectively account for more than 90% of all venison sales by estates. There are approximately 150 licensed game dealers in Scotland, of which the majority deal only occasionally in venison, supplying local demand. The concentration of the market structure is illustrated in Figure 3.1.

Figure 3.1 Venison Market Concentration

The main outlet for Scottish venison is on the Continent and is the preserve of the 4 main dealers. Because the small dealers, despite
their numbers, do not have the facilities to deal with the large quantities of venison which are exported, an oligopsonistic situation has developed in which the estates are price takers with very little, if any, variation in price between dealers in any one season. The dealers, however, do not themselves have any great degree of control over the prices which they offer; these are largely determined by the prevailing conditions in the Continental markets.

Each of the estates has to accept the price offered by the dealers because it is too small relative to the total supply to do anything else. Even the largest estates supply, at most, only 2-2.5% of the total venison produced (see Figure 3.2). In this oligopsonistic situation, the estates can sell as much or as little of their product as they wish, at the ruling price. It should, however, be noted that any given price may not rule for longer than a season. Until recently, the ruling price was fixed for the season by contract; this has now been abandoned in favour of a basic price, which is adjusted depending upon the competitive position in the Continental markets.

Estates do not, however, go out of business when the price is low, neither do new estates enter the industry as venison producers when prices are high. Because the estates need to control the size of the deer herds to prevent damage to agricultural land, a proportion

1 There is no legal onus on the landowner to control the deer on his land but in so far as he has the overall "right to kill" the R.D.C. consider there is a moral obligation on him to do so. If he fails in this, the Commission may authorise anyone they consider competent to go on his land to kill the deer doing the damage and to take the carcase away.
Fig. 3.2 Total output of Scottish estates (1973). Source: R.D.C.
of the population is culled every year irrespective of the ruling price. In addition, many estates obtain a proportion of their income from the letting of shooting rights. In order to protect this income, they have to maintain the quality of the herds by regular culling. Thus a stable level of production is maintained by the majority of the estates; production may be increased or even decreased but rarely, if ever, ceases altogether.

The movement of prices over the last 12 years is shown in Figure 3.3 in which we can see that the price of venison increased steadily, if unspectacularly, up to 1972. The increases were mainly due to the rapid exploitation of an export market in Western Europe. Between 70% and 75% of the venison produced in Scotland was sold on the Continent, with the Federal Republic of Germany accounting for the bulk of this. In addition to reflecting the increased demand for venison in the F.R.G., the price rises were a result of increased competition among game dealers themselves. As more dealers began operating in the venison market, the competition to obtain supplies of venison, and thereby increase sales, grew. As prices rose so too did output, mainly as a result of increased culling rates of hinds. When the ceiling to physical output was approached the dealers resorted to offering higher prices to the estates to induce them to enter into contracts with them. Thus although demand on the Continent was increasing the supply of venison was constrained as the limits of production were approached.

The dramatic increase in price in the early '70s was due in part
to the dealers' expectation of a general meat shortage and subsequent high retail prices. High prices were also offered to the estates to induce them to change dealers, which in turn resulted in counter offers to retain estates, thus leading to a price spiral. The expected shortage, however, failed to materialise as a result of increased output from Eastern European exporters. The dealers, who had contracted to buy venison at high prices, were forced to sell at a loss their large stocks of venison; stocks which had been augmented by the attractive prices offered. As a result, price levels dropped in the following season and were further adversely affected by the introduction of stricter hygiene regulations by the F.R.G. in 1975.

**Import Regulations into the F.R.G.**

Since the beginning of 1975 new regulations for the import of game into the F.R.G. have been in force. The measures introduced at that time were a preliminary to the imposition of stricter control as from January 1976. Their objective is to ensure a high degree of hygiene both after killing on the estates and during transportation and processing by the dealers. During the interim period of 1975 it was sufficient for the premises to be approved by a Ministry Veterinary Officer. His place was eventually taken by a German health official, whose approval of the premises and his presence during processing was necessary if venison was to be exported to the F.R.G. The immediate effect of these regulations on the price of venison was drastic. Prior to their introduction, the estates had been receiving 55-70p/kg, by February 1975 this had fallen to 20-30p/kg.
Fig. 3.3 Venison prices 1964 - 1976

![Graph showing venison prices from 1964 to 1976.](image)

Fig. 3.4 Venison imports by the F.R.G. (1973)

![Bar chart showing venison imports by exporter in 1973.](image)
In addition to the controls on the processing operation, further restrictions were imposed upon both processors and estates. In view of the storage and transport facilities available to them, they were faced with a difficult task complying with them. The regulations state that immediately after shooting and evisceration the animal must be stored in such a way that allows it to cool thoroughly. If the ambient temperature is not low enough to reduce the internal temperature of the game to 7°C it must be taken as quickly as possible, and at most within 10 hours of its death, to a suitable cooling facility. Venison must be submitted within 24 hours to a licensed game exporting firm officially inspected appraised and marked accordingly. The extra cost of meeting these requirements has been estimated by the dealers to be in the region of 12-15p/kg of carcase.

Scotland is not, however, the only supplier of venison to the Continental and German markets. There are several competing countries, the most notable of which is New Zealand (see Figure 3.4). The new regulations, therefore, apply in all these countries too. The ease with which they are able to comply with the regulations will depend upon the sophistication of the cropping and processing systems in use. Thus the financial burden of meeting the regulations will vary from country to country. In New Zealand, for example, where a highly mechanised production system utilising helicopters has been in use for some time, the extra cost may be lower than the 12-15p/kg in Scotland, where transport on the estates is mainly by pony and Land Rover. Although the immediate effect of the new
regulations upon the venison industry in Scotland was economically unfavourable, in the long term, they can only improve the quality of the venison sold both at home and abroad.

The Venison Market in the F.R.G.

As we have already noted, the F.R.G. is the major European outlet for Scottish venison. Thus any changes in the price levels there will have a direct effect upon the prevailing prices offered to the estates in Scotland. Venison production in the F.R.G. is currently 10-12 million kgs annually, (Deutsche Jagdschutsverband Handbuch) which compares with 1.5 million kgs produced in Scotland. The proportion of this which comes onto the open market is, however, considerably smaller. Much of the shooting is carried out by clubs and private individuals and the carcases are retained by the hunters. Thus although general price levels for Scottish estates are determined by the prevailing price in Germany, the actual price which can be obtained depends to a great extent upon the price being offered by other exporting countries. The main venison exporters such as New Zealand and Eastern Europe have a considerable influence on the price level of venison imported by the F.R.G. Because of their relatively small contribution to Continental sales, the Scottish dealers have to follow the prices being offered by the major exporters. Thus, in effect, the prices offered to the Scottish estates by the dealers reflect more the situation in the major venison producing countries and the Continental markets than that in Scotland.
The Retail Market for Venison in the U.K.

The market for venison in the U.K. has never received the same attention from the game dealers as has the Continental market. The over-riding reason for this has been the relative ease of entry into an already established market on the Continent, where game is more readily accepted by consumer. In addition, meat price levels on the Continent are considerably higher than in this country, further increasing the attractiveness of these markets.

The majority of the venison sold in this country is processed; only a small proportion of the total production is sold as fresh or fresh-frozen meat through the retail and catering trades. The recent increase (1969-74) in the market share of the retail meat trade by multiple chain stores opened a relatively new channel for the sale of venison in the U.K. In line with general practice in the retail meat trade, the large multiple buyers, supermarkets and catering buyers introduced stricter hygiene requirements, with which wholesalers are expected to comply. Venison marketed through these channels is governed by these same conditions. Typical of these are requirements that cuts be prepared within a specified time of delivery at the processing plant; carcase meat be moved to a chiller within a specified time of slaughter and that all meat must be delivered in insulated vehicles. In one form or another, the above requirements must be met for export to the German and other E.E.C. markets, and should thus present few extra difficulties for the dealers.
Venison Production in Scotland

The statutory close season for red deer in Scotland is from October 21st to June 30th for stags and from February 16th to October 20th for hinds. Thus venison "production" is confined to 7½ months of the year: 4 months for hinds and 3½ for stags.

The policy of most estates is to cull approximately \( \frac{1}{6} \) of the population annually. A certain proportion of this is undertaken by sporting tenants who pay for the rights to stalk (mainly stags); the remainder of the cull is carried out by the estate employees and/or owners. During the shooting season the estates employ temporary labour, such that during periods of high activity as many as 20 men may be engaged in venison production on the larger estates.

When an animal is shot on the hill it is the duty of the stalker or his assistant to "gralloch" (eviscerate) the beast within 1 hour to prevent any contamination of the carcase. The carcases are subsequently uplifted by pony or vehicle, depending on the terrain, and brought to the estate "larder" where they are hung to await collection by the game dealers. Prior to the introduction of stricter hygiene regulations, the larders on most estates were very rudimentary offering only the minimum protection against contamination by flies etc.

Hygiene and care of the meat nevertheless must begin on the hill from the moment a stalker sets his sights upon a particular animal. Any meat that is spoiled is generally spoiled on the hill or on the way to the larder. There are many ways in which this can happen, e.g. the stalkers not washing knives with which they gralloch the animals
not carrying out the gralloch quickly enough, not getting the carcase to the larder quickly enough, allowing the carcase to come into contact with hot engines while being transported and not attending to personal hygiene. Once these problems are eliminated, the meat can be maintained at a consistently high quality with little trouble.

The present arrangement between estates and game dealers is such that the responsibility for transportation of carcases from the estates to the processing plant rests with the dealers. Because of the high cost of transport, efforts have been made to encourage neighbouring estates to rationalise collection procedures. Further efforts are also being made to rationalise larder facilities between estates due to the introduction of the new hygiene regulations, which call for a great improvement in the standards of storage facilities.

**Production Costs and Output on the Estates.**

A recent marketing survey (Paluchowski 1974) contains information regarding outputs and production factors on a number of Scottish estates. In this section, we shall utilise this data to construct a production function for the venison "industry". A brief account of the collection procedures and a description of the data are included in Appendix A.

As we have already noted, revenue from deer on the Scottish estates is obtained not only from the direct sale of venison, but also from the leasing of shooting rights. As our main concern is with the economics of venison production, we shall restrict our analysis to estates which derive over 90% of their income from deer in the form of venison sales revenue. In this way we hope to avoid any
discrepancies which may occur due to the different management policies on the different types of estate.

A similar study undertaken by Beddington (Beddington et al. 1975) of a considerably smaller sample of 9 estates, found that the Cobb-Douglas production function modelled the situation with a high degree of accuracy. The model proposed was of the form shown below in equation 3.1:

\[ C = KL^{\alpha_1} P^{\alpha_2} A^{\alpha_3} \]  

(Eqn. 3.1)

where \( C \) is the number of animals cropped, \( L \) is the number of stalkers employed per year, \( A \) is the area of the estate and \( P \) is the deer population level. \( K, \alpha_1, \alpha_2, \alpha_3 \) are constants. As we can see capital input does not play any significant part in this model. In the main, capital investment is restricted to the provision of transport facilities and rudimentary storage facilities. However, these were considered unlikely to affect the total crop taken.

With one exception, the model we utilised for the analysis of production is identical to the above. The labour input, \( L \), was given in man days spent stalking rather than numbers of employees per year. It was considered that a greater level of accuracy could thus be obtained in regard to labour productivity. Equation 3.1 was modified to the log linear form below:

\[ \ln C = \ln K + \alpha_1 \ln L + \alpha_2 \ln A + \alpha_3 \ln P \]  

(Eqn. 3.2)

and a linear regression performed to obtain estimates of the constants. The following values were obtained:

\[ K = 3.511, \alpha_1 = 0.607, \alpha_2 = -0.316, \alpha_3 = 0.519 \]

The analysis of variance for the regression is shown in Table 3.2.
Table 3.2 Analysis of Variance

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Degrees of Freedom</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>22</td>
<td>10.4955</td>
<td>0.4771</td>
<td></td>
</tr>
<tr>
<td>Regression</td>
<td>3</td>
<td>9.7709</td>
<td>3.2569</td>
<td>85.406</td>
</tr>
<tr>
<td>Error</td>
<td>19</td>
<td>0.7246</td>
<td>0.0381</td>
<td></td>
</tr>
</tbody>
</table>

The production function for the wild red deer system therefore takes the following form:

\[
C = \frac{3.511L^{0.607}P^{0.519}}{A^{0.316}} \quad \text{(Eqn. 3.3)}
\]

From this equation we may obtain the marginal productivities of each of the factor inputs.

In the system under consideration, however, we are restricted in our ability to vary these inputs. Very little, if anything, can be done to alter the area over which the animals graze, or, in the short term, to increase the population density. Thus only the labour input may be varied to change the level of the crop on any estate. The marginal productivities of the three factors are given below:

\[
\begin{align*}
\frac{\partial C}{\partial L} &= 0.607 \frac{C}{L} \quad \text{(Eqn. 3.4a)} \\
\frac{\partial C}{\partial P} &= 0.519 \frac{C}{P} \quad \text{(" 3.4b")} \\
\frac{\partial C}{\partial A} &= -0.316 \frac{C}{A} \quad \text{(" 3.4c")}
\end{align*}
\]

As we can see the marginal productivity of the land is negative, which is what we might have intuitively expected. That is to say with fixed labour and population the crop will be inversely proportional
to the area searched. Thus on good terrain the stalker does not search over the whole area of the estate, but sweeps out a number of paths to encompass the land. The fewer the sweeps, the lower the value of $\alpha_2$. On the other hand, in densely wooded terrain, for example, $\alpha_2$ values approaching 1.0 may be obtained.

The extent of the economies of scale of the system is given by the sum of the $\alpha$-values. In this case we have $\Sigma \alpha = 0.810$ indicating decreasing returns to scale. However, because of the limited control which estates have over the input factors, we shall consider this no further, but concentrate our attention on the response of the output to changes in manning levels.

**Output Response Analysis**

Due to the relative inability of the estates to regulate the input factors of area and carrying capacity, for any given estate these may be regarded as constant. The equation 3.3 thus becomes a production function with a single variable input, viz. manpower, assuming that the crop is balanced by population productivity.

$$C = f(L) \quad \text{(Eqn. 3.5)}$$

Our objective is therefore to determine the manpower input and the crop level which will maximise the contribution from venison sales on a given estate. In order to do this we must estimate the margin net of operating costs other than labour, which is available from the sale of venison.

**Transport Costs**

Although the capital investment in transport facilities plays no
direct part in determining cropping levels, the operating costs attributable to transport on the estate constitute a high proportion of total operating costs. Data on transport collected from a number of estates (Paluchowski 1974) are included in Appendix A. As might be expected, transport costs were directly related to the level of the crop and to a lesser extent to the area of the estate. They are given in the following equation:

$$T = 8.66C + 0.0027A - 94.65 \quad \text{(Eqn. 3.6)}$$

where $T$ is the total cost of transport on the estate.

**Optimal Operating Conditions**

Given that our objective is to maximise contribution margins from venison sales, the objective function takes the following form:

$$\Pi = M.C - P_L.L \quad \text{(Eqn. 3.7)}$$

where $\Pi$ is the operating profit, $P_L$ is the labour cost/man day and $M$ is as shown below:

$$M = P_c - T/C \quad \text{(Eqn. 3.8)}$$

where $P_c$ is the venison selling price per carcase and $T/C$ is the unit transport cost.

$$\Pi = (P_c - P_T).C - P_L.L - \beta \quad \text{(Eqn. 3.9)}$$

where $\beta = (0.0027A - 94.65) \text{ and } P_T = 8.66 \text{ (see Eqn. 3.6)}$

In order to obtain the optimal manning levels we take the first derivative of the objective function.

$$\frac{\partial \Pi}{\partial L} = (P_c - P_T) \frac{\partial C}{\partial L} - P_L \quad \text{(Eqn. 3.10)}$$

To maximise profit we set $\frac{\partial \Pi}{\partial L}$ equal to zero and solve for $L$. The necessary second order condition ($\frac{\partial^2 \Pi}{\partial L^2} < 0$) holds automatically as we have diminishing returns to labour (see Eqn. 3.4a).
From equation 3.3 we obtain the following:

\[ C = KL a_1 Q \]  \hspace{1cm} \text{(Eqn. 3.11)}

where \( Q = A a^2 P^{a_3} \), and \( P \) and \( A \) are constant for any given estate.

Thus:

\[ \frac{\partial C}{\partial L} = a_1 KL (a^1-1) Q \]  \hspace{1cm} \text{(Eqn. 3.12)}

and \[ \frac{\partial I}{\partial L} = (P_C - P_T) [a_1KL(a^1 - 1).Q] - P_L \] \hspace{1cm} \text{(Eqn. 3.13)}

the optimal manning level, \( L \), is therefore given by:

\[ L = \left[ \frac{(P_C - P_T) a_1 K Q}{P_L} \right] \left( \frac{1}{1 - a_1} \right) \]  \hspace{1cm} \text{(Eqn. 3.14)}

Model Parameter Values

Average larder carcase weights obtained in Scotland are 45kg (calculated on average carcase weights of each sex and the proportion killed - R.D.C. 1972). At the current venison price of £1.33/kg the value of the average carcase, \( P_C \), is thus £59.85. From this we must subtract the average cost of transporting the carcase on the estate. We have already noted in Eqn. 3.6 that in 1974 this was approximately £8.66. In order to obtain current costs, however, this figure must be increased by 72% (Department of Trade & Industry - Vehicle & Transport Prices Index 1976) i.e. current costs are in the region of £14.90/carcase. Thus we obtain a value for \( (P_C - P_T) \) of approximately £45. Labour costs on the estate were taken as the average of agricultural wages in Scotland i.e. £2790 (Department of Employment 1977) which for a 250 day working year is £11.16 per man-day.
Fig. 3.5  Optimal labour inputs for varying area, stocking density, labour cost, $P_L$, and margin, $(P_C - P_T)$.

(a) Stocking Density 3:100

(b) Stocking Density 6:100
Fig. 3.6 Proportion of population culled at optimal manning levels

(a) Stocking Density
3:100

(b) Stocking Density
6:100

Proportion of population culled at optimal manning levels
Figures 3.5a and b illustrate the optimal labour inputs, L, for a number of estate areas and population densities, over a range of values of \((P_c - P_T)\) and \(P_L\). In Figures 3.6a and b we show the corresponding proportional cull obtained at the optimal manning levels over the same range of costs.

The cull rates shown in Figures 3.6a and b are those determined on economic grounds. However, the estates are also governed by the ecological restrictions imposed by the biotic potential of the red deer. If we consider the cull rate recommended by the R.D.C. based on average population growth rates, i.e. approximately 15-20%, we can see that only a small proportion of estates will be operating near their economically optimal outputs. What is more important, is that there is little incentive for the larger estates to comply with these recommended culling rates, when by doing so they reduce profits. Nevertheless our intention here is not to advocate strategies which are consistent both economically and ecologically; it is rather to illustrate the differences which exist between them.

Ultimately each estate aims to adopt a policy which allows it to obtain the highest sustainable profits. Any measures which will accomplish this will be governed by the ability of the estate to modify input productivities. In the short term, only labour is within the control of the estate. On a longer term basis, however, it is possible to modify the population density and productivity by the use of more intensive husbandry methods. In such a way the estates may indirectly achieve some degree of control over a further production variable, viz. carrying capacity.
In the following chapters we shall study the population growth process in the wild and examine the areas which afford the greatest scope for improvement. With this knowledge of the ecological constraints on the venison production system we shall re-examine the production function developed above.
An Analysis of the Productivity of Red Deer

A study of the effects of management policies on a red deer population may be accomplished in two ways; either by experimenting with a live population or by constructing a model of the system and utilising this to predict the outcome of changes to population parameters. The obvious restriction of time precludes our experimentation with live herds. Any such analysis must therefore be concentrated upon the mathematical modelling of deer systems. In this chapter we discuss our choice of model, which is based upon the Leslie matrix model utilised by Pennycuick et al (1968). The adaptation of this model to the study of red deer populations carried out by Beddington (1973) is described in detail. The model is then used to study the sensitivity of the population growth rate to changes in population parameters. Due to the paucity of data from other sources, the analysis is centred upon the red deer population of Rhum. It is not the objective of this study to obtain a general model for all red deer populations in Scotland. This cannot be done. What we are concerned with here is the impact of changes in population characteristics, as a basis for the evaluation of the intensification of production techniques in the wild.

The Mathematical Model

A model is a formulation that portrays a real-world phenomenon. In its simplest form it may be verbal or graphic. Ultimately, however, if it is to be used to make quantitative predictions, a
model must be statistical and mathematical. The importance of a model thus hinges on its ability to predict probable outcomes as parameters in the model are changed, new parameters added or old ones removed. Although models are selective abstractions of real systems, they are extremely powerful tools because tentative answers and predictions regarding critical matters are more important than precise treatment of unimportant details. We are thus faced with the decision regarding the degree of precision which is required from our model.

The brute force approach would be to set up a mathematical model which is a faithful one-to-one reflection of the complex system. This could involve the use of hundred of simultaneous partial differential equations with time lags, measuring hundreds of parameters, solving the equations to get numerical predictions and measuring these against the real system. There are, however, too many parameters to measure, some of which are still only vaguely defined and others which would take many years to measure. The equations are analytically insoluble and even if soluble, the results expressed in the form of quotients of sums of products of parameters would have no meaning for us. Thus the model must be simplified in such a way that the essential features of the problem are preserved.

It is, of course, desirable to work with manageable models which maximise generality, realism and precision towards the goals of understanding predicting and modifying the real system. This,
however, cannot be done and some other strategies have to be evolved. The choice of strategy depends solely on the reason for which the model has been constructed. Generality refers to the breadth of applicability of the model (i.e. the number of different situations in which it can be applied). Realism refers to the degree to which the mathematical formulation corresponds to the biological concepts which it is to represent. Precision is the ability of the model to mimic the data on which it is based. The above properties can be paired in three ways -

1. Sacrifice generality to realism and precision.
2. Sacrifice realism to generality and precision.
3. Sacrifice precision to generality and realism.

depending on the goals of our model. Having simplified the model to more manageable proportions, we should always bear in mind the possibility that the results from the model owe more to the details of our simplifying assumptions than to the essentials of the model.

The fact that we are dealing with a specific animal, the red deer, and wish to obtain precise testable predictions from our model leads us to adopt the first of the above strategies and sacrifice generality for realism and precision. Once we have chosen the entities which compose the system, the number of animals, we can go on to specify the mechanism by which changes in the system states occur.

In the following sections we shall review the development of such a mechanism, the Leslie Matrix Model, and its adaptation for use with a red deer population.
Mathematical Models of Population Processes

In his "Essay on Population" (1798) Malthus calculated that although the numbers of organisms can increase geometrically, as shown in equation 4.1 below, their food supply may never increase faster than arithmetically -

\[ N_t = N_0 e^{(b-d)t} \]  \hspace{1cm} (Eqn. 4.1)

where \( b \) and \( d \) are the birth and death rates per individual respectively. The arithmetic rate of increase in food production seems to be somewhat arbitrary, and Malthus may have presented this rate as a maximum supposition (Flew 1957). The great disproportion between the two powers of increase led Malthus to infer that reproduction must eventually be checked by food production.

In the simple birth and death process, it is assumed that the probability that an organism will reproduce or die remains constant and is unaffected by the size of the population to which it belongs. This can, of course, be true if the time period in question is very small or if the population is so small that there is no interference among its members. When a population is growing in a restricted environment the density gradually rises until the growth is restricted by a shortage of resources. The stage is eventually reached where the demands made by the existing population attains its "saturation level", a value determined by the carrying capacity of the environment. Thus Verhulst (1838) assumed the growth rate per individual to be a function of \( N \), the population size, for all values of \( N \), i.e.

\[ \frac{dN}{dt} = Nf(N) \]  \hspace{1cm} (Eqn. 4.2)
If the inhibitory effect on growth is to increase with increasing population size, \(df(N)/dN\) must be negative. Assuming \(f(N)\) linear, i.e. \[f(N) = a - bN \quad (a, b > 0)\]
we get
\[
\frac{dN_t}{dt} = N_t \left( a - bN_t \right) \quad \text{(Eqn. 4.3)}
\]
The above equation 4.3 is commonly known as the Verhulst-Pearl logistic equation. The equilibrium level of the population occurs when \(N_t = a/b\).

The above models have several major restrictions; the most important one from the point of view of describing the dynamics of a red deer population, is that the population is regarded as homogeneous. This could lead to a certain degree of error in the case of red deer where there is a considerable variation in the population parameters for different age groups. Pennycuick et al. (1968) noted that the difficulty of incorporating age structure and density dependent effects in the same population model, could be overcome using the model developed by Leslie (1945, 1948, 1959) and Williamson (1959).

**Leslie Matrix Model**

In arriving at the logistic equation allowance was made for density dependence, i.e. a decrease in birth rate and an increase in mortality rate as the population grew larger. For the purpose of simplification, we shall disregard that phenomenon and assume that an individual animal’s chances of reproducing and dying are a function of its age and not of the population size. The discussion is further simplified if only females are considered. These restrictions on the model can, of course, be removed at a
Given an arbitrary age distribution, age specific survival and fecundity rates for a group of females at time $t$ the model utilises simple difference equations to predict subsequent age distributions. Using Leslie's notation:

- $n_{x,t}$ = the number of females alive in the age group $x$ to $x+1$ in time $t$
- $p_x$ = the probability that a female aged $x$ to $x+1$ at time $t$ will be alive in the age group $x+1$ to $x+2$ in time $t+1$
- $F_x$ = the number of female offspring born in the interval $t$ to $t+1$ per female alive aged $x$ to $x+1$, which will be alive in the age group 0 to 1 at time $t+1$.

After one time interval, $t$ to $t+1$, the new age distribution will be given by

$$
\sum_{x=0}^{m} F_x n_{x,t} = n_{0,t+1}
$$

$$
p_0 n_{0,t} = n_{1,t+1}
$$

$$
P(m-1)n(m-1),t = n_{m,t+1}
$$

where $m$ is the maximum age considered. The matrix notation is shown in Figure 4.1 below.
The matrix $M$ is square of order $m+1$ with all the elements equal to zero except those in the first row and in the sub-diagonal immediately below the principal diagonal. The $P_x$ figures all take values between 0 and 1, while the $F_x$ figures take values $> 0$.

Two results follow immediately from the above basic model.

Since $M_n = n_{t+1}$ and $M_{n+1} = n_{t+2}$ etc. it can be seen that after $k$ time periods

$$n_{t+k} = M^k n_t$$  \hspace{1cm} (Eqn. 4.4)

further, since the matrix $M$ is square with $m+1$ rows and columns it follows that there are $m+1$ eigenvalues and eigenvectors which satisfy the equation

$$M\mathbf{a} = \lambda \mathbf{a}$$ \hspace{1cm} (Eqn. 4.5)

where $\lambda$ is any eigenvalue and $\mathbf{a}$ is the eigenvector associated with $\lambda$.

The algebraic properties of matrices such as $M$ have been well researched. Sykes (1969) has shown that such matrices are
non-negative irreducible and therefore are subject to the theorem of Perron and Frobenius, which states that the matrix has a positive eigenvalue $\lambda_0$ which is a simple root of the characteristic equation of the matrix and which is not exceeded by the modulus of any other eigenvalue of the matrix. Corresponding to this eigenvalue is an eigenvector having all its elements real and non-negative. $\lambda_0$ is the only eigenvalue of the matrix for which the corresponding eigenvector, $\mathbf{a}$, can be chosen with all its elements positive. From the biological point of view, the above implies that the Leslie matrix model will always determine a meaningful age structure for the population, i.e. no negative or imaginary numbers of animals will result from the application of the model.

The Two Sex Model

The algebraic properties of the simple matrix of type $M$ also hold true for more complex matrices which include both sexes as long as the elements of $M$ remain positive. Such an extension has been described by Williamson (1959) and is shown in Figure 4.2. The notation follows that of the matrix $M$ with the following additions:

- $F_{m,x}^{\text{male}}$ = the number of offspring born in the interval $t$ to $t+1$ per female alive aged $x$ to $x+1$ at time $t$, which are alive in age group 0 to 1 at time $t+1$.
- $F_{f,x}^{\text{female}}$ = the number of offspring born in the interval $t$ to $t+1$ per female alive aged $x$ to $x+1$ at time $t$, which are alive in age group 0 to 1 at time $t+1$.
- $P_{m,x}^{\text{male}}$ = the probability that a male aged $x$ to $x+1$ at time $t$ will be alive in the age group $x+1$ to $x+2$ at time $t+1$.
- $P_{f,x}^{\text{female}}$ = the probability that a female aged $x$ to $x+1$ at time $t$ will be alive in the age group $x+1$ to $x+2$ at time $t+1$.
- $n_{m,x,t}^{\text{male}}$ = the number of males alive in the age group $x$ to $x+1$ at time $t$.
- $n_{f,x,t}^{\text{female}}$ = the number of females alive in the age group $x$ to $x+1$ at time $t$. 
Figure 4.2 illustrates the matrix form of the model

\[ M'_{nt} = M_{t+1} \]  
(Eqn. 4.6)

**Seasonality in Population Dynamics**

Further sophistications to the basic model were proposed by Skellam (1967), whose contention it was that time should not be regarded as homogeneous as evidenced by the cyclical phenomena (diurnal, seasonal) which occur.

If the basic time unit of 1 year is divided into \( n \) parts (\( n = 4, 13 \) etc.) it is possible to make each matrix \( M_s \) appropriate to the time of year with which we are concerned and to represent seasonal changes by repeating the matrices in cyclical order.

Time is given by \( t = j + s/n \) where \( n \) is the number of divisions
in the year and \( j \) is an integer. Thus adapting equation 4.6 we have:

\[
\begin{align*}
\frac{n_j}{n} + \frac{1}{n} &= M_{n_j} \\
\frac{n_j}{n} + \frac{2}{n} &= M_{1n_j} + \frac{1}{n} = M_{1M_{n_j}} \\
\frac{n_j}{n} + \frac{1+s/n}{n} &= M_{s-1M_{s-2} \ldots M_{0n-1} \ldots M_s} n_j + s/n \\
\end{align*}
\]

where \( G_s = M_{s-1} \ldots M_{0n-1} \ldots M_s \) is a square matrix

According to Skellam, the \( n \) possible square matrices \( G_s \) have the same characteristic equation and therefore the same dominant eigenvalue, \( \lambda_0 \). The population vectors associated with a particular season \( (s = \text{fixed}) \) acquire a constancy of form characteristic of that season and undergo multiplication annually by \( \lambda_0 \). The vectors associated with different seasons are tied rigidly together by relations of the type

\[
\frac{n_j}{n} + \frac{s/n}{n} = M_{s-1} \ldots M_{0n_j}
\]

If any of them acquires constancy of form, the remainder do so automatically and the approach to stability may be regarded as simultaneous.

**Density Dependent Population Processes**

We stated at the beginning of this discussion of the Leslie matrix model that fecundity and survival rates were assumed to be independent of the total population size. The next step in the development of the Leslie model is therefore the re-introduction of density dependence. It was Leslie who first proposed such a
modification (1948) and added to it in (1959). It was from this that Pennycuick et al. (1968) developed the computational procedure for the inclusion of density dependence in the matrix model (see Figure 4.3).

Figure 4.3

Flow Diagram for the Computation of Population Projections

In that particular study the variation of the factors with population size was defined arbitrarily by choosing a suitably shaped curve and fitting an equation to it. As far as red deer are concerned, density dependent relationships have been developed for the deer population of Rhum (Beddington 1973). Beddington also proposed further modifications to the model described above, which improved
Figure 4.4 Transition Process of the Female Population.
the realism of the model. These are described in the following sections.

A Matrix Model of the Red Deer Population of Rhum

Research carried out on populations of red deer in various parts of Scotland has shown a great disparity between the fecundity schedules of lactating and yeld hinds. The disparity was shown to be statistically significant (Mitchell et al 1971) and could not, therefore, be regarded as a chance occurrence. For this reason the following modifications to the model were proposed. (Given that the age of first breeding on Rhum was 3 years, the first possible groups of lactating hinds would be aged 4). The transitions which occur for the female population are shown in Figure 4.4 below. The resulting matrix, including both sexes, is shown in Figure 4.5.
|          | 0 | 0 | 0 | 0 | 0 | FY₃/2 | 0 | FY₄/2 | FL₄/2 | 0 | FY₅/2 | NM₁,ₜ | 0 | 0 | 0 | 0 | FY₃/2 | 0 | FY₄/2 | FL₄/2 | 0 | FY₅/2 | NM₂,ₜ |
|----------|---|---|---|---|---|-------|---|-------|-------|---|-------| NY₁,ₜ | 0 | 0 | 0 | 0 | FY₃/2 | 0 | FY₄/2 | FL₄/2 | 0 | FY₅/2 | NM₃,ₜ |
| SM₁      |   |   |   |   |   |       |   |       |       |   |       | NY₂,ₜ | 0 | 0 | 0 | 0 | FY₃/2 | 0 | FY₄/2 | FL₄/2 | 0 | FY₅/2 | NM₄,ₜ |
| SY₁      |   |   |   |   |   |       |   |       |       |   |       | NY₃,ₜ | 0 | 0 | 0 | 0 | FY₃/2 | 0 | FY₄/2 | FL₄/2 | 0 | FY₅/2 | NM₅,ₜ |
| SM₂      |   |   |   |   |   |       |   |       |       |   |       | NY₄,ₜ | 0 | 0 | 0 | 0 | FY₃/2 | 0 | FY₄/2 | FL₄/2 | 0 | FY₅/2 | NM₆,ₜ |
| SY₂      |   |   |   |   |   |       |   |       |       |   |       | NY₅,ₜ | 0 | 0 | 0 | 0 | FY₃/2 | 0 | FY₄/2 | FL₄/2 | 0 | FY₅/2 | NL₆,ₜ |
| SM₃      |   |   |   |   |   |       |   |       |       |   |       | NY₆,ₜ | 0 | 0 | 0 | 0 | FY₃/2 | 0 | FY₄/2 | FL₄/2 | 0 | FY₅/2 | NL₆,ₜ |
| SY₃(1-FY₃) |   |   |   |   |   |       |   |       |       |   |       |       |   |   |   |   |       |   |       |       |   |       |       |
| SY₃FY₃   |   |   |   |   |   |       |   |       |       |   |       |       |   |   |   |   |       |   |       |       |   |       |       |
| SM₄      |   |   |   |   |   |       |   |       |       |   |       |       |   |   |   |   |       |   |       |       |   |       |       |
| SY₄(1-FY₄) |   |   |   |   |   |       |   |       |       |   |       |       |   |   |   |   |       |   |       |       |   |       |       |
| SY₄FY₄   |   |   |   |   |   |       |   |       |       |   |       |       |   |   |   |   |       |   |       |       |   |       |       |
| SL₄(1-FL₄) |   |   |   |   |   |       |   |       |       |   |       |       |   |   |   |   |       |   |       |       |   |       |       |
| SM₅      |   |   |   |   |   |       |   |       |       |   |       |       |   |   |   |   |       |   |       |       |   |       |       |
| SY₅(1-FY₅) |   |   |   |   |   |       |   |       |       |   |       |       |   |   |   |   |       |   |       |       |   |       |       |
| SY₅FY₅   |   |   |   |   |   |       |   |       |       |   |       |       |   |   |   |   |       |   |       |       |   |       |       |

**Note:**
- The ratio of males to females at birth is 50:50
- Proportion of lactating hinds breeding in their i<sup>th</sup> year of life.
- Survival rate of lactating hinds surviving their i<sup>th</sup> year of life
- Number of lactating hinds in their i<sup>th</sup> year at time t
Seasonality in the Red Deer Model

The transitional process from one year to the next of the Scottish red deer can be divided into 3 main stages when changes in the structure of the herd occur. These are as follows:

1. From summer to winter
2. From winter to spring
3. From spring to summer

The first of these stages involves the culling of the herd, which in Scotland takes place during autumn and early winter. If we start with the summer population vector, $n_{t, su}$, the winter population is given by:

$$n_{t, w} = M_{c} n_{t, su}$$  \hspace{1cm} (Eqn. 4.8)

where $M_{c}$ is the diagonal matrix whose elements contain the proportion of each age group of the population which survive the cull. The bulk of the natural mortality occurs in the period following the cull through to spring. The spring population is thus given by

$$n_{t, sp} = M_{n} n_{t, w}$$  \hspace{1cm} (Eqn. 4.9)

In this case, $M_{n}$ is the diagonal matrix of age specific survival coefficients. The final transition from spring to summer is given by

$$n_{t + 1, su} = M_{b} n_{t, sp}$$  \hspace{1cm} (Eqn. 4.10)

The matrix $M_{b}$ produces the new calf input and is of the form shown in Figure 4.6. It will be noted that there is no mortality included in this transitional stage. The overall transition from
<table>
<thead>
<tr>
<th></th>
<th>( F_{3/2} )</th>
<th>( F_{3/2} )</th>
<th>( F_{4/2} )</th>
<th>( F_{4/2} )</th>
<th>( F_{5/2} )</th>
<th>( F_{5/2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_{3/2} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( F_{3/2} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( F_{4/2} )</td>
<td>( F_{4/2} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( F_{4/2} )</td>
<td>( F_{4/2} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( F_{5/2} )</td>
<td>( F_{5/2} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( F_{5/2} )</td>
<td>( F_{5/2} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Matrix \( M_b \)
\[
\begin{array}{cccccc}
0 & 0 & 0 & 0 & 0 & FY_3/2 \cdot SY_3 \cdot 0 \\
0 & 0 & 0 & 0 & 0 & FY_4/2 \cdot SY_4 \cdot FL_4/2 \cdot SL_4 \cdot 0 \\
SM_1 & \cdot & \cdot & \cdot & \cdot & FY_5/2 \cdot SY_5 \\
\cdot & SY_1 & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & SM_2 & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & SY_2 & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & SM_3 & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & SY_3(1-FY_3) \\
\cdot & \cdot & \cdot & \cdot & \cdot & SY_3FY_3 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & SM_4 \\
\cdot & \cdot & \cdot & \cdot & \cdot & SY_4(1-FY_4) \cdot SL_4(1-FL_4) \\
\cdot & \cdot & \cdot & \cdot & \cdot & SY_4FY_4 \cdot SL_4FL_4 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & SM_5 \\
\cdot & \cdot & \cdot & \cdot & \cdot & SY_5(1-FY_5) \\
\cdot & \cdot & \cdot & \cdot & \cdot & SY_5FY_5 \\
\end{array}
\]
the summer of one year to the following summer is given by

\[ n_{t+1, su} = M_t M_{1,c} n_{t, su} \]  

(Eqn. 4.11)

Rewriting we get

\[ n_{t+1, su} = M_o n_{t, su} \]  

(Eqn. 4.12)

where the matrix \( M_o \) is of the form shown in Figure 4.7.

Comparing the matrix in Figure 4.7 with that of Figure 4.5 we can see that the fecundity rates have been modified by the inclusion of the cull and natural survival rates. The discrete nature of the birth and death processes in a red deer population require such a modification to the model if the calf input is not to be overestimated.

Density Dependent Birth and Death Rates of the Rhum Population

From the data obtained by Lowe (1969) of the Rhum herd, Beddington (1973) noted that the survival of males and females aged 2 to 8 varied little from year to year and was uniformly high. Thus the question of density dependent survival rates was seen to be one of varying survival in animals aged 1 and 9 and over. In addition to population density, weather also played a part in controlling survival. The following mathematical model was proposed to determine the survival rates for the relevant age classes.

\[ S_{i,t} = \sin (a \ln N_t + bR + C) \]  

(Eqn. 4.13)

where \( S_{i,t} \) is the survival of the age group \( i \) in period \( t \), \( N_t \) is the population density in early winter of year \( t \) and \( R \) is the level of
summer rainfall (June-Sept.); a, b, and c are constants.

Beddington obtained the following values for these constants (see Table 4.1).

Table 4.1 Age Specific Survival Constants (Beddington 1973)

<table>
<thead>
<tr>
<th>MALES</th>
<th>Age</th>
<th>Constant</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>1</td>
<td>-0.3772</td>
<td>-0.0156</td>
<td>4.4722</td>
</tr>
<tr>
<td>9</td>
<td>-2.3739</td>
<td>-0.0070</td>
<td>19.3342</td>
</tr>
<tr>
<td>10</td>
<td>-3.8969</td>
<td>-0.0153</td>
<td>31.0812</td>
</tr>
<tr>
<td>11</td>
<td>-3.6574</td>
<td>-0.0082</td>
<td>28.8191</td>
</tr>
<tr>
<td>11+</td>
<td>-6.4585</td>
<td>-0.0162</td>
<td>49.9780</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>FEMALES</th>
<th>Age</th>
<th>Constant</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>1</td>
<td>-0.7569</td>
<td>-0.0100</td>
<td>7.1948</td>
</tr>
<tr>
<td>9</td>
<td>-1.7421</td>
<td>-0.0191</td>
<td>14.9486</td>
</tr>
<tr>
<td>10</td>
<td>-6.9998</td>
<td>-0.0239</td>
<td>54.6813</td>
</tr>
<tr>
<td>11</td>
<td>-5.0559</td>
<td>-0.0123</td>
<td>39.7717</td>
</tr>
<tr>
<td>11+</td>
<td>-4.4184</td>
<td>-0.0014</td>
<td>34.4807</td>
</tr>
</tbody>
</table>

A similar analysis of the fecundity data was undertaken to determine which age classes showed density dependent variations. It was found that 2 groups of age classes showed marked variation; yield hinds aged 3 and yield hinds aged 10 and over. The variation was considered to be of the form shown below -

\[ F_{i,t} = a \ln N_t + b \]  

(Eqn. 4.14)

where \( F_{i,t} \) is the fecundity of age class \( i \) in year \( t \), \( N_t \) is the population size in late summer of year \( t \) and \( a \) and \( b \) are constants, whose values are given in Table 4.2.
Table 4.2  
Age Specific Fecundity Constants (Beddington 1973)

<table>
<thead>
<tr>
<th>Age</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>-6.2886</td>
<td>47,856</td>
</tr>
<tr>
<td>10</td>
<td>-8.5351</td>
<td>64,953</td>
</tr>
<tr>
<td>10+</td>
<td>-9.0083</td>
<td>68,554</td>
</tr>
</tbody>
</table>

Thus where Pennycuick et al (1968) utilised arbitrary density dependent relationships, Beddington obtained specific density dependent relationships for the Rhum herd. We thus have the framework which will allow us to study the effects of changes in the population parameters of a specific red deer population.

The Effects of Variation of Population Parameters on Population Productivity.

In the following sections we shall examine the sensitivity of the population growth rate to changes in the density dependent population parameters, in order to establish which age classes exert a dominant influence upon herd productivity and whether this is mediated through mortality or through fecundity. The effects of the other main factor controlling population size in Rhum, viz. rainfall, are also examined.

Density Dependent Fecundity

The following table (4.3) shows the fecundity schedule obtained for the Rhum herd (Lowe 1969).
Table 4.3  

Mean Fecundity Rates (1957-1966)

<table>
<thead>
<tr>
<th>Age (Years)</th>
<th>Lactating</th>
<th>Proportion Breeding</th>
<th>Yeld</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>-</td>
<td>0.42</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.50</td>
<td>0.86</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.35</td>
<td>0.90</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.44</td>
<td>0.95</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.67</td>
<td>0.95</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.47</td>
<td>0.93</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.54</td>
<td>0.99</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.50</td>
<td>0.77</td>
<td>0.82</td>
</tr>
<tr>
<td>11</td>
<td>0.38</td>
<td>0.82</td>
<td></td>
</tr>
<tr>
<td>11+</td>
<td>0.38</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As we have already seen, those age classes in parentheses above were subject to density dependent variations, the equations for which are given in Table 4.2. The general shape of these fecundity rate curves is illustrated in Figure 4.8.

Figure 4.8  

Fecundity v's Population Density

Loge Population Size
FR_{m,i} is the maximum attainable fecundity rate of yield hinds aged i, N_{0,i} is the total population density above which fecundity for age class i begins to decline, and N_{m,i} is the density above which reproduction ceases.

Density Dependent Survival

The survival data obtained on Rhum is shown below in Table 4.4 (Lowe 1969).

<table>
<thead>
<tr>
<th>Age (Years)</th>
<th>Stags Proportion Surviving</th>
<th>Hinds Proportion Surviving</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[.89]</td>
<td>[.90]</td>
</tr>
<tr>
<td>2</td>
<td>.97</td>
<td>.98</td>
</tr>
<tr>
<td>3</td>
<td>.99</td>
<td>.99</td>
</tr>
<tr>
<td>4</td>
<td>.99</td>
<td>.99</td>
</tr>
<tr>
<td>5</td>
<td>.99</td>
<td>.99</td>
</tr>
<tr>
<td>6</td>
<td>.97</td>
<td>.99</td>
</tr>
<tr>
<td>7</td>
<td>.97</td>
<td>.96</td>
</tr>
<tr>
<td>8</td>
<td>.97</td>
<td>.96</td>
</tr>
<tr>
<td>9</td>
<td>[.98]</td>
<td>[.92]</td>
</tr>
<tr>
<td>10</td>
<td>[.91]</td>
<td>[.85]</td>
</tr>
<tr>
<td>11</td>
<td>[.81]</td>
<td>[.88]</td>
</tr>
<tr>
<td>11+</td>
<td>[.65]</td>
<td>[.85]</td>
</tr>
</tbody>
</table>

Those age classes subject to density dependent variation are shown in parentheses. The relationship between survival rate and population density for these age classes is shown below in Figure 4.9. SR_{m,i} is the maximum attainable survival rate of animals aged i, N_{0,i} is the population level above which survival declines from the maximum and N_{m,i} is the level at which there is no survival of animals aged i.
Density Dependence and Population Growth Rates

The effect of the various density dependent relationships upon the growth of the population was examined using the model of equation 4.12. Because the density dependent equations derived from the Rhum population were obtained from a population which had been artificially restricted by culling, these equations are only valid for the same boundary conditions. We do not know what values would have been obtained had the population been allowed to find its natural level. Our study must therefore be based upon the model which includes the cull rates and limits the population size to those levels observed on Rhum. The following Table 4.5 illustrates the average cull rates utilised during the period 1957-66 in Rhum.
Table 4.5 Average Cull Rates 1957-66

<table>
<thead>
<tr>
<th>Age (Years)</th>
<th>Proportion Surviving Cull</th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>.99</td>
<td>.99</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>.99</td>
<td>.96</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>.97</td>
<td>.90</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>.97</td>
<td>.88</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>.89</td>
<td>.78</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>.85</td>
<td>.82</td>
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<td>7</td>
<td></td>
<td>.80</td>
<td>.83</td>
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<tr>
<td>8</td>
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<td>.72</td>
<td>.70</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td>.74</td>
<td>.84</td>
</tr>
<tr>
<td>11+</td>
<td></td>
<td>.71</td>
<td>.84</td>
</tr>
</tbody>
</table>

1. No Density Dependence

The first set of calculations was carried out with the constant fecundity and survival rates shown in Tables 4.3 and 4.4. This resulted in an exponential increase in numbers (see Figure 4.10a). In order to facilitate comparisons of the various density dependent survival rates, a constant rainfall value of 28.75 ins. was utilised (this is the mean summer rainfall for the period 1958-75). With fecundity and survival rates density and rainfall dependent, the population growth pattern obtained was that shown in Figure 4.10b. The density dependent fecundity and survival equations were then tested separately and in a number of combinations and their effects upon population numbers were noted.

2. Density Dependent Survival

If only the survival rates are made density dependent, the
Fig. 4.10  Population growth pattern - density effects

- a - no density dependence
- b - survival & fecundity density dependent

Fig. 4.11  Population growth pattern - density effects.
No density dependent fecundity.
equilibrium level is considerably higher than that of 4.10b, stabilising at approximately 2300. If the survival rate of only the one year old animals is made density dependent, equilibrium occurs at a higher level. However, if this is done for the survival of only the mature animals, an exponential growth occurs and stability is not achieved (see Figure 4.11, (1)-(3)).

3. **Density Dependent Fecundity**

When only the fecundity of the relevant age groups is made density dependent, the population rapidly attains its equilibrium level of approximately 1930 animals (Figure 4.12, (1)). This is almost identical to the level attained when all density dependent relationships operate (see Figure 4.10b). If only the fecundity of the 3 year old yeld hinds is made density dependent, the equilibrium level attained is 1927 animals. If only the yeld hinds aged 10 and over are subjected to density dependence, population growth is exponential and does not reach stability (see Figure 4.12 (2) and (3)). From these growth patterns we see that the main factor affecting population growth, at the densities studied on Rhum, is the fecundity of the first breeding yeld hinds.

4. **Rainfall Dependent Survival**

It was shown in equation 4.13 that the survival rates of 1 year olds and animals aged 9 and over was not only dependent upon population density but also on the summer rainfall (June-September). In the above calculations this had been given a constant value.
Fig. 4.12  Population growth pattern - density effects. No density dependent survival.

Fig. 4.13  Population growth pattern - rainfall effects. Fecundity & survival density dependent.
However, as rainfall varies from year to year, it is necessary to examine the effect of this variation on the population growth patterns.

From the data available (see Appendix B) summer rainfall was assumed to vary according to a normal distribution with mean 28.75 in. and standard deviation 6.91 in. Rainfall values were randomly generated using the above parameters and the following population projection was obtained (see Figure 4.13). It can be seen that as the population is growing the effect of variations in the level of rainfall is dominated by the density dependent effect. Thus until the 12th year as the population approaches its maximum, there is little difference between the growth curves for constant and varying rainfall. Thereafter the rainfall effect is dominant and changes in the population size are governed by it. A further five sets of randomly generated rainfall values were utilised with the model and in each case a similar population growth pattern was recorded.

Conclusions

From the analysis of the density dependent schedules, we can see that the young animals have the greatest controlling effect upon the population growth rate; of these, it is the density dependent fecundity rates which dominate. In addition, fluctuations in summer rainfall levels only cause significant variations in population size when the equilibrium level is attained. In the following section, we shall see how the actual productivity of the herd is affected by these density dependent parameters.
Population Growth and Herd Productivities

The effect of density on birth and death rates was illustrated in Figures 4.8 and 4.9. At lowest densities births and deaths per unit of breeding stock are at their maximum and minimum respectively. As the density increases due to the positive net differences between birth and death rates, birth rates decrease slowly and death rates increase. The increase in density is slowed down and when births and deaths per unit of breeding stock are equal, the population stops growing. The interaction between population size, birth rate and death rate gives a dome shaped yield curve. Thus at lowest densities, although net production per unit of breeding stock is at its highest, the net number of animals produced by this population is small because of its small size. As the size of the breeding stock increases and the net production per unit decreases a point is reached at which the breeding stock size and net production rate combine to give a maximum net input of young into the population. Further increases in density result in rapid decreases in the net production rate and the net input of animals produced by the population declines from the maximum value.

We have already seen the population growth curves which resulted from the survival and fecundity being subjected to density dependence (Figure 4.10b). We now have the conceptual framework which integrates birth rates, death rates, population density and yield. It now remains to be seen how yield varies with population density for the Rhum herd.
In order to calculate the net productivity of the herd at various population densities, we must determine the rate of increase of the herd. This is given by the dominant eigenvalue of the matrix -

\[ M = M_b M_n \]  \hspace{1cm} (Eqn. 4.15)

The method by which the eigenvalues are calculated is illustrated in Appendix C.

Figure 4.14 shows the relationship between population density and the rate of increase for a constant value of rainfall. The population densities for which these rates are calculated were restricted to the range given in Figure 4.10b i.e. between approximately 1500 and 2000 animals. As we noted earlier, it is necessary to restrict the population density to within the boundary conditions for which the fecundity and survival schedules are valid. The net productivity of the herd is given by

\[ H = \left[ \frac{\lambda - 1}{\lambda} \right] N \]  \hspace{1cm} (Eqn. 4.16)

where \( \lambda \) is the calculated rate of increase and \( N \) is the total summer population size.

Figure 4.15 illustrates the relationship between net productivity and population density. From this it appears that maximum productivity occurs between 1700 and 1750. A more detailed examination revealed that maximum productivity of 278 is obtained at a population density of 1724. This is more than 200 below the equilibrium population level; thus the effects of varying rainfall upon the cropping strategy should be negligible.
Fig. 4.14  Population size v's rate of increase of the population

Fig. 4.15  Population size v's yield
Thus far we have seen how the population density affects the growth rate of the herd and its net productivity. In the following chapter we shall examine some of the factors which affect reproduction and survival of red deer both on Rhum and in other areas.
Chapter 5

Management Strategies for Increased Productivity in the Wild

In this chapter we summarise some general observations in the natural mortality and fertility among a number of red deer populations in Scottish deer forests and examine their implications for increased productivity on the estates. We examine how modifications to the threshold levels, $N_{o,i}$ and $N_{m,i}$, affect optimal stocking rates and how modifications to the age structure of the herd influence the harvest. Our prime objective is to define a boundary of permissible cull rates which can be achieved in the wild, outlining the areas which have the greatest impact upon productivity. Finally, the production function generated in Chapter 3 is re-examined in light of the above analysis of productivity.

Factors Affecting Reproduction and Survival in Red Deer Populations

Characteristics of Natural Mortality

From studies carried out in a number of areas in Scotland, including Rhum, it appears that there is no obvious areal variation in the kind of natural mortality suffered by red deer (Mitchell et al. 1971(a)). Apart from accidental deaths - avalanches, falls in rough terrain and entanglement in loose wire - most natural deaths affecting all age classes occur between mid-January and late April. In addition, the effects of predators such as foxes and eagles are negligible.

As we have already seen (Table 4.4) on Rhum most natural mortality occurs among calves and the upper age classes and least in the period of early sexual maturity. Average annual mortalities
amount to 3-5% of adults and 10-12% of calves. One of the most striking characteristics of natural mortality among adults is the high incidence of dental abnormalities, which render it difficult for the animals to feed normally (op. cit.). Thus when no soft plant material is available these animals cannot cope with the more resistant plant tissues and subsequently their condition deteriorates. Diseased jaw bones, caused by infection during premolar replacement, are also common in natural deaths among adults. Both the above conditions were much in evidence among poor quality animals shot during the autumn. It appears, therefore, that one of the major causes of death is due to physiological ageing, which is aggravated by the scarcity of suitable plant material during winter. As far as calves are concerned, low fat reserves during the first winter of life account for the high mortality rates. The fat reserves are much lower and the natural mortality rate much higher in calves than in animals of either sex up to about 10 years of age (Mitchell et al 1976).

Exceptionally heavy mortalities have occurred on Rhum and in other areas during periods of adverse weather conditions (Mitchell et al 1971(a); Mitchell & Staines 1976). We have already noted that on Rhum natural mortality was closely related to the level of summer rainfall. The periodicity of growth and food intake also contributes to the unfavourable effect which heavy rainfall during summer has upon the natural mortality of the deer. During the summer months food intake is increased considerably in order to build up fat reserves for the following winter. If, however, these energy reserves have to be utilised before the onset of
winter, as is the case during very wet summer weather, the probability of survival during winter is decreased.

The situation is aggravated in the case of stags, which expend a tremendous amount of energy during the rut while reducing their normal food intake to almost nil. During this period they lose large amounts of their fat reserves and any deterioration in condition is thus made more acute. A marked reduction of intake has also been found in captive stags, isolated from hinds, during the period from November to March, even when given access to unlimited amounts of high quality fodder (Blaxter et al 1974). Such a reduction also occurs for hinds (Mitchell et al 1976). Thus the weight loss and deterioration in condition shown by deer in the wild may be governed as much by inappetance as by the deficiencies in the available forage.

Heavy snow falls also have an adverse effect upon the survival of the deer by hindering the progress of the animals to feeding areas and rendering shrubs and grasses inaccessible. In overcoming such difficult conditions the deer incur a negative energy balance, that is to say, the extra effort needed to move through deep snow and forage successfully cannot be balanced by the food intake. If these conditions persist for any length of time the effects on survival can be catastrophic (Watson 1971).

As far as infectious diseases and parasites are concerned, it appears that they do not play any significant part in the mortality of the red deer in Scotland (Mitchell et al 1976).
Reproductive Performance

The principle that the dynamics of an animal population are dependent upon the availability and quality of foodstuffs has been long established. For more than 100 years the provision of supplementary fodder during winter has been a fundamental husbandry technique for large game animals both in this country and abroad. The theory behind such a practice was that the herd size was largely dependent upon mortality which in turn was a direct consequence of a shortage of suitable fodder during the winter months. More recent research, however refutes the above theory (Jezierski 1976). As the preceding analysis of the Rhum herd indicated, it is not the mortality which acts as the controlling force under normal circumstances, but the fecundity of the hinds.

This does not mean that supplementary feeding does not have an important role to play in determining the growth rate of the red deer population. It has been shown that the probability of a hind breeding successfully is directly related to its weight and condition (Mitchell et al 1971; Mitchell & Brown 1974). Thus the provision of supplementary fodder, by improving the condition of the animal, increases the productivity of the herd. The effects of such a policy on the growth of the population are not however infinite. At some point during the population growth process intrapopulation regulation mechanisms come into play and the rapid growth in numbers declines.

Whether a hind conceives during the rut depends largely on her
body weight and condition and it is not possible to separate the effects of these two factors because of their high correlation (Mitchell 1973). The lower pregnancy rate in lactating hinds than yeld hinds can be explained by these factors, although other factors may be involved. Differences of body weight and condition between the two classes of hinds may be attributed to the consequences of lactation and pregnancy. The fact that differences between yeld and lactating hinds of comparable age and physiological characteristics are lowest immediately after the period of parturition and thereafter increase in favour of yeld hinds, indicates that lactation is the main controlling factor. Since pregnant hinds become lactating hinds and non-pregnant hinds become yeld hinds after the parturition period, it appears from the results that a large proportion of mature hinds are lactating one year and yeld the next (op. cit.). The existence of yeld hinds in any great numbers in a population is indicative of a poor nutritional environment (Mitchell et al 1976).

Management Implications of Reproductive and Mortality Characteristics

The primary objective of the game manager should be to stabilise the population size at an "optimal" level depending upon the objectives of the enterprise. Since the overall performance and productivity of a red deer population can be manipulated through its age and sex composition, data on fecundity and survival will hold certain implications for the game manager. Thus if meat production is the primary management objective, a short age distribution of hinds must favour a high rate of production of calves. Selective culling fulfils a dual function by adjusting
the population to suit the objective of high meat production; those animals most likely to die of natural causes are eliminated first. The characteristics of natural mortality indicate where such efforts should be concentrated. Although in the short term such policies will produce poorer quality carcasses, continued selective shooting is likely to upgrade the quality progressively. As far as supplementary feeding is concerned, no general conclusions can be drawn; each case must be regarded on its own merits. We can, however, examine the effects which such programmes might have upon herd productivity. This we shall do in the following sections.

**Modifications to Density Dependent Relationships**

In Figures 4.8 and 4.9 we illustrated the relationships between fecundity and survival rates and population density. The threshold levels \( N_{o,i} \) and \( N_{m,i} \) were respectively defined as those population densities above which fecundity and survival rates decline from their maximum and cease altogether. In this section we examine the effects which increasing the upper threshold levels \( N_{o,i} \) of various age classes has upon herd productivity.

1. \( N_{o,3} \) (fecundity) increased.

The threshold level, \( N_{o,3} \), was increased from the original 1720 animals to the upper boundary of 2000 animals. The effect of this on the yield curve is shown in Figure 5.1(2). Maximum crop is increased from the original 279 to 291 animals at an "optimal" population density of 1830. Thus we can see that the
Fig. 5.1  The effects upon yield of increases to the threshold levels of density dependence.

<table>
<thead>
<tr>
<th>Yield Curve</th>
<th>Increased Threshold Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$F_Y_3$, $S_Y_1$, $S_M_1$</td>
</tr>
<tr>
<td>2</td>
<td>$F_Y_2$</td>
</tr>
<tr>
<td>3</td>
<td>$S_Y_1$, $S_M_1$</td>
</tr>
<tr>
<td>4</td>
<td>$F_Y_{10-15}$, $S_Y_{9-15}$, $S_M_{9-15}$</td>
</tr>
<tr>
<td>5</td>
<td>None</td>
</tr>
</tbody>
</table>
other density dependent relationships dominate at densities above 1830.

2. $N_{0.3}$ (fecundity) $N_{0.1}$ (survival) increased

Both threshold levels of calves (survival) and 3 year old hinds (fecundity) were once again increased to 2000 animals. Although the maximum permissible crop was increased to 301 animals, the optimal population level was raised by only 18 to 1848 (Figure 5.1(1)). Thus we can see that it is the negative effect of the density dependent relationships of the mature animals which reduce the available crop. It should also be noted that there has only been a marginal increase in culling % age from 16.1 to 16.3 as a result of the increase in threshold level $N_{0.1}$ (survival). Productivity is increased largely as a result of increased "optimum" population level. By removing the mature animals from the population structure, we may reduce the negative effect of the density dependent survival and fecundity relationships upon the permissible crop. Optimal population levels will then be determined solely by the characteristics of the younger animals. There is a further advantage to be gained from such a strategy, which we shall examine in the following section.

**Modification of Herd Structure as a Means of Increasing Productivity**

Research carried out both in this country and on the continent (Dzięciotowski 1969; Mitchell et al 1971) shows that red deer increase their body weight up to the 9th year of life at which stage, maximum weights are achieved; thereafter they suffer a decline in weight. Figure 5.2 illustrates the relationship
Fig. 5.2 Growth rates of red deer in the wild.

- Stag (Poland)
- Hinds (Poland)
- Stag (Scotland)
- Hinds (Scotland)
between carcase larder weight and age for stags and yeld hinds (op. cit.) Dzięciotowski obtained a similar relationship for a number of red deer herds in Southern Poland (see equations 5.1a & b).

**Stags**

\[ \text{Weight} = 45.28 + 16.79t + 1.019t^2 - 0.1532t^3 \]  
(Eqn. 5.1(a)

**Hinds**

\[ \text{Weight} = 44.45 + 12.06t - 0.8986t^2 + 0.0127t^3 \]  
(Eqn. 5.1(b)

\( t \) = age in years

We shall therefore examine the effect upon the venison output of culling all animals in their 9th year. The rate of growth of the herd with this modified age structure is calculated as before (see Chapter 4) and used to determine the proportional crop taken from each age group 1 to 8. From Figure 5.3 we can see that the cropping pattern in terms of total carcase numbers, is very similar to that obtained when the threshold levels, \( N_{0,i} \), of the mature animals were raised to 2000 (see Figure 5.1(4)). The output in terms of carcase weights, however, is considerably higher (11%), as can be seen in Table 5.1 below:

<table>
<thead>
<tr>
<th>Age (Years)</th>
<th>Output (kg)</th>
<th>Original</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>524</td>
<td>716</td>
</tr>
<tr>
<td>2</td>
<td>657</td>
<td>878</td>
</tr>
<tr>
<td>3</td>
<td>754</td>
<td>941</td>
</tr>
<tr>
<td>4</td>
<td>761</td>
<td>912</td>
</tr>
<tr>
<td>5</td>
<td>752</td>
<td>898</td>
</tr>
<tr>
<td>6</td>
<td>639</td>
<td>765</td>
</tr>
<tr>
<td>7</td>
<td>653</td>
<td>653</td>
</tr>
<tr>
<td>8</td>
<td>545</td>
<td>545</td>
</tr>
<tr>
<td>9+</td>
<td>3763</td>
<td>1836</td>
</tr>
<tr>
<td>Total (kg)</td>
<td>9048</td>
<td>8144</td>
</tr>
</tbody>
</table>

Table 5.1 Venison Output for Modified (1) and Original (2) Age Structures
Fig. 5.3 The effects upon yield of modifying the population structure

1 - Original yield curve

2 - Yield curve obtained when all 9-yr olds are removed annually and a proportion of the remaining age classes, determined by the population growth rate, is harvested.
The above productivities are calculated per 1000 animals.

The problem of optimal cropping strategies in the wild has been investigated in some detail by Beddington (1974). However, in a system in which the identification of specific age classes is well nigh impossible, a more sophisticated selective cropping policy may prove to be impracticable.

We noted earlier in this chapter that breeding success rates were related to the body weight and condition of the hinds. Thus if 2 year old yield hinds can achieve a sufficiently high condition, they too may breed successfully (Mitchell et al 1971; Blaxter et al 1974). In the following section we shall quantify the effects of such a modification to the age of puberty upon the growth rate of the population.

**Modifications to the Age of First Breeding**

In order to examine the effects of such a measure, a range of fecundity values from 0.30 to 0.90 was assigned to the 2 year old yield hinds. The fecundity of the 3 year old yield hinds was set at 1.00 and a further range of fecundity values from 0.60 to 0.90 assigned to the 3 year old lactating hinds. The following table illustrates the maximum permissible cull rates.

<table>
<thead>
<tr>
<th>Fecundity Rate 3 year Lactating Hinds</th>
<th>0.60</th>
<th>0.70</th>
<th>0.80</th>
<th>0.90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fecundity Rate 2 yr Yeld Hinds</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.30</td>
<td>17.5</td>
<td>17.5</td>
<td>17.6</td>
<td>17.7</td>
</tr>
<tr>
<td>0.45</td>
<td>18.0</td>
<td>18.1</td>
<td>18.3</td>
<td>18.4</td>
</tr>
<tr>
<td>0.60</td>
<td>18.5</td>
<td>18.7</td>
<td>18.9</td>
<td>19.1</td>
</tr>
<tr>
<td>0.75</td>
<td>19.0</td>
<td>19.3</td>
<td>19.5</td>
<td>19.8</td>
</tr>
<tr>
<td>0.90</td>
<td>19.6</td>
<td>19.9</td>
<td>20.2</td>
<td>20.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fecundity Rate 3 yr Yeld Hinds</th>
<th>1.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.60</td>
<td>19.0</td>
</tr>
<tr>
<td>0.75</td>
<td>19.5</td>
</tr>
<tr>
<td>0.90</td>
<td>19.8</td>
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<tr>
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<tr>
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<tr>
<td>0.90</td>
<td>19.8</td>
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<td>19.5</td>
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<td>0.90</td>
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<td>19.5</td>
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<tr>
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<tr>
<td>0.90</td>
<td>19.8</td>
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<tr>
<td>0.90</td>
<td>19.8</td>
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<tr>
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<td>19.5</td>
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<tr>
<td>0.90</td>
<td>19.8</td>
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<tr>
<td>0.75</td>
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<tr>
<td>0.90</td>
<td>19.8</td>
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</tr>
<tr>
<td>0.75</td>
<td>19.5</td>
</tr>
<tr>
<td>0.90</td>
<td>19.8</td>
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</tbody>
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<table>
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<tbody>
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</tr>
<tr>
<td>0.75</td>
<td>19.5</td>
</tr>
<tr>
<td>0.90</td>
<td>19.8</td>
</tr>
</tbody>
</table>
Net productivity on the estate will, of course, be determined by both the permissible cull rate and the optimal density as we saw in Chapter 4. Thus although the maximum rate illustrated above is 20.5%, it can only serve as a boundary condition for cropping rates and need not necessarily be the rate adopted for maximum productivity.

Any improvement in the condition of the younger animals is unlikely to be restricted to these age groups alone; the effects would be carried through, to some extent, as the animals aged. We have already seen that survival rates for animals aged 2 to 8 are consistently high as are the fecundity rates of yeld hinds aged 4 to 9. Thus only the fecundity of the lactating hinds leaves any scope for further improvement. To what extent this can be achieved without some reinforcement through further supplementary feeding, we cannot say. At this stage, however, we are concerned with the magnitude of the effects of any increase in the fecundity of lactating hinds. The benefits can then be quantified and used as a guide for any investment in supplementary feeding.

**Increase in Fecundity of Lactating Hinds**

The fecundity rates of lactating hinds were increased in steps of 25% up to 100% and the increase in permissible culling rates noted. The following Table 5.3 illustrates the original and improved fecundity rates together with the corresponding maximum permissible cull rates. As a comparison, the corresponding yeld fecundity rates are included.
Table 5.3

<table>
<thead>
<tr>
<th>Fecundity Rate</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>Cull Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>0.50</td>
<td>0.35</td>
<td>0.44</td>
<td>0.67</td>
<td>0.47</td>
<td>0.164</td>
</tr>
<tr>
<td>+ 25%</td>
<td>0.63</td>
<td>0.44</td>
<td>0.55</td>
<td>0.84</td>
<td>0.59</td>
<td>0.172</td>
</tr>
<tr>
<td>+ 50%</td>
<td>0.75</td>
<td>0.53</td>
<td>0.66</td>
<td>1.00</td>
<td>0.70</td>
<td>0.180</td>
</tr>
<tr>
<td>+ 75%</td>
<td>0.88</td>
<td>0.61</td>
<td>0.77</td>
<td>1.00</td>
<td>0.82</td>
<td>0.187</td>
</tr>
<tr>
<td>+ 100%</td>
<td>1.00</td>
<td>0.70</td>
<td>0.88</td>
<td>1.00</td>
<td>0.94</td>
<td>0.195</td>
</tr>
<tr>
<td>Yeld</td>
<td>0.86</td>
<td>0.90</td>
<td>0.95</td>
<td>0.95</td>
<td>0.93</td>
<td>-</td>
</tr>
</tbody>
</table>

The modified maximum cull rate is increased by approximately 5% for each 25% increase in the fecundity. Comparing the modified figures (+100%) with those of the corresponding yeld hinds, we can see that such improvements are well within the potential of the animals. Indeed, fecundity rates of lactating hinds similar to those above (+100%) have been observed at a number of sites in Scotland (Mitchell et al 1971 ). The following Table 5.4 illustrates the effects of the aggregated improvements upon the cull rates. It should be noted again, however, that the actual cull rate will be influenced by any density dependent fecundity or survival relationships. These figures do, however, give some indication of the limit of culling rates on estates in Scotland.
Table 5.4  
Overall Increased Culling Rates (%) 

<table>
<thead>
<tr>
<th>$FY_2$</th>
<th>$FL_3$</th>
<th>Increases in $FL_4 - FL_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.60</td>
<td>0.70</td>
</tr>
<tr>
<td>0.30</td>
<td>18.4</td>
<td>18.4</td>
</tr>
<tr>
<td>0.60</td>
<td>19.4</td>
<td>19.6</td>
</tr>
<tr>
<td>0.90</td>
<td>20.6</td>
<td>20.9</td>
</tr>
<tr>
<td>0.30</td>
<td>19.2</td>
<td>19.2</td>
</tr>
<tr>
<td>0.60</td>
<td>21.1</td>
<td>21.3</td>
</tr>
<tr>
<td>0.90</td>
<td>21.5</td>
<td>21.8</td>
</tr>
<tr>
<td>0.30</td>
<td>20.0</td>
<td>20.0</td>
</tr>
<tr>
<td>0.60</td>
<td>21.1</td>
<td>21.3</td>
</tr>
<tr>
<td>0.90</td>
<td>22.3</td>
<td>22.7</td>
</tr>
<tr>
<td>0.30</td>
<td>20.8</td>
<td>20.8</td>
</tr>
<tr>
<td>0.60</td>
<td>22.0</td>
<td>22.2</td>
</tr>
<tr>
<td>0.90</td>
<td>23.3</td>
<td>23.7</td>
</tr>
</tbody>
</table>

$FY_i$ = Fecundity rate i year old yearling hinds  

$FL_i$ = Fecundity rate i year old lactating hinds  

Conclusions  

Although the above analysis of productivity was based, in the main, on the performance of one population, i.e. the Rhum herd, the red deer on the island are not atypical in terms of body weight and reproductive performance of Scottish red deer (Mitchell 1973). Thus any conclusions we reach on the basis of this study will have implications for red deer populations in general.
We have shown that the fecundity threshold level of the first breeding hinds plays the major role in determining net productivity. Survival of calves was relatively insensitive to changes in population density under the conditions prevailing. These conclusions are corroborated to a great extent by the results of research on several deer herds (including deer other than C. Elaphus) abroad (O'Roke et al 1948; Halloran 1962; Murphy 1962; Jezierski 1976). In these studies survival rates were seen to change very little with increasing density. The population size was regulated by changes in fecundity. It was seen that lower threshold \( N_{m,i} \) depended not only upon the availability and quality of fodder but also on various social factors. Overcrowding, even in areas of high food quality and availability, was seen to have a detrimental effect upon the breeding performance of the animals. The upper threshold level, \( N_{o,i} \), allowed the greatest scope for improvement in productivity (Gross 1972), as the results of our analysis of the Rhum herd have shown. Because of the sensitivity of the system productivity to changes in the breeding success rate of the first breeding hinds, it is important that their fecundity is maintained at a high level. On Rhum, it was the upper threshold level, \( N_{o,3} \), of the first breeding hinds which determined the optimal stocking density. By the same token, reducing the age of first breeding resulted in a productivity increase of up to 25%. This can be compared with an increase in productivity of 19% which resulted when the fecundity of all lactating hinds was increased by 100%. However, as we noted, these improvements should not be regarded in isolation from each other. Measures taken to enhance productivity are carried over as the animals age.
The removal of animals aged 9 and over from the population serves a dual purpose. Not only do we increase the absolute quantities of venison which may be produced, but we also remove those animals most sensitive to adverse changes in food availability and population density. In the transitional period from one age structure to the other, productivity will be reduced. However, in the long term, both quantity and quality of venison output will be increased.

Of the improvements we have examined above, only the selective culling policy does not have any cost penalty associated with it, assuming, of course, that labour productivity is not adversely affected by the imposition of such a constraint. The interdependence of the other improvements precludes their being regarded in isolation when considering strategies which are of most benefit to the system as a whole. Certain observations can, however, be made regarding the provision of supplementary fodder. Feeding should be carried out on a selective basis with the youngest animals being given the greatest attention followed by the hinds. The advantages of this are twofold. Firstly, the age of puberty may be reduced, and, secondly, the carryover effect is spread over a longer age span. Selective feeding in the wild may be accomplished by constructing feeding troughs which exclude antlered animals, thereby allowing calves and hinds to feed without interference. (Research has shown that if fodder is supplied freely to all classes, stags dominate and actively discourage hinds and calves from feeding (G.J. Wiersema 1974))
Further size restrictions can limit feeding to calves alone (Dziegielewski 1973). There is, of course, one further advantage to be gained from supplementary feeding, namely an increase in the carcase weight.

The decision to undertake any measures to improve productivity must, of course, be made on the basis of expected costs and benefits. In equation 3.7 profit was given as -

\[ \Pi = MC - PL \]

where \( C \) was given by equation 3.1

\[ C = K L^{a_1} p^{a_2} A^{a_3} \]

If we assume that the aim of an estate is to keep the population size constant at the optimal density, the crop will be a certain proportion of the population level. Thus rewriting equation 3.1 we obtain -

\[ C = z P = KL^{a_1} p^{a_2} A^{a_3} \quad (Eqn. 5.2) \]

This can be rewritten so that -

\[ L = \left( \frac{zp^{1 - a_2}}{KA^{a_3}} \right)^{1/a_1} \quad (Eqn. 5.3) \]

which defines the manning level required to maintain a constant proportional cull \( z \) for a given area and population. Incorporating the above relationship into the profit equation we obtain:

\[ \Pi = M \cdot z P - PL \left[ \frac{zp^{1 - a_2}}{KA^{a_3}} \right]^{1/a_1} \]

This relationship between profit, culling rate, estate size and
population density is illustrated in Figures 5.4 - 6. As the estate area increases, the profit increase per unit of population decreases. The effect of increasing culling rates, $z$, as a consequence of increased herd growth rate, affords only a small increase in profit - 10% for a 33% increase in cull rate - and this only on the smaller estates (<40,000 acres). On the other hand, increasing stocking density provides for an increase in profit across the whole range of estate areas. However, any measures taken with the specific intention of increasing optimal stocking densities are likely to have added implications for herd growth rates. On the smaller estates, this is, in fact, a desirable concomitant. On those estates, however, where the environmentally dictated cull rate is above the economically optimal rate, any increase in herd growth does not directly increase profitability. This only occurs at high stocking densities (see Figures 5.5 and 5.6).

In the previous chapter we described the relationship between stocking density, population growth rate and net productivity. Using this model we were able to calculate the environmentally optimal stocking density and corresponding culling rate for the Rhum herd. As we have seen above, it is this rate which determines the upper limit of the permissible crop on the smaller estates. It is, however, difficult to establish such optimal stocking densities and cropping policies with any degree of accuracy on the estates. As we have already seen this requires considerable data on density dependent relationships. Thus any management strategies
Fig. 5.4 Estate characteristics & profit.

PROFIT $000's

AREA $000's Acres

Stocking Density

- 15% Cull
- 20% Cull

3.9
3.3
3.0

3.9
3.3
3.0
**Fig. 5.5** Culling rates & profit.

Stocking Density 3:100

Culling Rate

- 15%
- 20%
- 25%

Profit (in 1000's)

AREA (in 000's of Acres)

**Fig. 5.6** Culling rates & profit.

Stocking Density 6:100

Culling Rate

- 20%
- 15%
- 25%

Profit (in 1000's)

AREA (in 000's of Acres)
must always involve a high degree of risk due to uncertainty. Fluctuating venison prices also introduce further areas of uncertainty into the system; we saw in Chapter 3 how the economically optimal cropping policy varied with venison price. The result of this is that estates tend to adopt a "safe" policy of cropping from the environmental point of view. That is to say, rather than over-cull the population, they are culling below what is recognised as the average environmentally regulated cull rate. Average cull rates during the period 1970-75 were between 12%-13% (R.D.C.)

However, leaving aside the above sources of uncertainty, one other basic property of the wild deer system mitigates against the adoption of a more intensive approach to venison production in the wild. Because the animals are generally free to move between estates, there can be no guarantee that the benefits from any measures undertaken on an estate to increase herd productivity will accrue to that particular estate. On larger estates this may not present such a problem if the total range of the herd/s is contained within a single estate. However, on the smaller estates, deer herds may be "shared" by neighbouring estates and without some form of co-operation there is no incentive for a manager to invest in supplementary feeding without the certain knowledge that he alone will benefit.

This difficulty may be overcome if the estate can achieve a high degree of control over all the production inputs, which includes the containment of the population. To make this transition from the system of culling in the wild to that of intensive
husbandry under controlled conditions, a new "technology" must be developed. Because of the problems associated with the acquisition of new production techniques, the decision was made in 1970 to establish an experimental deer farm under the auspices of the Hill Farming Research Organisation (H.F.R.O) and the Rowett Research Institute in Aberdeen. In the following chapters we shall discuss the basic concepts underlying deer farming operations and examine the experimental research at Glensaugh and its implications for the viability of commercial deer farming in Scotland.
Chapter 6

The Glensaugh Project and the Development of Red Deer Farming

In the previous chapters we examined the circumstances which lead up to the evolution of intensive husbandry systems for the production of venison. We noted then, that in making a transition from one production system to another, a new "technology" had to be acquired. To this end, the experimental deer farm at Glensaugh was established. The choice of the site for the experimental farm was governed to a great extent by the fact that the H.F.R.O. already possessed a research facility at the Glensaugh farm. The actual area selected for the project was one of the remote hill grazings belonging to the Glensaugh farm. There were several advantages to be gained from the selection of this site, not least of which was its isolated position. In addition, the land had been previously owned by the North of Scotland College of Agriculture, thus records of previous stocking and management policies for a period of 30 years were available (Blaxter et al 1974). A view of the farm is shown in Plate 1.

The aim of this chapter is twofold. Firstly, we discuss the basic principles underlying the concept of deer farming and, secondly, we examine the operations at Glensaugh and discuss their implications for the establishment of commercial deer farming in general.

Stocking Density

In the previous chapter we saw how the need for control over the population density was a crucial factor in determining productivity.
The concept of carrying capacity as applied to a deer farm, however, must of necessity be different to that of a wildlife grazing system. In using the term carrying capacity, we are singling out one species in a community thereby assigning the others to a supporting role. As far as natural systems are concerned, such a term is misleading. On the farm, however, where the manager has a high degree of control over the natural relationships between the vegetation and the grazing species, the grazing species may become the prime determinant of the system as a whole. It is in this context that the term carrying capacity can be applied. Because different grazing systems involve different inputs and degrees of management, the process underlying carrying capacity will change. These processes may be illustrated by considering a number of grazing systems of varying intensity.

In a highly intensive grazing system, the inputs can be varied to give high levels of productivity of digestible nutrients. This can be effected by the sowing of specific herbage species in conjunction with the use of fertilisers or by controlling the grazing pressure on existing herbage. To allow the pasture plants to attain maturity, grazing pressure must be low during the growing season. Unless, however, the grazing pressure is subsequently increased, the feeding value rapidly declines. Dead material soon accumulates and dilutes the quality of forage intake. Close grazing, however, reduces the scope for selection and consequently the grazing pressure necessary to maintain pasture quality often results in a lowering of animal performance (T. Eadie 1970).

This conflict is expressed in the following diagram, Figure 6.1,
which provides a conceptual framework for considering carrying capacity. The advantage of the highly intensive system is that, in the short term, the vegetation may be heavily exploited. There is no need to utilise the animal in a non-productive way to maintain pasture quality when this can be done by the input of fertiliser and/or new seed.

Figure 6.1 Changes in Production per Animal and Production per Unit Area with Increasing Grazing Intensity.

In the case of a less intensive system, carrying capacity is expressed in animals per unit area. Management is accomplished principally by varying the number of animals and time of grazing to control plant growth. Thus the animal is not only the economic product, but also the principal management tool. Such a management system must be based upon a knowledge of the limitations which soil and climate exert upon the natural and semi-natural plant types, which exist on a site. Stocking rate influences the way in which
the available nutrients in the vegetation are utilised and determines the relationship between individual animal performance and output per unit area (Figure 6.1). Whereas in the highly intensive system the floristic composition could be regarded as static on the time scale involved, animals grazing the less intensive system are, in the long term, faced with an ever-changing forage source as they themselves alter the floristic composition. In such a dynamic vegetational system, it is impracticable to separate the long- and short-term effects of stocking density on the relationships between output per animal and output per unit area (I.A. Nicholson 1975).

Reconsidering the wild grazing system we can see that management is limited in the main, to the control of the population size by culling; the main objective being the maximisation of meat yields. The principal management implication of the optimum yield concept in the wildlife system (see Chapter 4) is that maximum annual production of young in most deer herds can be obtained by keeping population densities down below those which other management policies might dictate (e.g. maximise population size). We have noted, however, that such policies may, in the case of large estates, be in conflict with those dictated by economic considerations. Nevertheless, the optimum yield concept has significant implications for the management of the land resource. The foremost deer management problem is the maintenance of a balance between the population and its food supply. Population sizes which produce optimum yields are below densities where birth and death
rates equilibrate. Thus the optimum yield policy allows not only efficient management of the deer as a species but also of the natural or semi-natural vegetation (op. cit.).

**Stocking Density at Glensaugh**

In determining the stocking capacity of the experimental farm at Glensaugh, the final estimate relied upon general agricultural principles relating the herbage resources available to the herd's requirements. These varied with the number of males, breeding females, breeding rates, replacement rates, cull rates and growth rates of the fattening animals. The availability of forage during winter was seen to be the main constraint upon herd size, although this could be relieved by the supply of supplementary fodder. Ultimately, upper limits on herd size may be imposed by the occurrence of disease and general decline in herd productivity.

The carrying capacity at Glensaugh was estimated from data obtained when the land had carried sheep (5 acres per ewe for 8 months of the year). The final winter stocking of the farm was calculated using this data in conjunction with estimated fecundity and survival rates based on data obtained from feral herds. The herd structure is given below (Blaxter et al 1974) in Table 6.1.

<table>
<thead>
<tr>
<th></th>
<th>Stocking Policy at Glensaugh (winter)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Breeding Hinds</td>
<td>50</td>
</tr>
<tr>
<td>Growing &amp; Breeding Stage</td>
<td>15</td>
</tr>
<tr>
<td>Replacement Hinds</td>
<td>10</td>
</tr>
<tr>
<td>Calves &amp; Meat Stock</td>
<td>40</td>
</tr>
</tbody>
</table>

115 on 530 acres
Taming Stock

Central to the concept of an intensive grazing system is the requirement that the stock be amenable to the regular handling which such a policy entails. Whereas in the wild no attempt is made to control grazing, under the intensive system the animals must be herded from one grazing area to another, fed and slaughtered. Identification of individual animals is also greatly facilitated if there is a degree of tameness. The development of tame stock thus constitutes the first "technological" innovation in the venison production system. In this section we examine the taming programme undertaken at Glensaugh and its implications for deer farming in general.

The various methods used to tame the deer are described in detail in the first report of the H.F.R.O. and R.I. (Blaxter et al 1974). In general, these rely upon hand feeding of calves and continuous reinforcement by regular handling of all stock. These measures were not, however, undertaken solely as a taming programme, they were also intended to improve the breeding capability of the animals. It should be noted that a considerable amount of biological investigation of the herd was undertaken throughout the period of study, which undoubtedly acted as a reinforcing agent for the taming programme, which itself relied upon the familiarisation of the animals with the handlers and handling facilities. It is, however, impossible to quantify the effect as no control groups of animals were used.
Initial tameness, however, is not an end in itself, endurance from year to year in the animal and from generation to generation in its offspring is of paramount importance. This factor has been the object of intensive study on the farm at Glensaugh, the main findings of which are described below.

Newly born calves captured in the wild formed the nucleus of the experimental herd. These animals were all bottle fed and therefore came into constant contact with their handlers. As a result they proved to be the most tractable and presented no difficulties during handling in subsequent years. Experiences with second and third generation calves born on the farm have, however, been mixed, depending upon the time of weaning and degree of contact with the handlers. The sooner the calves were weaned and came into contact with their handlers, the more tractable they were. Without regular contact, even though weaned early and supplied with fodder throughout the first winter, calves proved troublesome. Thus it can be seen that early and frequent contact with humans is necessary if calves, which are suckled by their dams, are to be tamed. The degree to which stock at Glensaugh has accepted the presence of humans, even strangers, is illustrated in Plate 2, where the author, in the presence of other visitors to the farm, has no disturbing effect upon the stock. This is the case even when the hinds are suckling their young (Plate 3).

One problem which came to light on the farm and could have a significant influence on the stocking policy of a commercial farm,
concerns the introduction of new stock to the farm. It appears that up to the age of about 6 months new stock can be readily introduced and integrated into the herd, with little difficulty. Older animals, on the other hand, tend not to integrate and, even if tame at the outset, become very difficult to handle. Thus if any new stock is to be introduced to the farm, only animals of 6 months and younger should be considered.

As far as the tractability of the older animals is concerned, the hinds have presented little or no problems. Stags, however, have proved to be extremely unpredictable especially when the velvet is fraying off the antlers and before the rut. As a safety measure, the antlers are removed as soon as they have come out of velvet. In Plate 4 we can see the development of the antlers, which are formidable weapons, on stags aged 3 and 4 years. For this reason it is recommended that stags aged over 4 years are not kept on the farm (pers. comm. W.J. Hamilton). Any advantage which may be gained from the ability of these older stags to mate larger numbers of hinds must be weighed against the problems of controlling them.

Nevertheless, taming is not an end in itself. The objective of any such programme must be to improve the ability to control the stock and regulate grazing. In the following section we discuss grazing patterns and examine the policy adopted on the Glensaugh farm.

**Herd Management and Grazing Control**

All ranges are best maintained in good condition if grazing takes place at times when the physiological shock to the main forage plants
is at a minimum. The most critical periods for grasses and heather are usually at the start of growth when the plants are dependent on stored food reserves. However, under practical livestock range management it is not always possible to graze in complete accordance with the plant growth requirements because grazing during the "green feed" period is also optimal for animal production. For this reason, the intensity and the season of use should be adjusted so that the effects of grazing are not beyond the tolerance of the important forage plants.

It is theoretically possible by means of carefully managed, moderately heavy grazing to maintain a heather stand more or less indefinitely in a productive condition, without any need for burning (Gimmingham 1972). However, this is generally a balance almost impossible to strike in practice. Should the grazing be so heavy that the productivity of heather is reduced in comparison with other species in the community, a rapid change in community structure may occur. Where grazing promotes replacement of heather by bent grasses, such a change is advantageous. There is, nevertheless, always a danger that overgrazing may permit the entry and spread of bracken, which is of no value whatsoever as forage. The abundance of this species in areas formerly occupied by heath-land is often attributed to the effects of management for sheep grazing in conjunction with indiscriminate burning (Tivy 1973).

Grazing at Glensaugh

In order to make the best use of the herbage available to the animals and to provide small areas where hand feeding could take
place, the total area of the Glensaugh farm was divided into 10 smaller plots of differing size and vegetational composition. Details of the vegetation and paddock configuration are given in the report (Blaxter et al 1974) and will not be repeated here. We do, however, examine the implications which various developments on the farm could have for the establishment of commercial deer farms.

In order to facilitate the movement of animals from one grazing area to another, without the need for complicated sequences of herding through vacant paddocks, the handling area should be located so that the paddocks are radially distributed about it. The handling area will be used as a collection point prior to slaughter and for monitoring and treating the animals at various times during the year. This should not, however, be as frequent as at Glensaugh where monthly tests of the herd were carried out for the purpose of scientific studies. Treatment of the deer is a simple operation carried out to prevent infestation by parasites. Weighing and any medical treatment required may be carried out simultaneously.

The number of pens in the handling area depends upon the size of the herd and its age and sex structure. They will be required during the rut to assemble breeding groups and as a means of separating the various classes of animals during treatment and at slaughter. The pens need not be of great size as the animals should not be in the handling area for any length of time. For short periods 12sq. feet per adult and 3sq. feet per calf under 6 months were found to be sufficient at Glensaugh. The
facilities at Glensaugh also included 2 larger feeding pens which were used to familiarise weaned calves brought to the farm with the surroundings. They were also used to hold animals before transfer into the smaller pens prior to observation and weighing. Such a system of handling through pens of decreasing size greatly facilitated the movement of the animals.

Apart from serving as a control on grazing pressure, the paddocks were constructed to keep breeding groups separate during the rut. The groups consisted of between 12 and 16 hinds and one or two stags. Their relatively small size necessitated the construction of a large number of paddocks, which, from the point of view of grazing management were unnecessary. The paddocks were also used to separate hinds during the calving season. That is to say, hinds with newly born calves were kept separate from hinds which had not yet given birth. This measure did not, however, entail the construction of any additional paddocks.

We have described above the effects which varying intensity has upon the operating techniques employed for venison production. The Glensaugh Experimental Farm has furnished us with details of the techniques involved in the operation of an intensive production system. In addition, we have obtained data regarding the costs of such an operation, which are examined in the following sections.

**Deer Farming Costs**

**Capital Investment**

For the purpose of our study we shall assume that there is zero
opportunity cost associated with the land to be utilised for farming the deer. This can be justified on the grounds that we are attempting to find a use for marginal hill land, which under other circumstances would not be utilised. Capital investment will therefore be restricted to the provision of those facilities required to facilitate the intensive production of venison. It should, however, be noted that this may not always be the case. Thus where some alternative use might be found for the site of a proposed deer farm, the appropriate opportunity cost should be taken into account. In the following sections we shall not include a study of the provision of roads or other transport facilities which will be peculiar to each individual site. The main capital cost is therefore associated with the provision of fencing and handling facilities.

**Fencing Costs**

As fencing will be the single most important investment on a deer farm, every effort should be made to minimise the requirement - commensurate with efficient husbandry practices. In this section we discuss how the fencing is utilised and explore ways in which costs may be kept to a minimum. As far as perimeter fencing is concerned, little can be done to reduce the length required to enclose a given area. At best a favourable topography may be chosen. It is the internal fencing, sub-dividing the farm into paddocks, which offers most scope for rationalisation.

**Internal Fences**

Work on the Glensaugh farm has already shown that as far as
handling the animals is concerned, the most efficient configuration of paddocks is one in which they are radially distributed around the central handling area. The number of paddocks required depends on the stocking density and the size of the breeding groups during the rut. It is given by:

\[ n = \frac{640 \cdot SD}{BG} \]  

(Eqn. 6.1)

where \( n \) is the number of paddocks per sq.mile, \( SD \) is the stocking density in animals/acre and \( BG \) is the breeding group size.

Experiments carried out at Glensaugh to determine the optimal size of breeding group indicate that a single stag is capable of mating up to 16 hinds successfully (Blaxter et al 1974).

The fencing requirement for a range of farm areas was calculated and is shown in Figure 6.1(a). From this we can see that the radial distribution results in a rapid growth of internal fencing. Modifying the fencing configuration in such a way that the paddocks are no longer all contiguous to the handling area reduces the fencing requirement to that shown in Figure 6.1(b). The lower the paddock requirement and the smaller the area of the farm the lower the discrepancy in fencing requirement between the two configurations. Thus for smaller farms, the radial configuration may prove to be preferable due to the advantages afforded when herding the animals. In Figure 6.2 we show how fencing requirement varies with the area of the farm for a range of paddock densities. It should be noted that the increasing unit requirement of internal fencing is counterbalanced by the decreasing unit requirement of perimeter fences.
Fig. 6.1 Internal fencing requirements on the farm.

Fig. 6.2 Unit fencing requirement (perimeter & internal).
Fencing Types and Costs

Several types of fences were utilised at various stages of the development of the farm at Glensaugh. These are described in full detail in the report (Blaxter et al. 1974). The perimeter fences were designed to be highly deer proof and as such were all 2.1m high. Internal fences, which were not required to provide such a secure barrier, ranged from 1.50 to 1.8m in height. Experience on the farm has shown, however, that even some of the higher internal fences (1.8m) could serve as adequate perimeter fences. As the cost of the fence increases with its height, this could offer some saving in outlay.

In general, the different age classes require different types of fencing. Thus a fence which may contain an adult might not present a barrier to a calf, and vice-versa. This should be taken into account when planning the layout of a farm. The following Table, 6.2, illustrates the fence types used at Glensaugh, their cost and special applications.

<table>
<thead>
<tr>
<th>Fence Type</th>
<th>Height (m)</th>
<th>Cost (£/mile)</th>
<th>Special Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perimeter</td>
<td>2.10</td>
<td>1750</td>
<td>-</td>
</tr>
<tr>
<td>Perimeter</td>
<td>2.10</td>
<td>2350</td>
<td>Light - low wind resistance</td>
</tr>
<tr>
<td>Perimeter</td>
<td>2.10</td>
<td>2350</td>
<td>Fine mesh to contain calves</td>
</tr>
<tr>
<td>Internal</td>
<td>1.80</td>
<td>1750</td>
<td>Light - can serve as perimeter</td>
</tr>
<tr>
<td>Internal</td>
<td>1.50</td>
<td>1450</td>
<td></td>
</tr>
<tr>
<td>Internal</td>
<td>1.68</td>
<td>1570</td>
<td></td>
</tr>
<tr>
<td>Internal</td>
<td>1.80</td>
<td>1930</td>
<td>Fine mesh to contain calves</td>
</tr>
</tbody>
</table>
The cost of internal fencing may be reduced by utilising a cheaper type of fence, such as electric fencing, and/or by reducing the length of fencing required. As yet, no experiments have been undertaken at Glensbaugh with electric fences, although they have been utilised in the wild for the protection of plantations from marauding deer (pers. comm. I.S. Patterson I.T.E.). Tests have, however, been carried out to determine the possibility of reducing the paddock requirement.

We noted earlier that one of the reasons for the high number of paddocks at Glensbaugh was to keep the breeding groups separate during the rut. Subsequently, during the rutting season of 1975 a number of breeding groups were introduced into the same paddock once the stags had established dominance over their hinds in the handling area. It was found that groups did not interact and that breeding was successfully completed (pers. comm. W.J. Hamilton). At this stage no firm conclusions can be drawn regarding the success of this measure, but if subsequent trials are successful only the grazing constraint will govern the size and number of paddocks required. This should considerably reduce the paddock and hence internal fencing requirement. We can see in Figure 6.2 the effect that such a reduction has upon fencing needs.

In addition to the fences, gates must be erected between adjacent paddocks and between paddocks and handling facilities at a cost of approximately £34 for gate and posts. The paddock fences contiguous to the handling area will serve as a perimeter to these facilities.
The area will be sub-divided into the required number of pens using fences as for the paddocks. Because of its small size, the fencing requirement will be nominal. We have already seen that 12sq. feet per adult animal is sufficient space in these pens.

**Operating Costs**

Up to this stage in the development of the Glensaugh farm, operating costs have been restricted to the supply of fodder, medication and labour. Very little maintenance of fences has been required. Nevertheless, as they age, repairs will have to be carried on both internal and perimeter fences.

**Labour**

As a result of the amount of scientific study undertaken on the Glensaugh project, no precise labour requirements, from the point of view of commercial farming, are available. However, from the experience gained during the 6 years of operation a tentative figure of 1 man year per 500 breeding animals plus offspring has been suggested (pers. comm. W.J. Hamilton), as a typical manning level on a commercial farm. A more detailed breakdown by sex and age class has not been proposed. Because of the high ratio of hinds to stags which will prevail on a farm, we are primarily concerned with the labour required to attend to the hinds and their calves. On this basis, any error due to over-estimation of manpower requirement for stags will be negligible. The average manpower cost per animal is calculated below on the basis that the average annual cost of a Stockman (wages, insurance etc.) are £2800 (Department of Employment 1976) and a ratio of adult hinds to stags of 16:1 is maintained.
Total Adult Animals  500
Total Hinds  470
Calves (Hinds x 0.9)  430

Total Animals  930
Average Cost/Animal  £3.0

Feeding

Supplementary feeding was undertaken primarily to improve the overall breeding performance of the herd. The greatest efforts were directed at the hind calves during the period from October to April. The objective was to increase the growth rate of these hinds so that they would be in a condition to breed at 16-17 months and calve at 2 years.

Experiments undertaken on the farm indicate that to ensure successful breeding at this age, body weights in excess of 60-65kg must be attained by the 16th month of life. At Glensaugh, this necessitated feeding at a rate of between 0.75kg - 1.00kg/head/day. It should be noted that these feeding rates are peculiar to the conditions which exist at Glensaugh; they cannot be regarded as typical for all commercial farms. They do, nevertheless, bearing in mind the characteristics of the experimental site, give some indication of the relative quantities of fodder which might be required. The following table details the feeding rates utilised on the farm and their results in terms of breeding success.
The effects of supplementary feeding did not, however, manifest themselves only in the first breeding age class. The fecundity of subsequent age classes was also affected as shown below.

As a result of the supplementary feeding even lactating hinds on the farm have shown consistently high fecundity rates which compare favourably with those of yield hinds in the wild. (Relationship between body weight and condition and breeding success - Mitchell & Brown 1974). Additional supplementary fodder was given to hinds during the final stages of pregnancy and the immediate post-natal period at a rate of 0.5kg/head/day. The same high quality fodder as that given to the calves was used. Fodder was also given to the animals during adverse weather conditions towards the end of the winter period. The average amount supplied was 1.0kg/head/day of hay for 100 days. The following table gives details of the costs of the above measures.

<table>
<thead>
<tr>
<th>Feed Rate (kg/head/day)</th>
<th>Breeding Rate - 2 yr. yield hinds</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>0.90</td>
<td>0.91</td>
</tr>
<tr>
<td>0.75</td>
<td>0.80</td>
</tr>
<tr>
<td>0.50</td>
<td>0.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Feed Rate (kg/head/day)</th>
<th>FY2,t</th>
<th>FY3,t+1</th>
<th>FY3,t+1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>NONE</td>
</tr>
<tr>
<td>0.90</td>
<td>0.91</td>
<td>0.94</td>
<td>1.00</td>
</tr>
<tr>
<td>0.75</td>
<td>0.80</td>
<td>0.84</td>
<td>1.00</td>
</tr>
<tr>
<td>0.50</td>
<td>0.00</td>
<td>NONE</td>
<td>0.70</td>
</tr>
</tbody>
</table>
Table 6.5 Average Annual Cost/Head of Supplementary Fodder

<table>
<thead>
<tr>
<th>Class</th>
<th>Cost (£)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female Calf</td>
<td>14.90</td>
</tr>
<tr>
<td>Male Calf</td>
<td>5.50</td>
</tr>
<tr>
<td>Stag</td>
<td>4.40</td>
</tr>
<tr>
<td>Hind</td>
<td>7.40</td>
</tr>
</tbody>
</table>

One other way by which the quality of fodder intake was increased at Glenshaugh was by improving the quality of the pasture. An area of land was cleared of heather, limed and re-seeded with a grass mixture of cocksfoot, timothy, rye grass and clovers. The total cost of such an operation is approximately £90 per acre. This area is illustrated in Plate 5, in which animals in their second year are being fattened prior to slaughter in October. In addition, higher winter stocking rates may be achieved on this land. The desirability of such measures on a commercial farm will, of course, be dependent upon the grazing quality of a particular site.

**Miscellaneous Costs**

In addition to the above operating costs, allowances must be made for treatment against parasitic infections, veterinary inspection and other miscellaneous items. Typical costs for such measures are £2-£3 per animal per year.

Finally, an estimate of the cost of buying in new stock and rearing up to weaning age must be made. Data from Glenshaugh are available on the cost of fodder required; this is approximately £9 per calf and includes the cost of 20-25kg of ewe-milk substitute, 30-40kg of concentrates and 20-25 kg of roughage. These quantities should apply in general to any commercial venture. As far as the
procurement of stock is concerned, there is as yet no accepted "market" price. In the long term, it is likely that the price of stock will reflect the value of the venison which may be obtained from the sale of weaned calves. We shall examine the effects of varying prices at a later stage in this study.

Conclusions

In this chapter we have shown how the production techniques and factor inputs vary with increasing production intensity. The Glenshaugh project has been used to illustrate the changes in "technology" that are required under intensive conditions. In reviewing the progress on the experimental farm we have been able to draw some general conclusions regarding the establishment of future commercial deer farms. These include policies for stocking the farm, taming stock, handling stock, feeding and erecting facilities. There were, however, measures undertaken at Glenshaugh which were peculiar to the particular conditions existing at that site. Thus, in examining the policies adopted for supplementary feeding, we must emphasize that although the objectives of feeding will be the same on all commercial farms, the quantities of fodder utilised will be dependent upon the characteristics of each individual site.

In the following chapter we shall examine the development of the population at Glenshaugh and discuss the data obtained. We have already noted that some data from Glenshaugh merely reflect the conditions which prevail at that site. This is equally true of the population statistics. Nevertheless, as a convenient starting point for analysis, these data are most valuable.
Chapter 7

Population Dynamics at Glenshaugh

The Establishment of the Herd

The first steps to set up the herd were taken in 1970 when 9 calves were captured in the wild and brought to the farm. In the ensuing years further calves were captured and added to the herd. No attempt was made to increase the stock by capture in 1973, but 1974 saw the resumption of this measure. The present herd thus consists partly of captured calves and partly of animals born on the farm. Occasionally, animals kept at the Rowett Institute for research purposes were introduced to the herd; likewise animals were removed from the farm for slaughter or study. Details of the herd's development are shown in Table 7.1.

Table 7.1

<table>
<thead>
<tr>
<th>Year</th>
<th>Hinds</th>
<th>Age</th>
<th>Class</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1971</td>
<td>(55)</td>
<td>(2)2</td>
<td>2</td>
</tr>
<tr>
<td>1972</td>
<td>(24)</td>
<td>3</td>
<td>50</td>
</tr>
<tr>
<td>1973</td>
<td>27</td>
<td>26</td>
<td>49</td>
</tr>
<tr>
<td>1974</td>
<td>(30)</td>
<td>34</td>
<td>24</td>
</tr>
<tr>
<td>1975</td>
<td>(19)</td>
<td>43</td>
<td>53</td>
</tr>
</tbody>
</table>
Table 7.1 (contd)

<table>
<thead>
<tr>
<th>Year</th>
<th>Age</th>
<th>Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sept</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1971 (6)</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>1972 (9) 3</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>1973 26</td>
<td>11</td>
<td>6</td>
</tr>
</tbody>
</table>

(  ) = animals bought  [  ] = animals removed

Fecundity and Survival Rates

Table 7.2 shows the overall reproductive performance for each year from 1972-75. The drop in overall fecundity in 1975 is a result of the failure of the 2 year old hinds to breed. This can be seen in Table 7.3 where the reproductive performance is given for individual age classes. If we examine the feeding rates used it can be seen that the failure to breed corresponds to the decrease in feeding rate. The average weight of the non-breeding 2 year old yield hinds was 51kg which compares with 64-66kg for previous groups of 2 year olds reared on a higher plane of nutrition. The same reduced ration was given to calves during the period October, 1974 to April, 1975. In September, 1975, at the beginning of the rut, the 15 month old hinds averaged 48.5kg and again all failed to conceive. The successful breeding of these animals thus depends upon their condition and body weight at the time of the rut. For breeding to be successful, body weights in excess of 60-65kg must be attained by this time.
The rate of supplementary feeding required on commercial farms will, of course, depend upon the conditions which exist at that site.

Table 7.2  Overall Reproductive Performance.

<table>
<thead>
<tr>
<th>Year</th>
<th>Hinds put to Stag and surviving to calve (1)</th>
<th>Live Births (2)</th>
<th>%Age (2)/(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1972</td>
<td>6</td>
<td>6</td>
<td>100</td>
</tr>
<tr>
<td>1973</td>
<td>57</td>
<td>53</td>
<td>93</td>
</tr>
<tr>
<td>1974</td>
<td>81</td>
<td>74</td>
<td>91</td>
</tr>
<tr>
<td>1975</td>
<td>102</td>
<td>73</td>
<td>72</td>
</tr>
</tbody>
</table>

Table 7.3  Age Specific Reproductive Performance

<table>
<thead>
<tr>
<th>Year</th>
<th>$F_2$ (n)</th>
<th>$F_3$ (n)</th>
<th>$F_4$ (n)</th>
<th>$F_5$ (n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1972</td>
<td>1.00 (4)</td>
<td>1.00 (2)</td>
<td>1.00 (2)</td>
<td></td>
</tr>
<tr>
<td>1973</td>
<td>0.91 (49)</td>
<td>1.00 (4)</td>
<td>1.00 (2)</td>
<td>1.00 (2)</td>
</tr>
<tr>
<td>1974</td>
<td>0.80 (26)</td>
<td>0.94 (48)</td>
<td>1.00 (7)</td>
<td></td>
</tr>
<tr>
<td>1975</td>
<td>0.00 (24)</td>
<td>0.84 (25)</td>
<td>0.98 (47)</td>
<td>1.00 (6)</td>
</tr>
</tbody>
</table>

n = number of hinds put to stag and surviving to calve.

Note: The incidence of yieldness on the farm was negligible. Those mature hinds which failed to breed in any year always bred successfully the following year, i.e. the fecundity of mature yield hinds was 1.00

Table 7.4  Supplementary Feeding Rates

<table>
<thead>
<tr>
<th>Year</th>
<th>Feeding Rate (kg/head/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970/71</td>
<td>1.00</td>
</tr>
<tr>
<td>1971/72</td>
<td>0.90</td>
</tr>
<tr>
<td>1972/73</td>
<td>0.75</td>
</tr>
<tr>
<td>1973/74</td>
<td>0.50</td>
</tr>
<tr>
<td>1974/75</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Note: Calves fed in period t/t+1 breed in period t+2.
Table 7.5 illustrates the age specific fecundity rates obtained from a number of wild herds (Mitchell et al 1971). The results obtained at Glenshaugh for lactating hinds compare well with those of yield hinds in the wild. Breeding efficiency on the experimental farm varied from 72% to 100% during the period 1972-75, which compares with a typical breeding efficiency in the wild of approximately 40-50% (Mitchell 1973).

Table 7.5  Fecundity of Yield Hinds in the Wild

<table>
<thead>
<tr>
<th>Age Class</th>
<th>Rhum</th>
<th>Glen Feshie</th>
<th>Glen Fiddich</th>
<th>Invermark</th>
<th>All Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.00(38)</td>
<td>0.00(48)</td>
<td>0.20(10)</td>
<td>0.43(27)</td>
<td>0.11(123)</td>
</tr>
<tr>
<td>3</td>
<td>0.42(125)</td>
<td>0.62(104)</td>
<td>0.87(19)</td>
<td>0.71(63)</td>
<td>0.57(311)</td>
</tr>
<tr>
<td>4</td>
<td>0.86(144)</td>
<td>0.85(82)</td>
<td>1.00(9)</td>
<td>1.00(43)</td>
<td>0.88(278)</td>
</tr>
<tr>
<td>5</td>
<td>0.90(155)</td>
<td>0.90(70)</td>
<td>1.00(20)</td>
<td>1.00(71)</td>
<td>0.93(316)</td>
</tr>
<tr>
<td>6</td>
<td>0.95(103)</td>
<td>0.91(54)</td>
<td>1.00(10)</td>
<td>0.89(57)</td>
<td>0.93(224)</td>
</tr>
<tr>
<td>7</td>
<td>0.95(84)</td>
<td>0.93(43)</td>
<td>1.00(9)</td>
<td>0.93(57)</td>
<td>0.94(193)</td>
</tr>
<tr>
<td>8</td>
<td>0.93(60)</td>
<td>0.97(33)</td>
<td>1.00(5)</td>
<td>0.90(47)</td>
<td>0.93(145)</td>
</tr>
<tr>
<td>9</td>
<td>0.99(89)</td>
<td>0.88(41)</td>
<td>1.00(6)</td>
<td>1.00(3)</td>
<td>0.96(139)</td>
</tr>
</tbody>
</table>

As far as survival is concerned (see Table 7.6 a and b) there is no significant difference between the farmed animals and those in the wild. The high mortality rate on the farm was due in part to "black loss" which occurred when newly born calves were drowned in the bogs on the wetter parts of the farm. This can be avoided by ensuring that these animals are excluded from such areas. Even so, survival rates of calves compare favourably with those in the wild.
Table 7.6 Survival on the Farm and in the Wild

(a) Glensaugh

<table>
<thead>
<tr>
<th>Age Class</th>
<th>Survival Rate</th>
<th>Survival Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Stags</td>
<td>Hinds</td>
</tr>
<tr>
<td>1</td>
<td>0.88(89)</td>
<td>0.88(173)</td>
</tr>
<tr>
<td>2</td>
<td>0.97(36)</td>
<td>0.99(104)</td>
</tr>
<tr>
<td>3</td>
<td>0.90(21)</td>
<td>0.98(81)</td>
</tr>
<tr>
<td>4</td>
<td>0.89(9)</td>
<td>0.98(59)</td>
</tr>
<tr>
<td>5</td>
<td>1.00(2)</td>
<td>0.91(11)</td>
</tr>
<tr>
<td>6</td>
<td>-</td>
<td>1.00(2)</td>
</tr>
</tbody>
</table>

(b) Rhum

<table>
<thead>
<tr>
<th>Age Class</th>
<th>Survival Rate</th>
<th>Survival Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Stags</td>
<td>Hinds</td>
</tr>
<tr>
<td>1</td>
<td>0.88</td>
<td>0.90</td>
</tr>
<tr>
<td>2</td>
<td>0.96</td>
<td>0.98</td>
</tr>
<tr>
<td>3</td>
<td>0.98</td>
<td>0.99</td>
</tr>
<tr>
<td>4</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>5</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>6</td>
<td>0.97</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Growth Rates

From the comparisons of fecundity and survival on the farm with those in the wild, we saw that the main advantage of the farming operation was the considerable increase in breeding efficiency. However, the intensive management techniques employed to accomplish this have further advantages from the point of view of growth and development of the animals. The following tables, 7.7a and b illustrate the average larder carcase weights attained by animals on the farm and in the wild. Because different feeding regimes were used for the various groups of yeld calves, we have shown the growth rates for both the high and low planes of nutrition.
Table 7.7  Larder Carcase Weights on the Farm and in the Wild

Note: Larder carcase weight = Live weight x 0.7

(a) Glensaugh

<table>
<thead>
<tr>
<th>Age Class</th>
<th>Stags (kg)</th>
<th>Hinds (High Plane) (kg)</th>
<th>Hinds (Low Plane) (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>27.2</td>
<td>25.7(173)</td>
<td>25.7(173)</td>
</tr>
<tr>
<td>2</td>
<td>48.7(83)</td>
<td>45.6(94)</td>
<td>34.4(52)</td>
</tr>
<tr>
<td>3</td>
<td>58.9(35)</td>
<td>49.1(81)</td>
<td>40.2(22)</td>
</tr>
<tr>
<td>4</td>
<td>68.9(18)</td>
<td>53.6(59)</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>71.8(8)</td>
<td>56.7(11)</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>-</td>
<td>61.6(6)</td>
<td>-</td>
</tr>
</tbody>
</table>

(b) Glen Feshie

<table>
<thead>
<tr>
<th>Age Class</th>
<th>Stags (kg)</th>
<th>Yield Hinds (kg)</th>
<th>Lactating Hinds (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24.8(48)</td>
<td>22.9(44)</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>37.3(26)</td>
<td>35.8(48)</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>48.0(5)</td>
<td>46.3(107)</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>64.9(16)</td>
<td>50.6(82)</td>
<td>43.7(7)</td>
</tr>
<tr>
<td>5</td>
<td>71.6(40)</td>
<td>52.7(72)</td>
<td>46.6(13)</td>
</tr>
<tr>
<td>6</td>
<td>-</td>
<td>52.7(59)</td>
<td>46.6(13)</td>
</tr>
</tbody>
</table>

The above growth rates are not, however, strictly comparable in that in each case the quality of the grazing differs. To some extent we can see the effect of fodder inputs for calves in the difference between the weights of 2 year old yield calves on high and low planes of nutrition. Any true comparison would have to be made on the basis of the performance of the animals at Glensaugh without fodder inputs against that with fodder inputs. Nevertheless, in all cases, the weight of the lactating hinds on the farm compares well with that of the corresponding yield hinds in the wild. This is what we might have expected on the basis of the fecundity rates achieved. As far as the stags are concerned, the inputs
of supplementary fodder are minimal for mature animals, thus any advantage the farmed animals have over their wild counterparts does not persist.

Because of the short period of time which the experimental farm has been in operation, performance data is incomplete in a number of areas, especially among the older animals. We have, however, noted certain similarities between the data obtained from a number of feral herds and that obtained from the farm. In this section we shall attempt to fill these gaps in the farm data using our knowledge of the performance in the wild.

1. Fecundity.

Throughout this discussion we have referred to the relationship between body weight and condition and reproductive performance. Because of the inputs of supplementary fodder on the farm, fecundity rates of lactating hinds have been consistently high. Indeed, we noted that they compared favourably with those of yeld hinds in the wild. For this reason we use the average fecundity rates of the feral yeld hinds shown in Table 7.5 to complete the fecundity schedule for lactating hinds on the farm. This is shown below in Table 7.8.

<table>
<thead>
<tr>
<th>Proportion Breeding</th>
<th>Age Class</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3  4  5  6  7  8</td>
</tr>
<tr>
<td>Lactating Hinds</td>
<td>1.00 0.98 1.00 (0.93) (0.94) (0.93)</td>
</tr>
</tbody>
</table>

Note: Figures in parentheses are obtained from feral herds. The above data apply to the high feeding plane for calves.
2. Survival

As was the case with fecundity, survival rate data on the farm was available only for animals aged up to 5 years (female) and 4 years (male). Because the survival rates in the wild are consistently high, it was decided that where necessary those data from Table 7.6(b) could supplement data from the farm.

The completed survival schedule is shown below:

<table>
<thead>
<tr>
<th>Age Class</th>
<th>Proportion Surviving (Stasa)</th>
<th>Proportion Surviving (Hinds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.88</td>
<td>0.88</td>
</tr>
<tr>
<td>2</td>
<td>0.97</td>
<td>0.99</td>
</tr>
<tr>
<td>3</td>
<td>(0.99)</td>
<td>0.98</td>
</tr>
<tr>
<td>4</td>
<td>(0.99)</td>
<td>0.98</td>
</tr>
<tr>
<td>5</td>
<td>(0.99)</td>
<td>(0.99)</td>
</tr>
<tr>
<td>6</td>
<td>(0.97)</td>
<td>(0.98)</td>
</tr>
<tr>
<td>7</td>
<td>(0.96)</td>
<td>(0.95)</td>
</tr>
<tr>
<td>8</td>
<td>(0.97)</td>
<td>(0.94)</td>
</tr>
</tbody>
</table>

3. Growth Rates

As far as the growth rates are concerned, extrapolation for the older age groups may prove unnecessary. It has been suggested that the effects of supplementary feeding on growth do not affect the ultimate size of the animal, (Blaxter et al 1974, R.N.B. Kay pers. comm.); rather the animals attain maximum size at an earlier age. However, those hinds on a low plane of nutrition did not exhibit such a rapid growth and it seems likely that their growth pattern will be more akin to that of the wild animals. For this reason we use the growth curve analysed in
Chapter 4 to approximate that of the farmed deer on a low nutritional plane. The following table, 7.10, shows the larder carcase weights obtained:

<table>
<thead>
<tr>
<th>Age Class</th>
<th>Weight (kg) Stags</th>
<th>Weight (kg) Hinds (High Plane)</th>
<th>Weight (kg) Hinds (Low Plane)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>27.2</td>
<td>25.7</td>
<td>25.7</td>
</tr>
<tr>
<td>2</td>
<td>48.7</td>
<td>45.6</td>
<td>34.4</td>
</tr>
<tr>
<td>3</td>
<td>58.9</td>
<td>49.1</td>
<td>40.2</td>
</tr>
<tr>
<td>4</td>
<td>68.9</td>
<td>53.6</td>
<td>(47.7)</td>
</tr>
<tr>
<td>5</td>
<td>71.8</td>
<td>56.7</td>
<td>(51.8)</td>
</tr>
<tr>
<td>6</td>
<td>71.8</td>
<td>61.6</td>
<td>(54.5)</td>
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<td>7</td>
<td>71.8</td>
<td>61.6</td>
<td>(56.1)</td>
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<td>71.8</td>
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<td>(56.9)</td>
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<td>9</td>
<td>71.8</td>
<td>61.6</td>
<td>(56.8)</td>
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**Summary**

In Chapter 4 we developed a model which allowed us to examine the productivity of deer herds in the wild using fecundity survival and growth data of the same form as that above. At that stage no attempt was made to propose management policies which relied upon the identification of individual classes of animals. Such policies were seen to be impracticable in the wild. On a farm, however, these restraints upon the operations no longer apply. In the following chapter we shall describe how the model developed previously may be incorporated into a mathematical programming model, to enable us to study optimal strategies under a number of operating conditions.

On a number of occasions we have drawn attention to the fact that
111.

much of the data obtained at Glensaugh reflected not only the particular characteristics of the site but also the nature of the experimental work. Thus any analysis of a farming operation which utilises this data, not only mirrors these non-commercial aspects but also the skill and experience of the operational staff.

Nevertheless, the above data constitute a valuable starting point for the further analysis of commercial deer farming economics.
Chapter 8

A Linear Programming Model of the Farm Operation

In Chapters 4 and 5 we examined the effects which changes in breeding and survival rates have upon the production of venison in the wild. The analytical techniques used to study the population dynamics of the feral herds were based upon the assumption that these populations had attained stable age structures. Whereas such an assumption is not unreasonable in the case of long established wild populations, it is patently not the case on the deer farm where the starting population may consist almost entirely of calves and is therefore far removed from the stable configuration. That is not to say, however, that some form of stability is not attained on the farm. However, the effects of natural mortality and birth rates are modified by harvesting and stock purchase. Thus the problem of stabilisation of the population structure at the economic optimum depends not only on the rate changes induced by the natural phases of the population growth cycle, but also on the management strategies which modify it.

Our objective at this stage is thus to apply those analytical techniques to the basic structure of the matrix model discussed in Chapter 4, which will allow us to evaluate the operating strategies at both the stable and unstable states. Our aim is not merely to obtain the "optimal" solution; it is rather to determine which factors exert the greatest influence upon these operating strategies, bearing in mind that current decisions not
only affect present outcomes but also those of future periods. We are, essentially, trying to solve a production scheduling type of problem. The general programming problem with which we are concerned may be formulated in the following way.

We wish to determine values for n variables, \( x_1 \ldots x_n \), which satisfy the m constraints

\[ g_i(x_1 \ldots x_n) < b_i \quad i = 1 \ldots m \]

while maximising the function

\[ z = f(x_1 \ldots x_n) \]

where the \( x_i \) are the levels of activity which define the numbers of animals bought, harvested, and left to breed of each age group. The solution technique most appropriate to this type of problem will depend upon the form which the functions \( 'g' \) and \( 'f' \) take.

We have already seen that the matrix model used simple difference equations to describe the population dynamics. We can thus see that the function \( 'g' \) will be linear in form. If, therefore, the objective function \( 'f' \) is linear also, the problem may be formulated as a general linear programming problem. If the costs and selling prices associated with the function \( 'f' \) are constant multiples with the amount of the activity \( x_i \), the objective function is linear. We saw in Chapter 3 that the venison dealers in Scotland operate a basic price regardless of estate output, in any one season. Thus as far as selling prices are concerned, the assumption of linearity is valid. On the other hand, feedstuff costs are subject to quantity discounts.
These, however, are not continuous but are applied in discrete steps; thus over the operating range feedstuff prices may be regarded as constant. Under this assumption, the objective function may be regarded as linear.

Linear programming has been widely used in the field of industrial production planning and inventory control (Williams 1967, Gass 1969, Wagner 1970). In this chapter we shall examine how L.P. is related to the classical economic technique of marginal analysis and develop the production planning model of the red deer farm, to which the L.P. analysis will be applied.

Production Economics: Linear Programming and Marginal Analysis

Although L.P. is essentially a mathematical technique, it can be explained in straightforward economic terms (Baumol 1965). Basically L.P. is a reformulation of the standard economic problem of marginal analysis. "The essential simplification achieved in L.P. is the replacement of the notion of the production function by the motion of the process." (Dorfman, 1958). In this section we shall briefly discuss the similarities of marginal analysis to L.P. and their relationships to production economics.

The conventional linear homogeneous production function in short run marginal analysis is generally depicted as a smooth surface. In L.P. this production surface is approximated by a series of processes (vectors) from the origin. The assumption in this system is that for each vector, production coefficients, marginal products and rates of substitution are constant.
Isoprodut functions on the two surfaces are also different. Whereas on the typical surface they are smooth and differentiable (see Figure 8.1) on the surface approximated by L.P. analysis each isoprodut function can be envisaged as consisting of a series of joined linear segments each with a different slope, Figure 8.2 (Baumol, 1965, Yuan-li Wu and Ching-wen Kwang, 1960)

Figure 8.1  Isoprodut Curves

Isoprodut curves (P\textsubscript{1}, P\textsubscript{2}, P\textsubscript{3}) showing varying levels of production. Expansion path E denotes that the production coefficients, marginal products and rates of substitution are constant.
Surface of isoproduct functions \((P_1, P_2)\) approximated from L.P. analysis. Straight line expansion paths \((E_1, E_2, E_3, E_4)\) denote that production coefficients, marginal products and rates of resource substitution are constant.

The differences in the surface depicted by the two methods arise because in L.P. only a finite number of potential alternative combinations of inputs are considered. The same accuracy could be achieved with a complete set of input-output relationships if data were available for their construction.

Marginal analysis can be applied to any problem where a maximum or minimum goal is established, whether the functional changes in the dependent variable are infinitesimal or are finite.

Where inputs and management alternatives are limited in numbers, the problem of finding maximum profit for a specified output is easily solved by linear programming.
Factor price ratios required to make the decision are no problem since the assumption of constant unit prices, regardless of quantity obtained, is made.

The assumption of linear segments in the use of L.P. presents no problems. The net result is an abrupt substitution in inputs when relative prices pass critical levels, followed by no change in inputs as their relative prices continue to change in the same direction. Where the number of alternative inputs and/or management alternatives is large, L.P. simplifies the processing of data to arrive at a solution. The solution is termed an optimal selection of inputs and management alternatives in terms of cost minimisation or profit maximisation. L.P. merely selects the optimum from among a number of alternatives in a relatively efficient manner.

As with any research tool, attention is directed to defining and formulating the problem, establishing the conceptual framework of analysis, selecting method and gathering appropriate data. L.P. is merely a tool for treating a specific type of data in a well-defined study area. In the previous chapters we have been constructing just such a framework for analysis.

Linear programming has found numerous applications in the producing industries for the scheduling of the production process. Although each industry may have different products and components and different specifications for them, many can be grouped together and common mathematical models can be used to describe
the operational systems. Production can be scheduled to cover several plants in a single time period, or alternatively, a single plant operation schedule may be extended over several time periods. In the following sections we shall construct a multi-time period model for venison production systems.

**Deer Production Relationships**

If the L.P. model is to be meaningful in that it provides an accurate representation of the real situation, all the significant factors and relationships which affect deer production must be formally described and incorporated. These include the factors of fertility, mortality, breeding requirements, harvesting and the amount of money, land and labour available. In Chapters 4 and 5 we described the development of models incorporating the fecundity and survival of the deer, which portrayed the population dynamics. We shall now incorporate these into the L.P. format for analysis. Each major relationship will be separately identified and stated as a linear equation.

In matrix notation the basic L.P. problem may be expressed as follows:
Maximise

subject to

and

\[ x \geq 0, \ b \geq 0 \] where

\[ x = x_1, x_2, \ldots, x_n \]

is the column vector of activities

in the primal L.P. problem

\[ q = q_1, q_2, \ldots, q_n \]

is the row vector of costs and revenues in the objective function

\[ A = a_{1,1}, \ldots, a_{n,1} \]

\[ \ldots \]

\[ a_{m,1}, \ldots, a_{m,n} \]

is the matrix of coefficients

\[ b = b_1, \ldots, b_m \]

is the column vector of resource constraints

**Activity Variables**

The management activities are the variables around which the production relationships are developed. The following is a list of the activity variables used in the model of venison production on the farm:
In Chapter 4 we described the basic Leslie Matrix model which depicts the progression of the population from one time period to the next (Figure 4.7). In this section we show how the matrix may be incorporated into the L.P. model. The following equations show the number of animals by sex and age class in the period \( t \) in relation to the surplus animals following harvest in period \( t - 1 \).

**Calves**

\[
BM_{1,t} = \text{male calves bought in year } t \\
BY_{1,t} = \text{female calves bought in year } t \\
RM_{i,t} = \text{male aged } i \text{ remaining after cull in year } t \\
RY_{i,t} = \text{yeld hind aged } i \text{ remaining after cull in year } t \\
HL_{i,t} = \text{lactating hind aged } i \text{ remaining after cull in year } t \\
HM_{i,t} = \text{male aged } i \text{ harvester in year } t \\
HY_{i,t} = \text{yeld hind aged } i \text{ harvested in year } t \\
HL_{i,t} = \text{lactating hind aged } i \text{ harvester in year } t
\]

In Chapter 4 we described the basic Leslie Matrix model which depicts the progression of the population from one time period to the next (Figure 4.7). In this section we show how the matrix may be incorporated into the L.P. model. The following equations show the number of animals by sex and age class in the period \( t \) in relation to the surplus animals following harvest in period \( t - 1 \).

\[
\begin{align*}
\text{Male calves bought in year } t & = \text{BRAT} \left[ \sum_{i=2}^{m-1} RY_{i,t-1} FY_{i,1} + \sum_{j=3}^{m-1} RL_{j,t-1} FL_{j,1} \right] + BM_{1,t} \quad \text{Eqn. 8.1} \\
\text{Female calves bought in year } t & = (1-\text{BRAT}) \left[ \sum_{i=2}^{m-1} RY_{i,t-1} FY_{i,1} + \sum_{j=3}^{m-1} RL_{j,t-1} FL_{j,1} \right] + BY_{1,t} \quad \text{Eqn. 8.2}
\end{align*}
\]

\[
\begin{align*}
\text{Male aged } i \text{ remaining after cull in year } t & = RM_{i-1,t-1} SM_{i-1} \quad i = 2, m \\
\end{align*}
\]

\[
\begin{align*}
\text{lactating hind aged } i \text{ remaining after cull in year } t & = \text{BY}_{1,t} \quad \text{Eqn. 8.3}
\end{align*}
\]
Hinds

\[ \text{Yeld}_{2,t} = RY_{1,t-1} SY_{1} \quad \text{Eqn. 8.4} \]
\[ \text{Yeld}_{3,t} = RY_{2,t-1} SY_{2}(1-FY_{2}) \quad \text{Eqn. 8.5} \]
\[ \text{Yeld}_{i,t} = RY_{i-1,t-1} SY_{i-1}(1-FY_{i-1}) + RL_{i-1,t-1} SL_{i-1}(1-FL_{i-1}) \quad \text{i=4,m Eqn. 8.6} \]
\[ \text{Lact}_{3,t} = RY_{2,t-1} SY_{2} FY_{2} \quad \text{Eqn. 8.7} \]
\[ \text{Lact}_{i,t} = RY_{i-1,t-1} SY_{i-1} FY_{i-1} + RL_{i-1,t-1} SL_{i-1} FL_{i-1} \quad \text{i=4,m Eqn. 8.8} \]

where \( m \) is the maximum longevity of animals and BRAT is the proportion of the calf input which is male.

Continuity Equations

The number of animals at the beginning of a period, \( t \), must equal the sum of the animals harvested and those left to carry over at the end of that period. The above equations 8.1 - 8.8 may be rewritten for inclusion in the L.P. as follows:

Calves

\[ \text{Male} \]
\[-\text{BRAT} \sum_{i=2}^{m-1} RY_{i,t-1} FY_{i} SY_{i} + \sum_{j=3}^{m} RL_{j,t-1} FL_{j} SL_{j} = BM_{1,t} + RM_{1,c} + HM_{1,t} = 0 \quad \text{Eqn. 8.9} \]

\[ \text{Female} \]
\[ (\text{BRAT-1}) \sum_{i=2}^{m-1} RY_{i,t-1} FY_{i} SY_{i} + \sum_{j=3}^{m} RL_{j,t-1} FL_{j} SL_{j} = BY_{1,t} + \text{RY}_{1,t} + HY_{1,t} = 0 \quad \text{Eqn. 8.10} \]

Stags

\[ - \text{RM}_{i-1,t-1} SM_{i-1} + \text{RM}_{i,t} + HM_{i,t} = 0 \quad \text{i=2,m-1 Eqn. 8.11a} \]
\[ - \text{RM}_{m-1,t-1} SM_{m-1} + HM_{m,t} = 0 \quad \text{Eqn. 8.11b} \]
Hinds

\[ -RY_{1,t-1}S_{1} + RY_{2,t} + HY_{2,t} = 0 \]  
\[ (Eqn. 8.12) \]

\[-RY_{2,t-1}S_{2}(1-FY_{2}) + RY_{3,t} + HY_{3,t} = 0 \]  
\[ (Eqn. 8.13) \]

\[-RY_{i-1,t-1}S_{i-1}(1-FY_{i-1}) - R_{i-1,t-1}S_{i-1}(1-FL_{i-1}) + R_{i,t} + HY_{i,t} = 0 \]  
\[ (Eqn. 8.14a) \]

\[ i = 4, m-1 \]

\[-RY_{m-1,t-1}S_{m-1}(1-FY_{m-1}) - R_{m-1,t-1}S_{m-1}(1-FL_{m-1}) + HY_{m,t} = 0 \]  
\[ (Eqn. 8.14b) \]

Lactating

\[-RY_{i-1,t-1}S_{i-1}F_{i-1} - R_{i-1,t-1}S_{i-1}L_{i-1}F_{i-1} + R_{i,t} + H_{i,t} = 0 \]  
\[ (Eqn. 8.15a) \]

\[ i = 4, m-1 \]

\[-RY_{m-1,t-1}S_{m-1}F_{m-1} - R_{m-1,t-1}S_{m-1}L_{m-1}F_{m-1} + H_{m,t} = 0 \]  
\[ (Eqn. 8.15b) \]

Thus the coefficients of the activity variables of the equations 8.9 - 8.15b constitute the herd identity matrix which describes the progression of the population from one time period to the next.

There are, however, further constraints imposed upon the system which have to be incorporated into the matrix. These we describe below.

Constraints

1. **Stocking Density**

The constraint upon the carrying capacity of the farm is expressed as shown below -

\[ \sum_{i=2}^{m-1} R_{i} + R_{i} + R_{i} < X \]  
\[ (Eqn. 8.16) \]

where \( X \) is the total carrying capacity of the farm. Unlike the Rhum system, however, we have been unable to study optimal
stocking policies in regard to population densities and density dependent birth and death rates. As yet there is insufficient data from the experimental farm to warrant such an analysis. Nevertheless, our analysis of the Rhum system (Chapters 4 and 5) indicated that it was the survival rates of calves and the fecundity rates of the first breeding hinds which had the greatest influence upon stocking policy. The latter of these was the controlling factor. Studies on the experimental farm have shown that this factor can be controlled by the input of supplementary fodder. Thus by varying these parameters we may evaluate the sensitivity of the system outputs to these factors.

Breeding Ratio

A further constraint upon the progression of the population is imposed by the breeding ratio of stags to hinds. We noted in the previous chapters that a ratio of hinds to stags of up to 16:1 could be maintained successfully. We can express the constraint on the maximum ratio of hinds to stage in the following way -

$$\sum_{i=3}^{i=m-1} Q_{i}RM_{i} + \sum_{i=3}^{i=m-1} Q_{i}HM_{i} - \sum_{i=2}^{i=m-1} RY_{i} - \sum_{i=3}^{i=m-1} RL_{i} \geq 0$$

(Eqn. 8.17)

where $Q_{i}$ is the number of hinds which a stag aged $i$ can serve successfully.

In addition to these "natural" constraints upon the population growth, there are imposed a number of economic or market limitations.
Stock Purchase

The first of these may be regarded as a somewhat arbitrary restriction in that, at present, there is no information defining possible boundary conditions, for the number of calves which can be bought to supplement stock on the farm. Whether stock will continue to come from captured new-born wild calves, or from some form of farm-stock breeding pool is immaterial. Regardless of the source, there will be some physical limit to the numbers of animals available. This will, in turn, have implications for the management strategies on the farm. For this reason we wish to investigate the effects of constraints upon availability.

These are expressed below -

\[ BM_{1,t} \leq W_{1,t} \quad BY_{1,t} \leq W_{2,t} \]  

(Eqn. 8.18 a & b)

where \( W_1 \) and \( W_2 \) are the availabilities of new stock.

Finance and Labour Restrictions

We may introduce two further constraints on the system which will affect the operating strategies. These, again, are of a somewhat arbitrary nature in that they express some notional limits upon the availability of cash to finance running costs and labour to implement the operating strategies. Such limitations do occur in practice and their influence upon the operating strategies should be examined. They are expressed in the following manner -

\[
\sum_{i=1}^{m-1} C_{i,t} \cdot RM_{i,t} + \sum_{i=2}^{m-1} C_{i,t} \cdot RY_{i,t} + \sum_{i=3}^{m-1} C_{i,t} \cdot RL_{i,t} + C_4 (BM_{1,t} + BY_{1,t}) \leq V_t \]  

(Eqn. 8.19)

where \( C_{n,i} \) is the cost associated with each activity and \( V_t \) is the total cash available in any period \( t \).
where \( L_i \) is the unit labour requirement for each activity and \( U_t \) is the total workforce available in any period \( t \). The constraints of equations 8.16 to 8.20 in conjunction with the equality constraints of equations 8.9 to 8.15b constitute the constraint vector, \( b \). Only the vector \( q \), of costs and revenues, which comprises the objective function, remains to be defined. The model developed in equations 8.9 to 8.20 describes the future operating strategies available to the manager. The objective is to develop the best operating strategy within the resource constraints. The objective function is thus a mathematical statement of this management aim, the general form of which is given below:

\[
\text{Contribution} = \sum_{t=1}^{T} \left( \sum_{i=1}^{m} S_{i,HM_i} + \sum_{i=1}^{m} S_{i,HY_i} + \sum_{i=3}^{m} S_{i,HL_i} - \sum_{i=1}^{m} C_{i,RM_i} + \sum_{i=1}^{m} C_{i,RY_i} - \sum_{i=1}^{m} C_{i,RL_i} - C_4(BM_1 + BY_1) \right) t
\]  

(Eqn. 8.21)

where \( T \) is the length of the total planning period under consideration.

For the L.P. solution technique the equations detailed above must be set up in a matrix format, the general structure of which is shown in Figure 8.3.
The partitioned uni-period matrix above comprises a number of submatrices and sub-vectors which contain the parameters for each operation described in equations 8.1 to 8.20.

**OBJ:** is the sub-vector listing the management costs and revenues

**B:** is the final value (max) of the objective function

**A:** is the sub-matrix of continuity coefficients linking the population in period t to that in period t + 1

**Y:** is the null sub-vector resulting from the continuity equations. In the first period the parameters of Y may be used to define the starting herd.

**B:** is the row sub-vector linking stocking density with its constraint
X : is the sub-vector defining the stocking density limit
C : is the row sub-vector defining the breeding ratio constraint
N : is the null sub-vector
D : is the sub-matrix linking the activity of stock purchase with its constraint
W : is the sub-vector of stock purchase limits
E : is the row sub-vector linking management activities with the financial constraint
V : is the sub-vector defining the limit of cash availability
F : is the row vector linking management activities with the labour constraint
U : is the sub-vector defining the limit of labour availability
G : is the sub-matrix which describes the population progression from period t to t + 1

The above partitioned matrix is itself a sub-matrix of the model for the total planning period, T. This is shown in Figure 8.4
## Figure 8.4 Total Planning Period Matrix

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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
It is this matrix which is analysed using the L.P. technique. The coefficients utilised are those discussed in Chapter 7. A full listing of the data input and the input procedures is given in Appendix D.

In the following chapters we shall examine the results obtained using the above model for a range of constraint and population parameter values.
Chapter 9.

Model Output Analysis - The Effects of Economics on Operating Strategies.

In this and the following chapter we analyse the operating strategies which are proposed by the L.P. model. Our primary objective in the first instance is to examine the age structure obtained when the herd attains the maximum size dictated by the stocking density constraint. We examine the way in which the growth pattern develops through the transitional stage and analyse the effects which altering the stock purchase constraints and the stocking capacity constraints has upon the cropping strategy. We next investigate how the imposition of an upper bound constraint on the operating strategies obtained in these preliminary analyses affects the growth up to and age structure at equilibrium. The consequences of varying labour and cash availability upon the operating strategies is examined and, finally, we inspect the effects which varying the cost and revenue parameters has upon the management plan.

The Model Output

In terms of herd structure, the output from the L.P. model can be divided into five distinct phases. These are as follows:

1. The build-up of the herd to maximum density
2. Transition from the building structure to equilibrium structure
3. Equilibrium structure
4. Transition from equilibrium structure to phase-out
5. Removal of all animals.
The occurrence of each of these phases depends upon three main factors, viz. the length of the planning horizon, the level of resources available and the capacity constraints. Thus if the time horizon and resources do not allow the population to attain an equilibrium structure, we may move directly from the growth phase to elimination. It is, therefore, important that we examine the sensitivity of our solution to changes in the above factors. Ideally, we would wish to propose an unbounded planning horizon. However, such a model would be intractable for solution by the L.P. method. In order to obtain solutions which allow us to study the first three phases, we must compromise between the length of the planning horizon and the cost of computing the optimal solution. For this reason a time horizon of 20 years is adopted for the analysis of the herd structure.

Equilibrium Herd Structure

For the purpose of this first analysis constraint levels were set out at the values shown in Table 9.1.

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stocking density</td>
<td>250 (adult animals)</td>
</tr>
<tr>
<td>Breeding ratio</td>
<td>16:1 (hinds:stags)</td>
</tr>
<tr>
<td>Stock purchase</td>
<td>100 (female calves)</td>
</tr>
<tr>
<td>Stock purchase</td>
<td>10 (male calves)</td>
</tr>
<tr>
<td>Working capital</td>
<td>£10,000</td>
</tr>
<tr>
<td>Labour</td>
<td>150 (man days)</td>
</tr>
</tbody>
</table>

Costs and revenues utilised in this analysis are given in Table 9.2.
Table 9.2  

<table>
<thead>
<tr>
<th>Activity</th>
<th>Obj. Fn. Coefficient (£)</th>
<th>Activity</th>
<th>Obj. Fn. Coefficient (£)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RM_{1}</td>
<td>10.5</td>
<td>HM_{1}</td>
<td>36.0</td>
</tr>
<tr>
<td>RY_{1}</td>
<td>19.9</td>
<td>HY_{1}</td>
<td>34.2</td>
</tr>
<tr>
<td>RM_{1}</td>
<td>9.4</td>
<td>HM_{2}</td>
<td>64.8</td>
</tr>
<tr>
<td>RY_{1}</td>
<td>12.4</td>
<td>HY_{2}</td>
<td>60.6</td>
</tr>
<tr>
<td>RL_{1}</td>
<td>12.4</td>
<td>HM_{3}</td>
<td>78.6</td>
</tr>
<tr>
<td>BM_{1}</td>
<td>36.0</td>
<td>HY_{3\text{HL}_{3}}</td>
<td>65.4</td>
</tr>
<tr>
<td>BY_{1}</td>
<td>36.0</td>
<td>HM_{4}</td>
<td>91.8</td>
</tr>
<tr>
<td>i = 2,8</td>
<td></td>
<td>HY_{4\text{HL}_{4}}</td>
<td>71.4</td>
</tr>
<tr>
<td>j = 3,8</td>
<td></td>
<td>HM_{5}</td>
<td>95.4</td>
</tr>
<tr>
<td>i = 6,9</td>
<td></td>
<td>HY_{5\text{HL}_{5}}</td>
<td>75.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>HM_{6}</td>
<td>95.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>HY_{6\text{HL}_{6}}</td>
<td>82.2</td>
</tr>
</tbody>
</table>

Revenue from the harvested stock is the product of the average larder carcase weights (Table 7.10) and the current (1976/77) venison price of £1.33/kg. Replacement stock costs, including the cost of feeding up to weaning age, were assumed to be equal to the expected value of a weaned male calf. The model objective was to obtain the management strategy which yields the maximum contribution margin over the 20 year planning period. Under these conditions, the female population structure obtained at equilibrium is that shown in Figure 9.1.

The preliminary run suggests that we adopt a cyclical cropping pattern of the 1 and 6 year old hinds, which is repeated every fifth year. This is best illustrated in Figure 9.2a which shows the annual variation in the numbers of 1 year old hinds left.
Fig. 9.1 Female population structure at equilibrium.

Fig. 9.2 Numbers of 1 year old hinds in the population.
following the cull. Since no restriction was placed on the size of the herd after the 20th year, the total herd is, predictably, harvested in the final year. Because of this, the harvest and herd structure during the last three or four years is ignored when we interpret the results. The cropping strategy for the stags is constant from year to year. A sufficient number of two year old stags, determined by the breeding ratio constraint are retained in the herd; the remainder are all harvested before the rut as are all three year old stags, following the rut.

Up to this stage we have considered that revenue in period $t$ is equivalent to that in period $t + n$, ignoring the time profile of the net cash flow. In order to take this into account the objective function of equation 8.21 must be modified to the form shown below.

$$CONTRIBUTION = \sum_{t=1}^{T} \left[ \sum_i (ES_iHM_i + ES_iHY_i + ES_iHL_i - EC_iRM_i - EC_iRY_i - EC_iRL_i - Ca(BM + BY)) \right] (1 + r)^t$$

Eqn. 9.1

where $r$ is the specified discount rate. Utilising the above objective function with a range of discount values up to 20% produced virtually no change in the cropping strategy, as can be seen from Figure 9.2b. All further analysis is undertaken using the above objective function with a discount rate of 10%.

We noted at the beginning of this chapter that both the time horizon and the capacity constraints could affect the optimal management strategy. Our choice of planning horizon of 20 years
was intended to give the system the opportunity to stabilise so that we could examine the various phases of the population development. From the above analysis, it appears that this is sufficient time. The next stage of the analysis was to examine what effect increasing the stocking capacity constraint has upon the management policy at equilibrium, and to investigate the way in which the oscillating cropping strategy is developed.

Initial stocking capacity was varied between 250 through 475 and the subsequent cropping policies examined. Once again, the management strategy involved a cyclical cropping pattern of the one year old hinds removing all six year hinds from the herd. No change occurred in the male cropping strategy. Figure 9.3 (a-d) shows the numbers of one year old hinds remaining in the population following the harvest, for a number of stocking capacities. Thus we can see that although the proportion of the one year old hinds removed varies with changing stocking capacity, the general strategy remains the same.

As our choice of stocking replacement availability had been purely arbitrary, we next examined what effect varying this constraint had upon the the operating policy. Values for the availability of female calves were ranged from 25 up to 175, while those for males were varied proportionately. The stocking capacity constraint remained constant at 250 adult animals. Under these conditions, the optimal management policy once more dictated the removal of all six year old hinds and a varying proportion of the one year old hinds.
Fig. 9.3 Numbers of 1 year old hinds in the population with varying stocking capacity.

Stock Purchase Constraint = 100

Stocking Capacity

<table>
<thead>
<tr>
<th>Stocking Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
</tr>
<tr>
<td>325</td>
</tr>
<tr>
<td>400</td>
</tr>
<tr>
<td>475</td>
</tr>
</tbody>
</table>

Number of 1 Year Old Hinds vs. Period
Fig. 9.4 Numbers of 1 year old hinds in the population with varying stock purchase availability.

Stocking Capacity Constraint = 250

a

b

c

d

Number of 1 year old hinds

Period

Stock Purchase

25

75

125

175
The variation in numbers of one year old hinds remaining in the population following harvest is illustrated in Figure 9.4 (a - d).

If we examine the population structures illustrated in Figures 9.3 and 9.4, we can see that in each case the pattern developed while the herd is in the building phase continues to dominate when the equilibrium level is attained. The amplitude of the oscillation, however, is slowly attenuated through time. These patterns are dictated by the need to maximise the return over the planning period. To accomplish this the solution dictates that we attain the maximum stocking density in as short a time as possible. Stock purchase thus continues until the population attains the level at which calf input from existing stock is at a sufficiently high level for the population to achieve the maximum stocking capacity. At this level any excess stock is removed; the least cost-effective animals, in this case the one year old hinds, are removed first.

The relationships between the annual increment in the female population and the maximum stocking capacity during the transitional period from growth to equilibrium structure determines the cropping pattern. Thus in Figure 9.3c we can see that in the fourth period stock purchase is reduced to 10 and the bulk of the input consists of calves born to mature hinds on the farm. In Figure 9.3d, however, the population capacity is higher and the number of calves purchased in the fourth period must in turn be increased to accommodate the difference. In the following, fifth, period maximum stocking density is achieved and the excess calves are removed. Thus in these first periods an impulse is introduced which has set up a "wave". The fact that the
amplitude of this is reduced as we progress, suggests that such a cyclical pattern of harvesting, imposed by the impulse, is not a permanent feature. Through time we may approach a constant cropping strategy. An abrupt change to such a strategy would, however, result in a reduction in the value of the objective function, thus the model refrains from such a policy. We may, however, introduce a further constraint upon the system to attenuate the amplitude of oscillation more rapidly. Should the value of the objective function not vary significantly from the optimal undamped situation, we may assume that the cyclical pattern is not a vital feature of any management policy.

In the first instance, the damping effect was accomplished by specifying an upper bound of 55 to the numbers of one year old hinds which could be harvested ($HY_{1,t}$). All other constraints were as in the original model. Under these conditions, the herd structure which developed is that shown in Figure 9.5, in which the fluctuation in the proportion of one year old hinds harvested has been considerably damped. The damping effect is illustrated in Figure 9.6 (a - d), for various buying capacity constraints, in which we show the numbers of one year old hinds remaining in the population in the damped (1) and undamped (2) situations. It is noticeable that the damping effect observed in the unconstrained model is still present, although to a lesser degree, indicating that there is a continuing tendency to reduce further the amplitude of oscillation. As before, the cropping strategy requires that all six year old hinds be removed from the population. The value
Fig. 9.5 Female population structure at equilibrium with upper bound harvesting limit.

Fig. 9.6 Numbers of 1 year old hinds in the population with (1) upper bound & (2) unbounded harvest.
of the objective function under this modified operating strategy was reduced by between 0.4% and 0.6% for the four cases illustrated.

Thus for practical purposes a more stable cropping policy may be implemented with greater facility. If we examine the annual labour requirements for the two cropping strategies it is evident that the cyclical pattern of cropping is reflected in the manning levels. (See Figure 9.7).

The cyclical pattern may be removed entirely by constraining the harvesting of one year old hinds further still. This was achieved by setting the activities \( HY_{1t} (t = 1,19) \) at the lower limit level of zero, thereby preventing any cropping of the one year old hinds. The resultant herd structure is illustrated in Figure 9.8, in which cropping is now aimed at the four and six year old hinds. However, the value of the objective function is decreased by 4.2%, which suggests that under the existing conditions the most favourable cropping strategy should include some proportion of one year old hinds.

Up to this stage we have been concerned with the age structure at equilibrium levels; in the following section we shall examine the factors which influence the growth phase.

**Growth Phase**

During the building phase the policy is to purchase as much stock as the buying constraint allows, until the maximum stocking density is attained. This, however, is dependent upon the cost of the replacement stock in relation to the shadow prices of the one year old hinds and stags. As long as these are greater than or equal to
Fig. 9.7 Manning levels for (1) bounded & (2) unbounded strategies.

Fig. 9.8 Female population structure obtained when harvesting of 1 year old hinds is restricted.
stock purchase costs, additional stock is acquired. Table 9.3 illustrates the shadow prices of one year old hinds for varying stock purchase costs.

Table 9.3  Variation in Shadow Price of one Year Old Hinds with Stock Replacement Cost

<table>
<thead>
<tr>
<th>Stock Replacement Cost (£)</th>
<th>Shadow Price (£)</th>
<th>Period (Years)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>33.0</td>
<td>58.2</td>
<td>46.6</td>
</tr>
<tr>
<td>36.0</td>
<td>60.8</td>
<td>48.5</td>
</tr>
<tr>
<td>43.2</td>
<td>69.0</td>
<td>54.9</td>
</tr>
<tr>
<td>50.4</td>
<td>73.8</td>
<td>58.5</td>
</tr>
<tr>
<td>57.6</td>
<td>73.3</td>
<td>58.1</td>
</tr>
</tbody>
</table>

(Note: Stock replacement cost includes the cost of purchase and rearing up to weaning age).

The shadow prices associated with the original variables are indicative of the rate of change in the value of the objective function when the product requirements or resource availabilities are changed. In mathematical terms, the shadow price is the partial derivative of the objective function with respect to changes in the resource availability - in this case the numbers of one year old hinds. These changes apply over at least a small range of availability of the resource. Because in the early growth phase the number of one year old hinds present in the population is dependent upon the availability of replacement stock, the shadow price of these animals is given by the sum of the shadow price and the actual cost of replacement stock. Stock is purchased up to
the point at which the shadow price of the one year old hinds is greater than or equal to the cost of stock i.e. while the shadow price of replacement stock is $\geq 0$. Where the shadow price is less than the cost of stock but greater than the sales revenue no stock is bought nor animals of that age group cropped.

As stock replacement capacity increases the shadow price of the one year old hinds decreases. Similarly, increasing the stocking capacity constraint while holding replacement stock availabilities constant, increases the shadow price. This is illustrated in Table 9.4 below, for a range of replacement availabilities and stocking capacity constraints, from which we can see the importance of the ability to achieve maximum stocking capacity in the shortest possible time.

<table>
<thead>
<tr>
<th>Replacement Availability = 100</th>
<th>Stocking Capacity</th>
<th>S. Price (£)</th>
<th>Replacement Availability</th>
<th>S. Price (£)</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>60.8</td>
<td>25</td>
<td>135.1</td>
<td></td>
</tr>
<tr>
<td>325</td>
<td>74.3</td>
<td>75</td>
<td>74.4</td>
<td></td>
</tr>
<tr>
<td>400</td>
<td>76.8</td>
<td>125</td>
<td>57.4</td>
<td></td>
</tr>
<tr>
<td>475</td>
<td>76.8</td>
<td>175</td>
<td>47.5</td>
<td></td>
</tr>
</tbody>
</table>

In all the above cases, the general cropping strategy at equilibrium levels remained unchanged.
Thus far we have examined only one population transition in which the age structure at the end of the growth phase is longer than the optimal age structure at equilibrium. This was the case when the stock replacement constraint was set at a level of 25 female calves. In Figure 9.4(a) we can see that in the seventh year stock purchase is reduced and in the eighth harvesting of one year old hinds begins. In addition, all surplus six, seven and eight year old hinds are removed from the population.

In order to determine the effects of increasing the length of the growth phase, we reduced the buying capacity constraint for female calves to 25 and increased the maximum stocking capacity to 750 adult animals. The fecundity and survival schedules of Tables 7.8 and 7.9 were augmented by the inclusion of three further age classes. As before, the data were obtained from feral herds (Mitchell et al 1971; Lowe 1969). The additions to the original survival and fecundity schedules are given in Table 9.5.

<table>
<thead>
<tr>
<th>Age (Years)</th>
<th>Survival Rate</th>
<th>Fecundity Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hinds</td>
<td>Stags</td>
</tr>
<tr>
<td>9</td>
<td>0.92</td>
<td>0.92</td>
</tr>
<tr>
<td>10</td>
<td>0.85</td>
<td>0.91</td>
</tr>
<tr>
<td>11</td>
<td>0.88</td>
<td>0.81</td>
</tr>
</tbody>
</table>

The resulting population structure during the transition from growth to equilibrium is illustrated in Figure 9.9. Unlike in the previous example described above, when maximum stocking capacity is approached
Fig. 9.9 Transition of the female population from growth to equilibrium structure. Stocking capacity = 750, Stock purchase constraint = 25.
it is not the one year old hinds which are removed first from the population, but the less cost-effective ten year old hinds. In the following two periods the population age structure is once again truncated by the removal of all the hinds aged six to ten. Thereafter the original cropping strategy is adopted. It would appear therefore that under the original objective function and population parameter values the "best" cropping strategy is one in which a proportion of the one year old and all the six year old hinds are removed. If it is desired to crop a constant proportion of the one year old hinds to maintain a constant age structure, the proportion cropped should be approximately 40%. In the following section we shall examine how the cropping strategy is affected by changes in the value of the objective function coefficients.

Changes to Objective Function Coefficients
Stockholding Costs.

From the above analysis it appears that the overwintering and "stockholding" cost of the one year old hinds plays an important part in determining the cropping strategy. For this reason, in this part of the analysis we concentrate upon changes to the cost of holding these animals. For the first analysis the cost was increased over a range of 45% in steps of 15%. The cropping pattern remained unchanged for increases up to 30%. At the 45% level, however, the cropping strategy was altered and the herd structure lengthened to include seven year old hinds. This is shown in Figure 9.10, in which a proportional crop is taken from the one and the six year olds and all seven year olds are removed. The
Fig. 9.10 Female population structure at equilibrium with stock-holding cost of 1 year old hinds increased by 45%
Fig. 9.11 Female population structure at equilibrium with stockholding cost of 1 year old hinds reduced by 25%

Fig. 9.12 Female population structure at equilibrium with stockholding cost of 1 year old hinds reduced by 37.5%
male cropping strategy remains unchanged.

The cost was next reduced in steps of 12.5% down to 50%. The first change in strategy was observed when costs were reduced by 25%; the resulting age structure is shown in Figure 9.11. The cropping pattern is identical to that obtained when the original harvesting strategy was constrained to prevent the removal of one year old hinds from the population. Thus we can see that a reduction in stockholding cost of the one year old hinds leads to a rapid stabilisation in cropping pattern.

A further reduction in the cost by 12.5% to 62.5% resulted in yet another change in herd structure. The shadow price of the one year old yield hinds now exceeded the cost of stock replacement and the operating strategy included the annual purchase of replacement stock. The herd structure illustrated in Figure 9.12 was constrained by the labour availability, which prevented the purchase of further stock. When this constraint was relieved, the optimal strategy dictated that only one and two year old animals be overwintered and that three year old hinds be harvested after their offspring are weaned. Harvested three year olds are then replaced by purchased stock.

In order to determine to what extent the cost of holding one year old hinds predominates, the cost of retaining all female stock aged two to eight was varied over the range ± 50% while the former cost remained constant at the original level of £19.9. There was no change in the cropping policy from the original. That is to say, a proportion of one year olds and all six year olds were cropped.
However, when all stockholding costs were varied over the same range, the resulting management strategies were virtually identical to those obtained when only the cost of holding one year olds was varied. It appears, therefore, that the marginal revenue from the one year old hinds is the dominating factor which determines the management strategy.

In order to determine the importance of changes in the population age structure dictated by changes in the stockholding cost of one year old hinds, we carried out the following analysis. Costs were increased by 50% while the original cropping strategy was retained. The objective function value was compared with that in which the strategy was free to adjust to changes in operating cost. The "optimal" objective function value was less than 1% above that for which the cropping strategy was unchanged. It appears, therefore, that even for large increases in the stockholding cost of one year old hinds, the benefits to be gained from adjusting the cropping policy are very small. At the other end of the scale, however, holding the strategy constant while costs were decreased by 50%, the difference between the original and the optimal strategy was over 10%. Thus as the marginal revenue from one year old hinds is increased it becomes increasingly important to alter the cropping strategy.

In addition to changing stockholding costs, variation in venison revenue may affect the stocking and culling policies. In the following section, therefore, we examine how changes in price and/or venison yield influence the management strategies.
Venison Revenue

In estimating the range over which venison revenues should be varied, we took into account changes which might occur in both price and venison yield. It was considered unlikely that any downward change in revenue would exceed 25 - 30%. At this level, venison yield would be below even the poorest recorded yields in the wild (Mitchell et al 1971), and venison price at £1.00/kg would be at the price level of 1972/73. Because of the high quality of venison, in comparison with the wild, which might be expected from the farm, no difficulties should be experienced in meeting even the strictest existing hygiene regulations. Thus price levels should not be subject to any appreciable decline. At the other end of the scale, venison yields higher than those at Glensaugh have been recorded under favourable conditions in the wild (op.cit). In addition, because of the quality premium available to farmed venison, due to the elimination of shooting losses, price levels may be increased by 20-25% (on average 20% of saleable meat is lost due to poor shooting). In order to allow for these increases and any increases in the real price of venison, revenue was ranged upwards by up to 60%. As it is likely that stock replacement costs will vary with venison prices, the cost of buying stock was varied proportionately.

No significant change in the optimal cropping strategy was recorded for variations in revenue of ± 25%. At the +40% level, however, the cropping strategy was modified to that shown in Figure 9.13(a). Cropping is directed at the four and six year old hinds, as was the case when costs were reduced by 25% for the one year old hinds (see Figure 9.14). In this case, however, replacement stock is
Fig. 9.13 Female population structure at equilibrium with revenue increased by (a) 40% & (b) 60%
purchased up to the level permitted by the labour constraint (150 man days). If the constraint is relieved, stock purchase is increased to the level that permits a constant proportional crop of four year olds and the removal of five year olds; annual stock purchase levels are then equalised. If the availability of replacement stock is restricted, the cropping strategy illustrated in Figure 9.11 is adopted.

Further increases in revenue up to 60% results in the cropping strategy being altered once again. In this case it is identical to that obtained when the stockholding cost of one year old hinds was reduced by 37.5%. As before, the labour and cash availability constraints restrict the ability to purchase stock. When they are lifted, the optimal policy dictates that only one and two year old hinds are overwintered and all three year olds are removed after their calves are weaned and are replaced the following year by purchased stock.

Increasing the revenue has the effect of increasing the margin on the one year old hinds, as was the case when the overwintering costs of these animals was reduced. It is this single factor which dominates the cropping strategies. As the marginal revenue of the one year old hinds is increased so the cropping pattern changes and the younger animals are retained in the population.

We noted in Chapter 7 that the growth pattern of the farmed animals was modified by the inputs of supplementary fodder so that they attained maximum bodyweight at an earlier age. In order to investigate the effects of a continuation in growth, we modified the schedule given in Table 7.10 by extrapolating the growth rates
obtained in the wild (see Figure 5.2) to the farm situation.

The modified values are shown in parentheses in Table 9.6 below.

<table>
<thead>
<tr>
<th>Age Class</th>
<th>Weight (kg)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Stags</td>
<td>Hinds</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>27.2</td>
<td>25.7</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>48.7</td>
<td>45.6</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>58.9</td>
<td>49.1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>68.9</td>
<td>53.6</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>71.8</td>
<td>56.7</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>(78.0)</td>
<td>61.6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>(81.9)</td>
<td>(63.4)</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>(84.2)</td>
<td>(63.8)</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>(85.1)</td>
<td>(64.0)</td>
<td></td>
</tr>
</tbody>
</table>

There was, however, no change in the operating strategy for the modified growth schedule.

From the above analyses it appears that the original optimal strategy is relatively stable for changes in revenue between -25% and +30%. Further upward changes result in a shortening of the age structure of the herd, dependent upon the availability and cost of replacement stock.

In Chapter 5 we noted that changes in venison prices altered the economically optimal harvesting level. Although uncertainty regarding future price levels was one of the factors which led to the estates adopting conservative cropping policies, in a production system where there are severe limitations on the control, such uncertainties were only of secondary importance. On a farm, however, a much higher level of control over the production
possibilities may be achieved and price variation may become an important factor in determining cropping strategy. In the following section we shall examine what effect variation in venison price levels has upon cropping policy.

Fluctuating Venison Prices

In order to simulate a 20 year series of prices, a random number generator based on a normal distribution of mean £1.33/kg and standard deviation £0.33/kg was utilised. (Venison yields given in Table 7.10 were utilised). Stock replacement costs were assumed to vary with venison price. All other parameters were as for the original model (see Table 9.1). Under these conditions five replicated runs over the 20 year planning period were carried out. In each case the management strategy did not tend towards a stable policy, but fluctuated as the venison price varied. The general strategy, predictably, was to stock up when prices were low, harvesting when they were high.

The management strategy based on constant price (£1.33/kg) was compared with each of the strategies which adjusted to product price changes. The contribution margin for the 20 year period of each random run was compared with that which would have been obtained had the constant harvesting strategy been adopted for the random prices. The resulting labour utilisations for the fluctuating harvesting strategies ranged in each case from approximately 70 to 190 man days, which, as we noted before, may prove impracticable. The labour availability was therefore restricted to the range utilised in the constant price strategy (see Figure 9.6(a))
and the runs repeated. The resulting contribution margins for both restricted (2) and unrestricted (3) labour usage are shown below in Table 9.7.

<table>
<thead>
<tr>
<th>Run. No.</th>
<th>Constant Strategy (1)</th>
<th>Variable Strategy (2)</th>
<th>Variable Strategy (3)</th>
<th>% Age Difference (2) - (1)</th>
<th>% Age Difference (3) - (1)</th>
<th>Average % Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>35.22</td>
<td>38.96</td>
<td>46.94</td>
<td>9.6</td>
<td>25.0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>29.91</td>
<td>31.45</td>
<td>33.85</td>
<td>4.9</td>
<td>11.6</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>37.29</td>
<td>40.24</td>
<td>44.12</td>
<td>7.3</td>
<td>15.5</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>26.42</td>
<td>28.40</td>
<td>31.26</td>
<td>7.0</td>
<td>15.5</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>37.86</td>
<td>44.21</td>
<td>53.95</td>
<td>14.4</td>
<td>29.8</td>
<td></td>
</tr>
</tbody>
</table>

The above results suggest that the penalty associated with imperfect knowledge of prices is, at 8.6%, not of serious proportions. The original optimal strategy, which does not rely upon constant purchase of replacement stock, affords a more stable and practicable solution to management policy.

We have seen that in a number of the operating strategies described above, the cash and labour availability constraints have been instrumental in determining stock replacement strategies. This occurred in those cases where annual stock replacement was included in the optimal strategy. In the following section, we shall examine the outcome of reducing the availability of the above factors upon the original cropping strategy.

**Reduction in Availability of Working Capital and Labour**

In the first analysis, labour availability was reduced to 100 man
days. Because of the interdependence of labour and cash, constraining the labour supply resulted in an increase in the degree of slack in the cash utilisation.

In the case where the labour constraint is not limiting, the annual utilisation of these resources is approximately £5000 - £5500 and 120 - 130 man days respectively, once equilibrium is attained. Under the above constraints, the female cropping policy remained substantially unchanged; a proportion of the one year old hinds and all six year old hinds were removed. However, the male cropping policy was modified so that a proportion of the one year olds was removed annually; sufficient numbers of two year olds were retained to meet the breeding ratio constraint, and, following the rut, all three year olds were removed. The same cyclical pattern observed in the original model was repeated. However, in this case, the proportion of one year old males also varied as shown in Figure 9.14. Maximum stocking capacity was attained.

The next analysis was undertaken with the cash availability constraint reduced to £3000, which effectively limited the level of labour which could be utilised to approximately 80 man days. At the above constraint level, the ability to purchase stock was severely limited; in addition, maximum stocking density was not attained. Under these conditions the cropping policy for both males and females was greatly modified. The female population structure was lengthened to include seven year old hinds; thus a proportion of the one year old hinds and all seven year old hinds were removed annually. Once equilibrium was attained, all stags surplus to breeding requirements were removed in their first year, none were cropped as two year olds.
Fig. 9.14 Numbers of 1 year old animals in the population at equilibrium when labour availability is restricted.
and all three year olds were removed after the rut.

From the above analyses, we can see that when production factors are limited, the cropping strategy of the males is altered before any adjustments are made to the structure of the female population. In the following section, we shall examine by-product revenues from the sale of antlers to see whether increasing the revenue from stags has any effect upon the female population structure.

By-Product Revenues

In addition to venison sales, a further source of revenue is available from the sale of antlers, skins, lights etc. With the exception of antlers, there is little differential in the value of the above products for the two sexes. We shall therefore confine this study to the revenue available from the sale of antlers.

Antlers from wild red deer have been utilised in the manufacture of cutlery handles, buttons and local souvenirs for many years. The exploitation of such markets for the antlers of farmed red deer has already been investigated at the experimental farm. It was found, however, that due to the rapid development of the animals on the farm, the density of the antlers is too low for them to be of any use in cutlery or button manufacture. Nevertheless, there is a thriving market for antlers in velvet in the Far East, especially China, Taiwan and Hong Kong. The main suppliers are the USSR (Medexport, Moscow) and the deer ranches of New Zealand (Financial Times, 1977).

The main requirements for this industry, which may be met with
little difficulty by stags aged two to four years on the farm, are that the antlers be removed while still in velvet and carry between four and six tines. There is, however, one difficulty associated with the removal of the antlers, in that while in velvet they are living organisms, necessitating veterinary supervision during their removal. Nevertheless, in New Zealand antlers are removed with little or no complications even without the aid of anaesthetic, which suggests that with the greater degree of control available on a farm such an operation should be viable. On this basis, we examined the effects of increasing the revenue from two, three and four year old stags.

The number of i year old stags in any period t is given by

\[ R_{i,t} + HM_{i,t}. \]

In order to include the revenue from the sale of antlers, we must increase the objective function coefficients of the above activities by the value of the antlers. Typical prices, which vary with weight and condition are in the region of £20 - £30 for a pair. (pers. comm. Goldenwell Development Co. Ltd., Taipei, Taiwan). In plates 4 and 5 (Chapter 6) we can see the type of product which attracts the highest premium. Even stags at fifteen to sixteen months of age (plate 5) develop perfectly acceptable antlers. The following Table 9.6 shows the modified objective function coefficients utilised.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Coefficient Value (£)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HM_{2,t}</td>
<td>84.8</td>
</tr>
<tr>
<td>HM_{3,t}</td>
<td>103.6</td>
</tr>
<tr>
<td>HM_{4,t}</td>
<td>121.8</td>
</tr>
<tr>
<td>RM_{2,t}</td>
<td>11.6</td>
</tr>
<tr>
<td>RM_{3,t}</td>
<td>16.6</td>
</tr>
<tr>
<td>RM_{4,t}</td>
<td>21.6</td>
</tr>
</tbody>
</table>
Under the above conditions, although the contribution margin was increased by over 50%, no change in the cropping strategy occurred. Even when all antler revenues were increased to £30, neither the male nor female population structures were altered. Thus, although there is a possibility that contribution margin may be greatly enhanced by the additional revenue from the sale of antlers, no change should be made to the management policy of the population as a whole.

Deviation from "Optimal" Strategies

We have seen that the cropping strategy outlined above, in which a constant proportion of the one year old and all the six year old hinds are removed, is "optimal" over a wide range of operating conditions. However, outside these ranges the model suggests changes to the management plan.

As far as costs are concerned, the changes induced by increases were seen to offer little advantage. When costs were increased by 50%, the modified strategy yielded an increase in contribution margins of less than 1% over the original plan. At the other end of the scale, however, as costs were decreased the penalties for not adopting the modified strategies became greater. Thus when costs were reduced by 50%, the modified cropping strategy, which resulted in a shorter age structure, yielded an increase in contribution of 10%.

Increasing revenue also induced a change in the optimum cropping policy. However, in this case the penalties incurred in not adopting the modified plan were less severe than in the previous
case. As revenues were increased by 40% and 60% the modified strategy resulted in an increase in margin of 1.5% and 3.3% respectively over the original plan. Thus we can see that the penalties incurred by maintaining the original cropping policy are not overwhelming. Indeed, the burden of implementing the changes may outweigh any advantages. For reductions in revenue of 25%, no changes in operating strategy occurred. In the hypothetical case where there is perfect knowledge of the future price levels, adjusting the cropping strategy to varying prices results in an increase in contribution of 8.6%. Once again, however, even if such foreknowledge was available, the difficulties associated with the fluctuating cropping policy may nullify the advantage.

Thus far we have been concerned solely with the "external" factors which affect the population management strategies. These include stock replacement costs and availabilities, stocking capacity constraints, working capital and labour availabilities, venison and by-product revenues and stockholding costs. In the following chapter we shall examine how changes in the population parameters, the fecundity and survival rates, influence management plans.
Chapter 10.

Model Output Analysis – The Effects of Population Parameter Changes on Operating Strategies.

From the preceding analyses it is evident that the young animals have the greatest impact upon the optimal cropping policies. For this reason, in the first part of this chapter we analyse the effects of changes in the fecundity and survival rates of these age classes. This is followed by an analysis of the fecundity of the mature animals and the overall fecundity schedule. Finally, we review the findings of these two chapters and discuss our conclusions.

Fecundity Rates of First Breeding Hinds

We noted in the previous chapter that the cost of overwintering and feeding the one year old hinds in relation to their value was the factor which determined the management policy on the farm. The objective of the intensive feeding programme, which these animals undergo, is to enhance their ability to achieve breeding condition in their second year. Results on the Glensaugh farm indicate that the success of this measure is highly dependent upon the level of feeding undertaken. Where previously we varied the cost of the feeding programme and maintained the level of breeding success at the maximum, in this section we shall examine the effects of varying both the breeding rate and the cost of overwintering and feeding.

For the first analysis, we maintained the stockholding cost at the original level ($RY_{1,t}$ - Table 9.1) and varied the breeding success rate of the two year old hinds. All other parameters were kept
at their original values. No significant change in operating strategy was observed as the fecundity was reduced from 1.00 to 0.80, although the value of the objective function was reduced by 32% as the number of yield hinds in the population was increased. At the 0.70 level of fecundity, however, the herd structure was altered to include seven year old hinds (see Figure 10.1). The cropping policy was to remove a proportion of one and six year old hinds and all seven year olds. The male cropping policy remained unchanged from the original.

The cyclical pattern of cropping observed in the previous chapter was again present in this situation, although, as before, the amplitude of oscillation was damped through time. Imposing an upper bound of 50 on the level of permissible crop of one year old hinds \( (HY_{1,t}) \), rapidly attenuated the amplitude of oscillation. The value of the objective function was reduced by 1.04%. As before, the attenuation of the oscillation resulted in a more uniform manning level. The population structure during the transitional and equilibrium phases is illustrated in Figure 10.2, in which the equilibrium cropping policy dictates the removal of all seven year old hinds and a constant proportion of 10% and 47% of the six and one year old hinds respectively.

Predictably, as a result of the change in margin of the one year old hinds, reducing the fecundity rate of the first breeding hinds has a similar effect upon the age structure to increasing the stockholding costs of the one year old hinds. In order to counteract the decline in cost effectiveness of these young animals, the herd structure expands to allow us to utilise the relatively "cheaper" older animals.
Fig. 10.1  Female population structure at equilibrium when the fecundity of 2 year old hinds is reduced to 0.70

Fig. 10.2  Female population structure at equilibrium when the fecundity of 2 year old hinds is reduced to 0.7 and an upper limit of 50 is specified for the harvest of 1 year old hinds
However, even for reductions in breeding rate of 30%, or, as we saw before, increases in feeding costs of one year old hinds of up to 45%, only one additional age class is incorporated into the herd structure.

So far we have dealt only with the situation in which supplementary feeding is supplied at a level sufficient to induce breeding in the two year old hinds. Results at Glensaugh show that at feeding rates of 0.5 kg/head/day hinds attained average weights of only 49 kg in their fifteenth-sixteenth month and consequently failed to breed. The outcome of adopting such a policy of low supplementary feeding is therefore examined, using the modified fecundity schedule and objective function coefficients shown below in Tables 10.1 and 10.2. It should be noted that in order to carry out the taming programme some degree of supplementary feeding must be undertaken. It is therefore not possible to discontinue this activity altogether.

Table 10.1 Modified Lactating Hind Fecundity Schedule for Low Feeding Plane

<table>
<thead>
<tr>
<th>Proportion Breeding</th>
<th>Age (Years)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3</td>
</tr>
<tr>
<td>Lactating Hinds</td>
<td>[0.00]</td>
</tr>
</tbody>
</table>

Note: Figures in parentheses are taken from Table 6.4, the remainder from Table 7.8
Table 10.2

<table>
<thead>
<tr>
<th>Activity</th>
<th>Obj. Fn. Coefficient (£)</th>
<th>Activity</th>
<th>Obj. Fn. Coefficient (£)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RY₁</td>
<td>10.5</td>
<td>HY₅HL₅</td>
<td>60.0</td>
</tr>
<tr>
<td>RY₂</td>
<td>9.4</td>
<td>HY₆HL₆</td>
<td>64.8</td>
</tr>
<tr>
<td>HY₂</td>
<td>45.6</td>
<td>HY₇HL₇</td>
<td>67.2</td>
</tr>
<tr>
<td>HY₃</td>
<td>53.4</td>
<td>HY₈HL₈</td>
<td>67.8</td>
</tr>
<tr>
<td>HY₄HL₄</td>
<td>56.4</td>
<td>HY₉HL₉</td>
<td>67.8</td>
</tr>
</tbody>
</table>

Note: The values for venison revenue are the product of venison yields (low plane) given in Table 7.10 and the average venison price of £1.33/kg. All other objective function values are as given in Table 9.2.

The resulting population configuration is shown in Figure 10.3, in which the age structure is lengthened to include seven, eight and nine year old hinds. The cropping strategy includes a varying proportion of one and eight year old hinds and the total removal of nine year old hinds. Here again we observe the policy of utilising the more cost-effective older animals at the expense of the younger age classes. The introduction of a further non-productive age class (three year old yield hinds) is the major cause of the extension in the age structure. The contribution margin under these conditions is 44% of that for the original cropping strategy utilising the high feeding plane. As a further comparison, it is 55% of that achieved for the high feeding plane with stockholding costs increased by 45% or breeding rate of two year olds reduced to 0.70. If we take the case in which the fecundity of the two year olds is 0.70 and increase the cost of stockholding by 50%, assuming that the optimal strategy does not change, the low feeding plane contribution margin is still, at best, only 73% of the above. (Any change in optimal strategy for the above combination of breeding rate and stockholding cost will, of course increase the contribution margin). If we consider that
Fig. 10.3 Female population structure at equilibrium when the low feeding plane is adopted. The resulting fecundity of the 2 & 3 year old hinds is 0.0 & 0.70 respectively.
the proportion of the stockholding cost $R_{1,t}$ which is due to feeding is only 75%, an increase in overall stockholding cost of 50% allows an increase in feeding from the original 1kg/head/day to 1.67kg/head/day. At such high levels, the assumption of a 70% breeding success rate for two year old hinds may be regarded as a conservative estimate. It would therefore appear that to adopt a management plan which does not include the use of supplementary feeding to encourage early breeding, incurs a severe penalty. The level of feeding to be utilised on any estate will not only depend upon the characteristics of the natural vegetation and its feeding value, but also upon the relationship between supplementary feeding cost and fecundity rate. As yet, however, there are insufficient data from the Glensaugh project to allow us to examine this relationship in more detail.

One Year Old Survival Rate

As far as young animals are concerned, the only other major area of sensitivity is the survival rate of the one year olds. We noted in Chapter 6 that a high proportion of the mortality at Glensaugh was ascribed to "black loss". However, this was seen to be peculiar to the area utilised by the hinds and their calves during the period prior to weaning. There is therefore the possibility that the survival rates of the young may be improved.

The survival rate of both male and female calves was increased from 0.88 to 0.92. Although there was an appreciable increase in the value of the objective function of 13% there was no change in the general cropping strategy from that of the original model.
further increase in the survival rate up to 0.95, resulted in a modification to the female cropping strategy to that shown in Figure 10.4. The value of the objective function was raised to 23% over the original. The modified cropping strategy results in a decrease in the proportion of the one year old hinds cropped, so that only 11% are removed every second year. This cyclical pattern is, once again, an artefact of the structure imposed during the growth phase. The removal of this fluctuating pattern of cropping results in no one year olds being harvested; the harvested is then concentrated solely on the four and six year olds.

A noticeable feature of the cropping strategy illustrated in Figure 10.4 is the way in which the five year old hinds are retained and the harvest is taken only from four and six year olds. We have already observed a similar pattern when the stockholding cost of one year old hinds was reduced or venison revenues increased (see Figures 9.10 and 9.12a). If we examine the original fecundity and survival schedules (Tables 7.8 and 7.9) we can see that in each case the values for the five year old hinds are higher than that of both four and six year olds. During the transitional stage to equilibrium, because of their poorer performance, the four year old hinds are removed in preference to the five year olds. As a result, the fluctuating cropping pattern of four and six year old hinds persists.

In order to examine the importance of such a pattern, we imposed an upper bound constraint upon the numbers of five year old hinds which could be retained in the population. The resulting "optimal"
Fig. 10.4  Female population structure at equilibrium when the survival rates of the 1 year old animals are increased to 0.95
cropping strategy dictated the removal of a constant proportion of four year old hinds and, of course, all five year olds. The objective function value was reduced by 0.2%, an insignificant amount. The L.P. solution technique, however, takes account of such "insignificant" amounts and the solution proposed is the "best" on a purely mathematical basis. For practical purposes, the adoption of the "sub-optimal" solution obtained from the bounded model, which advocates a uniform cropping pattern is to be preferred.

From the above analysis, it is evident that the effects of increasing the survival rate are identical to those of reducing the stock-holding cost. However, although costs have to be reduced by approximately 25% to effect any change in cropping strategy, survival rates need only be increased by 8% to produce the same change. The reason for this is that a reduction in costs merely improves the cost effectiveness of the one year old hinds, whereas an increase in survival also increases the overall productivity of the population.

**Modifications to Fecundity of Mature Hinds**

We noted above the sensitivity of the model to slight differences in the fecundity and survival rates of the animals aged four to six and observed that the adjustments made to the herd structure to accommodate them afforded no significant advantages in terms of contribution margin. In estimating the range over which fecundity values for the age classes are likely to change, we should take into account the way in which the fecundities of each age class are related to one another. We have observed that both lactating hinds on the farm and mature yield hinds in the wild exhibit uniformly high
fecundity rates. Under the conditions which would exist on a farm, with regard to the availability of supplementary fodder and general husbandry practices, we would not expect any violent fluctuation in breeding rate between the age classes four to eight. We have also observed that where the policy is to aim for a reduction in age of first breeding to two years, the success rate of three year old hinds is equally high. In order to determine the effects of any fluctuation in the fecundity of the three year old animals, we reduced the breeding rate from 1.00 to 0.90. Apart from the increase in the number of four year old yeld hinds and the reduction in herd productivity, there was no change to the original herd cropping strategy for either males or females.

Although it is unlikely that any great difference in breeding rate between the age groups would occur on the farm, the overall breeding rate may well vary between sites due to differences in vegetation, geography and weather. Reducing the overall breeding rate does not, however, significantly affect the cropping strategy as can be seen from Figure 10.5. Due to the lowered fecundity rate of lactating hinds (0.80) the numbers of yeld hinds of each age class are increased. Consequently the six year old yeld hinds are retained in the population and cropped in their seventh year, after their calves are weaned. These animals are retained as a result of their superior breeding performance; 1.00 for yeld hinds compared to 0.80 for lactating and first breeding hinds. Where there was little or no difference between the breeding success rates of neighbouring age classes of five years and over, no attempt was made to increase the age structure (Figure 10.6).
Fig. 10.5 Female population structure at equilibrium with the fecundity of the 2 year old and all lactating hinds = 0.8. The fecundity of the remaining yield hinds, other than the 1 year olds, = 1.0.

Fig. 10.6 Female population structure at equilibrium with the fecundity of yield and lactating hinds aged 6 and over = 1.0.
From the above analysis it appears that with a relatively uniform breeding performance in the mature age classes, there is no impetus to increase age structure. Even where fluctuations do exist and the "optimal" solution dictates an increase in length, the difference in contribution obtained by the modification is of no great significance. It is, however, unlikely that such fluctuations would be a feature of the fecundity schedule of an intensively husbanded stock.

**Breeding Ratio**

The productivity of the adult population as a whole is also governed by the numbers of "non-productive" males which have to be retained to allow the herd to breed. Thus far we have maintained a constant breeding ratio of one male to sixteen females, which is the best ratio recorded at the experimental farm. In this section we examine the outcome of varying the breeding ratio and the relative ability of the stags to mate successfully.

Reducing the breeding ration from 16:1 through 10:1 resulted in an increase from 15 to 23 in the number of two year old stags retained in the population to provide the breeding stock, thereby reducing the available capacity of the hinds from 235 to 227. As before, the three year old stags formed the nucleus of the breeding stock, although the retained two year olds could also be utilised, either as an incentive to spur on the older stags or even as breeding animals themselves. The three year old stags are then harvested immediately after the rut. From the evidence of breeding trials at Glensaugh (Blaxter et al 1974) such a ratio should prove more
than adequate.

As far as the cropping strategy of the hinds is concerned, there is no change in the general management policy. The changed breeding ratio merely restricts the number of breeding hinds in the population and, consequently, the calf input. The value of the objective function at the 10:1 breeding ratio is 4.3% below that at the 16:1 ratio.

Model Validity

In Chapter 8 we outlined the relationships which affect venison production on the farm and defined the objective of our model. The ability of the model to represent the relationships between the factors in the real system depends upon the assumptions we made regarding the inclusion of the system variables and their interactions. One of these assumptions was that there is no variation in the population parameters from year to year as a result of population density or climatic effects. As we saw in the previous chapters which dealt with wild deer systems, both the above factors play an important part in the population dynamics. We have also assumed that through time, the grazing population does not alter the vegetational composition on the farm and hence the relationships between management inputs and productivity remain constant.

As far as density dependent effects are concerned, we may justify our assumption on the basis that at equilibrium the population density on the farm is constant. Thus any errors which may arise should be minimised. The effects of extraneous variables such as climate cannot as yet be included in the model due to the lack of
data. However, we saw in Chapter 4, Figure 4.13 that the rainfall dependent variation in survival on Rhum appeared to exert an influence only at population densities above the optimal for maximum productivity. Thus rainfall effects on the farm may prove to be negligible. The degree of variation on the farm due to other climatic factors, such as snowfall, may also be reduced as a result of the ability of the manager to vary production inputs. Thus the variation in climate may result in a variation in inputs rather than population productivity. At this early stage in the development of deer farming, however, there are insufficient data to allow us to formulate any such relationships.

The relationships which we have included are based almost entirely on the information obtained from the experimental farm at Glensaugh, as a result of which the model may merely reflect the particular characteristics of that operation. We have attempted to overcome this problem by examining the model output over a wide range of population parameters and operating conditions. The final judgement regarding the usefulness of the model, however, must rest upon the extent to which it helps the manager to improve the operation of the farm.

Conclusions

The objective of this and the preceding chapter has been to examine how the optimal operating strategy on the deer farm varies with changes in costs, revenues, the availability of resources and population parameters. In choosing the ranges over which the
above parameters were varied, we aimed to take into account a comprehensive range of "expected" perturbations. As far as population parameters are concerned, these expectations were based upon our analysis of the productivity on Rhum, the results of numerous studies of feral herds and the experimental results from Glensaugh. Using the L.P. technique we obtained a number of optimal solutions for various combinations of the above parameters. We do not, however, suggest that these are "the optimal" management strategies. Indeed, we have shown that in some cases the mathematically optimal solutions proposed may be "improved" on the grounds of ease of implementation. Thus, by specifying further bounds or constraints upon the model to reflect our operational requirements, we may compare the optimal solutions obtained under various operating conditions. The value of the L.P. method lies in its ability to compare the "ideal" optimal solution with the modified practicable solution.

As far as costs are concerned, it was the stockholding cost of the one year old hinds which had the greatest impact upon the management strategy. Increasing these by 50% resulted in a lengthening of the age structure to include seven year old hinds. However, it was found that the benefits to be gained from such an adjustment (less than 1.0% increase in contribution margin over the 20 year planning period) might not warrant the effort required. As the cost of holding one year old hinds was decreased, the age structure was shortened and cropping concentrated first on the four and five year olds (25% decrease) and then on to the three year olds (37.5% decrease). In this case, when costs were reduced by 50%, the change in cropping strategy from the original resulted in a 10%
increase in contribution.

In general, as costs are decreased the tendency is to shorten the age structure so that cropping may be concentrated on those age classes which have the highest growth rate. In the limit, only one breeding age class, the two year old hinds, is retained. The cropping policy is then to remove the three year old hinds after their calves are weaned and replace them the following year with purchased stock.

Changing the revenue available for each age class, either as a result of price changes or changes in yield, over the range -25\% to +30\% had no effect upon the cropping strategy. Further increases led to a shortening of the age structure to that obtained when costs were reduced. It appears, therefore, that the original cropping strategy, in which a fixed proportion of approximately 40\% of the one year old and all six year old hinds are removed is the "optimal" over a wide range of costs and revenues. Even in the hypothetical situation where we had perfect knowledge of future price changes, the benefits to be gained from varying the management strategy were not, at 8.6\%, overwhelming. The single dominating factor throughout has been the marginal cost of the one year old hinds. Where this was greater than the value of the venison revenue, the young animals were retained in the population and a shorter age structure adopted.

Under conditions in which the availability of working capital or labour was restricted, we saw that it was the male population which was adjusted first. Only the minimal numbers required for breeding purposes were retained. Neither was any adjustment required to the female population structure when the revenue from the stags was increased as a result of the exploitation of antlers.
In many respects, the effects of changes to the population parameters were similar to those above. Assuming that fecundity schedule of the mature animals was relatively uniform, we saw that variations in the overall schedule had no effect upon the cropping pattern over the range examined (1.00 to 0.80). It was, once again, the young animals which controlled the management strategies. Changes to both the fecundity of the first breeding hinds and the survival rates of all one year old animals induced changes in the herd structure.

Decreasing the fecundity of the two year old hinds by up to 20% had no effect upon the cropping pattern. By reducing the breeding rate to 70%, a change in the optimal harvesting strategy to include a further age class (seven year olds) could be induced. However, as was the case with the cost increases discussed above, the benefits to be gained from such an extension in the age structure were only marginal.

Although contribution margins were greatly enhanced by the increase in survival rate of the one year old animals from 0.88 to 0.92, no change in the cropping strategy resulted. Even when a change did occur at the 0.95 level, the increase in objective function value over the original strategy was of no great significance. Likewise, changes in the breeding ratio, predictably, produced no alteration to the harvesting plan.

While the great majority of the above analysis dealt with the situation in which the management objective was to induce early breeding in the two year old hinds, we did examine the effects of
adopting a "low feeding" strategy. The results of our analysis leave no doubt that the inclusion of a further non-breeding group of animals, together with the reduced venison yield, cannot be balanced by the accompanying reduction in feeding cost. The optimal policy should include an intensive feeding programme.

**Recommendations for Prospective Deer Farmers**

As a result of the above analyses we may propose some recommendations regarding the operating strategies which prospective deer farmers might adopt.

1. **Growth Phase.**

To enable maximum stocking capacity to be attained rapidly, no females should be removed from the populating during the growth phase. The male cropping strategy, which remains the same throughout the operation of the farm, is to remove all three year old stags after the rut and leave only sufficient numbers of two year old stags to meet breeding ratio constraints. The remaining two year olds are removed prior to the rut. In the event of a shortage of manpower or cash, the stags should be removed before the first winter leaving only sufficient numbers for breeding purposes.

2. **Equilibrium Phase**

Once maximum stocking density is attained excess females aged six and over should be removed and the population structure adjusted such that a uniform cropping strategy may be adopted. Thereafter the "optimal" strategy entails the removal of approximately 40% of the one year old hinds and all hinds aged six years. During both the growth and
equilibrium phase the policy should be to supply sufficient fodder to the one year old hinds during the winter and early spring to enable them to attain breeding status in their second year. The reduction in productivity due to the failure to breed at two years far outweighs any savings which may be made in feeding costs. As far as breeding ratios are concerned, the marginal productivity of increasing the ratio rapidly declines above 10:1. At levels in excess of this some account should be taken of the risk due to the failure of the stags to attain expected performance.

In the following chapter we shall examine in more detail the costs, revenues and cash flow situation of the above management plans.
Chapter 11

Deer Farming - An Investment Appraisal

In the preceding chapters we concluded that the "best" operating strategy to adopt, once equilibrium was attained, was one in which all six year old and a proportion of one year old hinds were removed annually. In this chapter we examine the financial aspects of the farming operating based on such a strategy. We study the effects of changing investment in fencing upon the rate of return and the cash flow situation. In addition we investigate how changes in the population parameters affect these factors.

Uniform Harvesting Strategy

The equilibrium age structure of the female population may be determined as shown below -

\[
RF_i = \frac{SCAP \times BR}{BR+1} \sum_{i=1}^{n-1} \prod_{i=1}^{n} SF_i
\]

(Eqn. 11.1)

where \( RF_1 \) is the number of one year old females retained in the population, \( SF_i \) is the survival rate of the \( i \)-year old hinds, \( BR \) is the breeding ratio (number of hinds per stag), \( SCAP \) is the maximum stocking capacity of the farm and \( n \) is the length of the age structure in years. The number of females in the remaining age classes, 2 to \( n \), is given by

\[
RF_i = RF_{i-1} \times SF_{i-1} \quad (i=2, n)
\]

(Eqn. 11.2)

The proportion of yeld and lactating hinds present in each age class is determined by the respective fecundity rates.

As we saw in the previous chapters, the "optimal" length of age
structure was five years. The proportional structure of the female population with maximum longevity of five years is shown in Figure 11.1 (survival and fecundity data are taken from Tables 7.8 and 7.9). Because survival rates are uniformly high for hinds aged two to eight, the proportions of the age classes two to five are almost identical. The overall breeding rate of the hinds is 97% with 40% of the one year olds being removed annually. The transition to such an equilibrium structure is illustrated in Figure 11.2, in which the stocking capacity, breeding ratio and buying capacity of one year old hinds are 250, 16:1 and 100 respectively. The cropping pattern of the male animals remains unchanged i.e. all three year olds are removed following the rut and only those numbers of two year olds determined by the breeding ratio constraint are retained; the remainder of the two year olds are removed prior to the rut. In the following sections we shall examine the returns using the strategy illustrated in Figure 11.2.

Investment Appraisal - The Discounted Cash Flow Method (D.C.F.)

In Figure 11.2 we illustrated the operating strategy for a projected deer farm. This strategy will provide a series of cash outflows - the cost of purchasing and maintaining the herd - and cash inflows - sales revenue from the harvested animals. In order to generate the earnings, however, an investment must be made for the provision of fencing. With such an investment we cannot simply take the sums of money spent and received at their face values; money one year or ten years hence is not worth the same as money today. This problem may be overcome by the use of the discounted cash flow (DCF) technique, which takes into account both the timing and the size of the receipts and payments associated with the investment.
Fig. 11.1 Proportional age structure of the female population as a percentage of the total number of mature females retained following the harvest.

Proportion of remaining mature females %

- Harvested
- Remaining Lactating
- Remaining Yeld

AGE CLASS

<table>
<thead>
<tr>
<th>Age Class</th>
<th>Harvested</th>
<th>Remaining Lactating</th>
<th>Remaining Yeld</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30</td>
<td>25</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td>20</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Fig. 11.2  Transition to equilibrium population structure

(a) Females  (b) Males

- Purchased
- Harvested
- Remaining Yld
- Remaining Lact
The reason for using the DCF technique may be illustrated by considering the way in which investment is financed. Money is raised, at a cost, by paying interest to the lender. This money is then used to buy assets, in our case fences and stock, to generate a cash flow, which must be sufficient to repay the initial outlay and to pay an adequate rate of interest on the balance outstanding at any time. It is this interest rate which the DCF method measures and which may be used to express the worth of the investment.

The procedure is to find a rate of interest that will make the present value of the cash proceeds expected from the investment equal to the present value of the cash outlays required by the investment. The purpose of discounting the cash flows is to determine whether the investment yields more cash than alternative uses of the same amount of money borrowed at the same cost. Two important points should be noted. Firstly the DCF method takes account of depreciation automatically and, secondly, cash disbursed for interest repayments is excluded from the cash flow computation used in analysing the investment. This then is the method we use in the following sections to evaluate the deer farming project of Figure 11.2.

**DCF Calculations**

In this section we illustrate the method of calculation of the cash flow for a particular level of fencing and stock investment. This constitutes our base level, around which the subsequent sensitivity analysis is undertaken. In all cases a tax rate of 52% is used and the stock is treated on a herd basis (I.C.T.A. 1970 Sch. 6, Sch.D) for taxation purposes. That is to say, the initial cost of the herd
and the cost of any animals added to it are not deducted as expenses. Similarly, for the purpose of calculating taxable income, a depreciation rate of 10% per annum is used. As can be seen in Table 11.4, the depreciation is not included in the cash flow; it is required only for the estimation of tax liability. For this analysis the effects of inflation on future prices and costs are ignored; all calculations are carried out on the basis of present costs and prices i.e. those given in Chapters 6 and 7. Unless otherwise stated, it is assumed that taxation allowances may be offset against taxable profits (agricultural) from other activities. The effects of changes to the above assumptions on the rate of return are examined in the subsequent sensitivity analysis.

The Base Case

Cash Flow

Investment in fencing is assumed to occur over a two year period to coincide with the growth in the size of the herd. Because the availability of stock is constrained, stock is purchased over a three year period. For the purpose of this calculation, the total cost of fencing was taken to be £23,000. The unit fencing costs utilised (£/mile) were the mean of the values for the fence types given in Table 6.2.

<table>
<thead>
<tr>
<th>Investment</th>
<th>Cost (£)</th>
<th>Date of Cash Outlay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fencing</td>
<td>11500</td>
<td>( t_{10n}, t_{10n+1} ) ( n=0,1,2, \text{ etc} )</td>
</tr>
<tr>
<td>Stock</td>
<td>2600</td>
<td>( t_1, t_2 )</td>
</tr>
<tr>
<td>Stock</td>
<td>600</td>
<td>( t_3 )</td>
</tr>
<tr>
<td>Stock purchase cost</td>
<td>£24/head</td>
<td>(see Table 8.1)</td>
</tr>
</tbody>
</table>
Table 11.2  
Projected Sales Revenue

<table>
<thead>
<tr>
<th>Date of Cash Receipts</th>
<th>Expected Sales (£)</th>
</tr>
</thead>
<tbody>
<tr>
<td>t₃</td>
<td>470</td>
</tr>
<tr>
<td>t₄</td>
<td>2740</td>
</tr>
<tr>
<td>t₅</td>
<td>10300</td>
</tr>
<tr>
<td>t₆⁺</td>
<td>12800</td>
</tr>
</tbody>
</table>

Sales = Price (£1.33/kg) x Output (kg) (see Figure 11.2 a & b)

Table 11.3  
Projected Operating Costs

<table>
<thead>
<tr>
<th>Dates</th>
<th>Labour</th>
<th>Fodder</th>
<th>Veterinary &amp; Miscellaneous</th>
<th>Total Cash Outlay (£)</th>
</tr>
</thead>
<tbody>
<tr>
<td>t₁</td>
<td>640</td>
<td>2500</td>
<td>210</td>
<td>3350</td>
</tr>
<tr>
<td>t₂</td>
<td>930</td>
<td>3220</td>
<td>410</td>
<td>4560</td>
</tr>
<tr>
<td>t₃</td>
<td>970</td>
<td>2590</td>
<td>590</td>
<td>4150</td>
</tr>
<tr>
<td>t₄</td>
<td>1200</td>
<td>3250</td>
<td>800</td>
<td>5250</td>
</tr>
<tr>
<td>t₅</td>
<td>1280</td>
<td>3420</td>
<td>850</td>
<td>5550</td>
</tr>
<tr>
<td>t₆⁺</td>
<td>1300</td>
<td>3440</td>
<td>860</td>
<td>5600</td>
</tr>
</tbody>
</table>

Note: Operating costs are those given in Chapter 6 and Table 9.1 (£/animal) for each age and sex class. Total cost is the product of the above unit costs and the numbers of animals in the population (see Figures 11.2 a & b)

Project Life and the Rate of Return

For this analysis it is assumed that fences are renewed every ten years. The rate of return is calculated on the basis that the deer farmer will continue in operation indefinitely, renewing fences as required. It is assumed that maintenance costs of fencing will
<table>
<thead>
<tr>
<th>Year</th>
<th>Sales Revenue</th>
<th>Operating Costs</th>
<th>Gross Margin (1)-(2)</th>
<th>Depreciation</th>
<th>Taxable Profit (3)-(4)</th>
<th>Tax (52%)</th>
<th>Investment Fences</th>
<th>Stock</th>
<th>Cash Flow (3)-(6)-(7)-(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1150</td>
<td>-1150</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-11500</td>
</tr>
<tr>
<td>1</td>
<td>3350</td>
<td>-3350</td>
<td>1150</td>
<td>-4500</td>
<td>11500</td>
<td>2600</td>
<td>-17400</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>4560</td>
<td>-4560</td>
<td>2300</td>
<td>-6860 -2340</td>
<td>2600</td>
<td></td>
<td>-4820</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>470</td>
<td>4150</td>
<td>-3680</td>
<td>2300</td>
<td>-5980 -3570</td>
<td>600</td>
<td>-710</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2740</td>
<td>5250</td>
<td>-2510</td>
<td>2300</td>
<td>-4810 -3110</td>
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<td>600</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>10300</td>
<td>5550</td>
<td>4750</td>
<td>2300</td>
<td>2450 -2500</td>
<td></td>
<td>7250</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>12800</td>
<td>5600</td>
<td>7200</td>
<td>2300</td>
<td>4900 1270</td>
<td></td>
<td>5930</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>12800</td>
<td>5600</td>
<td>7200</td>
<td>2300</td>
<td>4900 2550</td>
<td></td>
<td>4650</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

D.C.F. Rate of Return = 6.7%
Fig. 11.3  Net cash flow pattern.

NET CASH FLOW

NET CASH FLOW

TIME yrs
be minimal over this period. The net cash flow calculations are given in Table 11.4 and the cash flow pattern is illustrated in Figure 11.3. The DCF rate of return for this base case is 6.7%.

In the following sections we shall examine the sensitivity of the above return to changes in the operating conditions and our assumptions.

**Offsetting Taxation Allowances**

The above rate of return was calculated on the assumption that the investor was able to offset negative taxes against profits from other agricultural operations. Should this not be possible, the allowances may be set against future profits from this project. Under these conditions, the return is reduced from 6.7% to 6.3%.

**Fence Life Increased**

Increasing the useful life of the fences from ten to twelve years, assuming still that repair costs are negligible, the return is increased from 6.7% to 7.5%.

**The Effects of Inflation**

In the preceding analyses we chose to ignore the effects of inflation when calculating future cost and revenues. In this section we shall therefore discuss the effects of inflation on the viability of the project.

Where the inflation rate of costs and prices is synchronised, i.e. all costs and prices are rising at the same rate, net revenues will increase and hence returns will go up. However, as interest rates are a function of future anticipated price changes, the
required rate of return on the project must take account of future inflation (Schofield et al 1973). Assuming synchronised geometric growth in the level of prices and costs, an approximation for the DCF return may be obtained by adding the rate of inflation to the return under no inflation. Thus for an inflation rate of 10% per annum the return on our base case would be increased from 6.7% to approximately 17%. It should be noted, however, that the cost of capital will also reflect the market's expectation of the same inflation rate. Only when the prospective investor feels that the market is being either too optimistic or pessimistic about future inflation, should he adjust his cost of capital otherwise.

When different elements in the revenue/cost structure rise at different rates, the situation becomes more complicated. The only general conclusion that may be drawn is that while revenue is increasing at a faster rate than costs, returns will increase. For any other combination a detailed analysis of the projected cash flows must be undertaken. Such an analysis of future inflationary trends is, however, beyond the scope of this study. Suffice it to say, that these effects constitute a further degree of uncertainty in the appraisal of the project.

**Changes in Fencing Requirement**

In addition to the fencing cost illustrated in the base calculation, we examined the effects of varying fencing outlay over a range from £13,000 to £33,000 (see Table 11.5). The total fencing cost is dependent on a number of factors including unit cost, stocking ratio and paddock requirement. This is illustrated in Figure 11.4.
In Chapter 6 we discussed the relationship between the paddock requirement and breeding ratio. We noted that the paddock requirement could be reduced considerably if breeding groups were allowed to share paddocks. It was the breeding ratio rather than any grazing restriction which was the factor that determined the number of paddocks on the experimental farm. Whether it will continue to do so on commercial farms will depend upon the breeding success rate which unsegregated breeding groups can achieve.

If we take the example in which breeding ratio is reduced from the original 16:1 to 10:1 and calculate the rate of return, assuming that paddock requirement, and hence fencing investment, is determined by grazing management considerations, there is no significant change from the original rate of return. However, if the breeding ratio is the controlling factor determining the number of paddocks, any decrease in this ratio leads to an increase in paddock requirement and hence a decrease in the rate of return. Assuming a stocking rate of one adult animal to six acres we can see from Figure 11.4 that the respective fencing outlays for breeding ratios of 16:1 and 10:1 are £28000 and £33000 (16 paddocks and 24 paddocks). We can see in Table 11.5 that for these fencing costs the rate of return is

<table>
<thead>
<tr>
<th>Total Fencing Investment (£000)</th>
<th>DCF Rate of Return (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.0</td>
<td>10.0</td>
</tr>
<tr>
<td>18.0</td>
<td>8.0</td>
</tr>
<tr>
<td>23.0</td>
<td>6.7</td>
</tr>
<tr>
<td>28.0</td>
<td>5.6</td>
</tr>
<tr>
<td>33.0</td>
<td>4.4</td>
</tr>
</tbody>
</table>
Fig. 11.4 Total fencing cost for varying stocking ratios and paddock requirements.

Stocking Capacity = 250

Fence Type | Cost (£/ml)
---|---
Perimeter | 2050
Internal | 1690
reduced to 5.6% and 4.4% respectively. On the other hand, if paddock requirement is reduced to four, the fencing cost for a stocking ratio of one animal to six acres is reduced to £18000. At this level of investment the rate of return is 8%.

Thus we can see that where breeding groups have to be segregated, the breeding ratio is of great importance in determining the fencing outlay and hence the rate of return on the investment. Where segregation is unnecessary, the breeding ratio is relatively unimportant for ratios of 10:1 and above. The difference in productivity at these high levels is insignificant. Indeed, it may prove to be safer to adopt a lower breeding ratio than the maximum so far attained (16:1). Should even a small proportion of the stags fail to reach expectations the overall herd productivity may be severely impaired. At the lower levels there is more room for adjustment to such eventualities without any significant adverse effects upon productivity due to the small increase in the number of males retained. On the other hand, should segregation be a necessity to ensure successful breeding, some balance must be achieved between the risk due to mating failure at high breeding ratios and the increase in fencing investment required at low ratios.

Stock Purchase Costs

Thus far we have been concerned with the effects which varying fencing outlay has upon the rate of return. In this section we shall see how rates of return vary with changes in stock purchase costs. We saw in the previous chapters that no change occurred in the stock purchase pattern for increases in costs of up to 50%.
Thus the management strategy illustrated in Figure 11.2 may be utilised to calculate the D.C.F. rate of return over this range of costs. As we can see in Table 11.4 the proportion of the investment which is attributed to stock purchase is small in comparison with that for fencing. Thus even for cost increases of 25% and 50% the rate of return is reduced from the original 6.7% to 6.5% and 6.2% respectively. At this level of investment it appears that stock purchase costs are only a minor factor.

Operating Costs and Revenues

Returning to Table 11.4 we can see that feeding costs constitute the major proportion of the overall operating cost. At equilibrium fodder accounts for 61% of the total. Any changes in the amounts of fodder required or in fodder costs will thus have a great impact upon the gross margin and net cash flow. As these costs are likely to vary considerably from one site to another, they were varied over a range of ±25% to obtain the rates of return shown in Table 11.6 below.

As far as revenues are concerned, we have already noted that the improvement in useable meat yield due to the elimination of shooting losses offers some scope for a higher price being paid for farmed venison (prices are set on a dead weight basis). In addition, there is a further source of revenue available from the sale of antlers. On the other hand, however, venison prices may fall and yields on some commercial farms may not reach those levels attained at Glen-saugh. To obtain an indication of the sensitivity to changes, price (and hence revenue) was varied over the range ±25%. In
addition, we calculated the level by which revenues would need to be reduced to give a rate of return of 0%. The results are given in Table 11.6 below.

Table 11.6  D.C.F. Rate of Return and Varying Costs and Revenues

<table>
<thead>
<tr>
<th>Change in Costs (%)</th>
<th>Change in Revenue (%)</th>
<th>Rate of Return %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>+25</td>
<td>10.2</td>
</tr>
<tr>
<td>-25</td>
<td></td>
<td>8.8</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>6.7</td>
</tr>
<tr>
<td>+25</td>
<td>-25</td>
<td>4.9</td>
</tr>
<tr>
<td></td>
<td>-36</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Opportunity Cost of Land

In Chapter 6 we suggested that the opportunity cost of the land to be utilised for deer farming might be considered to be zero. Thus far our calculations of the return on investment have therefore excluded any allowance for such a cost. However, land that is suitable for deer farming may already support a stock of feral deer. In such circumstances the opportunity cost of utilising this land for farming will be equal to the operating profit foregone from the exploitation of the wild animals.

In Figures 5.5 and 5.6 we can see that at current venison prices and operating costs gross margins may vary from £0.05 to £0.40 per acre. The latter of these values would result in an opportunity cost of £600 per annum for the example illustrated in Table 11.4 (stock rate = one animal per six acres; total area = 1500 acres). If we set an upper limit of £1000 per annum for this cost, the rate of
return is reduced from 6.7% to 5.3%.

Grants and Subsidies

At present a number of grants and subsidies are available to hill farmers operating in the "Less Favoured Areas". A 50% capital grant is available for roads, fences, gates, reseeding and revivals of grassland and other land improvements, in addition to which, subsidies of £29 and £3.60 per head are payable on eligible hill cows and ewes respectively. As yet, however, deer farming has not been accepted as eligible for any of the above payments. Nevertheless, should the efforts of the H.F.R.O., the Rowett Research Institute and the H.I.D.B. prove that deer farming is a viable concern, some government assistance may become available to the commercial deer farmer. For this reason we have re-calculated the net cash flows illustrated in Table 11.4 and Figure 11.3, including a capital allowance for fencing of 50% and a breeding hind subsidy of £5.80 (one-fifth of the hill cow subsidy). The rates of return are shown in Table 11.7 below.

Table 11.7 The Effects of Grants and Subsidies on the Rate of Return

<table>
<thead>
<tr>
<th>Allowance</th>
<th>Rate of Return (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>6.7</td>
</tr>
<tr>
<td>Hind Subsidy (£5.80)</td>
<td>8.8</td>
</tr>
<tr>
<td>50% Grant</td>
<td>10.2</td>
</tr>
<tr>
<td>50% Grant + Subsidy</td>
<td>12.1</td>
</tr>
</tbody>
</table>

The Effects of Changes in Population Parameters

Gross margins and hence cash flows depend not only on the "economic" variables. As we saw in the previous chapter, the population parameters - the survival and fecundity rates - also affect the
output. For this reason we have calculated the rate of return for a number of fecundity and survival levels, the values of which are given in Table 11.8 below. The cash flow patterns are illustrated in Figure 11.5a & b for the situations in which the survival rate of the one year old animals is increased from 0.88 to 0.95 (a) and the fecundity of the lactating and first breeding hinds is reduced from the original schedule (Table 7.8) to 0.80 overall (b). In comparison, the average overall breeding rate attained at Glensaugh, when the high feeding plane was used, was in excess of 0.90.

<table>
<thead>
<tr>
<th>Modified Survival Rate 1 Year Olds</th>
<th>Modified Fecundity Rate Lactating &amp; First Breeding Hinds</th>
<th>Rate of Return %</th>
<th>Gross Margin (£000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.95</td>
<td>-</td>
<td>7.8</td>
<td>8.0</td>
</tr>
<tr>
<td>0.92</td>
<td>-</td>
<td>7.4</td>
<td>7.7</td>
</tr>
<tr>
<td>None</td>
<td>None</td>
<td>6.7</td>
<td>7.2</td>
</tr>
<tr>
<td></td>
<td>0.90</td>
<td>5.9</td>
<td>6.3</td>
</tr>
<tr>
<td></td>
<td>0.80</td>
<td>4.9</td>
<td>5.6</td>
</tr>
</tbody>
</table>

Increasing Stocking Capacity and Farm Size

Thus far we have restricted our analysis to the projected operation illustrated in Figure 11.2. We have used the above example to obtain some idea of the "typical" returns which may be obtained from deer farming. Naturally, our choice of stock purchase constraint influences the cash flow pattern during the build-up phase. However, as we change the availability of stock and alter the rate of expansion of the population, so we alter the timing of investment in fencing. In the long term, for a given stocking capacity, the equilibrium
Fig. 11.5  Net cash flow patterns for varying fecundity & survival rates.

a - Survival 1 year olds = 0.95
b - Fecundity lactating &
   2 year old bumbs = 0.80
c - Original (fig. 11.3)
structure is the same regardless of the time taken for the population to attain equilibrium. In this section we shall examine the effects of increasing stocking capacity at equilibrium and discuss the factors which might alter the production relationships under these conditions.

If we assume that the production function is linear, as the equilibrium population size changes so the costs and revenues change proportionately. However, as the area of the farm increases fencing costs per unit area decrease. Thus the ratio of gross margin to fencing investment increases with farm size. This is illustrated below in Table 11.9.

**Table 11.9 Population Size, Gross Margin and Fencing Investment**

<table>
<thead>
<tr>
<th>Population Size (Adult Animals)</th>
<th>Gross Margin (£000)</th>
<th>Fencing Investment (£000)</th>
<th>Ratio (1)/(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>7.2</td>
<td>22.8</td>
<td>0.316</td>
</tr>
<tr>
<td>500</td>
<td>14.4</td>
<td>32.4</td>
<td>0.444</td>
</tr>
<tr>
<td>1000</td>
<td>28.8</td>
<td>45.8</td>
<td>0.629</td>
</tr>
<tr>
<td>2000</td>
<td>57.6</td>
<td>64.8</td>
<td>0.889</td>
</tr>
</tbody>
</table>

Note: Stocking rate = 1 animal/6 acres
Paddock requirement = 8 for all farms.

Increasing the population size not only increases the earnings ratio, but also increases the absolute value of these earnings. Theoretically, it would be possible to expand the farm size indefinitely, thereby further increasing the returns. In reality, however, there are likely to be a number of restrictions to such an expansion. Firstly the market for venison is not unlimited; thus as output increases so prices may fall. Secondly, as the farm is expanded, the land which it occupies may have more than a notional opportunity
cost - thus costs may increase. Finally, our experience of the
deer farming operation has been restricted to only small farming
units, thus we cannot say, with any degree of certainty, how the
production relationships behave as we increase the scale of the
operation. Transport on the farm may become an important factor
in the operation as the distances increase. As the paddock size
increases the animals may become more difficult to control, so
necessitating higher labour inputs. Thus, although at first sight
the advantages of the large scale operation may seem to be over­
whelming, we require more knowledge of the farming system at these
levels before we can draw any firm conclusions.

**Summary and Conclusions**

The objective of this chapter has been to examine the finances of
a proposed deer farming operation based on the "optimal" management
strategy derived in the preceding chapters.

Predictably, the cash flow situation in the early years of the project
demands a high level of working capital during the early life of the
farm as we can see in Figures 11.4 and 11.5. Positive net cash
flows occur only in the fourth year after the initial investment in
fencing. Although fencing costs account for the major proportion of
the outgoings in the early years, the cost of buying stock and
bringing it up to breeding age accounts for over one third of the
cash outflow in this period. The provision of grants and/or sub­
sidies may, however, ease the situation to some extent. This sit­
uation results from the necessity to start the farming operation
using immature animals. There is a possibility that mature wild
animals may be captured and used to provide the input of calves for
taming, thereby providing some revenue to offset the expenditure in the early years. As yet, however, no work has been undertaken to investigate the viability of such a measure, and it is unlikely that in the near future tame mature stock will be available in any numbers. It would appear, therefore, that if deer farming is to become a viable proposition, the pioneer farmers must be prepared to shoulder the burden of high outlays and no returns for an initial period of four to five years.

Returning to the question of fencing outlay, we have noted that the ability to mate several breeding groups in a single paddock reduces the fencing investment. Under these conditions the breeding ratio is no longer a prime consideration and lower breeding ratios, say 10:1, than the maximum 16:1 may be adopted without any significant reduction in productivity. At the lower breeding ratio, the risk due to unsuccessful mating is greatly reduced. Should it prove impracticable to utilise unsegregated breeding groups on the farm, some balance must be reached between high breeding ratios, which reduce fencing costs but may incur a higher risk of mating failure, and low ratios, which have a negligible risk but increase the cost of fencing. In addition, some consideration might be given to the investigation of artificial insemination techniques to eliminate breeding groups altogether.

As we noted in Chapter 6, there is a need to investigate the alternative fences which may be utilised to construct paddocks regardless of the way in which breeding groups are contained. Indeed, this is one of the aims of the H.I.D.B. on their pilot commercial farm at Rahoy (H.I.D.B. 1977). Just as the cost of fencing is of paramount
importance in regard to cash flow, stock purchase costs should prove to be a relatively minor consideration. The majority of the outflows in the early years are employed in the process of bringing the stock up to breeding status.

In addition to examining the levels of finance required, we have obtained some estimates of the rate of return which may be achieved on the projected farm under a variety of conditions. Our analyses have been restricted to the small scale operation due to the uncertainties regarding production relationships on farms of a scale far removed from the size of operation at Glensough. Assuming that costs and revenues did not change markedly from the projected values, we showed that returns on capital invested varied from 4.4% at a level of fencing investment of £33,000 to 10% for a fencing outlay of £13,000. In both cases the equilibrium herd size was 250 adult animals.

As might be expected, changes in revenue had the greatest effect upon the rate of return. Unfortunately, it is in the area of venison prices that we have the least control or information. Much of the variation in price levels over recent years may be attributed to the imposition of stricter hygiene regulations in the German markets (Chapter 3). The high quality of the farmed venison should overcome any problems in this area in the event of an imposition of further restrictions. Thus to some extent the degree of price uncertainty may be reduced. Nevertheless, prices will still be influenced by foreign competition and other vagaries of the market. In addition, yields will vary from site to site depending on the quality of the vegetation. This may, however, be counterbalanced by
the opportunity cost of the land, which is likely to be higher on the better quality pastures and vice-versa. The sensitivity to changes in revenue was illustrated by the change in return from 6.7% to 2.9% and 10.2% for a change in price of ± 25%. When prices were reduced by more than 36% a negative return resulted. That is to say, at a price level of £0.85/kg the rate of return was 0%.

Revenue levels are also affected by changes in the survival and fecundity characteristics of the population. Reducing the breeding rates of the lactating hinds from the original (given in Table 7.8) to 0.80 overall resulted in a decrease in return from 6.7% to 5.6%, while increasing the survival rate of the one year old animals from 0.88 to 0.95 raised the return to 8.0%. Comparing gross margins given in Table 11.8, we can see that at equilibrium, the reduction in breeding and the increase in survival lead to changes in gross margin of -22% and +11% respectively.

As far as production inputs are concerned the degree of control is somewhat greater than that over revenues and the effects of changes in costs less drastic, as can be seen in Table 11.8.

The most obvious means by which earnings and returns from deer farming may be enhanced is by an expansion in the scale of the operation. Whether the improved earnings ratios suggested in Table 11.9 may be realised will depend upon the extent to which the production relationships in the small scale system are applicable to the expanded system. In addition, the effects of increased output upon the venison market and the utilisation of land which
may have alternative uses might affect the profitability of the larger enterprise. As far as production factors are concerned, the Rahoy farm owned by the H.I.D.B., which is expected to stock a minimum of 400 breeding hinds and extend to over 3000 acres, should provide further information regarding the viability of large-scale farms.

To obtain a further perspective against which deer farms may be gauged we may compare some of the above results with the returns obtained from hill farming in general. In order to remove the uncertainty of fencing investment the following comparison is made on the basis of the returns on total tenants' capital (machinery, livestock and working capital) which are defined as the gross output less total inputs as a percentage of total tenants' capital. (Nix 1976). On this basis the returns from hill sheep farming, for farms over 250 acres, averaged 17% with a premium level of 30% (i.e. the average of the most profitable 50%). On the same basis, the returns calculated for the projected operation ranged from 15% to 38% at equilibrium. The above figures may be inflated due to the low rental charges (opportunity costs) attributed to the deer farm. Nevertheless, one of the main objectives of deer farming is to utilise marginal or sub-marginal land which has little or no opportunity cost.

It is not, however, the objective of this study to make recommendations regarding investment opportunities in deer farming; each case must be regarded in its own context and on its own merits. Our aim is to outline the possible returns which may be achieved from a "typical" farm and indicate those factors which may have the
greatest effect upon them. In the final analysis, each individual must reach a decision based upon his own objectives and the alternative opportunities available to him.
Chapter 12.

Summary and Conclusions

The preceding chapters have provided an insight into the development of venison production systems over the years. They have shown how the evolution of the red deer in Scotland has been influenced by a combination of economic, social, political and ecological factors and examined the problems which arise when an improvement in the methods of exploitation is sought. This chapter reviews the analysis of the production systems and the factors which influence their viability.

In Chapter 3 a production function for the wild system was developed and the relationship between the economically optimal cropping levels and those governed by the biotic potential of the animal was examined. It was seen that over a wide range of estate sizes, the economically optimal culling rate was below that recommended by the Red Deer Commission (R.D.C.), based upon the average population growth rates in Scotland. It appeared, therefore, that in many cases there was a disincentive to crop at the ecologically defined level. In order to examine the relationships between the population growth rate and population parameters, a mathematical model of the production system was developed based on the red deer population of Rhum. Thus it was possible to determine those factors which had the greatest impact upon productivity.

The results of the above analyses showed that there were a number of ways in which productivity could be increased. The first of
these involved a change in the emphasis of the harvesting policy by concentrating upon those animals which were most susceptible to the negative effects of adverse weather conditions and increased population density, i.e. those aged over 9 years. Under the boundary conditions of the model, the population growth pattern on Rhum was largely determined by the density dependent effects upon the fecundity of the first breeding hinds (i.e. three year olds). Further, the greatest improvement in productivity resulted from the reduction of the age of first breeding rather than from any improvement in the breeding performance of mature hinds. It appeared, therefore, that any input of supplementary fodder should be directed primarily at the youngest animals to enable them to attain breeding status at an earlier age. In addition to reducing the proportion of non-breeding hinds in the population, the carry-over effect of feeding these young animals may lead to an improvement in the body-weight and hence breeding ability of the animals as they age.

The applicability of the results obtained from the model of the Rhum herd to the red deer population of the mainland are in many ways restricted. Under the boundary conditions studied the range of population density was severely restricted and the density dependent relationships were modified by the culling strategies employed. Nevertheless, the model may be used to compare the effects of changes in the population parameters and their relative importance in determining productivity.

There are, however, several practical difficulties associated with the implementation of any improvement plans suggested by the
above analysis. Thus any cropping policy which requires the ability to differentiate between age and sex classes must be severely handicapped in the wild system due to the problems of recognition. Further, any detailed prescription for harvesting implies a sound knowledge of the fecundity and survival rates of the population and its structure, which in practice is not available. As far as supplementary feeding is concerned, problems arise from two sources. Firstly the selective nature of the feeding programme requires that the more dominant animals be excluded from the feeding points. In this case difficulties may be overcome to some extent by the design of the feeding stalls. Secondly, and more importantly, there is no assurance that the estate on which the animals are fed during winter is the one on which they will be culled. As a result there is little incentive for individual estates to follow an improvement policy in isolation from their neighbours.

Although the intensive husbandry techniques of deer farming overcome many of the above difficulties they, in turn, generate a further series of problems. The second part of this thesis therefore examined the evolution of the intensive production system and attempted to develop some management prescriptions for commercial deer farms using when available the experience of the development of the experimental deer farm at Glensaugh. The greater part of the analysis was accomplished using a modified version of the model of the wild deer system incorporated in a linear programming (L.P.) format. Despite the reservations there may be regarding the validity of the model, its usefulness must be judged in relation to the extent to which it
assists the further development of deer farming.

The most obvious advantage which the intensive system affords the manager is the high degree of control over the animals and the level of knowledge of their performance. Thus where previously any prescriptions for a harvesting policy were based almost entirely upon guesswork, the farm manager may apply more exact techniques to determine cropping strategies. The L.P. analysis determined just such strategies.

These proposed that in order to allow maximum stocking capacity to be attained rapidly no females should be removed (up to the boundary limit of twelve years of age) during the expansion phase. Once equilibrium is attained, approximately 40% of the one year old and all six year old hinds should be removed annually. Extensive sensitivity analysis indicated that it was the operating margin on the one year old hinds which dominated the management policy. As the marginal revenue from the youngest hinds increased, so the management strategy dictated a reduction in the age structure of the population. Cropping was then concentrated upon the four and five year old hinds. At the other end of the scale, however, increases in the marginal cost, either as a result of reduced breeding efficiency of the first breeding hinds or an increase in operating costs, had little or no effect upon the harvesting plan. There was no incentive to increase the length of the age structure over a wide range of population parameters and operating conditions.

Due to the early stage of the development of the experimental farm the data available on the relationship between supplementary feeding
rates and the breeding performance of the two year old hinds is not complete. It was possible, however, to examine productivity at the extremes of fecundity based upon the existing data. From this it appeared that on a low feeding plane, the cost savings were outweighed by the loss of output due to reduced overall breeding efficiency. The general policy of supplying sufficient quantities of fodder to the female calves to permit them to attain breeding status as two year olds achieved the higher contribution margin over a wide range of feeding costs.

The harvesting strategy adopted for the stags involved the removal of all males aged three after the rut. Only sufficient numbers of two year olds required to make up the breeding groups and form the nucleus of the breeding males in the following year were retained; all remaining two year olds were removed prior to the rut. As far as breeding ratios were concerned, the marginal productivity of increasing the ratio rapidly declined above the 10:1 level. However, breeding ratios may have an important effect upon the level of fencing investment required, thus this factor should not be regarded in isolation.

Turning to the problems of investment outlays and cash flows, the disbursements in the early years of the farming operation may prove to be an obstacle to the prospective deer farmer. Fencing accounts for the bulk of this, although the working capital required to bring stock up to breeding status may account for up to 30% of the outflow in this period. It is unlikely that this situation will alter in the near future. Until tame mature stock may be acquired
"en masse", farm stock will have to be reared and tamed by each prospective farmer. As yet the situation regarding the supply of calves for taming is unresolved. Although the experimental farm was given permission to capture new-born calves in the wild by the R.D.C. to form the nucleus of its breeding stock, there is no guarantee that a similar agreement will be reached with the aspiring commercial deer farmer. However, stock may be available from establishments such as Glensaugh or the H.I.D.B. farm at Rahoy or other confined wild herds. Regardless of the source of stock, the cash flows in the early years of any farm will be decidedly negative.

Because of the importance of the fencing investment, the relationship between fencing requirement, paddock requirements and breeding ratio was examined in some detail. It was evident that if breeding groups were required to be kept apart during the rut, the breeding ratio would be of paramount importance in determining the paddock requirement and hence investment in internal fences. The higher the breeding ratio, the lower is the outlay in fencing. With high breeding ratios, however, difficulties may be experienced in the mating performance of stags: thus a compromise may have to be accepted between the fencing utilisation efficiency and herd productivity. As yet, however, there are insufficient data to permit a more detailed study of these effects. Should it be possible to permit breeding groups to share paddocks without adversely affecting performance, the breeding ratio ceases to be a controlling factor and paddock requirement is decided on the basis of grazing control. Within the model, the effect upon productivity
of reducing the breeding ratio from, say, 16:1 to 10:1 was negligible. At the lower level, however, the risk due to the possibility of mating failure is likely to be greatly reduced.

In Chapter 11, an investment appraisal of a deer farming project, based upon the optimal strategies developed previously, was undertaken. No attempt was made, however, to define levels of acceptable or unacceptable returns. The intention was rather to determine the effects which changes in the economic and environmental conditions had upon the investment potential so as to provide some basis against which the future viability of the farming system might be assessed.

Recommendations for Further Study

In the course of the preceding analyses a number of points were raised which indicated areas where more research might be profitably undertaken. The majority of these were in the context of the deer farming operation, although some aspects of the wild system were also included.

So far as the wild system is concerned, if any intensification of production is to be undertaken without the use of fences, there must be a high degree of co-operation between neighbouring estates sharing common deer herds. Under such circumstances, it is necessary that a knowledge of population movements, in addition to the data on survival and breeding performance, be obtained so that a common management policy may be determined. The adoption of supplementary feeding as part of this strategy requires research
into the ways in which selective feeding may be accomplished and a monitoring of its effects.

The analysis of the farming system was undertaken using the assumption that population density effects were negligible. However, as the efficiency with which fencing is utilised must depend upon stock capacity, the effects of disease and other intrapopulational factors upon the productivity at varying densities must be determined before an estimate of optimal stocking policies may be derived. In regard to fencing utilisation, we have already noted the need for further research into the aspects of breeding ratios and mating performance of stags. A further line of research which might be considered, should it prove impracticable to allow unsegregated breeding groups is the adoption of artificial insemination techniques. This, after all, is only a further extension of the basic concept of deer farming, which proposes the adoption of conventional husbandry techniques for the exploitation of red deer.

It was also suggested that extending the area of the farm might improve fencing utilisation. There is, however, a need to determine the intrapopulational effects and production relationships at the increased scale of operation. Allied to these problems is the relationship between supplementary feeding and breeding performance. Although some conclusions have been reached based on the results of experiments to date, the importance of the relationship between the supplementary feeding policy and the marginal revenue from the one year old hinds, the factor which largely determines management strategy, suggests that a more extensive study should be undertaken of these factors.
Turning now to factors outwith the boundary of the production system, one of the major sources of uncertainty in regard to the future viability of deer farming lies in the behaviour of the venison market. A detailed study of the factors which affect price levels, both in this country and abroad, might go some way to defining the levels of uncertainty.

In conclusion, this study has examined the factors which affect the production of venison in Scotland and shown which are the most critical. It has also provided some tentative solutions as to how improvements might be implemented. These should not, however, be regarded in any way as final solutions. As knowledge of the production systems improves, the solutions to the problems and the problems themselves will change.
APPENDIX A.

Production Inputs in the Wild.

The following data on production inputs in the wild are taken from a questionnaire survey of the members of the Scottish Landowners Federation undertaken in 1974 (Paluchowski 1974). Only those estates which derived over 90% of their income from deer in the form of venison sales revenue are included below (Table A.1).

Transportation costs on the estates were also obtained from a reduced sample of estates, as part of the above survey. These were calculated as the proportion of total estate transport cost which was attributable to venison production activities. All prices are as in the summer of 1974 (Table A.2).
<table>
<thead>
<tr>
<th>Crop (No. of Carcases)</th>
<th>Estate Area (000's Acres)</th>
<th>Population (No. of Animals)</th>
<th>Labour Input Man-Days</th>
<th>Estate Location (County)</th>
</tr>
</thead>
<tbody>
<tr>
<td>601</td>
<td>90.0</td>
<td>5000</td>
<td>1200</td>
<td>PERTH</td>
</tr>
<tr>
<td>493</td>
<td>48.0</td>
<td>3500</td>
<td>900</td>
<td>INVERNESS</td>
</tr>
<tr>
<td>320</td>
<td>18.0</td>
<td>3000</td>
<td>340</td>
<td>ANGUS</td>
</tr>
<tr>
<td>266</td>
<td>35.0</td>
<td>1200</td>
<td>400</td>
<td>ABERDEEN</td>
</tr>
<tr>
<td>250</td>
<td>66.0</td>
<td>3000</td>
<td>350</td>
<td>ABERDEEN</td>
</tr>
<tr>
<td>140</td>
<td>20.0</td>
<td>1750</td>
<td>150</td>
<td>INVERNESS</td>
</tr>
<tr>
<td>135</td>
<td>13.0</td>
<td>700</td>
<td>200</td>
<td>PERTH</td>
</tr>
<tr>
<td>130</td>
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<td>700</td>
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<td>PERTH</td>
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<td>110</td>
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<td>INVERNESS</td>
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<td>33.4</td>
<td>900</td>
<td>200</td>
<td>W. ROSS</td>
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<td>12.0</td>
<td>1500</td>
<td>100</td>
<td>INVERNESS</td>
</tr>
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<td>W. ROSS</td>
</tr>
<tr>
<td>86</td>
<td>12.1</td>
<td>600</td>
<td>140</td>
<td>ABERDEEN</td>
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<td>80</td>
<td>9.0</td>
<td>700</td>
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<td>INVERNESS</td>
</tr>
<tr>
<td>77</td>
<td>9.5</td>
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<td>40</td>
<td>ARGYLL</td>
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<tr>
<td>76</td>
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<td>PERTH</td>
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<td>69</td>
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<td>550</td>
<td>60</td>
<td>ARGYLL</td>
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<td>PERTH</td>
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<td>26.0</td>
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<td>100</td>
<td>ARRAN</td>
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<tr>
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<td>580</td>
<td>70</td>
<td>Sutherland</td>
</tr>
<tr>
<td>50</td>
<td>13.0</td>
<td>430</td>
<td>70</td>
<td>ARGYLL</td>
</tr>
</tbody>
</table>

CROP = \( \frac{\text{MAN DAYS}}{0.607} \) \( \frac{\text{POPULATION}}{0.519} \) \( \frac{\text{ESTATE AREA}}{0.316} \)
Table A.2. | Estate Transportation Costs for Venison Production

<table>
<thead>
<tr>
<th>Crop (No. of Carcases)</th>
<th>Estate Area (000's Acres)</th>
<th>Total Transport Cost (£)</th>
</tr>
</thead>
<tbody>
<tr>
<td>601</td>
<td>90.0</td>
<td>5420</td>
</tr>
<tr>
<td>320</td>
<td>18.0</td>
<td>2670</td>
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<td>151</td>
<td>50.0</td>
<td>1260</td>
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<tr>
<td>140</td>
<td>20.0</td>
<td>860</td>
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<tr>
<td>135</td>
<td>13.0</td>
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<td>610</td>
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<tr>
<td>80</td>
<td>9.0</td>
<td>840</td>
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<td>65</td>
<td>4.0</td>
<td>510</td>
</tr>
<tr>
<td>50</td>
<td>18.1</td>
<td>400</td>
</tr>
</tbody>
</table>

Total Cost = 8.65 Crop + 0.0027 Area - 9416 (P < 0.01)

As might be expected, transport costs are largely determined by the size of the crop. These facilities are only used when a kill has been made, although the area of the estate has some influence on the distance travelled to uplift the carcase.
APPENDIX B.

Rainfall Data

The following graph, Figure B.1 shows the distribution of summer rainfall recorded on the island of Rhum for the period in which official rainfall data are available, i.e. 1958-1975 (Met. Office Statistics).

![Rainfall Data Graph]

Total rainfall June-Sept. (inches)

Mean rainfall = 28.75 in.
Standard rainfall = 6.91 in.

From the above Figure B.1 it appears that the rainfall data approximate a Normal Distribution. For this reason the random generation of rainfall for the population model of Chapter 4 is based upon the Normal Distribution with mean 28.75" and standard deviation 6.91".
APPENDIX C

Calculation of Dominant Eigenvalues

The calculation was originally intended to be undertaken using a Numerical Algorithm Group (NAG) standard computer routine. It was found, however, that the time taken to calculate the eigenvalues for the population matrix (45 x 45) was prohibitive and only four sets of eigenvalues - and thus the four dominant eigenvalues - were obtained. For this reason the method of obtaining the dominant eigenvalue was modified.

Various theorems are to be found concerning the existence of a largest or dominant eigenvalue of a matrix. These provide methods for finding the largest eigenvalue without having to solve the characteristic equation. They shall not be discussed here (Frazer, Duncan & Collar 1952) but the method used for calculating \( \lambda \), will be outlined briefly below.

It may be shown (op cit) that if the dominant eigenvalue of a matrix \( M \) is raised to a sufficiently high power, \( k \), so that \( \lambda^k \), predominates to a large enough extent over the other eigenvalues \( \lambda_2^k \ldots \lambda_n^k \), the dominant eigenvalue is given by

\[
\lambda_1 = \frac{M^k x}{M^{k-1} x} \tag{Eqn. C.1}
\]

where \( x \) is a non-null vector (in our case the population vector).

The accuracy of the above relationship depends upon the degree to which \( \lambda_1^k \) exceeds \( \lambda_2^k \ldots \lambda_n^k \).

To check whether such an approximation could be used to calculate the eigenvalues of the population matrix in Chapter 4, the eigenvalues obtained using the NAG routine were compared with the
values obtained using the above method. The comparison is shown in Table C.1.

<table>
<thead>
<tr>
<th>Run No.</th>
<th>NAG Routine</th>
<th>Approximate Method</th>
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</thead>
<tbody>
<tr>
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<td>0.92031</td>
</tr>
<tr>
<td>4</td>
<td>1.24039</td>
<td>0.92985</td>
</tr>
</tbody>
</table>

$\lambda_1$ = dominant eigenvalue  
$\lambda_2$ = 2nd largest eigenvalue  
k = 40

The values obtained for $\lambda_1^k$ and $\lambda_2^k$ from the NAG routine indicate that the approximation should yield a sufficiently accurate estimate. Indeed, the "approximate" values obtained are identical to five places of decimals. The computer programme listing used to calculate $\lambda_1$ is given in Figure C.1.
FIG. C.1. Calculation of Eigenvalues.

REAL LAMDA
REAL P(45,1),A(45,45),FY(15),SM(15),SY(15),FL(15)

REAL P2(45,1)
REAL Q(45,1)
INTEGER TIME,OUT

<table>
<thead>
<tr>
<th>DATA</th>
<th>A/2</th>
<th>0.0, 0.0, 0.72, 42*0.0,</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3*0</td>
<td>0.0, 0.56, 41*0.0,</td>
</tr>
<tr>
<td>2</td>
<td>4*0</td>
<td>0.0, 0.96, 40*0.0,</td>
</tr>
<tr>
<td>3</td>
<td>5*0</td>
<td>0.0, 0.98, 39*0.0,</td>
</tr>
<tr>
<td>4</td>
<td>6*0</td>
<td>0.0, 0.96, 38*0.0,</td>
</tr>
<tr>
<td>5</td>
<td>2*0</td>
<td>0.445, 0.0, 2<em>0, 0.445, 0.36</em>0.0,</td>
</tr>
<tr>
<td>6</td>
<td>9*0</td>
<td>0.0, 0.0, 35*0.0,</td>
</tr>
<tr>
<td>7</td>
<td>2*0</td>
<td>0.426, 0.0, 2<em>0, 0.426, 0.33</em>0.0,</td>
</tr>
<tr>
<td>8</td>
<td>2*0</td>
<td>0.245, 0.0, 2<em>0, 0.245, 0.33</em>0.0,</td>
</tr>
<tr>
<td>9</td>
<td>12*0</td>
<td>0.0, 0.33*0.0,</td>
</tr>
<tr>
<td></td>
<td>2*0</td>
<td>0.446, 0.0, 2<em>0, 0.446, 0.33</em>0.0,</td>
</tr>
<tr>
<td>1</td>
<td>2*0</td>
<td>0.173, 0.0, 2<em>0, 0.173, 0.33</em>0.0,</td>
</tr>
<tr>
<td>2</td>
<td>15*0</td>
<td>0.0, 0.97, 29*0.0,</td>
</tr>
<tr>
<td>3</td>
<td>2*0</td>
<td>0.470, 0.0, 2<em>0, 0.470, 0.27</em>0.0,</td>
</tr>
<tr>
<td>4</td>
<td>2*0</td>
<td>0.216, 0.0, 2<em>0, 0.216, 0.27</em>0.0,</td>
</tr>
<tr>
<td>5</td>
<td>18*0</td>
<td>0.0, 0.95, 26*0.0,</td>
</tr>
<tr>
<td>6</td>
<td>2*0</td>
<td>0.451, 0.0, 2<em>0, 0.451, 0.24</em>0.0,</td>
</tr>
<tr>
<td>7</td>
<td>2*0</td>
<td>0.316, 0.0, 2<em>0, 0.316, 0.24</em>0.0,</td>
</tr>
<tr>
<td>8</td>
<td>21*0</td>
<td>0.0, 0.97, 23*0.0,</td>
</tr>
<tr>
<td>9</td>
<td>2*0</td>
<td>0.457, 0.0, 2<em>0, 0.457, 0.21</em>0.0,</td>
</tr>
<tr>
<td></td>
<td>2*0</td>
<td>0.221, 0.0, 2<em>0, 0.221, 0.21</em>0.0,</td>
</tr>
<tr>
<td>1</td>
<td>24*0</td>
<td>0.0, 0.37, 20*0.0,</td>
</tr>
<tr>
<td>2</td>
<td>2*0</td>
<td>0.165, 0.0, 2<em>0, 0.165, 0.18</em>0.0,</td>
</tr>
<tr>
<td>3</td>
<td>2*0</td>
<td>0.178, 0.0, 2<em>0, 0.178, 0.18</em>0.0,</td>
</tr>
<tr>
<td>4</td>
<td>27*0</td>
<td>0.0, 0.85, 17*0.0,</td>
</tr>
<tr>
<td>5</td>
<td>2*0</td>
<td>0.430, 0.0, 2<em>0, 0.430, 15</em>0.0,</td>
</tr>
<tr>
<td>6</td>
<td>2*0</td>
<td>0.430, 0.0, 2<em>0, 0.430, 15</em>0.0,</td>
</tr>
<tr>
<td>7</td>
<td>30*0</td>
<td>0.0, 0.82, 14*0.0,</td>
</tr>
<tr>
<td>8</td>
<td>2*0</td>
<td>0.410, 0.0, 2<em>0, 0.410, 12</em>0.0,</td>
</tr>
<tr>
<td>9</td>
<td>2*0</td>
<td>0.312, 0.0, 2<em>0, 0.312, 12</em>0.0,</td>
</tr>
<tr>
<td>10</td>
<td>33*0</td>
<td>0.0, 0.78, 11*0.0,</td>
</tr>
<tr>
<td>1</td>
<td>2*0</td>
<td>0.405, 0.0, 2<em>0, 0.405, 9</em>0.0,</td>
</tr>
<tr>
<td>2</td>
<td>2*0</td>
<td>0.308, 0.0, 2<em>0, 0.308, 9</em>0.0,</td>
</tr>
<tr>
<td>3</td>
<td>36*0</td>
<td>0.0, 0.72, 8*0.0,</td>
</tr>
<tr>
<td>4</td>
<td>2*0</td>
<td>0.365, 0.0, 2<em>0, 0.365, 6</em>0.0,</td>
</tr>
<tr>
<td>5</td>
<td>2*0</td>
<td>0.277, 0.0, 2<em>0, 0.277, 6</em>0.0,</td>
</tr>
<tr>
<td>6</td>
<td>39*0</td>
<td>0.0, 0.61, 5*0.0,</td>
</tr>
<tr>
<td>7</td>
<td>2*0</td>
<td>0.340, 0.0, 2<em>0, 0.340, 3</em>0.0,</td>
</tr>
<tr>
<td>8</td>
<td>2*0</td>
<td>0.253, 0.0, 2<em>0, 0.253, 3</em>0.0,</td>
</tr>
<tr>
<td>9</td>
<td>42*0</td>
<td>0.0, 0.362, 0*0.0,</td>
</tr>
<tr>
<td></td>
<td>2*0</td>
<td>0.265, 0.0, 2*0, 0.265,</td>
</tr>
<tr>
<td>1</td>
<td>2*0</td>
<td>0.201, 0.0, 2*0, 0.201,</td>
</tr>
<tr>
<td>2</td>
<td>45*0</td>
<td>0.0,</td>
</tr>
<tr>
<td>3</td>
<td>45*0</td>
<td>0.0,</td>
</tr>
<tr>
<td>4</td>
<td>45*0</td>
<td>0.0,</td>
</tr>
</tbody>
</table>
**C** **C** **C** **C** INSERT DATA CARDS FOR MATRICES A,FY,SY
DATA FL/3=0.0,0.0,0.5,0.35,0.44,0.67,0.47,0.54,0.50,5*0.38/
DATA SY/0.90,0.98,4*0.99,0.95,0.94,0.88,2*0.77,4*0.68/
DATA FY/2=0.0,0.0,0.5,0.85,0.90,0.95,0.99,0.93,0.5,0.77,5*0.82/
DATA SM/0.88,0.96,0.98,0.99,0.99,0.97,0.96,0.97,0.91,0.91,0.0/38
DATA P2/120,0.138,0.84,0,121,0,90,0,119,0,73,0,49,0/18
2 35.0,91.0,23.0,57.0,80.0,51.0,32.0,80.0/
3 20.0,47.0,53.0,14.0,42.0,31.0,11.0,15.0/
4 12.0,3.0,7.0,10.0,4.0,5.0,9.0,3.0/
5 4.0,5.0,2.0,2.0,7.0,1.0,1.0,3.0/
6 2.0,3.0,1.0,1.0,2.0/18
IN = 37
OUT = 32
NDIVS = 45
JFAIL = 0
MAXTIM = 40
R = 20.0
M = 10
L = 1
DO 100 N = 1600,2000,100
PTOT = N
WRITE(OUT,5007) PTOT
5007 FORMAT(4T1,TOTAL POPULATION = 'F10.0')
18 PLOG = ALOG(PTOT)
TIME = 0
C
C 1-YEAR MALES
C
TEMP = (4.4722 - 0.0156*0)*M/10.0 = 0.3772*PLOG
TEMP = RANGE(TEMP,0.0,1.571)
A(3,1) = SIN(TEMP)
C
C 1-YEAR HINDS
C
TEMP = (7.1948 - 0.01*R)*M/10.0 = 0.7569*PLOG
TEMP = RANGE(TEMP,0.0,1.571)
A(4,2) = SIN(TEMP)
C
C 3-YEAR YIELD HINDS
C
TEMP = 47.656*L = 6.2886*PLOG
TEMP = TEMP/2.0
FY(3) = RANGE(TEMP,0.0,0.5)
A(1,6) = SY(3) *FY(3)

A(2,6) = A(1,6)
A(5,6) = SY(3) *(1.0 - 2.0*FY(3))
A(9,6) = A(1,6)*2.0
C
C 9-15 YEARS
C
C MALES
C
9-15 YEARS

MALES

TEMP = 19.3342 - 2.3739 * PLOG - 0.0070 * R
TEMP = RANGE(TEMP, 0.0, 1.571)
SM(9) = SIN(TEMP)
TEMP = 31.0612 - 3.6968 * PLOG - 0.0153 * R
TEMP = RANGE(TEMP, 0.0, 1.571)
SM(10) = SIN(TEMP)
TEMP = 28.8191 - 3.6574 * PLOG - 0.0082 * R
TEMP = RANGE(TEMP, 0.0, 1.571)
SM(11) = SIN(TEMP)
TEMP = 49.9780 - 6.4585 * PLOG - 0.0162 * R
TEMP = RANGE(TEMP, 0.0, 1.571)
SM(12) = SIN(TEMP)

DO 109 I = 13, 15
109 SH(I) = SM(12)

HINDS

TEMP = 14.9486 - 1.7421 * PLOG - 0.0191 * R
TEMP = RANGE(TEMP, 0.0, 1.571)
SY(9) = SIN(TEMP)
TEMP = 54.6813 - 6.9998 * PLOG - 0.0239 * R
TEMP = RANGE(TEMP, 0.0, 1.571)
SY(10) = SIN(TEMP)
TEMP = 39.7717 - 5.0559 * PLOG - 0.0129 * R
TEMP = RANGE(TEMP, 0.0, 1.571)
SY(11) = SIN(TEMP)
TEMP = 34.4807 - 4.4184 * PLOG - 0.0014 * R
TEMP = RANGE(TEMP, 0.0, 1.571)
SY(12) = SIN(TEMP)

DO 116 I = 13, 15
116 SY(I) = SY(12)
TEMP = 64.953 - 8.5351 * PLOG
TEMP = TEMP / 2.0
FY(10) = RANGE(TEMP, 0.0, 0.5)
TEMP = 68.554 - 9.0083 * PLOG
TEMP = TEMP / 2.0
FY(11) = RANGE(TEMP, 0.0, 0.5)

DO 127 I = 12, 15
127 FY(I) = FY(11)

FILL IN MATRIX A FOR 9-16 YEARS

J = 25
K = 22
DO 136 I = 9, 15
A(J,K) = SM(I)
K = N + 1
A(1,K) = FY(I)*SY(I)
A(2,K) = A(1,K)
A(J+1,K) = (1,0 - 2,0*FY(I))*SY(I)
A(J+2,K) = A(1,K)*2,0
K = K + 1
A(1,K) = FL(I)*SY(I)*0,5
A(2,K) = A(1,K)
A(J+1,K) = (1,0 - FL(I))*SY(I)
A(J+2,K) = A(1,K)*2,0
J = J + 3
K = K + 1
136 CONTINUE
DO 170, NN = 1, NDIVS
P(NN,1) = P(NN,1)
170 CONTINUE
19 CALL PRODXX(0,A,P,45,45,1)
DO 169, NI = 1, NDIVS
P(NI,1) = Q(NI,1)
169 CONTINUE
TIME = TIME + 1
IF(TIME .LT. MAXTIM) GO TO 19
C C SUM POPN, DISTBN, TO GIVE TOTAL POPN IN YEAR MAXTIM
C POPTOT = 0,0
DO 27, I = 1, NDIVS
POPTOT = POPTOT + P(I,1)
27 CONTINUE
CALL PRODXX(0,A,P,45,45,1)
C C SUM POPN, DISTBN, TO GIVE TOTAL POPN, IN YEAR MAXTIM + 1
C POPMAX = 0,0
DO 28, I = 1, NDIVS
POMAX = POPMAX + Q(I,1)
28 CONTINUE
LAMDA = POPMAX/POPTOT
CROP = ((LAMDA - 1,0)/LAMDA)*PTOT
WRITE(OUT,7002) LAMDA
WRITE(OUT,7003) CROP
7002 FORMAT(//LAMDA = ',',F10.5//)
7003 FORMAT('CROP = ',',F10.5//)
100 CONTINUE
STOP
END
FUNCTION RANGE(X,BOT,TOP)
Z = AMAX1(X,BOT)
Z = AMIN1(Z,TOP)
RANGE = Z
RETURN
END
APPENDIX D

Generation of Linear Programming Input Matrix

Because the single period matrix (Figure 8.3) forms the sub-diagonal of production relationships of the total planning period matrix (Figure 8.4), we need only supply the data for this single period and use a computer programme to generate the total matrix. All data input requirements and operating instructions for the IBM Mathematical Programming System (MPS 360) are detailed in the IBM Users Manual (H20 - 0476 - 2). The following programme listing is that used to generate the data matrix in the form required by the MPS 360 (See Figure D.1.)

```plaintext
REAL FF(12)
REAL WT1(25), WT2(25), WT3(25)
REAL D(3,47)
REAL A(66,47), C(5,8)
REAL F(16)
REAL *B RANGES/AHRANGES
REAL *B TERM/AHENDATA
REAL *B COLS/8ICOLUMNS
REAL *B FARM/AHFARMADAT
REAL *B HARV1/AHARVEST1
REAL *B BOUNDS/AHBOUNDS
REAL *B REMD1/AREMAIN1
REAL *B REMD2/AREMAIN2
REAL DCODE/4HLD
REAL UCODE/4HUP
REAL BRED/4HBRED
REAL STOCK/4HSDEV
REAL MONEY/4HDONY
REAL LABOR/4HLABR
REAL LACT/4HLCYT
REAL YELD/4HYELD
REAL DENSIT/4HDENY
REAL MALE/4HMALE
REAL RHAND/4HHAND
REAL RAW/4HRAWS
REAL NAME/4HNAME
REAL NCODE/4H N
REAL ECODE/4HE E
REAL LCODE/4H L
REAL GCODE/4H G
REAL REMAL/4HRM
REAL REYEL/4HRY
REAL RELAC/4HRL
REAL HAMAL/4HHA
REAL HAYEL/4HHY
REAL HALAC/4HHL
REAL OBJ/4HVEN
REAL RHS/4HRHS
REAL BOYEL/4HBY
REAL BOTYD/4BYEL
REAL SOMAL/4HSM
REAL BOTML/4HBSM
INTEGER Q
INTEGER DQ,FINAL
INTEGER IN
INTEGER OUT
OUT = 8
IN = 5
111 FORMAT(A4,10X,A8,57X,1H)
222 FORMAT(A4,75X,1H)
333 FORMAT(A4,75X,1H)
444 FORMAT(A7,72X,1H)
555 FORMAT(A3,76X,1H)
666 FORMAT(A4,73X,1H)
1000 FORMAT(12F5.1)
1001 FORMAT(F10.2)
1002 FORMAT(10F8.0)
1111 FORMAT(1X,A2,1X,A3,12,70X,1H)
1119 FORMAT(1X,A2,1X,A8,2X,I2,A2,1I,5X,F12.2,43X,1H)
2222 FORMAT(1X,A2,1X,I2,A4,1I,58X,1H)
```
4444 FORMAT(4X,I2,A2,I1,5X,A3,I2,5X,F12,2,3X,A3,I2,5X,F12,2,18X,1H)
3332 FORMAT(4X,I2,A2,I1,5X,I2,A4,11,5X,F12,2,43X,1H)
3333 FORMAT(4X,I2,A2,I1,5X,I2,A4,11,3X,F12,2,3X,I2,A4,11,3X,
1 F1P,2,18X,1H)
4443 FORMAT(4X,I2,A2,I1,5X,A3,I2,5X,F12,2,43X,1H)
4442 FORMAT(4X,A4,I2,4X,I2,A4,11,3X,F12,2,43X,1H)
5555 FORMAT(4X,I2,A2,I1,5X,I2,A4,11,5X,F12,2,43X,1H)
6666 FORMAT(4X,A2,I1X,12,A4,49X,1H)
7777 FORMAT(4X,I2,A2,I1,5X,I2,A4,11,3X,F12,2,3X,I2,A4,4X,F12,2,18X,
1 1H)
8888 FORMAT(4X,I2,A2,I1,5X,I2,A4,4X,F12,2,3X,I2,A4,11,3X,F12,2,18X,
1 1H)
9999 FORMAT(4X,A4,12,4X,I2,A4,4X,F12,2,43X,1H)
9997 FORMAT(4X,I2,A2,I1,5X,I2,A4,4X,F12,2,43X,1H)
9996 FORMAT(4X,A4,6X,I2,A4,4X,F12,2,43X,1H)

C

VENISON YIELD DATA INPUTS (LBS)

DATA WT1/60.57,108.101.131.109,109,153,119,119,
1 159,126,126,173,137,137,182,141,141,187,
2 143,143,190,143,143/.
DATA WT2/60.57,108,75,131,89,89,153,94,94,
1 159,100,100,173,104,182,122,112,112,147,
2 113,113,190,113,113/.
DATA WT3/60.57,108,75,131,89,89,153,94,94,
1 159,100,100,159,108,159,112,112,159,
2 113,113,159,113,113/.
DATA F/12x.56,0.1,0.25,1.36/.
DATA A/65,47,0.0/.

C

CONSTRAINT DATA INPUTS

DATA C/250.150,9000,100,10,1,150,9000,25,25/.
1 250,150,3000,100,10,250,50,9000,100,10/.
2 250,150,9000,25,25,1,1,5000,0.0,0/.

C

READ IN COST DATA & GENERATE OBJECTIVE

FUNCTION PARAMETERS

READ(IN:1000)(D(I,J),J=1,22)
DO 6 J = 1,22
D(I,J) = D(1,J)
D(J,J) = D(I,J)
6 CONTINUE
L = 1
PRICE = 0.60
DO 3 I = 1,25
SALE = WT3(I)*PRICE
J = I+22
D(L,J) = SALE
3 CONTINUE
L = L+1
DO 4 I = 1,25
SALE = WT2(I)*PRICE
J = I+22
D(L,J) = SALE
4 CONTINUE
L = L+1
DO 5 I = 1, 25
SALE = WT3(I)*PRICE
J = I+22
D(L, J) = SALE
5 CONTINUE

C DATA INPUT FOR POPULATION PROJECTION
C MATRIX & CONSTRAINT EQUATION COEFFICIENTS
C
DO 400 I = 13, 34
J = I-12
400 A(I, J) = 1.0
DO 401 I = 13, 37
J = I+10
401 A(I, J) = 1.0
DO 407 J = 3, 22
407 A(38, J) = 1.0
READ(IN, 1000)(A(I, J), I=39, 43, J=1, 22)
DO 402 J = 23, 47
402 A(39, J) = 0.01
DO 403 I = 44, 47
J = I+43
403 READ(IN, 1001)A(I, J)
DO 404 I = 48, 66
J = I+44
404 READ(IN, 1001)A(I, J)
DO 405 I = 50, 65, 3
J = I+43
405 READ(IN, 1001)A(I, J)
DO 406 I = 51, 63, 3
J = I+45
406 READ(IN, 1001)A(I, J)
C INITIALISATION OF PARAMETERS
C FOR MATRIX GENERATION
C
JJ = 1
UPHAR1 = 55.0
DUREA1 = 70.0
DUREA2 = 60.0
RNGE = 5.0
FINAL = 30
NOB9 = 19
NOB10 = 20
NOB11 = 21
NOB12 = 22
C THE FOLLOWING INSTRUCTIONS OUTPUT THE
C ABOVE DATA IN A FORM SUITABLE FOR INPUT
C TO THE IMA360 LINEAR PROGRAMMING PROCEDURE
C
WRITE(OUT, 111)NAME, FORM
WRITE(OUT, 333)ROWS
DO 100 K = 11, 22
WRITE(OUT, 1111)NCODE, OBJ, K
100 CONTINUE
DO 101 J = 11, 30
NA2 = N-10
IF (NA2 .GT. 9) NA2 = 9
DO 102 NAGE = 1, NA2
WRITE (OUT, 2222) CODE, N, MALE, NAGE
WRITE (OUT, 2222) CODE, N, YIELD, NAGE
IF (NAGE .LT. 3) GO TO 102
WRITE (OUT, 2222) CODE, N, LACT, NAGE
102 CONTINUE
IF (N .EQ. 11) GO TO 523
WRITE (OUT, 6666) CODE, N, STOCK
523 WRITE (OUT, 6666) CODE, N, LABOR
522 WRITE (OUT, 6666) CODE, N, MONEY
WRITE (OUT, 6666) CODE, N, BQY
IF (N .EQ. 11) GO TO 101
WRITE (OUT, 6666) CODE, N, BRED
101 CONTINUE
DO 300 N = 11, 30
DO 300 J = 1, 47
A(1, J) = D(1: J)/(1.10)**(N-1)
A(2, J) = D(1: J)/(1.11)**(N-1)
A(3, J) = D(1: J)/(1.12)**(N-1)
A(4, J) = D(1: J)/(1.13)**(N-1)
A(5, J) = D(1: J)/(1.14)**(N-1)
A(6, J) = D(1: J)/(1.15)**(N-1)
A(7, J) = D(1: J)/(1.16)**(N-1)
A(8, J) = D(1: J)/(1.17)**(N-1)
A(9, J) = A(1, J)
A(12, J) = A(1, J)
300 CONTINUE
A(11, 2) = D(1, 2)/(1.10)**(N-1)
FF(1) = F(1)/(1.10)**(N-1)
FF(2) = F(1)/(1.11)**(N-1)
FF(3) = F(1)/(1.12)**(N-1)
FF(4) = F(1)/(1.13)**(N-1)
FF(5) = F(1)/(1.14)**(N-1)
FF(6) = F(1)/(1.15)**(N-1)
FF(7) = F(1)/(1.16)**(N-1)
FF(8) = F(1)/(1.17)**(N-1)
FF(9) = F(1)/(1.18)**(N-1)
FF(10) = FF(1)
NAGF = 1
J = 2
K = N+1
DO 105 L = 1, 7, 2
Q = L+10
QQ = L+11
LL = L+1
WRITE (OUT, 4444) N, REMAL, NAGE, OBJ, Q, A(L, 1), OBJ, QQ, A(LL, 1)
105 CONTINUE
WRITE (OUT, 4444) N, REMAL, NAGE, OBJ, NOB39, A(9, 1)
WRITE (OUT, 7777) N, REMAL, NAGE, N, MALE, NAGE, A(13, 1), N, LABOR, A(39, 1)
IF (N .EQ. 30) GO TO 501
500 WRITE (OUT, 8888) N, REMAL, NAGE, N, MONEY, A(49, 1), K, MALE, J, A(44, 1)
GO TO 150
501 WRITE (OUT, 9999) N, REMAL, NAGE, N, MONEY, A(49, 1)
150 DO 251 L = 1, 7, 2
Q = L+10
QQ = L+11
LL = L+1
WRITE (OUT, 4444) N, REMAL, NAGE, OBJ, Q, FF(L), OBJ, QQ, FF(LL)
251 CONTINUE
WRITE (OUT, 4444) N, BOYEL, NAGE, OBJ, NOB10, FF(L), OBJ, NOB12, FF(L)
WRITE (OUT, 7777) N, BOYEL, NAGE, N, MALE, NAGE, F(L), N, LABOR, F(L)
WRITE (OUT, 9999) N, BOYEL, NAGE, N, MONEY, F(L), N, BOTTLE, F(L)
DO 250 L = 1, 7, 2
Q = L + 10
LL = L + 1
WRITE (OUT, 4444) N, BOYEL, NAGE, OBJ, Q, FF(L), OBJ, Q, FF(L)
250 CONTINUE
WRITE (OUT, 4444) N, BOYEL, NAGE, OBJ, NOB10, FF(L), OBJ, NOB12, FF(L)
WRITE (OUT, 7777) N, BOYEL, NAGE, N, YELD, NAGE, F(L), N, LABOR, F(L)
WRITE (OUT, 9999) N, BOYEL, NAGE, N, MONEY, F(L), N, BOTTLE, F(L)
600 DO 106 L = 1, 7, 2
Q = L + 10
QQ = L + 11
LL = L + 1
WRITE (OUT, 4444) N, REYEL, NAGE, OBJ, Q, A(L, 2), OBJ, Q, A(LL, 2)
106 CONTINUE
WRITE (OUT, 4444) N, REYEL, NAGE, OBJ, NOB9, A(9, 2), OBJ, NOB11, A(11, 2)
WRITE (OUT, 7777) N, REYEL, NAGE, N, YELD, NAGE, A(14, 2), N, LABOR, A(39, 2)
IF (N.EQ.30) GO TO 503
502 WRITE (OUT, 8888) N, REYEL, NAGE, N, MONEY, A(40, 2), K, YELD, J, A(45, 2)
IF (N.EQ.11) GO TO 140
GO TO 601
503 WRITE (OUT, 9998) N, REYEL, NAGE, N, MONEY, A(40, 2)
501 NAGE = 2
J = 3
DO 107 L = 1, 7, 2
Q = L + 10
QQ = L + 11
LL = L + 1
WRITE (OUT, 4444) N, REMAL, NAGE, OBJ, Q, A(L, 3), OBJ, Q, A(LL, 3)
107 CONTINUE
WRITE (OUT, 4443) N, REMAL, NAGE, OBJ, NOB9, A(9, 3)
WRITE (OUT, 7777) N, REMAL, NAGE, N, MALE, NAGE, A(15, 3), N, STOCK, A(38, 3)
WRITE (OUT, 9999) N, REMAL, NAGE, N, LABOR, A(39, 3), N, MONEY, A(40, 3)
IF (N.EQ.30) GO TO 505
504 WRITE (OUT, 8888) N, REMAL, NAGE, N, BREED, A(41, 3), K, MALE, J, A(46, 3)
GO TO 602
505 WRITE (OUT, 9998) N, REMAL, NAGE, N, BREED, A(41, 3)
502 DO 108 L = 1, 7, 2
Q = L + 10
QQ = L + 11
LL = L + 1
WRITE (OUT, 4444) N, REYEL, NAGE, OBJ, Q, A(L, 4), OBJ, Q, A(LL, 4)
108 CONTINUE
WRITE (OUT, 4443) N, REYEL, NAGE, OBJ, NOB9, A(9, 4)
WRITE (OUT, 7777) N, REYEL, NAGE, N, YELD, NAGE, A(16, 4), N, STOCK, A(38, 4)
WRITE (OUT, 9999) N, REYEL, NAGE, N, LABOR, A(39, 4), N, MONEY, A(40, 4)
IF (N.EQ.30) GO TO 507
506 WRITE (OUT, 8888) N, REYEL, NAGE, N, BREED, A(41, 4), K, MALE, J, A(46, 4)
WRITE (OUT, 3553) N, REYEL, NAGE, K, YELD, J, A(43, 4), K, YELD, J, A(47, 4)
WRITE (OUT, 5555) N, REYEL, NAGE, K, LACT, J, A(43, 4)
IF (N.EQ.12) GO TO 140
GO TO 603
507 WRITE (OUT, 9998) N, REYEL, NAGE, N, BREED, A(41, 4)
503 I = 5
M = 17
MM = 49
ML = 50
MY = 51
MH = N - 10
TF (MA4, GT, 8) MA4 = 8
DO 109 MA4 = 31, MA4
J = NAGE + 1
DO 110 L = 1, 7, 2
Q = L + 10
QO = L + 11
LL = L + 1
WRITE (OUT, 4444) N, REMAL; NAGE; OBJ; Q; A (L, I); OBJ; QA; A (LL, I)
110 CONTINUE
WRITE (OUT, 4443) N, REMAL; NAGE; OBJ; NOB9; A (9, I)
WRITE (OUT, 7777) N, REMAL; NAGE; N, MALE; NAGE; A (M, I); N, STOCK; A (38, I)
WRITE (OUT, 9999) N, REMAL; NAGE; N, LABOR; A (39, I); N, MONEY; A (40, I)
IF (N, EQ, 30) GO TO 509
WRITE (OUT, 5555) N, REMAL; NAGE; N, BREED; A (41, I); K; MALE; J; A (W, I)
GO TO 504
504 I = I + 1
M = M + 1
DO 150 L = 1, 7, 2
Q = L + 10
QO = L + 11
LL = L + 1
WRITE (OUT, 4444) N, REYEL; NAGE; OBJ; Q; A (L, I); OBJ; QA; A (LL, I)
130 CONTINUE
WRITE (OUT, 4443) N, REYEL; NAGE; OBJ; NOB9; A (9, I)
WRITE (OUT, 7777) N, REYEL; NAGE; N, YELD; NAGE; A (M, I); N, STOCK; A (38, I)
WRITE (OUT, 9999) N, REYEL; NAGE; N, LABOR; A (39, I); N, MONEY; A (40, I)
IF (N, EQ, 30) GO TO 511
WRITE (OUT, 5555) N, REYEL; NAGE; K; YELD; J; A (43, I); K; YELD; J; A (ML, I)
WRITE (OUT, 5555) N, REYEL; NAGE; K; LACT; J; A (MY, I)
GO TO 605
511 WRITE (OUT, 9998) N, REYEL; NAGE; N, BREED; A (41, I)
505 I = I + 1
M = M + 1
DO 112 L = 1, 7, 2
Q = L + 10
QO = L + 11
LL = L + 1
WRITE (OUT, 4444) N, RELAC; NAGE; OBJ; Q; A (L, I); OBJ; QA; A (LL, I)
112 CONTINUE
WRITE (OUT, 4443) N, RELAC; NAGE; OBJ; NOB9; A (9, I)
WRITE (OUT, 7777) N, RELAC; NAGE; N, LACT; NAGE; A (M, I); N, STOCK; A (38, I)
WRITE (OUT, 9999) N, RELAC; NAGE; N, LABOR; A (39, I); N, MONEY; A (40, I)
TF (N, EQ, 30) GO TO 513
WRITE (OUT, 5555) N, RELAC; NAGE; N, BREED; A (41, I); K; MALE; J; A (42, I)
WRITE (OUT, 5555) N, RELAC; NAGE; K; YELD; J; A (43, I); K; YELD; J; A (ML, I)
WRITE (OUT, 5555) N, RELAC; NAGE; K; LACT; J; A (MY, I)
GO TO 606
513 WRITE (OUT, 9998) N, RELAC; NAGE; N, BREED; A (41, I)
506 I = I + 1
M = M + 1
MM = MM + 3
ML = ML + 3
MY = MY + 3
109 CONTINUE
140 I = 23
M = 13
DO 115 NAGE = 1, 2
DO 113 L = 1, 7, 2
Q = L+10
QQ = L+11
LL = L+1
WRITE(OUT,4444)N,HAMAL,NAGE,OBJ,Q,A(L,I),OBJ,QQ,A(LL,I)
113 CONTINUE
WRITE(OUT,4443)N,HAMAL,NAGE,OBJ,NOB12,A(12,I)
WRITE(OUT,7777)N,HAMAL,NAGE,N,MALE,NAGE,A(M,I),N,LABOR,A(39,I)
M = M+1
I = I+1
DO 114 L = 1, 7, 2
Q = L+10
QQ = L+11
LL = L+1
WRITE(OUT,4444)N,HAYEL,NAGE,OBJ,Q,A(L,I),OBJ,QQ,A(LL,I)
114 CONTINUE
WRITE(OUT,4443)N,HAYEL,NAGE,OBJ,NOB12,A(12,I)
WRITE(OUT,7777)N,HAYEL,NAGE,N,YE LD,NAGE,A(M,I),N,LABOR,A(39,I)
IF (N.EQ.11) GO TO 104
I = I+1
M = M+1
115 CONTINUE
IF (N.EQ.12) GO TO 104
NAS = N - 10
IF (NAS.GT.9) NAS = 9
DO 116 NAGE = 3, NAS
DO 117 L = 1, 7, 2
Q = L+10
QQ = L+11
LL = L+1
WRITE(OUT,4444)N,HAMAL,NAGE,OBJ,Q,A(L,I),OBJ,QQ,A(LL,I)
117 CONTINUE
WRITE(OUT,4443)N,HAMAL,NAGE,OBJ,NOB12,A(12,I)
WRITE(OUT,7777)N,HAMAL,NAGE,N,MALE,NAGE,A(M,I),N,LABOR,A(39,I)
I = I+1
M = M+1
DO 118 L = 1, 7, 2
Q = L+10
QQ = L+11
LL = L+1
WRITE(OUT,4444)N,HAYEL,NAGE,OBJ,Q,A(L,I),OBJ,QQ,A(LL,I)
118 CONTINUE
WRITE(OUT,4443)N,HAYEL,NAGE,OBJ,NOB12,A(12,I)
WRITE(OUT,7777)N,HAYEL,NAGE,N,YE LD,NAGE,A(M,I),N,LABOR,A(39,I)
I = I+1
M = M+1
DO 119 L = 1, 7, 2
Q = L+10
QQ = L+11
LL = L+1
WRITE(OUT,4444)N,HALAC,NAGE,OBJ,Q,A(L,I),OBJ,QQ,A(LL,I)
119 CONTINUE
WRITE(OUT,4443)N,HALAC,NAGE,OBJ,NOB12,A(12,I)
WRITE(OUT,7777)N,HALAC,NAGE,N,LACT,NAGE,A(M,I),N,LABOR,A(39,I)
I = I+1
M = M+1
116 CONTINUE
104 CONTINUE
WRITE(OUT,555)RHS
DO 210 K = 1, 8
M = K+10
DO 211 N = 11, 30
IF (N.EQ.11) GO TO 540
WRITE (OUT, 9997) RHAND, M, N, STOCK, C(1, K)
540 WRITE (OUT, 9997) RHAND, M, N, LABOR, C(2, K)
WRITE (OUT, 9997) RHAND, M, N, MONEY, C(3, K)
IF (K.GT.5) GO TO 211
WRITE (OUT, 9997) RHAND, M, N, B0TYD, C(4, K)
WRITE (OUT, 9997) RHAND, M, N, BOTHL, C(5, K)
211 CONTINUE
210 CONTINUE
WRITE (OUT, 666) RANGES
WRITE (OUT, 9996) DENSIT, FINAL, STOCK, RAGE
WRITE (OUT, 666) BOUNDS
DO 399 N = 11, 29
NAGE = 1
WRITE (OUT, 1119) UCODE, HARV1, M, HAYE1, NAGE, UPHAR1
399 CONTINUE
DO 499 N = 17, 29
NAGE = 7
WRITE (OUT, 1119) UCODE, REM02, N, RELAC, NAGE, DORE42
499 CONTINUE
WRITE (OUT, 222) TERM
STOP
END
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