THE VARIETIES OF PERCEPTUAL INDEPENDENCE IN IDENTIFICATION AND SIMILARITY JUDGEMENT TASKS

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CONTENTS

CONTENTS(Continued)

SUMMARY

The work reported here is an investigation of various forms of independence in identification and similarity judgement tasks for auditory stimuli.

Experiment 1A and 1B tested various independence properties (single factor and joint factor independence) on similarity data. Both experiments suggested that not only was a dimensional representation appropriate, but also that the 'differences' between two stimuli were combined additively.

The second sets of experiments (2A and 2B) explored dimensional independence in identification tasks for auditory stimuli presented in noise. It was found that for both pairs of dimensions tested (pitch and duration; and pitch and loudness) there was a lack of independence. Specifically, it was found, that the identifiability of a value on one dimension varied over different levels of the other. In other words there were interaction effects.

Another group of experiments (3A, 3B, 3C, and 3D) investigated both independence and metric properties on data derived from similarity judgements of auditory tones. Here the data was required to satisfy simultaneously, not only the dimensional requirements tested in experiments 1A and 1B but the various ordinal conditions which would allow it to be represented by a Minkowski metric. The four experiments established, for the particular values of pitch, duration, and loudness tested,

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that both a dimensional and metric representation of the data was tenable. One outcome from these experiments was that the similarity data could be represented by either a Euclidean, City Block or Dominance metric.

The last experiment, 4A, was in part a replication of Garner and Felfoldy's (1970) study. These authors had used metric concepts such as the Euclidean and City Block distance functions to account for the results of their identification experiment. It was noted that metric concepts were appropriate only for similarity data (under certain conditions),and their use,to explain results in identification data was entirely objectionable. It was suggested instead that Garner and Felfoldy's data could have arisen as a result of interaction between the stimulus dimensions. These interaction effects had already been investigated in experiments 2A and 2B. Experiment 4A, did indeed suggest that the results might be due to lack of independence of the dimensions rather than to their alleged metric properties.

CHAPTER I INTRODUCTION

Perceptual Independence is a notion that has enjoyed a fitful currency for some twenty years. Since Erikson and Hake (1955) first invoked it to predict multidimensional identification performance from a subject's unidimensional performance on the separate dimensions composing the complex stimulus, its meaning has undergone a variety of changes. To see this, one has only to consult Corcoran (1966), Corcoran (1967), Garner and Morton (1969), Garner (1974), and Kaufman and Levy (1971). The original notion was tied explicitly to the ideas of stimulus dimensions and their independence. This original notion was never explored very adequately and the work which followed Erikson and Hake's was mainly experimental and little attempt was made to clarify its conceptual basis.

Some exploratory work, in this direction, was attempted by Garner and Morton (1969) but in their treatment the original idea of stimulus independence had all but vanished only to be replaced by terms such as process interaction, state independence, integral dimensions, non-integral dimensions, process parity and a host of other technical terms. One theoretical paper which did continue the original theme was that by Kaufman and Levy.

The vicissitudes of Perceptual Independence in the hands of experimental psychologists are reminiscent of the recent past preoccupations in British philosophy. Gellner (1959) wrote that philosophers never tackled the 'big traditional' questions - or to lapse into the then current jargon 'the first order problems'. Instead, it was alleged, they concentrated exclusively on the second order problems. These philosophical practitioners who subsequently became known as 'ordinary language philosophers' viewed their work as merely preliminary - a sharpening up of the linguistic tools with which one might eventually be able to tackle the traditional "first-order" problems.

The recent career of experimental psychology is the reverse of that noticed in philosophy: there has been much speculation, the increase in technical terms testifies to this. Consider for instance: short term memory, working memory, process parity, and so on. With these terms go experiments, which, on close examination do not bear too much comparison with the original speculation. In psychology, it is true to say, that there has been an almost complete absence of the 'ordinary language approach' and a dearth of effort devoted to clarifying the concepts used in experimental work. In practice this has lead to the introduction of explanatory constructs and definitions which are tied not only to a particular small area in psychology but to a specific experimental design. A distressing consequence of the excessive preoccupation with the 'first-order' problems is, that although the same notions seem to reappear in different areas of psychology, very little effort is taken either to identify these recurring themes, or to place them in a broader context. It is argued here, that one such instance is

Independence: it appears and reappears so often in experimental psychology, and in different forms, that it inspires the hope that a thorough examination of these instances might reveal some similarities in the psychological processes, or in any event make some of them more apparent.

It is a temptation when embarking on an enterprise devoted to semantic and conceptual hygiene to adopt solely the style of the ordinary language philosopher: to attempt to clarify, or articulate the second order problems without recourse to the tiresome experimental work. If you like, to stand to experimental work much as the accountant does to the businessman: not to actually do the work, or make the money, or to otherwise engage in disagreeable trade, but simply to explain some of the rules which have to be observed before it can be said with any certitude whether or not a profit has been made. The temptation to become simply a probation officer for errant experimentalists is easily resisted. Indeed it will be seen that a conceptual approach in fact suggests new experiments; many of which should have been done long ago.

In this treatment some of the machinery employed to analyse independence will consist of measurement theory (Krantz, Luce, Suppes and Tversky 1971). In Chapter 2 a brief introduction will be given, and various types of independence which arise in conjoint structures will be discussed. Additive conjoint measurement - the two factor case (Luce and Tukey 1964), - is described, together with the various types of significant interaction effects, both ordinal and disordinal (Lindquist 1953), which arise in Analysis of Variance. Two experiments are then introduced both of which employ conjoint measurement techniques. The first study tests an additive difference model for similarity judgements of pure tones which vary in duration and loudness. This experiment is in reality a disguised test for additive independence. The second experiment is a disguised test for both single factor and joint factor independence of similarity data, for a set of auditory stimuli which vary on three dimensions: duration, loudness and pitch.

The main concern of Chapter 3 is independence in identification experiments i.e. perceptual independence. Some of the ideas derived from the measurement theoretic analysis in Chapter 2 are used to make distinctions between various types of independence. Briefly, two stimulus dimensions are said to be <u>perceptually independent</u> if the identifiability of particular levels of one dimension is independent of the levels of the other dimension. The experimental and theoretical work of this chapter relies fairly heavily on uncertainty analysis or information theory. This type of analysis is employed because it allows the logic of Analysis of Variance to be employed on data from single subjects.

Chapter 4 continues the theme of Chapter 2 with its emphasis on similarity judgements. However, independence in similarity judgements is now placed within the framework of the foundations of multidimensional scaling. (Beals, Krantz and Tversky 1968, Tversky and Krantz 1970). The 'additive difference' model tested in experiment 1A is examined for some possible composition rules when the additional constraints of representing the similarities by a distance metric are added. Two particular properties are discussed, and examined in the experiments - interdimensional additivity, and intradimensional subtractivity (Beals et al 1968). Beals, Krantz and Tversky showed that interdimensional additivity is implied by a kind of "context" independence property of the distances; which further implies that the dimensions do not interact in their effects. This kind of non-interaction or independence, while probably not being a necessary condition for a reasonable psychological metric, does seem like a desirable one. In view of this, and because of the close analogy with independence in similarity tasks to independence in identification experiments, non-interaction of the stimulus dimension provides the basis for an operational definition of independent perceptual dimensions.

Because of the importance attached to 'psychological metrics' by for instance: Shepard (1964), Torgerson (1958, 1965), Cross (1965), Garner (1970, 1971, 1974), Garner and Morton (1969), Lockhead (1970), and Levy and Hagbloom (1971), the two experiments reported in this chapter also attempt to diagnose whether the particular stimulus dimensions, duration, pitch and loudness fulfil those qualitative (i.e. ordinal) requirements which allow the similarity judgements to be represented by a distance metric.

The aim of the final chapter is not unlike the intention which usually animates the final scene of a restoration comedy, in which the confused lovers are disentangled from accidental unions and placed with their rightful partners. So, here, a meeting will be arranged between identification tasks and similarity judgement tasks. First, the use of distance metric terms like integral dimensions and non-integral dimensions to describe results in identification experiments (Garner and Felfoldy 1971) are disentangled and deprecated. In particular it is argued that 'stimulus integrality' and non-stimulus integrality, which are said to correspond to the Euclidean and City block metric respectively (Garner 1971), have been used to describe results in stimulus identification tasks in a quite objectionable manner. These terms when used to describe results in similarity judgement tasks, under certain circumstances (provided certain of the ordinal and qualitative conditions discussed in Chapter 3 are met) capture the psychological processes fairly well. In such cases, it seems sensible and even illuminating to discuss the type of distance metric which may correspond to particular methods of information processing (Garner 1971, Garner and Felfoldy 1971). However to employ concepts derived from theoretical work on <u>similarity</u> judgements to explain results in <u>identification</u> experiments seem to be quite illegitimate. Another consequence of the proposed meeting is to suggest that the notion of independence for both identification and similarity tasks serves to underpin similar psychological processes. In particular it is suggested that some of the results in sorting tasks (Egeth (1967) Lockhead (1966), Egeth and Pachella (1969))

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CHAPTER 2

Independence in Conjoint Structures: Applications to Similarity Data

2.00 The Representational Theory of Measurement Fundamental

Measurement of Length and Mass Extensive and Intensive

Measurement Conjoint Structures

Empirical Conjoint Structures

2.30 Additive and Non-Additive Independence in Conjoint Structures

Decomposability

Additive Independence - The Two Component case

Non-Additive Independence

The Luce & Tukey model of additive independence

2.40 Data Matrices

Additive Data Matrices

Examples of an Additive Data Matrix

Data Matrices, Analysis of Variance and Significant interaction

Ordinal and Disordinal interaction

Transformations, Additivity and Interaction

Analysis of Variance and Conjoint Measurement

Measurement and Model Testing

2.50 Independence in Conjoint Structures

Composition Rules and the Decomposition Hypothesis

Single Factor Independence

Joint Factor Independence

INDEPENDENCE IN SIMILARITY DATA

2.00 The Representational Theory of Measurement.

Measurement Theory is a logical analysis of the measurement process and one influential point of view of this enterprise has been referred to as the Representational Theory of Measurement (Adams 1966). In essence this outlook considers measurement to consist in assigning numbers to objects in such a way that certain operations on and relations among the assigned numbers come to correspond to or represent observable operations and relations on the things to which they are assigned. Much of the recent work on measurement in psychology has taken the representational view as its point of departure, for example, Suppes and Zinnes (1963), Luce and Tukey (1964), and Krantz, Luce, Suppes and Tversky (1971).

It is a commonplace observation that all of the quantitative laws of physics depend on the prior measurement of mass, length, and duration. However even these fundamental measurements are based on qualitative (non-numerical) laws which must hold before numbers can be assigned to these quantities.

2.10 The Representational Theory of Measurement may be summarised:

2.11 Measurement is the assignment of numbers to objects or phenomena according to some rule.

2.12 In the case of fundamental measurement certain empirical operations and empirical relations among objects come to correspond to, or be represented by, operations and relations among numbers.

2.13 The problem of the foundations of measurement is to "determine the conditions under which measurement is possible". This requires determining the empirical or qualitative laws which the objects must satisfy in order that it should be possible to assign numbers to them, such that the empirical operations and relations can be made to correspond to numerical operations and relations.

In practice representational measurement theorists require an empirical structure, a corresponding numerical structure, a Representation Theorem which states that if certain conditions hold in the empirical structure then this can be mirrored in the numerical structure, and a Uniqueness Theorem which defines the level of measurement obtained, that is, either nominal, ordinal, interval or ratio.

It might be as well to formalise these last remarks slightly. First there is the idea that measurement is the linking of mathematics represented as numerical systems with the world of empirical systems. Secondly three definitions are in order:

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2.14 An Empirical System; If E is a non-empty set of observed or empirical elements and Ri (i = 1, ... N) is a relation on ExE then the ordered (N + 1) tuple:

$$S_E = < E, R_1 \ldots R_N >$$

is called an empirical system.

As an example: S_E might consist of a set of occupational statuses; R_1 might be the relation "has greater prestige than" R_2 "has more tenure than".

2.15 Numerical System (or numerical relational system)

If A is any non-empty set of real numbers and R_i (i = 1 ... M) is a relation on AxA then the ordered (M + 1) tuple

$$S_M = < A_1, R_1 \ldots R_M, >$$

is a numerical system.

2.16 Measurement Function. If E is any non-empty set of observable elements and A is a subset of the real numbers, then a measurement of E is any function, f, on E to A and f is called a <u>measurement function</u>.

To illustrate this last definition suppose a set of voters V, are partitioned into 3 equivalence classes labelled Tory, Labour and SNP (T, L, and S respectively) then to establish a measurement f on these elements of V:

let f be given by the following:

$$f(x) = \begin{cases} (1) & \text{if } E \in T \\ (2) & \text{if } E \in L \\ (3) & \text{if } E \in S \end{cases}$$

2.20 Fundamental Measurement of Length and Mass.

Fundamental measurement in Physics is based on certain empirical observations which are made prior to the assignment of numbers. Moreover these observations are always of an ordinal nature. Consider, for example, the procedures involved in the fundamental measurement of length. They are based on the following empirical considerations:

A weak order relation, $\succsim$, "as least as long as" which can be empirically established by placing a rod A alongside a rod B.

An empirical concatenation or "addition" operation which can be empirically realised by placing rods end to end which can be signified by the symbol $\Box$.

Suppose there are rods A, A', B, B'

Suppose that Rod A extends beyond rod B

i.e. $A \succsim B$

Similarly $A' \succsim B'$

One of the sufficient qualitative conditions for fundamental measurement of length is that the following empirical observations hold:

If $A \succ B$

and $A' \succ B'$

2.21 Then $A \circ A' \succ B \circ B'$ (the combination length A with A' is greater in length than the combination of lengths B and B')

2.22 Similar qualitative laws for weight measurement can be obtained which are based on having a large collection of objects on an idealised equal arm pan balance. If a finite collection of objects is placed in each pan, and the pan which drops is noted, it is possible to 'order' by weight all finite collections of objects. Hölder (1901) obtained a number of empirical conditions sufficient to establish weight measurement.

One of these conditions is formally equivalent to the one noticed for the fundamental measurement of length. Hölder considered a weak ordering relation, 'as least as heavy as', $\gtrsim$, and an empirical concaternation operation, $\oplus$, which can be empirically realised by placing two objects in the same pan:

Suppose A, B, C and D are mutually disjoint objects such that:

if $A \gtrsim C$ (A is at least as heavy as C)

and $B \gtrsim D$ (B is at least as heavy as D)

then one of the sufficient conditions for the fundamental measurement of weight is that:

If $A > B$

and $A' > B'$

2.21 Then $A \sqcup A' > B \sqcup B'$ (the combination length A with A' is greater in length than the combination of lengths B and B')

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2.221 $A \oplus B \gtrsim C \oplus D$. (The combination of A and B is as least as heavy as the combination of C and D.)

Hölder in a classical theorem showed that if the behaviour of objects in a pan balance satisfied certain empirical conditions then numbers i.e. weights, can be assigned to these objects to represent their behaviour in the scale pan. From an abstract point of view the procedures for assigning numbers as measures of objects are identical for length, mass, and duration: the procedure is to concatenate many identical objects (metre lengths, grams, or pendulum periods) and then count how many identical objects are required to match approximately the object to be measured. Measurement based on an empirical concatenation operation is called extensive measurement. Many instances of measurement are like the measurement of mass, length, or duration inasmuch as they involve the construction of a real valued function that preserves the order and additive structure of an empirical system. All such structures are based ultimately on Hölder's theorem. In the measurement of mass for example, positive real numbers are assigned to objects so that the order of numbers reflects the order of objects as determined by a pan balance and the addition of real numbers corresponds to the combining of objects in a pan.

2.23 Extensive and Intensive Measurement.

Campbell (1920) distinguished between extensive properties or quantities and intensive properties or qualities.

Quantities were defined as properties for which there is an empirical combining operation analogous to the arithmetical operation of addition. Qualities however are characterised by an absence of an empirical combining or additive operation. Weight, length, and duration are examples of extensive quantities, whereas utility, response strength, and similarity are examples of intensive qualities. Campbell argued that only extensive properties - those for which there exists an empirical concatenation operation - can be measured on an interval or ratio scale. Since psychological attributes - for example, loudness, intelligence and thirst - have no empirical concatenation operation, interval scale measurement is not possible. So according to Campbell despite the fact that comparison relations like 'greater than' or 'as similar as' abound in psychology, because there is an absence of binary operations like $\oplus$, or $\Box$, then at most only ordinal measurement is possible.

Recently the view that the absence of an empirical 'addition' operation for intensive properties precludes anything stronger than ordinal measurement has been questioned by Luce & Tukey (1964). They have shown, as did Krantz (1964) also, that an empirical concatenation operation is not the only possible source of additional structure sufficient for interval measurement. Various other kinds of structure can combine with a comparison relation and if appropriate qualitative laws are satisfied interval or ratio scaled measurement results.

2.24 Conjoint Structures

One particular non-extensive structure with a corresponding system of empirical or qualitative laws which lead to interval or ratio scaled measurement can be characterised as follows:

(i) The existence of a comparison relation which orders objects with respect to some attribute

(ii) The objects to be compared can be regarded as elements of a cartesion product.

Such a structure is called an empirical conjoint structure and the ordering of any object with respect to the comparison relation depends on the value of two or more independently controllable factors. Luce and Tukey showed that measurement based on an empirical conjoint structure requires the simultaneous construction of scales for:

(a) the ordered variable (the independent variable) and

(b) the contributions of each of the factors to this ordering.

Luce and Tukey referred to this type of measurement as simultaneous conjoint measurement.

2.25 Example of an Empirical Conjoint Structure.

In order to present a reasonably concrete introduction to empirical conjoint structures an example provided by

Krantz et al. (1971) will be considered. This had to do with the measurement of response strength in rats (Hull 1952) and its dependence on three factors: drive, incentive, and habit strength labelled as D, K and H respectively.

The drive, D, in a rat was manipulated by varying the amount of food deprivation, in practice indexed by percentage decrease in body weight. Habit strength, or previous experience H, was manipulated by varying the amount of previous training. Incentive or food reward was varied by altering the concentration of sucrose solution offered to the rats at the end of the alley. Response strength was measured by the running speed of the rat.

Consider an actual experiment in which there are two levels each of habit strength, drive, and deprivation giving rise to a 2x2x2 factorial design - or an empirical conjoint structure. Two ways of representing this scheme of things are shown in Fig 2.51. The two levels of drive, (D) habit strength (H) and incentive (H) have been labelled by the lowercase letters d, d'; h, h' and k, k'; respectively. This experimental arrangement results in 8 different treatment combinations. For instance, each of the cells could, typically, contain the median running speed for four rats, at a particular level of drive, habit strength, and incentive. The particular treatment combination can be labelled in the general case by an ordered triple. For example. (d' h k'), represents the response strength at the corresponding levels of the independent variables.

Although the psychological attributes, drive, habit strength, incentive, and response strength are all 'defined' in terms of some physical quantity - loss in body weight, number of previous trials, volume of sugar, and running speed respectively - they do not constitute a definition of these attributes. All that can be said is that there is a monotonic relation between the physical measure and the level of the psychological attribute. These definitions of the psychological attributes in terms of the physical dimensions are operational definitions.

2.30 Additive and Non-Additive Independence in Conjoint Structures

Krantz et al (1971) (Chapter 6 page 246) in their discussion of independence in conjoint structures introduced two subsidiary ideas about the components or factors contributing to the ordering of an object. These were:

2.301 The Independent Realisation of the Components or factors. The idea being that the components or factors are separate entities.

2.302 Decomposability: The two or more components in the empirical conjoint structure contribute their effects independently to the attribute in question.

For the two-component case with dimensions A_1, A_2, and an ordering relation $\geq$, the above remarks prompt the question: Are there any qualitative 'laws' or conditions for the two-component conjoint structure generated by the triple $(A_1, A_2, \gtrsim)$ which are sufficient to permit the construction of real valued functions Φ_1, Φ_2 on A_1, A_2 respectively together with a function, F, from Re x Re such that:

$$(a,p) \gtrsim (b,q)$$

if and only if

$$F\left[\Phi_1(a), \Phi_2(p)\right] \geq F\left[\Phi_1(b), \Phi_2(q)\right]$$

where (a, p), (b, q), is some ordinal measure of the (a, p), (b, q) treatment combinations and

$$a, b \in A_1$$

$$p, q \in A_2$$

In words: Are there numerical scales Φ_1, Φ_2 on the two components A_1, A_2, respectively and a rule F for combining them such that the resultant measure preserves the qualitative ordering of the attribute? If such a representation exists the structure $(A_1, A_2 \gtrsim)$ is <u>decomposable</u>. F is a composition rule specifying the manner in which the dimensions of the stimulus are combined by the subject.

The form of the composition rule, F, determines different types of independence which it is convenient to classify broadly as <u>additive</u> and <u>non-additive</u> independence.

2.31 Additive Independence - The Two Component Case

Suppose that in a two-component conjoint structure, the composition rule, F, specifies that the two dimensions are combined additively.

Then (a, p) ≳ (b, q) implies:

$$\Phi_1 (a) + \Phi_2 (p) \gtrsim \Phi_1 (b) + \Phi_2 (q)$$

2.32 Non-Additive Independence

It is quite possible for the conditions expressed in 2.301 and 2.302 to hold, and for F to be a non-additive composition rule. For example consider a 'multiplicative' composition rule:

(a p) ≳ (b, q)

implies that

$$\Phi_1 (a) \cdot \Phi_2 (p) \geq \Phi_1 (b) \cdot \Phi_2 (q)$$

Procedures to determine the form of some simple composition rules for conjoint structures have been outlined by Krantz and Tversky (1971).

2.33 Additive Independence: Two Factor Case

Suppose $A_1 \times A_2$ is a weakly ordered product set obtained for example from a two way factorial design with typical elements:

$$(a, b) \in A_1 \quad (p, q) \in A_2$$

Suppose the following qualitative conditions obtain:

(i) $A = A_1 \times A_2$ has a weak order relation imposed on it.

(ii) A Solvability condition: any change in one factor can be exactly compensated by changes in another.

(iii) A cancellation condition. For all $a, b, c \in A_1$
$p, q, r \in A_2$

if $(a, q) \gtrsim (b, p)$

and $(b, r) \gtrsim (c, q)$

then $(a, r) \gtrsim (c, p)$

Luce and Tukey (1964) showed that these four conditions are sufficient to generate the following Representation Theorem which maps the empirical conjoint structure into a numerical relational system which allows measurement up to at least an interval scale.

2.34 Theorem

If $(A_1, A_2, \gtrsim)$ satisfy the above four qualitative conditions then there are real valued functions:

(i) Φ_1 on A_1

(ii) Φ_2 on A_2

and (iii) Φ on A

such that for all $a, b \in A_1$ and $p, q \in A_2$

$\Phi_1 (a) + \Phi_2 (p) \geq \Phi_1 (b) + \Phi_2 (q)$

if an only if

$(a, p) \gtrsim (a, q)$

Furthermore Φ_1 and Φ_2 are interval scales with a common unit.

By considering simultaneously the contribution of two factors, the Luce-Tukey model allows each factor to be measured separately. The important feature of the conjoint measurement 'axioms' or conditions is that these assumptions are stated entirely in terms of the observable properties of the data.

The four conjoint measurement 'axioms' or conditions do not explicitly contain the concatenation operation, as do the axioms for extensive measurement. In fact the four qualitative conditions postulate only a single observable binary relation on the ordered pair of factors or objects. The first condition assumes that the observable binary relation weakly orders the objects, whilst the cancellations condition is a form of transitivity condition. Luce and Tukey showed that the four conditions are <u>sufficient</u> though not <u>necessary</u> to establish the representation theorem whereby the effects of the two factors appear in a simple additive way.

2.40 Data Matrices

It is often the case that experimental work with conjoint structures give rise to a data matrix so that (a, p) again is an ordinal measure of the (a, p) treatment combination where

$a \in A$

$p \in P$

A, P, are two independent factors and $D = A \times P$.

A data matrix D, is said to be additive or to have an additive representation if there are functions: Φ_1, Φ_2, Φ, defined on A, P and $D = A \times P$ respectively, such that

(i) $\Phi(a, p) = \Phi_1(a) + \Phi_2(p)$

(ii) $\Phi(a, p) \geq \Phi(b, q)$

if and only if

$(a, p) \gtrsim (b, q)$

where $a, b \in A$

$p, q \in P$

Hence a data matrix is said to be <u>additive</u> if

(a) Its cell entries can be rescaled such that their order is preserved.

(b) Every rescaled entry is expressed as the sum of the row and column components.

If such a representation exists the two factors A and P can be regarded as independent in the sense that they contribute independently and additively to produce the joint effect.

Monotonic Data Matrix. A data matrix is said to be monotonic if all its entries are such that

(i) each row is monotonic and the rank order of the cell entries is the same for each row

$$a \ \varepsilon \ A$$
$$p \ \varepsilon \ P$$

A, P, are two independent factors and D = A x P.

A data matrix D, is said to be additive or to have an additive representation if there are functions: Φ_1, Φ_2, Φ, defined on A, P and D = A x P respectively, such that

(i) $\Phi\ (a, p) = \Phi_1\ (a) + \Phi_2\ (p)$

(ii) $\Phi\ (a, p) \geq \Phi\ (b, q)$

if and only if

$(a, p) \gtrsim (b, q)$

where $a, b \ \varepsilon \ A$

$p, q \ \varepsilon \ P$

Hence a data matrix is said to be <u>additive</u> if

(a) Its cell entries can be rescaled such that their order is preserved.

(b) Every rescaled entry is expressed as the sum of the row and column components.

If such a representation exists the two factors A and P can be regarded as independent in the sense that they contribute independently and additively to produce the joint effect.

Monotonic Data Matrix. A data matrix is said to be monotonic if all its entries are such that

(i) each row is monotonic and the rank order of the cell entries is the same for each row

FIG. 2.2 a CONSTRUCTING AN ADDITIVE DATA MATRIX BY RANK ORDERING THE ORIGINAL CELL ENTRIES.

Factor A	p	q	r
a	6·5	8·25	9·0
b	4·5	6·0	7·5
c	4·0	5·25	7·25

Factor P

A 3 x 3 MONOTONIC DATA MATRIX WHICH SATIFIES THE CANCELLATIONS CONDITIONS AND THEREFORE HAS AN ADDITIVE SOLUTION.

FIG. 2.2 b CONSTRUCTING AN ADDITIVE DATA MATRIX BY RANK ORDERING THE ORIGINAL CELL ENTRIES.

Factor A	p	q	r
c	5	8	9
b	2	4	7
a	1	3	6

Factor P

BECAUSE IT IS MERELY REQUIRED TO DERIVE SCALES WHICH REPRODUCE THE ORDER OF THE CELL ENTRIES THE MATIRX ON THE LEFT SHOWS THE RANK ORDERING OF EACH OF THE CELLS OF THE ORIGINAL DATA MATRIX.

FIG. 2.2 .c CONSTRUCTING AN ADDITIVE DATA MATRIX BY RANK ORDERING THE ORIGINAL CELL ENTRIES.

FACTOR A	Φ2(p)	Φ2(q)	Φ2(r)
Φ1(c)	6	10	13
Φ1(b)	1	5	8
Φ1(a)	0	4	7

FACTOR P

THE TRANSFORMED 'ADDITIVE' DATA MATRIX. EACH CELL ENTRY IS NOW AN ADDITIVE COMBINATION OF THEIR ROW AND COLOUMN COMPONENTS.

THE TRANSFORMED SCALES ARE:-

$\phi 1(a) = 0$	$\phi 2(p) = 0$
$\phi 1(b) = 1$	$\phi 2(q) = 4$
$\phi 1(c) = 6$	$\phi 2(r) = 7$

IN THE TRANSFORMED DATA MATRIX, [illegible]

For all a,b ∈ A and p,q ∈ P :

$$\Phi(a,p) = \Phi_1(a) + \Phi_2(p)$$

and (ii) each column is monotonic. the rank order of each column is identical.

Tversky (1967) showed that any data matrix satisfying the cancellation and solvability conditions has an additive representation that is unique up to a positive linear transformation. Hence if these two qualitative conditions hold there are functions Φ_1, Φ_2, such that for all

$$a, b, \in A, \qquad p, q, \in P$$

$$\Phi_1 (a) + \Phi_2 (p) \geq \Phi_1 (b) + \Phi_2 (q)$$

if and only if

$$(a, p) \quad \gtrsim \quad (b, q)$$

2.41 An Example of an Additive Data Matrix

Since data matrices and additive representations will figure in the experiments included in this chapter, a method of rank ordering the cell entries of the data matrix in order to obtain an additive representation is shown in Fig 2.2b. This method was used by the author in Experiment 1A. The transformed matrix of Fig 2.2c is such that each cell entry is an additive combination of their row and column components. Note that if the entries of this additive data matrix are rank ordered the matrix of Fig 2.2b will be obtained.

2.42 Two definitions are in order here:

Additivity A cell entry is additive in A and B (the factors) if its value can be reconstructed by adding

OUTCOME OF A FACTORIAL ANALYSIS OF VARIANCE.

	A1	A2	A3	Ur
B1	1	2	3	2
B2	2	3	4	3
B3	3	4	5	4
Uc	2	3	4	3

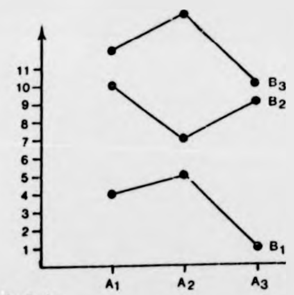
EACH CELL ENTRY IS THE MEAN OF SIX DIFFERENT SUBJECTS. IF THERE ARE NO INTERACTIONS, THEN EVERY CELL MEAN IN THE POPULATION IS DETERMINED BY ITS ROW AND COLUMN MEANS.

WHERE Urc = Ur + Uc − U. (Urc REPRESENTS THE CELL MEAN)

FIG. 2.3. ADDITIVITY IN ANALYSIS OF VARIANCE.

DIFFERENT TYPES OF SIGNIFICANT INTERACTIONS IN ANALYSIS OF VARIANCE.

SIGNIFICANT ORDINAL INTERACTION

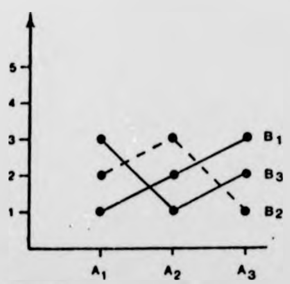


		A			
		A_1	A_2	A_3	Ur
B	B_1	4	5	1	3·3
	B_2	10	6	8	8·0
	B_3	12	15	11	12·6
Uc		8·7	8·7	6·7	8·0

FIG. 2·4a

Here B_3 is more effective than B_2, and B_2 is more effective than the B_1 treatment at level A_1. This rank ordering of the "effectiveness" of the B treatments persists for all levels of A even though B is differentially effective at different levels of A. In ordinal interaction the different B treatment effects have the same rank order for each level of A.

SIGNIFICANT DISORDINAL INTERACTION



	A_1	A_2	A_3	Ur
B_1	1	2	3	2·0
B_2	2	3	1	2·0
B_3	3	1	2	2·0
Uc	2·0	2·0	2·0	2

FIG. 2·4b

Here the different B treatment effects do not have the same rank order for each level of the A factor: the rank order of the different B effects change.

a row number and a column number and perhaps an extra constant. There is nothing new in this definition, the additivity assumption of analysis of variance is usually couched in these terms.

Monotonic Transformations: A transformation is called (monotonic) increasing if as in y = log x; y gets larger as x gets larger. Data transformation of interest to the psychologist are almost always either increasing or decreasing but not mixed. The term 'monotonic' covers both cases.

2.43 Data Matrices, Analysis of Variance and Significant Interaction. Additivity in Analysis of Variance

Consider the outcome of a 3 x 3 factorial analysis of variance as shown for instance in Fig 2.3, where each of the cell entries represent the mean of the scores of 6 persons. Such an experiment yields 3 sets of means:

i the row mean μ_r
ii the column mean μ_r
and iii the cell mean μ_r

If there is no interaction, then every cell mean in the population is determined by its row and column means in the following manner

2.431 $$\mu_{rc} = \mu_r + \mu_c - \mu$$

where μ represents the population mean. If this equality holds for every cell of the factorial experiment, all of the interaction effects are zero and the row and column effects are additive. If the equality does not hold for all cells the row and column effects are non-additive.

2.44 Significant Interations: Ordinal and Disordinal

Lindquist (1953) and Lubin (1961) have both distinguished between two classes of significant interaction effects: ordinal and disordinal interaction.

In ordinal interaction the rank order of the treatments is constant. Although a particular level of one factor is differentially effective at various levels of the other factor, that level of the first factor maintains its rank order of effectiveness at all levels of the second factor. Fig 2.4a shows an example of this case. The cell entries again can be considered as the means of six subjects scores (i.e. six different subjects to each cell).

However with disordinal interaction whether one level of the first factor is more effective than another level of that same factor depends on which level of the second factor is being considered. Fig 2.4b illustrates such a case.

2.45 Transformations Additivity and Interaction

Box and Cox (1964) and Box (1967) have classified significant interactions as transformable versus

TABLE 1

	A1	A2	A3	
B1	1	4	9	4·6
B2	4	9	16	9·6
B3	9	16	25	16·6
	4·6	9·6	16·6	10·3

TABLE 2

	A1	A2	A3	
B1	1	2	3	2
B2	2	3	4	3
B3	3	4	5	4
	2	3	4	3

IF TABLE I IS TRANSFORMED BY TAKING THE SQUARE ROOT OF EACH OF THE CELL MEANS IT YIELDS TABLE 2 WHICH IS ADDITIVE. THE SQUARE ROOT TRANSFORMATION IS MONOTONIC. HERE A SIGNIFICANT ORDINAL INTERACTION HAS BEEN REMOVED BY A TRANSFORMATION.

FIG. 2.5. AN EXAMPLE OF HOW A SIGNIFICANT ORDINAL INTERACTION CAN BE ELIMINATED BY A SUITABLE MONOTONIC TRANSFORMATION.

non-transformable. They showed that some significant interactions can be removed by an appropriate transformation, however these authors do not explicitly distinguish between discrdinal and ordinal interation.

A significant ordinal interaction can sometimes be eliminated or reduced by a suitable monotonic transformation. Fig 2.5 gives an example of such a case. Table I shows a significant ordinal interaction where a monotonic transformation (taking the square root of the cell entries) yields Table 2, which is additive. If the double cancellation axioms are applied to Table 1 this does suggest that an additive representation is possible.

A significant disordinal interaction can never be removed by a monotonic transformation. The rank order of the cell means is invariant under a monotonic transformation and therefore the rank order of each of the 'B' effects would still differ. It is also obvious that double cancellation in this case would fail.

2.46 Analysis of Variance and Conjoint Measurement

A significant ordinal interaction term in Analysis of Variance may possibly be eliminated by a monotonic transformation of the data but there is no systematic procedure to determine how this is done, or what significant ordinal interactions are transformable. Nor can testing the double cancellation conditions stipulate whether a significant ordinal interaction can be removed because the conjoint measurement 'axioms' are not both necessary and sufficient for establishing a simple additive representation; they are only sufficient. This means that rejection of the axioms for a particular data matrix does not necessarily imply that an additive representation does not exist.

In analysis of variance an additive representation is related to the absence of significant ordinal interaction and when testing for addivity the question is: Can the given scale values or cell means be described as an additive combination of their row and column components. In the additive conjoint measurement model the question becomes: Can the given scale values be monotonically transformed so that additivity would be satisfied by the transformed cell entries.

2.47 Measurement and Model Testing

Measurement concerns itself with assigning numerical values to objects so that laws relating the measured variables describe the empirical relations among the objects. Model testing has to do with whether a particular set of assumptions can account for a set of observed relationships.

Measurement as described here dealt with the case in which the response depends on the combined effect of two or more variables.

A typical situation has been the case where the physical stimuli x, and y with psychological values $\Phi(x)$, $\Phi(y)$ are combined under a composition rule F. This rule represents the psychological law or model which describes how the subjective values combine to form an overall index, Δ, of a psychological construct; for example, response strength, Hull (1952), or subjective expected utility Tversky [1967], or similarity Torgerson (1958). The overt response $\bar{R}xy$ is assumed to be a monotonic function (labelled D), of this psychological index Δ.

A psychological model on this scheme, then, resolves around three issues:

(i) Scaling

(ii) A theory of Composition or Combination, F

(iii) A Response Model D

Scaling: this has to do with finding an appropriate stimulus representation. This is a measurement problem, for in practice it requires the determination of the subjective values $\Phi(x)$, $\Phi(y)$.

The Composition Theory: this involves determining the composition rule F where:

$$\Delta = F\left[\Phi(x),\ \Phi(y)\right]$$

The Response Model: this necessitates finding the response function D between Δ and the response $\bar{R}xy$

i.e. $\bar{R}xy = D(\Delta)$

2.48 Psychological Models and Ordinal Tests

Algebraic models permit the simultaneous evaluation of a proposed composition rule and the scaling of stimuli. By considering the contribution of two or more factors simultaneously, it is possible to measure each factor separately. The resultant scale values are based upon a theory of composition and have a meaning with respect to the theory. Conjoint measurement for instance describes the conditions that ideal data would have to satisfy to be ordinally consistent with the composition theory F. As already noted, significant disordinal interactions would be ordinally inconsistent with additive models, for no monotonic transformation could make the data fit an additive model. However, ordinal violations of additivity will show up as significant interactions in analysis of variance; the problem arises when significant interactions occur in the absence of ordinal violations.

When this occurs there seem to be two strategies:

(i) assume the composition model to be valid and in keeping with the ordinal requirements of the data.

(ii) attempt a more fine grained analysis of the data, e.g. Anderson (1970), Birnbaum and Veit (1974).

If the additive composition rule is assumed to be valid despite the significant interactions then analysis of variance in essence, tests the linearity of D, the response function. A non-linear response function will of course produce significant interactions even though the underlying composition rule is additive. In the absence of ordinal violations of the composition rule it is possible to find a monotonic transformation D^{-1} which rescales the data to additivity. This is essentially the strategy of Krantz et al [1971] who take the view that when discrepancies from additivity can be removed by a monotonic transformation these discrepancies should be attributed to non-linearity in the response function D, rather than to non-additivity of the composition rule.

This strategy of model testing is a cause for uneasiness: whenever a monotonic transformation brings otherwise contradictory data into line with a proposed model the status of that model must always remain unclear, because this procedure simply assumes the validity of the composition rule. The existence of a transformation that removes interactions away cannot mean that the model is validated, yet this is what Krantz et al [1971, page 445] seem to suggest. These authors reanalysed the data of Sidowski and Anderson [1967] and they concluded that since the interactions could be eliminated by a monotonic transformation of the data the original authors were incorrect in attributing psychological significance to their findings. However Birnbaum and Veit [1974] have shown that if an experimenter had followed this same rescaling procedure with the data of their experiment they would have erroneously concluded that the additive model was an appropriate description.

2.50 Independence in Conjoint Structures

In section 2.30 additive independence in the two component case was examined. Since experiment 1B investigates various independence conditions for the three factor case involving similarity judgements of auditory tones, this section develops the theme, but in a more informal manner than that provided by Krantz et al. Moreover some consideration will be given to the possible forms of the various composition rules discussed in 2.30. Composition rules, it will be recalled, are theories that describe the relationships among several dimensions or factors. Conjoint measurement analyses provides methods for deciding among such rules using ordinal information only. The form of the composition rule, F, - whether additive, multiplicative, distributive, or dual distributive - imply certain testable properties that are not implied by the others. By examining the qualitative orderings of the independent variable in a 3 factor conjoint experiment it is possible to diagnose which, if any, of these four composition rules is applicable.

2.51 Consider the 3 factor conjoint structure discussed in 2.25. Suppose that eight matched groups of four rats

FIG. 2.51.

ILLUSTRATION OF AN EXPERIMENTAL DESIGN TO TEST WHETHER THE ORDERING OF THE JOINT EFFECT OF DEPRIVATION (D) AND REWARD (K) IS INDEPENDENT OF EXPERIENCE. JOINT INDEPENDENCE HOLDS WHEN THE ORDERING OF dk RELATIVE TO d^1k^1 IS THE SAME AT h AS AT h^1.

DEPRIVATION. D ↑	H → h	h
d^1	d^1hk	d^1hk^1
d	dhk	dhk^1
	k	k^1
	K →	

D ↑	H → h^1	h^1
d^1	d^1h^1k	$d^1h^1k^1$
d	dh^1k	dh^1k^1
	k	k^1
	K →	

are tested in each of the 8 treatment combinations shown in Fig 2.51. Each group then is subjected to different combinations of the 2 levels of drive (D), habit strength (H) and incentive (K). Suppose also that median running speed of the four rats in each of the treatment combinations is taken as a measure of the response strength.

Hull (1952) proposed a multiplicative composition rule (Krantz and Tversky 1971) to describe the effect of D, H, and K on response strength; that is to say, the ordering of the response strength is decomposable into the effect of 3 factors: D, H, and K.

If the ordered triples:

$$(dhk) > (d'h'k')$$

corresponds to the observation that the response strength in treatment combination dhk is greater than that in treatment combination d'h'k' then the decomposition condition of the independence hypothesis (section 2.30) asserts:

2.511 $$(dhk) > (d'h'k')$$

if and only if

$$F\left[\Phi_D(d),\ \Phi_K(k),\ \Phi_H(h)\right] \qquad F\left[\Phi_D(d'),\ \Phi_K(k'),\ \Phi_H(h')\right]$$

In words: If the following qualitative observation is true:

$$(dhk) > (d'h'k')$$

then there exists numerical scales Φ_D, Φ_K, and Φ_H, measuring drive, incentive and habit strength respectively together with a composition rule, F, for combining them such that the ordering of the scale values is identical to the empirical ordering. Hull's suggestion was that the rule was multiplicative.

2.52 The Composition Rules in Conjoint Structures

Krantz and Tversky (1971) and Krantz et al (1971) contain discussions and details of four different composition rules or models. These are set out here and illustrated with reference to Hull's formulation of response strength:

(1) The Multiplicative Model

Here (dhk) $\gtrsim$ (d'h'k')

if and only if

$$\phi_D(d).\ \phi_H(h).\ \phi_K(k) \geq \phi_D(d').\ \phi_H(h').\ \phi_K(k')$$

(2) The Distributive Model

(dhk) $\gtrsim$ (d'h'k')

if and only if

$$\left[\phi_D(d) + \phi_K(k)\right]\ I_H(h) \gtrsim \left[\phi_D(d') + \phi_K(k')\right]\ .\ \phi_H(h')$$

This model has been considered by Spence (1956) and more recently by Evans (1967) using statistical methods. Both authors were unable to decide between the two composition rules. Krantz and Tversky (1971) showed, however, that if it is assumed that the scale values are positive, more orderings are compatible with a distributive model than with a multiplicative one, and the multiplicative model implied testable ordinal properties which are not implied by the distributive rule (and vice versa). As yet, no empirical work employing measurement theoretic notions has yet been reported which decides the controversy between Hull and Spence.

(3) Additive Independence Model

$$(dhk) \geq (d'h'k')$$

if and only if

$$\Phi_D(d) + \Phi_H(h) + \Phi_K(k) > \Phi_D(d') + \Phi_H(h') + \Phi_K(k')$$

This composition rule corresponds to additive conjoint measurement for the case $N \geq 3$ (N is the number of factors). Luce and Tukey (1964) and Krantz (1964) both dealt with the ordinal requirements for additive conjoint measurement for the case $N = 2$. (See section 2.33.)

(4) Dual Distributive Model

$$(dhk) \geq (d'h'k')$$

if and only if

$$\Phi_D(d) \cdot \Phi_K(k) + \Phi_H(h) > \Phi_D(d') \cdot \Phi_K(k') + \Phi_H(h')$$

2.53. Independence in Conjoint Structures

In this section, the notation used to discuss independence is almost identical to that used in the two experiments reported at the end of the Chapter

The upper case letters F, A, and L denote variables $\Phi_F(F)$, $\Phi_A(A)$, $\Phi_L(L)$ denote the scales. The lower case letters f, f', a, a', and l, l' denote values (nominal) or levels on the 3 factors F, A, and L respectively. Fig 2.52 shows the 2x2x2 factorial structure so described.

Single Factor Independence

F is independent of A and L whenever
(f a l) ≥ (f' a l)

if and only if

(f a' l') ≥ (f' a' l')

2.54 The single factor independence of A from F and L, and L from A and F is similarly defined.

Single factor independence holds when the ordering of the effects of one factor is the same no matter what fixed levels are chosen in all other factors. The type of independence is really a fairly weak condition: it means, simply, that each factor can be ordered in such a way as to contribute monotonically to the overall effect. In other words, scales can be constructed, such that the rule of combination of the factors is monotonic in each variable, but no other constraint on the composition rule is imposed.

Violations of Single Factor Independence

One class of violations of single factor independence suggests a multiplicative combination of factors with positive, zero, and negative multipliers. A reversal in the sign of a multiplier reverses totally the ordering of the products. A zero multiplier produces a degenerate ordering. When the only violations of independence involve reversals of orderings or degenerate orderings we speak of sign dependence. This generalises very substantially the notion of independence, and McClelland (1972) has reported experimental work in which he has very thoroughly exploited the notion of sign dependence to explain the results from studies in impression formulation.

2.55 Joint Factor Independence

Independence can be applied to pairs of factors as well as to single factors. F and A are jointly independent of L whenever: [see Fig 2.52]

(f a l) $\geq$ (f' a' l)

if and only if

(f a l') $\geq$ (f' a' l')

Joint independence of the pairs of factors is defined similarly.

To gain some intuitive feel for joint independence consider Hull's experiment again and Fig 2.51.

FIG. 2.52A

GEOMETRICAL INTERPRETATION OF SINGLE FACTOR AND JOINT FACTOR INDEPENDENCE OF THE DIMENSIONS F, A, AND L. THIS ILLUSTRATION WILL BE HELPFUL IN UNDERSTANDING EXPERIMENT 1 (a) AND EXPERIMENT 1 (b).

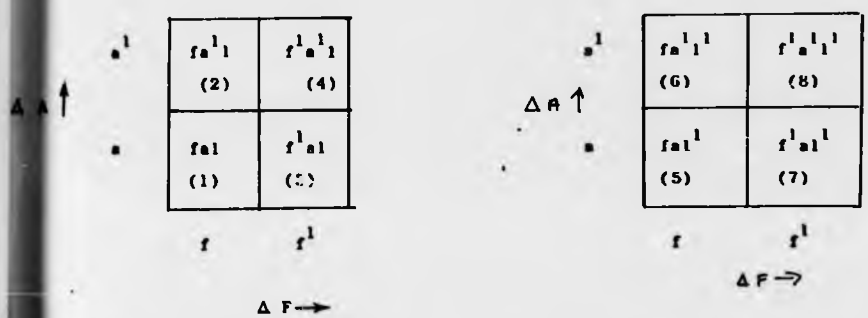


FIG.2.52B

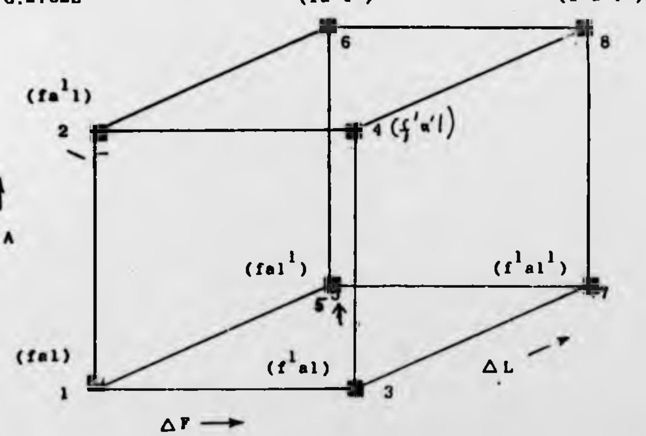


FIG. 2.52A AND FIG. 2.52B ARE ALTERNATIVE REPRESENTATIONS OF A 2 x 2 x 2 FACTORIAL STRUCTURE. IN FIG. 2.52A THE CELLS REPRESENT DIFFERENT TREATMENT COMBINATIONS WHEREAS IN FIG 2.52B IT IS THE VERTICES, THAT REPRESENT DIFFERENT TREATMENT COMBINATIONS.

FIG. 2.52A

GEOMETRICAL INTERPRETATION OF SINGLE FACTOR AND JOINT FACTOR INDEPENDENCE OF THE DIMENSIONS F, A, AND L. THIS ILLUSTRATION WILL BE HELPFUL IN UNDERSTANDING EXPERIMENT 1 (a) AND EXPERIMENT 1 (b).

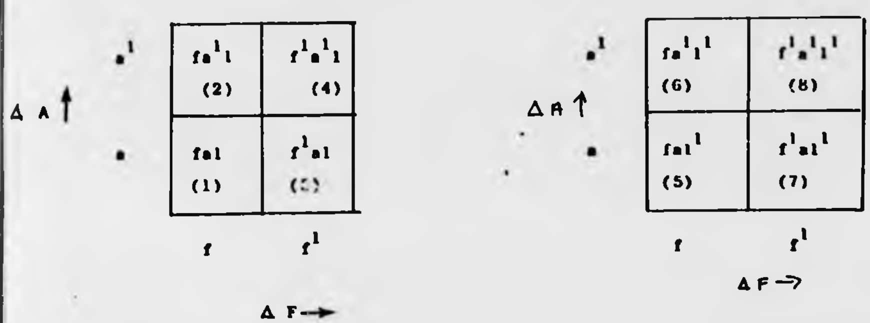


FIG.2.52B

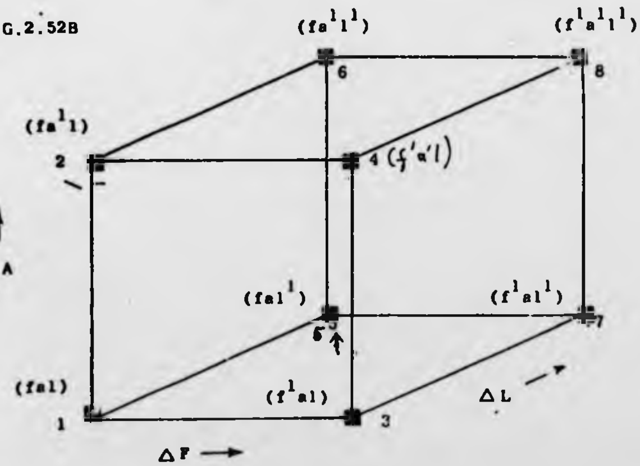


FIG. 2.52A AND FIG. 2.52B ARE ALTERNATIVE REPRESENTATIONS OF A 2 x 2 x 2 FACTORIAL STRUCTURE. IN FIG. 2.52A THE CELLS REPRESENT DIFFERENT TREATMENT COMBINATIONS WHEREAS IN FIG 2.52B IT IS THE VERTICES. THAT REPRESENT DIFFERENT TREATMENT COMBINATIONS.

Joint independence of D and K over H holds whenever

$$(d\ h\ k) \gtrsim (d'\ h\ k')$$

if and only if

$$(d\ h'\ k) \gtrsim (d'\ h'\ k')$$

In the first pair of observations the two treatment combinations have the same second component ('h': all rats have same amount of prior learning). If the effect of the d k combination is always greater than the effect of the d' k' combination whatever the fixed level of the H component ('h' in the second pair of observations) then D and K are jointly independent of H.

In contrast to single factor independence, joint factor independence is a very strong condition. When all possible joint factor independence laws hold for an empirical conjoint structure additive conjoint measurement is obtained. In the two factor case the only possible independence laws are the single factor ones.

2.56 Geometrical Interpretation of Single Factor and Joint Factor Independence.

Fig 2.52A and Fig 2.52B illustrate the identical 2x2x2 factorial structure. In Fig 2.52B the vertices represent treatment combinations, whereas in Fig 2.52A the cells do. Each vertex in Fig 2.53 is labelled by the appropriate lower case letter and numbered from 1 to 8.

Single Factor independence corresponds to the condition that end points of parallel edges are ordered in the same way for example

$$(8) \gtrsim (6)$$

if and only if

$$(3) \gtrsim (1)$$

Joint Factor independence correspond to the condition that end points of parallel diagonals are ordered in the same way.

e.g. $$(4) \gtrsim (1)$$

if and only if

$$(8) \gtrsim (5)$$

2.60 Information Processing Strategies and Conjoint Structures: The Anatomy of Similarity Judgments

Suppose two candidates have applied for a job and an interviewer has drawn up a list of attributes or dimensions required for the post: intelligence, humour, sociability, efficiency, and so on. How is the interviewer to choose between the two candidates?

One way would be to devise a series of role playing tasks that tap each of these attributes separately. The interviewer then would consider each applicant in turn and score this performance on each task. If this is done for both the candidates the one with the highest total score for all the tasks gets the job. Consider another interviewing technique: instead of seeing each candidate separately, suppose both are presented the same tasks as before but at the same time. For example the task designed to 'measure' sociability is presented to them both, followed by the task which taps efficiency, and so on. In this situation the interviewer probably employs a different evaluation strategy. Very likely he would evaluate the <u>differences</u> in scores on each of the attributes measured, and the person whom these differences favouredthe most number of times would get the job.

The first method of processing information is an <u>inter-dimensional</u> strategy: each object's dimensions are processed separately and each object is dealt with sequentially. The second processing strategy is a component-wise or <u>intradimensional</u> method: both objects are considered simultaneously and on each dimension a component-wise comparison is made.

Which is the best strategy to employ? There are a number of considerations:

(i) If the candidates are about equal in performance on all the relevant attributes but one, this will be immediately obvious using the intra-dimensional or component-wise strategy.

If the two alternatives are evaluated independently this dominance relation between the two will be obscured.

(ii) Intradimensional Evaluations are simpler than interdimensional ones because the compared quantities are expressed in terms of the same units. It is much simpler to evaluate the difference in intelligence between two people than to evaluate the combined effect of intelligence and sociability.

(iii) Suppose the two candidates are scored on N attributes or dimensions; if the applicants are judged independently then 2N inter-dimensional evaluations have to be made, whereas only N intradimensional comparisons have to be made if the second interviewing technique is adopted.

2.61 Similarity Judgment Tasks

The two accounts of information processing provide alternative models for similarity judgments. In such tasks the S is typically presented two stimuli and required to state how similar they are to one another; usually by assigning a number via a rating scale. Another method widely employed for making similarity judgments is the method of <u>paired comparisons</u>; here, one pair of stimuli is presented followed by another pair, and the S is asked which of the two pairs is more similar. The technique allows pairs of stimuli to be ordered with respect to their similarity to one other.

2.62 An informal sketch of a simple additive model for similarity judgments using the method of paired comparisons

Let $A = A_1 \times A_2 \times \ldots\ldots \times A_N$ be a set of multidimensional stimuli with typical elements of the form x, y whose co-ordinates in the N dimensional space are given by

$$x = (x_1 \ldots\ldots\ldots x_N),$$

$$y = (y_1 \ldots\ldots\ldots y_N),$$

where x_i, (i = 1, i .. N) is the level of the physical stimulus on dimension i.

Let $f_1 \ldots\ldots f_i \ldots\ldots f_N$ be psychophysical functions; then the subjective value of stimulus x, f(x), is

$$f(x) = (f_1(x_1), f_2(x_2) \ldots\ldots f_n(x_n)),$$

where $fi(x_i)$ is the subjective value of the stimulus on the ith dimension.

It is now assumed that in a paired comparison task the subjective values of x and y are computed separately and that these values on the dimensions are combined additively. According to the model of additive conjoint measurement we have

2.621 .... $$f(x) = \sum_{i=1}^{N} f_i(x_i),$$

2.623 .... $$f(y) = \sum_{i=1}^{N} f_i(y_i),$$

where $f(x)$ and $f(y)$ are defined to be the subjective values of the stimulus x and y respectively.

It is further assumed that after a pair of stimuli is presented the subject subtracts the larger <u>value</u> from the smaller <u>value</u> to obtain a dissimilarity index ($\delta(x,y)$). [The more similar x is to y the less will be the dissimilarity index.]

2.624 i.e. if

$$f(x) \geq f(y)$$

then $$[f(x) - f(y)] = \delta(x,y),$$

where $\delta(x,y)$ is the dissimilarity index of the stimuli x, and y and $f(x)$, $f(y)$ are their respective values.

For the second pair of stimuli p, q in the paired comparison task another dissimilarity index is computed $\delta(p,q)$.

The pair x,y is judged to be more dissimilar than the pair p,q if and only if:

$$\delta(x,y) \geq \delta(p,q)$$

So according to the simple additive model the stimulus pair x,y is more dissimilar than the stimulus pair p,q if and only if:

2.625 $$\sum_{i=1}^{N} f_i(x_i) - \sum_{i=1}^{N} f_i(y_i) \geq \sum_{i=1}^{N} f_i(p_i) - \sum_{i=1}^{N} f_i(q_i)$$

This simple additive model for similarity judgments captures the assumed psychological processes underlying the paired comparison method without the added complication of considering the 'distance' and metric assumptions of multidimensional scaling (MDS).

2.63 An informal sketch of an Additive Difference Model for similarity judgments in a paired comparison task.

An alternative strategy for assessing the similarity of pairs of stimuli in a paired comparison task is based on comparisons of component-wise differences on the respective dimension of the two stimuli.

Consider a quantity Δ_i defined by:

2.631 $$\Delta_i = f_i(x_i) - f_i(y_i) \text{ where } f_i(x_i) > f_i(y_i)$$

$f_i(x_i)$ is the subjective value of stimulus x on the ith dimension $[i = 1 \ldots N]$

i.e. Δi, corresponds to the subjective differences in value of the two stimuli x and y, on dimension i $\Delta i \geq 0$, and f_i is a psychophysical function as before.

Consider now a similarity function Φ_i such that

2.632 $$\Phi_i \left[f_i(x_i) - f_i(y_i) \right] \geq 0, (\Phi_i > 0).$$

that is, Φ_i determines the contributions of the subjective differences on the ith dimension to the over-all evaluation of similarity between the stimulus pairs in a paired comparison task.

If the quantities of the form 2.632 are summed over all the N dimensions and the index of dissimilarity $\delta(x,y)$ is obtained for each stimulus pair

$$\text{i.e.} \quad \delta(x,y) = \sum_{i=1}^{N} \Phi_i \left[f_i(xi) - f_i(y_i) \right]$$

Definition: A similarity structure satisfies the additive difference model if there exists real valued functions $f_1 \ldots\ldots f_i \ldots\ldots f_N$ such that

$$\delta(x,y) \geq \delta(p,q)$$

if and only if

$$\sum_{i=1}^{N} \Phi i \left[f_i(x_i) - f_i(y_i) \right] \geq \sum_{i=1}^{N} \Phi_i \left[f_i(p_i) - f_i(q_i) \right]$$

2.70 EXPERIMENT 1A: A test of the additive difference model

The first experiment is a test of the additive difference model of similarity judgments for a restricted range of stimuli. No attempt is made to examine the ordinal assumptions behind multidimensional scaling because the issue of the metric representation of similarity data is independent of the question of its dimensional representation. Indeed Tversky & Krantz (1970) and Beals, Krantz, and Tversky (1968) discuss both these aspects independently, and they moreover show that the metric requirements of MDS impose rather severe restrictions on the additive difference model. This experiment then is concerned to examine only the dimensional assumptions of similarity judgment tasks and in particular the additive difference model discussed in Section 2.60.

The heart of the general additive difference model for similarity judgments lies in three properties:

1. Decomposability
2. Intradimensional Subtractivity
3. Interdimensional Additivity

Decomposability requires that there is no <u>interaction</u> between the subjective dimensions; that is, each dimension contributes independently to the overall impression of dissimilarity. Intradimensional subtractivity specifies that on each subjective dimension the absolute value of the difference between corresponding 'co-ordinates'

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1. Decomposability
2. Intradimensional Subtractivity
3. Interdimensional Additivity

Decomposability requires that there is no <u>interaction</u> between the subjective dimensions; that is, each dimension contributes independently to the overall impression of dissimilarity. Intradimensional subtractivity specifies that on each subjective dimension the absolute value of the difference between corresponding 'co-ordinates' (or levels) is computed. These contributions are then combined additively after a monotonic transformation Φ_i is applied to each of these differences; by so doing, inter-dimensional additivity is satisfied. Each of these requirements was discussed in detail in Section 2.60.

Before the discussion assumes a more formal tone, a few remarks will be in order to highlight some of the intuitions which generated the experiment.

Suppose x and y are two stimuli which vary on just two dimensions, and suppose just two values on each of these discussions are considered:

$$\text{i.e.} \quad x = (x_1, x_2)$$
$$y = (y_1, y_2)$$

the corresponding 'psychological' values on these dimensions are

$$f(x) = (f_1\ (x_1),\ f_2\ (x_2))$$
$$f(y) = (f_1\ (y_1),\ f_2\ (y_2))\ ,$$

where f_1 and f_2 are psychophysical functions, f(x), f(y) are the corresponding psychological values of x and y.

According to the additivity - difference model the dissimilarity between x and y, $\delta(x,y)$ becomes:

$$\delta(x,y) = \Phi_1\ \left[f_1(f_1(y_1) - f_1(x_1)\right] + \Phi_2\ \left[f_2(y_2) - f_2(x_2)\right]$$

DESIGN FOR EXPERIMENT 1

Condition 1

Differences in Duration ΔL (milli. secs.) by Differences in Loudness ΔA (Decibels)

ΔL \ ΔA	0	10	20	30
750	A_0 A_0 L_0 L_2 1,3	A_0 A_1 L_0 L_2 1,4	A_0 A_2 L_0 L_2 1,9	A_0 A_3 L_0 L_2 1,10
500	A_0 A_0 L_1 L_2 2,3	A_0 A_1 L_1 L_2 2,4	A_0 A_2 L_1 L_2 2,9	A_0 A_3 L_1 L_2 2,10
250	A_0 A_0 L_0 L_1 1,2	A_0 A_1 L_0 L_1 1,5	A_0 A_2 L_0 L_1 1,8	A_0 A_3 L_0 L_1 1,11
0	A_0 A_0 L_0 L_0 1,1	A_0 A_1 L_0 L_0 1,6	A_0 A_2 L_0 L_0 1,7	A_0 A_3 L_0 L_0 1,12

Condition 2

ΔL (milli secs) by Differences in Loudness ΔA (Decibels)

ΔL \ ΔA	20	30	40	50
750	A_2 A_4 L_0 L_2 7,15	A_1 A_4 L_0 L_2 6,15	A_1 A_5 L_0 L_2 6,16	A_0 A_5 L_0 L_2 1,16
500	A_2 A_4 L_1 L_2 8,15	A_2 A_4 L_1 L_2 8,15	A_1 A_5 L_1 L_2 5,16	A_0 A_5 L_1 L_2 2,16
250	A_2 A_4 L_0 L_1 7,14	A_1 A_4 L_0 L_1 6,14	A_1 A_5 L_0 L_1 6,17	A_0 A_5 L_0 L_1 1,17
0	A_2 A_4 L_0 L_0 7,13	A_1 A_4 L_0 L_0 6,13	A_1 A_5 L_0 L_1 6,18	A_0 A_5 L_0 L_0 1,18

Fig 2.91 In each cell the pair of tones have a difference in duration and a difference in loudness. Example: the pairs A_0L_1 and A_2L_2 (tones 2 and 9 respectively, see Fig. 2.92) contribute a difference of 20 dbs. in loudness and 500 msecs. in duration. In each row cell each pair of tones contributes the same duration difference; likewise each column cell contains a pair of tones which contributes the same loudness difference. The pair of numbers identifies the tones shown in Fig. 2.92.

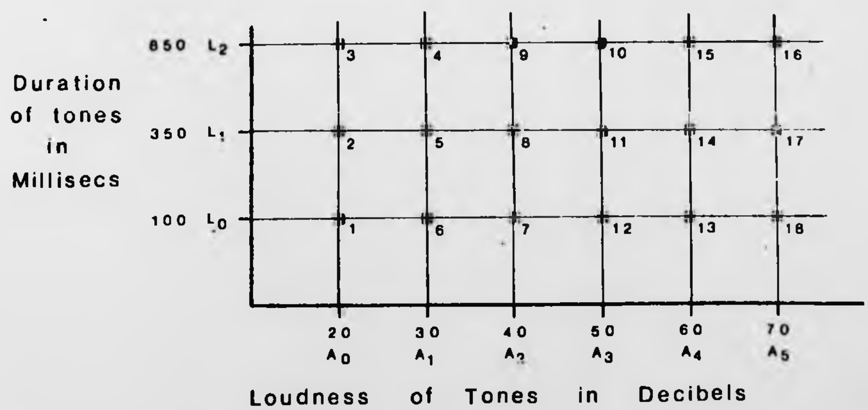


Fig 2.92 Each node with a number beside it represents a tone of a particular loudness and duration - all tones are of a frequency of 1200 Hz. For example, tone 7 (A_2L_0) is 40 decibels in loudness and 100 msecs. in duration. It is from this population of 18 tones that the stimulus pairs shown in Fig. 2.91 are drawn.

The quantities

$$\text{(i)} \quad (f_1(y_1) - f_1(x_1))$$

$$\text{(ii)} \quad (f_2(y_2) - f_2(x_2))$$

constitute the intra-dimensional subtractivity elements because each component-wise contribution to the dissimilarity score is the absolute value of the subjective scale difference.

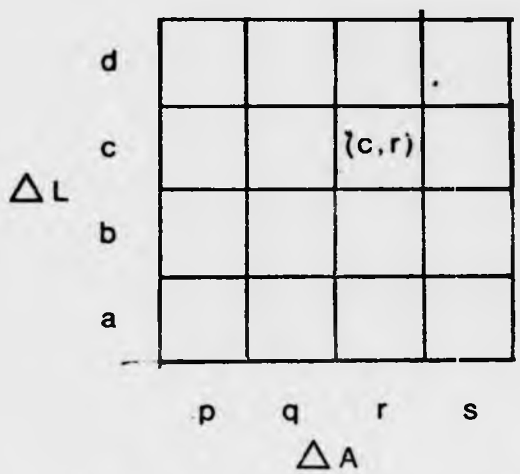
In the first experiment this intra-dimensional element was incorporated in the design and this can be verified from 2.91 which summarises the two conditions of the experiment. Both of these constitute a 4x4 factorial design: each cell contains a pair of tones which vary on two parameters: duration (in milliseconds) and loudness (decibels). The frequency remained constant throughout the experiment at 1200 Hz. Fig 2.92 shows values of the 18 different tones from which the pairs of stimuli were formed.

The two factor experimental arrangement is in fact a 4x4 factorial design in disguise. The two factors are the 'differences' on one of the two dimensions. The first factor, labelled 'duration differences', is ordered with respect to differences in duration; from a zero difference to a 750 m sec difference between pairs of tones in the cells. Each of the 4 rows has been so constructed that the duration differences between pairs of stimuli for every cell in that row is constant.

For example, all the pairs of stimuli in the cells of row 3 have a difference of 500 m secs duration between them. Likewise the second factor, labelled 'loudness differences', is ordered with respect to differences in loudness between pairs of tones, the size of difference again increasing monotonically. This means that all the cells of a particular column contain pairs of stimuli for which the differences in loudness between the pairs is constant: the cells of column 3, for instance, of condition 1 contain pairs of tones for which the loudness difference is 20db.

To summarise: each cell contains a pair of tones for which the subject is required to make a similarity judgment, the aim being to rank order all the pairs of stimuli presented to the subject in order of their dissimilarity to one another. Secondly, it is a two-factor experiment with four levels or values on each; each of these levels corresponds to the 'physical' differences between the stimuli of the pairs on just one of the dimensions.

Consider now a paired comparison task in which the pairs of tones in each cell of Fig 2.91 are presented to a subject. If these stimulus pairs are ordered with respect to the magnitude of their dissimilarities then a weak ordering relation can be obtained for the 4x4



△ A = (p,q,r,s,) REPRESENT DIFFERENCES IN LOUDNESS BETWEEN PAIRS OF TONE △ L = (a,b,c,d,) REPRESENTS DIFFERENCES IN DURATION.

FIG. 2.93. A 4 x 4 FACTORAL ARRANGEMENT WHERE $p<q<r<s$ THE MAGNITUDE OF THE LOUDNESS DIFFERENCE IS ORDERED. ALSO $a<b<c<d$ THE DURATION DIFFERENCES ARE MONOTONICALLY ORDERED.

factorial design to yield an empirical conjoint structure. If the resulting data matrix can be shown to possess an additive representation it will strongly support an additive difference model for similarity judgments in a paired comparison task.

The intuitive content of the previous section will now be formalised:

Consider a finite set of loudness differences, ΔA,

2.701 $\Delta A = \{p,q,r,s\}$

and a finite set of duration differences, ΔL, where

2.702 $\Delta L = \{a,b,c,d\}$

Let the cross product $\Delta L \times \Delta A$ denote the set of all pairs of differences in loudness and duration generated by the 'differences' in 2.801 and 2.802. Fig 2.93 shows a 4x4 factorial arrangement where

$\Delta A = \{p,q,r,s\}$ set of 4 loudness differences (dbs), and
$\Delta L = \{a,b,c,d\}$ set of 4 duration differences (m secs).

Let δ (c,r) be a measure of the dissimilarity between the stimuli of a pair of tones which have a physical difference of r decibals in loudness and a physical difference of c m secs in duration,

where $r \in \Delta A$
and $c \in \Delta L$

The additive-difference model of similarity judgments asserts that:

(i) $\delta(c,r) = \Phi_1(c) + \Phi_2(r)$ and Φ_1, Φ_2 are similarity functions

(ii) $\delta(c,r) > \delta(b,q)$

if and only if

$\Phi_1(c) + \Phi_2(r) > \Phi_1(b) + \Phi_2(q)$

for all c, b ε ΔA

and p, q, ε ΔL

2.71 Procedure

The double cancellation condition was used to test for additivity. The experiment had two conditions in which the ~~stimulus~~ stimuli shown in Fig 2.91 were presented to the subject. The principal aim of the study was to order these stimulus pairs with respect to their dissimilarity by giving the pair which was the most dissimilar (least similar) the highest rank and the least dissimilar pair the lowest rank. Such an ordering was obtained by the method of paired comparisons from which a data matrix was generated for each of the two conditions for each S.

2.72 Stimuli and Conditions

Each S was tested in two conditions (Fig 2.91). The tones used are shown as Fig 2.92. Altogether there were 8 loudness difference levels and 4 duration difference levels.

	CONDITION 1				CONDITION 2				TOTAL Nº OF TRIALS
	1	2	3	4	1	2	3	4	
S1	✓	✓	✓	✓	✓	✓	✓	✓	8
S2	✓	✓	✓	✓	✓	✓	✓	✓	8
S3	✓	✓	✓	✓	✓				5
								TOTAL	21

FIG. 2.94. EXPERIMENTAL PROCEDURE FOR THE THREE SUBJECTS S1, S2, AND S3. EACH TRIAL TOOK $3\frac{1}{2}$ HOURS.

The loudness difference levels were:

0, 10, 20, 30, db - Condition 1

20, 30, 40, 50, db - Condition 2

For each of the two conditions the duration differences levels were:

0, 250, 500 and 750 m secs.

The frequency of the tones was constant throughout the experiment at 1200 Hz.

For each condition the number of paired comparisons for 16 stimulus pairs is 256; if self comparisons are omitted this reduces to 240. To control for order effects, however, each experimental condition demanded 480 paired comparisons.

Each experimental session took 3½ hours with two breaks of 20 minutes. Each subject replicated each of the 2 conditions four times except for the third subject who completed four trials for condition 1 but only one for the second condition. Fig 2.94 summarises this.

Three Ss took part in the experiment, all of whom were highly practiced - each having spent 7 one-hour sessions in a similar auditory task reported in Chapter 4.

Equipment and Method

The 480 stimulus pairs in each of the two conditions were recorded on the two tracks of a tape recorder. The

	(1,10)	(1,9)	(1,4)	(2,10)	(2,9)	(1,3)	(2,4)	(2,3)	(1,11)	(1,8)	(1,12)	(1,5)	(1,7)	(1,2)	(1,6)	(1,1)	Dissimilarity Score	Rank
(1,10)	-	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	30	16
(1,9)	0	-	2	2	2	2	2	2	2	2	2	2	2	2	2	2	28	15
(1,4)	0	1	-	0	2	2	2	2	2	2	2	2	2	2	2	2	25	14
(2,10)	0	1	1	-	2	1	2	2	2	2	2	2	1	2	2	2	24	12.5
(2,9)	0	1	1	1	-	2	2	2	2	1	2	2	2	2	2	2	24	12.5
(1,3)	0	0	0	0	1	-	1	2	2	1	2	2	2	2	2	2	19	11
(2,4)	0	0	0	0	0	1	-	1	2	2	2	1	2	2	2	2	17	9.5
(2,3)	0	0	0	1	0	1	1	-	2	1	2	1	2	2	2	2	17	9.5
(1,11)	0	0	0	0	0	0	0	0	-	2	2	2	2	2	2	2	14	8
(1,8)	0	0	0	0	0	0	1	0	0	-	1	1	1	2	2	2	10	7
(1,12)	0	0	0	0	0	0	0	0	0	1	-	2	1	1	2	2	9	6
(1,5)	0	0	0	0	0	0	0	0	0	0	0	-	2	2	2	2	8	5
(1,7)	0	0	0	0	0	0	0	0	0	0	0	0	-	1	2	1	4	3.5
(1,2)	0	0	0	0	0	0	0	0	0	0	0	0	0	-	2	2	4	3.5
(1,6)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-	2	2	2
(1,1)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-	1	1

Fig 2·95 PAIRED COMPARISON DATA FOR S2 [Condition 1 (4th replication)] If the row pair is more dissimilar than the correspondi g column pair a 1 is entered, otherwise a zero. The row pair was presented first. For fuller details see text.

stimuli were delivered through earphones. Each experimental trial consisted of the following sequence of events:

(i) The first stimulus pair was presented to the S

(ii) A 2½ second delay

(iii) The second stimulus pair was presented

(iv) 10 second silence

After both the stimulus pairs had been presented, Ss responded by noting down on a prepared response sheet whether the first pair of tones or the second was the more similar.

Analysis of Results

For each of the conditions and replications the data for each subject was transformed into a dissimilarity ordering of the 16 pairs of tones. To aid understanding consider the second subject (S2), condition 1, (4th replication).

Fig 2.95 shows the result of the paired comparisons of the 16 pair of tones. If the pair of stimuli in the column was judged to be less dissimilar than the corresponding row pair a 1 was entered, otherwise a zero. Since each pair was presented twice to all other pairs (to control for order effects) the highest number that could appear in a row or a column was a 2. If the row scores are added the total for each row can be considered stimuli were delivered through earphones. Each experimental trial consisted of the following sequence of events:

(i) The first stimulus pair was presented to the S
(ii) A 2½ second delay
(iii) The second stimulus pair was presented
(iv) 10 second silence

After both the stimulus pairs had been presented, Ss responded by noting down on a prepared response sheet whether the first pair of tones or the second was the more similar.

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For each of the conditions and replications the data for each subject was transformed into a dissimilarity ordering of the 16 pairs of tones. To aid understanding consider the second subject (S2), condition 1, (4th replication).

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DISSIMILARITY DATA MATRIX FOR SUBJECTS

(Replication 4) Cell entries give dissimilarity scores

Differences in Duration ΔL (milli. secs)

750	19	25	28	30
500	17	17	24	24
250	4	8	10	14
0	1	2	4	9
	0	10	20	30

Decibels

Fig. 2.96 Differences in Loudness (ΔA)

FIG. 2.97 TRANSFORMED DISSIMILARITY DATA MATRIX

CELL ENTRIES GIVE THE RANK ORDERING OF THE DISSIMILARITY SCORES. IT IS ON THIS "DATA" MATRIX THAT DOUBLE CANCELLATION TESTS ARE CARRIED OUT.

Differnces in milli. secs.

750	11	14	15	16
500	9.5	9.5	12.5	12.5
250	3.5	5	7	8
0	1	2	3.5	6
	0	10	20	30

Differences in Decibels

as a dissimilarity score for each row pair. Fig 2.96 gives a Dissimilarity Data Matrix for this subject. Fig 2.97 shows the end result of ranking the dissimilarity scores of Fig 2.96 where the ties are treated in the usual way.

Tests for Additivity & Results

The double cancellation test was carried out on each of the transformed Dissimilarity Matrices (the data matrix with the ranked dissimilarity scores). Because the data matrices for both conditions were all 4x4 matrices and since the test for double cancellation involves 3 levels each of the L and F factors the total number of possible tests for double cancellation that have to be made are

$$\binom{4}{3} \times \binom{4}{3} = 16 \text{ possible tests for each data matrix}$$

Each test can have one of 3 outcomes (Krantz & Tversky 1971):

(a) Acceptance : the antecedents and conclusion hold

(b) Rejection : the antecedents hold but the conclusion does not

(c) No test : the antecedents do not hold

Ties sometimes occur in the ordering of pairs of stimuli with respect to similarity, and this can lead to the situation where there is an equality in the antecedents with the conclusions still holding. Levelt et al (1971) called this case a weak rejection.

Table 2.70 Results of Experiment 1A

SUBJECT 1

	Condition 1					Condition 2				
Replication:	1	2	3	4	%	1	2	3	4	%
Acceptance	13	16	16	4	85.93	16	16	16	12	93.75
Rejection:										
Strong	3	0	0	0	4.68	0	0	0	0	0.0
Weak	0	0	0	0	0.0	0	0	0	0	0.0
No test	0	0	0	0	0.0	0	0	0	0	0.0

SUBJECT 2

Replication	1	2	3	4	%	1	2	3	4	%
Acceptance	6	13	14	16	76.56	16	12	16	16	93.7
Rejection:										
Strong	0	0	0	0	0.0	0	0	0	0	0.0
Weak	1	0	0	0	1.56	0	0	0	0	0.0
No test	9	3	2	0	21.87	0	[illegible]	0	0	6.3

SUBJECT 3

Replication	1	2	3	4	%	1	2	3	4	%
Acceptance	14	16	16	16	96.91	10	-	-	-	62.5
Rejection:										
Strong	0	0	0	0	0.0	0	-	-	-	0.0
Weak	0	0	0	0	0.0	2	-	-	-	12.50
No test	2	0	0	0	3.13	4	-	-	-	25.00

Results

Table 2.70 shows the results of the double cancellation tests on the 21 data matrices. The justification for the demand that the hypothesis be accepted, i.e. that the matrices have an additive representation is a fairly powerful one for all Ss, except for S3 in condition 2 where there was only one replication. Hence the additive difference model for similarity judgments (using the method of paired comparison) is upheld, for these three subjects and for this particular population of auditory stimuli.

Appendix 2A gives the dissimilarity data matrices for each of the conditions and each of the replications. Examination of these matrices shows them to be in general monotonic data matrices: the ordering of the dissimilarity between pairs of stimuli depends on the particular levels of duration and frequency difference.

A more thorough analysis of the additive difference model for similarity data will be made in Chapter 4 but this is the first experiment, as far as the author is aware, in which the psychological processes underlying similarity judgments is spelled out and tested without the added complications of a metric representation.

EXPERIMENT 1B: Single Factor and Joint Factor Independence in a Similarity Judgment Task of Auditory Tones using the Method of Paired Comparisons

This study examines single factor and joint factor independence in a similarity judgment task involving paired comparisons of auditory stimuli.

In this experiment, as in the last, the strategy is to order the differences between tones of a pair, monotonically along the different factors. Only this time a three factor design was employed, the factors being ΔF, ΔA, and ΔL respectively, where:

ΔF corresponds to differences in frequency between the tones of a pair. This factor has two levels which represent differences of zero Hz and 250 Hz between tones. These will be denoted for convenience by the lower case letters f and f' respectively.

ΔA, corresponds to the factor in which the two levels represent the difference between tones of a pair in loudness of zero db and 20 db respectively. These two levels of difference along the ΔA factor are denoted by the lower case letters a and a' respectively.

ΔL corresponds to the factor in which the 3 levels represent the differences in duration between the tones of a pair of zero m secs,

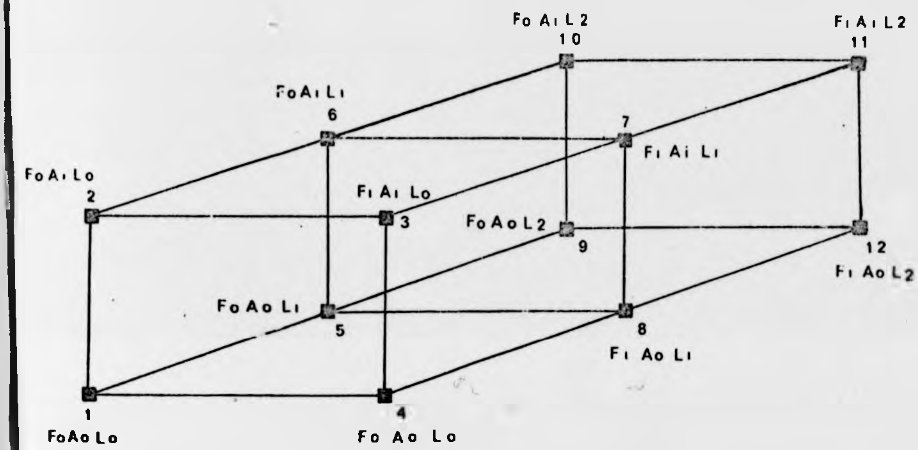


FIG 2.10 THE 12 STIMULI USED TO GENERATE THE DIFFERENCES BETWEEN THE STIMULI OF A PAIR IN FREQUENCY DURATION AND LOUDNESS.

Fo = 1000Hz F1 = 1250 Hz

Ao = 20 db A1 = 40 db

Lo = 100 msecs. L1 = 350 msecs L2 = 850 msecs

SUPPOSE THE TWO STIMULI (1 , 4) ie. Fo Ao Lo AND F1 Ao Lo ARE PRESENTED TO THE SUBJECT AND HE IS ASKED TO MAKE A SIMILARITY JUDGEMENT. THE PHYSICAL DIFFERENCE IN LOUDNESS AND DURATION ARE BOTH ZERO AND THE DIFFERENCE IN FREQUENCY IS 250 Hz. ACCORDING TO THE SIMPLE ADDITIVE DIFFERENCE MODEL OF SIMILARITY JUDGEMENTS THE PAIR OF STIMULI CAN BE REPRESENTED AS LEVELS OF A FACTORIAL DESIGN NAMELY $f^{1}al$. THE LOWER CASE LETTER f^{1} SIGNIFIES THAT THE PHYSICAL DIFFERENCE BETWEEN THE TONES IS 250 Hz WHILE a and l BOTH DENOTE THAT THE PHYSICAL DIFFERENCE IN LOUDNESS AND DURATION ARE BOTH ZERO.

250 m secs, and 500 m secs respectively. These three levels of duration difference along the ΔL factor are denoted by the lower case letters, l, l' and l" respectively.

In summary, ΔF, ΔA, and ΔL represent the factors of a 3 factor experiment. The levels correspond to the differences in frequency, loudness, and duration between the tones of a pair, thus generating a 2x2x3 factorial design.

The Stimuli generating these differences in Frequency, Loudness and Duration.

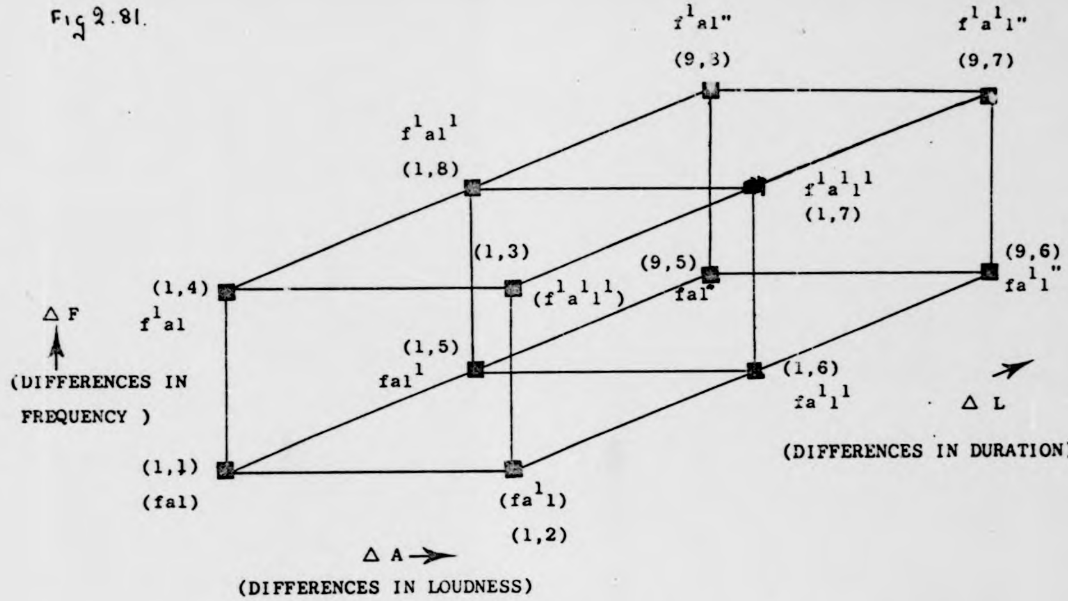
The 12 auditory stimuli used to generate the differences in frequency (0 Hz and 250 Hz); loudness (0 db and 20 db) and duration (0 m secs, 250 m secs and 500 m secs) between the tones of a pair varied on 3 dimensions: frequency, loudness and duration. The 12 tones are shown in Fig. 2.80 together with a number to identify them. The following notation is used:

(i) F_o, F_1 refer to the two values on the frequency dimension - 1,000 Hz and 1250 Hz respectively

(ii) A_o, A_1 refer to the two values on the loudness dimension - 20 db and 40 db respectively

(iii) L_o, L_1, L_2 refer to the 3 values on the duration dimension -

Fig 2.81.



THE 2 x 2 x 3 FACTORIAL STRUCTURE GENERATED BY DIFFERENCES IN LOUDNESS (ΔA) DURATION (ΔL) AND FREQUENCY (ΔF) BETWEEN TONES OF A PAIR. EACH OF THE VERTICES INDICATES BY A PAIR OF NUMBERS ONE OF THE STIMULUS PAIRS PRESENTED TO A SUBJECT. THE LOWER CASE LETTERS BESIDE THE PAIR OF NUMBERS INDICATES THE LEVEL OF DIFFERENCES BETWEEN THE STIMULI. THERE ARE TWO LEVELS OF DIFFERENCE IN FREQUENCY DENOTED BY f AND f^1. TWO LEVELS OF DIFFERENCE IN LOUDNESS a AND a^1 AND THREE LEVELS OF DIFFERENCE IN DURATION DENOTED BY l l^1 AND l'' RESPECTIVELY.

zero m secs ($\Delta L \rightarrow$)

$\Delta F \uparrow$	l	l
f^1 (250 Hz)	f^1al (1,4)	f^1a^1l (1,3)
f (0.0 Hz)	fal (1,1)	fa^1l (1,2)
	a 0 db	a^1 20 db

$\Delta A \rightarrow$

250 m secs (ΔL)

$\Delta F \uparrow$	l^1	l^1
f^1 (250 Hz)	f^1al^1 (1,8)	$f^1a^1l^1$ (1,7)
f (0.0 Hz)	fal^1 (1,5)	fa^1l^1 (1,6)
	a 0 db	a^1 20 db

$\Delta A \rightarrow$

500 m secs (ΔL)

$\Delta F \uparrow$	l''	l''
f^1 (250 Hz)	f^1al'' (9,8)	f^1a^1l'' (9,7)
f (0.0 Hz)	fal'' (9,5)	fa^1l'' (9,6)
	a 0 db	a^1 20 db

$\Delta A \rightarrow$

FIG. 2.82 ANOTHER REPRESENTATION OF THE 2 x 2 x 3 FACTORIAL STRUCTURE SHOWN IN FIG. 2.81 THE 12 TONES PRESENTED TO THE SUBJECT IN A PAIRED COMPARISION TASK ARE SHOWN ABOVE(INDICATED BY THE PAIRS OF NUMBERS THE IDENTITY OF WHICH CAN BE OBTAINED FROM FIG.2.80) EACH OF THE PAIRS OF TONES BELONG TO ONE OF THE CELLS OF A FACTORIAL DESIGN WHERE THE LEVELS OF THE FACTORS CORRESPOND TO DIFFERENCES BETWEEN THE TONES ON ONE OR MORE OF THE FACTORS. Δ F, Δ A, AND Δ L. FOR EXAMPLE THE TONES (1,3) HAVE A DIFFERENCE OF 250 Hz. IN FREQUENCY, 20db. IN LOUDNESS, AND ZERO DIFFERENCE IN DURATION. THIS TONE PAIR IS DENOTED BY THE LOWER CASE LETTERS f^1a^1l EACH OF THE LETTERS CORRESPONDS TO A PARTICULAR DIFFERENCE ON THE Δ F, Δ A, AND Δ L, FACTORS RESPECTIVELY.

Each tone is of the form $F_i\ A_j\ L_k$

where $i = 0, 1$

$j = 0, 1$

$k = 0, 1, 2.$

The 12 tones formed from these stimulus parameters are shown in Fig 2.80. The numbers serve to identify each tone. Thus for example, 7 refers to the tone $F_1\ A_1\ L_1$.

It is these 12 tones which are used to generate the difference along each of the 3 dimensions between the stimuli of a pair. This will be most easily understood by consulting Fig 2.81 which depicts a 2x2x3 factorial design whose 3 factors are ΔF, ΔA, and ΔL respectively. Each of the vertices indicates, by a pair of numbers, the stimulus pair presented to the subject in the experiment. Consider the front panel of the factorial design in Fig 2.82 with the stimulus pairs (1,1), (1,2), (1,3), and (1,4).

The notation (1,3), for instance, indicates that the pair of tones presented was $F_0\ A_0\ L_0$, and $F_1\ A_1\ L_0$; for which S was required to make a similarity judgment. The differences in frequency, loudness, and duration between these tones is 250 Hz, 20 db and zero m secs respectively. This pair is denoted by (f' a' 1): the f' indicating a 250 Hz difference in frequency between the tones, the a' indicating a 20 db difference in loudness, and the 1 a zero difference in duration between the stimuli. It is easy to see that the triple (f' a' l) defines a treatment combination in a factorial experiment. Consider again Fig 2.81, and the vertex which is labelled both by the number pair (1, 7) and the triple f' a' l'. This again corresponds to a treatment combination, where the differences in frequency, loudness and duration between the stimuli denoted by (1,7) are 250 Hz, 20 db, and 250 m secs respectively.

Thus Fig 2.81 which summarises the experimental design in reality illustrates a factorial structure whose vertices represent treatment combinations that are labelled by letters which denote the levels of the difference along the dimension between the two stimuli presented to the subject. These stimuli, of course, are denoted by the pair of numbers at each vertex.

2.81 TESTING FOR SINGLE FACTOR INDEPENDENCE

What does single factor independence mean in this case? Consider single factor independence of ΔF, (differences in frequency between tones of a pair) over ΔA, and ΔL.

Suppose it is true that:

$$f' \; a' \; l'' \geq f \; a' \; l''$$

which means that the dissimilarity generated by a pair of tones on the left hand side of the inequality is greater than that generated by the pair of tones on the right hand side i.e. the effect of a 250 Hz difference between two tones exceeds that of a zero difference in frequency between two other tones when the difference in the other components (loudness and duration) between both pairs is the same. If this is generally true, that is, the effect of a 250 Hz difference in frequency always exceeds that of a zero difference in frequency whenever the differences on the other components is fixed then there exists single factor independence of ΔF, over ΔA and ΔL.

For single factor independence to hold in this case,

then $f' \, a \, l \geq f \, a \, l$

if and only if

$f' \, a' \, l' \geq f \, a' \, l'$

if and only if

$f' \, a \, l'' \geq f \, a \, l''$

and so on.

(a) Single Factor Independence of ΔA over ΔF and ΔL. (From Fig 2.81 or 2.82)

If	$(1,2) \rightarrow (1,1)$
then	$(1,3) \rightarrow (1,4)$
and	$(1,6) \rightarrow (1,5)$
and	$(1,7) \rightarrow (1,8)$
and	$(9,6) \rightarrow (9,5)$
and	$(9,7) \rightarrow (9,8)$

where (1,2) → (1,1) means for example that if the dissimilarity between the pair (1,2) is greater than between the pair (1,1) or vice versa then the dissimilarity between the pair (1,3) will be greater than that between (1,4) or vice versa and so on. The ordering is consistent between all pairs.

(b) Single Factor Independence of ΔF over ΔA and ΔL:

If (1,4) → (1,1)
(1,8) → (1,5)
(9,8) → (9,5)
(9,7) → (9,6)
(1,7) → (1,6)
(1,3) → (1,2)

2.82 TESTING FOR JOINT FACTOR INDEPENDENCE

What does joint factor independence mean in this context? Consider the factors ΔF and ΔA; then these are jointly independent of ΔL whenever:

$(f\ a\ l) \geq (f'\ a'\ l)$

implies that

$(f\ a\ l') \geq (f'\ a'\ l')$ (see Fig 2.81 or 2.82)

The same order of the joint effects of ΔF and ΔA must hold for any fixed third component. In this context it means that if the dissimilarity produced by some combination of differences on two factors (ΔF, ΔA, in this example) exceeds that produced by some other combination of differences on these same two dimensions - the difference on the third dimension (here ΔL) remaining constant - then the same ordering holds as the constant difference on the third dimension varies.

(a) Joint Factor Independence of ΔL from ΔA and ΔF.
In Fig 2.81 this corresponds to diagonals having the same ordering; in particular, joint factor independence of ΔL from ΔA and ΔF means the following orderings hold:

if	(1.2) → (1,4)	similarly	(1,3) → (1,1)
then	(1,6) → (1,8)		(1,7) → (1,5)
and	(9,6) → (9,8)		(9,7) → (9,5)

In words:
If the dissimilarity between the tones (1,2) is greater than between (1,4) then this same ordering holds for (1,6) and (1,8), and (9,6) and (9,8). Fig 2.82 shows this more clearly by bringing out the factorial implications: f a' > f' a, irrespective of the level of ΔL (the difference in duration between tones).

(b) Joint Factor Independence of ΔA from ΔF and ΔL
(from Fig 2.81 or 2.82)

If	(1,4) → (1,5)	If	(1,8) → (1,1)
then	(1,3) → (1,6)	then	(1,7) → (1,2)

If	(1,8) → (9,5)	If	(9,8) → (1,5)
then	(1,7) → (9,6)	then	(9,7) → (1,6)
If	(1,4) → (9,5)	If	(9,8) → (1,1)
then	(1,3) → (9,6)	then	(9,7) → (1,2)

(c) Joint Factor Independence of F over A and L (from Fig 2.81 or 2.82)

If	(1,2) → (1,5)	(1,6) → (1,1)
then	(1,3) → (1,8)	(1,7) → (1,4)
If	(1,6) → (9,5)	(9,6) → (1,5)
then	(1,7) → (9,8)	(9,7) → (1,8)
If	(1,2) → (9,5)	(9,6) → (1,4)
then	(1,3) → (9,8)	(9,7) → (1,4)

Procedure

The method of paired comparisons was used to obtain a complete ordering of all the 12 stimulus pairs with respect to their dissimilarity to one another.

This means that 144 paired comparisons were required and omitting the self comparisons this reduced to 132. However, to control for order effects 264 paired comparisons are needed. In each experimental session this was repeated twice for each S, making 528 paired comparisons in all.

Subjects. 4 male undergraduates, all of whom had had extensive practice in making similarity judgments in a previous experiment. All subjects were paid. The complete experimental session of 528 presentations took about 4 hours to complete, including a short rest of 10 minutes on every hour.

Presentation of Stimuli. A pair of tones was presented via earphones to the subject followed by a second pair. The subject was required to indicate whether the first or the second pair were the most dissimilar. The presentation of stimuli followed the exact sequence reported in experiment 1A.

Apparatus. This was the same as that reported in the first experiment.

2.83 RESULTS

Tables 1B1, 1B2, 1B3, 1B4 appendix 2B gives the ranking of the stimulus pairs with respect to dissimilarity for each of the four subjects. The more dissimilar the stimuli of the pair are to each; other the higher the ranking. The paired-comparison tables for each subject from which Table 2.80 was generated are shown in the appendix. It was from Table 2.80 that the ordering of the stimulus pairs was established.

STIMULUS PAIR	(9,7)	(9,6)	(9,8)	(9,5)	(1,7)	(1,6)	(1,8)	(1,5)	(1,3)	(1,2)	(1,4)	(1,1)
S1	12	11	10	9	8	7	6	5	4	3	2	1
S2	12	10	8.5	6	11	8.5	7	5	4	3	2	1
S3	12	10	11	9	8	6	7	5	4	3	2	1
S4	12	11	10	9	8	7	6	5	4	3	2	1

TABLE 2.80 THE RANKING OF THE STIMULUS PAIRS BY THE 4 SUBJECTS. THE PAIRED COMPARISON TABLES FROM WHICH TABLE 2.84 WAS DERIVED ARE SHOWN IN THE APPENDIX (APPENDIX 28)

TABLE 2.81

	S1	S2	S3	S4
(1,2) ⟶ (1,1)	+	+	+	+
(1,3) ⟶ (1,4)	+	+	+	+
(1,6) ⟶ (1,5)	+	+	+	+
(1,7) ⟶ (1,8)	+	+	+	+
(9,6) ⟶ (9,5)	+	+	+	+
(9,7) ⟶ (9,8)	+	+	+	+

RESULTS: SINGLE FACTOR INDEPENDENCE OF △ A OVER △ F, AND △ L, FOR 4 SUBJECTS. THE + SIGN INDICATES THAT ORDERING BETWEEN PAIRS IS MAINTAINED. THE ⟶ '—' (negative) SIGN INDICATES THE ORDERING IS REVERSED.

	S1	S2	S3	S4
(1,4) → (1,1)	+	+	+	+
(1,8) → (1,5)	+	+	+	+
(9,8) → (9,5)	+	+	+	+
(9,7) → (9,6)	+	+	+	+
(1,7) → (1,6)	+	+	+	+
(1,3) → (1,2)	+	+	+	+

TABLE 2.82 SINGLE FACTOR INDEPENDENCE OF Δ F OVER Δ A AND Δ L.

	S1	S2	S3	S4
(9,5) → (1,1)	+	+	+	+
(9,6) → (1,2)	+	+	+	+
(9,8) → (1,4)	+	+	+	+
(9,7) → (1,3)	+	+	+	+
(1,8) → (1,4)	+	+	+	+
(1,5) → (1,1)	+	+	+	+
(1,7) → (1,3)	+	+	+	+
(1,6) → (1,2)	+	+	+	+
(9,8) → (1,8)	+	+	+	+
(9,5) → (1,5)	+	+	+	+

TABLE 2.83 SINGLE FACTOR INDEPENDENCE OF Δ L OVER Δ A AND Δ F.

Single Factor Independence

The results of the tests for single factor independence are shown in tables 2.81, 2.82 and 2.83.

In these tables the + sign indicates, (see table 2.81) that if the pair (1,2) is considered to be more dissimilar than the pair (1,1) then this ordering of dissimilarity is maintained between (1,3) and (1,4); (1,6) and (1,7) and so on. A'-'sign indicates that the ordering is reversed for this pair. The ordering was found to be consistent for all possible cases of single factor independence viz

(i) single factor independence of A over F & L
(ii) single factor independence of F over A & L
(iii) single factor independence of L over A & F

Since the tests of single factor independence does hold for all the factors then neither negative nor zero values are required. If it had failed for any of the factors then no composition rule with positive scales would have been compatible with the data i.e. either no composition rule is available or the composition rule must include some zero or negative scale values, Krantz & Tversky (1971).

Joint Factor Independence

The results of the test for joint factor independence are shown in tables 2.84, 2.85 and 2.86. In tables 2.84 and

TABLE 2.84 RESULTS JOINT FACTOR INDEPENDENCE FOR 4 SUBJECTS.

JOINT FACTOR INDEPENDENCE OF △ A FROM △ F AND △ L.

	S1	S2	S3	S4
(1,4) → (1,5)	+	+	+	+
(1,3) → (1,6)	+	+	+	+
(1,8) → (1,1)	+	+	+	+
(1,7) → (1,2)	+	+	+	+
(1,8) → (9,5)	+	+	+	+
(1,7) → (9,6)	+	+	+	+
(9,8) → (1,5)	+	+	+	+
(9,7) → (1,6)	+	+	+	+
(1,4) → (9,5)	+	+	+	+
(1,3) → (9,6)	+	+	+	+
(9,8) → (1,1)	+	+	+	+
(9,7) → (1,2)	+	+	+	+

N.B. THE + SIGN INDICATES, THAT FOR A SUBJECT, ORDERING IS CONSISTENT WITHIN PAIRS OF ORDERING. FOR EXAMPLE THE PAIR (1,4)IS LESS DISSIMILAR THAN THE PAIR (1,5). FOR JOINT INDEPENDENCE, THIS MEANS THAT (1,3) MUST BE LESS SIMILAR THAN THE PAIR (1,6). JOINT FACTOR INDEPENDENCE IS SATISFIED FOR ALL SUBJECTS.

TABLE 2.85 JOINT FACTOR INDEPENDENCE OF Δ F OVER Δ A AND Δ L.

	S1	S2	S3	S4
(1,2) → (1,5)	+	+	+	+
(1,3) → (1,8)	+	+	+	+
(1,6) → (1,1)	+	+	+	+
(1,7) → (1,4)	+	+	+	+
(1,6) → (9,5)	+	+	+	+
(1,7) → (9,8)	+	+	+	+
(9,6) → (1,5)	+	+	+	+
(9,7) → (1,8)	+	+	+	+
(1,2) → (9,5)	+	+	+	+
(1,3) → (9,8)	+	+	+	+
(9,6) → (1,4)	+	+	+	+
(9,7) → (1,4)	+	+	+	+

JOINT FACTOR INDEPENDENCE IS SATISFIED IN ALL INSTANCES FOR THESE 4 SUBJECTS.

TABLE 2.86 JOINT FACTOR INDEPENDENCE OF Δ L FROM Δ A AND Δ F.

		S1	S2	S3	S4
TRIPLE 1	(1,2) → (1,4)	+	+	+	+
	(1,6) → (1,8)	+	+	+	+
	(9,6) → (9,8)	+	+	+	+
TRIPLE 2	(1,3) → (1,1)	+	+	+	+
	(1,7) → (1,5)	+	+	+	+
	(9,7) → (9,5)	+	+	+	+

+ SIGN MEANS THAT ORDERING IN CONSISTANT WITHIN TRIPLES. JOINT FACTOR INDEPENDENCE IS SATISFIED FOR ALL SUBJECTS IN THIS INSTANCE.

2.85 the orderings are broken up into pairs. For example in table 2.84 we have:

(1,4) + (1,5)

(1,3) + (1,6)

which are orderings broken up into pairs. Also we have

(1,8) + (1,1)

(1,7) + (1,2)

which is another ordering broken up into pairs. The + sign indicates that within these pairs the orderings are consistent. It can be seen from tables 2.84 and 2.85 that the joint factor independence of A and F holds.

Table 2.86 gives the result of the test of joint factor independence of L from A and F. Here the + sign indicates that the ordering within the 2 triples is consistent. Thus joint factor inedpendence is satisfied for all three pairs of factors.

2.84 DISCUSSION

Again this is the only experiment of which the author is aware which applies conjoint measurement methods directly to look at single factor and joint factor independence in similarity data. The only other experiments seem to be those which test some of the axioms of multidimensional scaling, Tversky and Krantz (1969) and Wender (1971). However both these authors are essentially testing the requirements of a metric representation of similarity data, the two experiments reported here concentrate on the dimensional aspects. As well as vindicating to some extent the additive-difference model for similarity judgments this approach prompts some questions about the nature of psychological dimensions. In psychology, the term dimension is used inter changeably with the word attribute. Any set of mutually exhaustive event classes defines a dimension. Dimension can be classified in any number of ways, for example, in psychology, the term dimension is often used to denote a variable which can be manipulated experimentally, one use of the term dimension is a variable that can be manipulated experimentally, such as the frequency of a tone. This use of the term dimension simply refers to the way in which the stimuli are specified or generated physically and does not say anything about the way they are perceived.

Another common use of the term dimension is a trait or variable that cannot be observed directly, but can be expressed in terms of other measurable variables. The factor analytic definition of extraversion is an example of a dimension that is defined as a linear combination of some measurable variables such as test scores. The scaling of these dimensions does not depend on any testable psychological assumption: the attempt rather, is to express a large number of correlated variables in terms of a smaller number of uncorrelated ones. A third use of the term dimension refers to the factors along which stimuli are perceived and structured. In speaking of pitch, loudness, and duration as dimensions of auditory stimuli it is implied that they serve an organising principle in the perception of auditory structure. To accept such an interpretation, however, it is necessary to demonstrate the role played by these dimensions in the perception of tones. One way to do this is to construct a dimensional model and then to test whether any variable (specified physically e.g. frequency, loudness, duration etc.) acts like a dimension as defined in the model. In this approach, therefore, a psychological dimension is defined in terms of its formal characteristics. Consequently one can test which of several variables, if any, can be regarded as a dimension by studying its formal properties.

In the two experiments reported here this was precisely what was done. The formal characteristics of the model were, in the first experiment, that the 'difference' along the two dimensions should have an additive representation. In the second experiment these considerations were generalised: single factor independence and joint factor independence were added as further formal requirements of the model. Since, at least, for this small population of stimuli and subjects the physical dimensions frequency, loudness, and duration fulfilled the formal requirements of the model (the additive difference model) then thus far they can be considered as psychological dimensions as far as similarity judgments are concerned.

The notions of independence, and lack of interaction of dimensions, as defining attributes of a psychological dimension will be carried over to our consideration of independence in identification experiments.

INDEPENDENCE IN IDENTIFICATION TASKS

3.60 Further Methodological Points: The practice of 'normalising' confusion matrices and inferring independence from them.

3.70 The experiments

EXPERIMENT 2A (Pitch and loudness)

3.80 EXPERIMENT 2B (Pitch and Duration)

3.90 General Conclusions

3.60 Further Methodological Points: The practice of 'normalising' confusion matrices and inferring independence from them.

3.70 The experiments
EXPERIMENT 2A (Pitch and loudness)

3.80 EXPERIMENT 2B (Pitch and Duration)

3.90 General Conclusions

3.00 INTRODUCTION

In this chapter three experimental procedures which have been used to investigate independence in identification tasks are examined. The first two procedures have employed 'correlated' and orthogonal stimulus presentations respectively. The third method relies on testing for independence on confusion matrices which are themselves derived from larger ensembles of confusion data.

3.01 Correlated Dimensions

One method to investigate independence has been to arrange that the separate stimulus dimensions presented to the subject are correlated. An example will help explain this: if there are 10 values on one stimulus dimension to be identified and they are paired with 10 values of a second dimension in a perfectly redundant fashion there are still 10 stimuli which now vary on two dimensions and there are still 10 responses. This addition of redundant or correlated information from a second dimension can be effective in reducing the number of errors in an identification task compared with judging values on either unidimensional stimulus set (Erikson and Hake 1955).

One technique for introducing redundancy or correlated stimulus dimensions is as follows: a set of N distinct stimuli is selected from a unidimensional continuum and assigned a set of identification responses. A second set of N distinct stimuli is selected from another unidimensional continuum and is assigned the same set of identification responses as set 1. These stimuli are now paired, one to one with the first set to create a set of N distinct stimuli with a perfect 1:1 correlation between the two stimulus dimensions.

If identification performance is better with the correlated stimuli than with either of the separate sets of stimuli there is said to be a redundancy gain. Theoretical attempts to account for redundancy gains have assumed that the two dimension are psychologically independent (Erikson and Hake).

3.02 Orthogonal Combinations of Stimulus Dimensions

Another procedure used to investigate independence is to present orthogonal combinations of the values of the component dimensions for identification (Corcoran 1967, 1968). Those workers who have employed this paradigm, although invoking independent perceptual dimensions, have not always been clear as to the exact status of a dimension. The consequence is that these methods, when used to infer independence, raise questions of a methodological and theoretical nature. This issue is discussed in section 3.40.

3.03 Normalising Submatrices obtained from a Master Matrix

A third method has to do with the common practice of testing for independence, of one kind or another, on data derived from a 'master confusion' matrix by normalising the obtained submatrix (Smith 1972, Corcoran 1968). The actual procedure, itself, of normalisation in fact assumes independence and the practice of testing 'normalised' data for it therefore confuses the issue. This problem is discussed in section 3.60.

3.10 Independence in Identification Tasks: Correlated (or Redundant) Dimensions Presentations.

Probably the first mention of 'perceptual' independence was by Erikson and Hake [1955]. They were attemtping to predict a subject's identification performance in a multidimensional task from their performance on the component dimensions in an absolute judgment experiment. These authors employed the size, hue, and brightness of coloured patches as the three component stimulus dimensions with twenty levels or values on each. The response scheme was an arbitrary code number from 1 through 20. Each of the separate component dimensions was presented to the subject for identification in an absolute judgment task and the average value of the information transmitted, or contingent uncertainty, (Garner 1962) was found to be 2.75 bits. That is to say, the subjects could typically only make 7 different discriminations in the unidimensional presentation condition.

The second experimental condition consisted of a correlated bidimensional presentation of the component dimensions. That is, each level of the colour dimension was paired with another - fixed level - of the size dimension. Hue '1', with size '1', hue '2' with size '2' and so on. The following pairs of component dimensions were presented in a correlated manner two at a time:

(i) hue and size

(ii) hue and brightness

(iii) size and brightness .

In the bidimensional condition the subjects were again required to identify the stimulus by assigning them a number from 1 through 20. The average amount of information transmitted for these three pairs of dimensions was found to be 3.43 bits - an improvement of 0.68 bits over the unidimensional judgments.

In the third condition, all component dimensions were presented simultaneously, again in a perfectly correlated manner, so for instance, size 1, hue 1, brightness 1 were always presented together as was:

size i, hue i, and brightness i

where $1 < i \leq 20$

In the tridimensional correlated condition nearly perfect discrimination was achieved by the subjects. The authors reported, that on average, 4.11 bits of information were transmitted that is, about 17 or 18 discriminations could be made.

Erikson and Hake argued that the critical factor responsible for the improvement in discrimination as the number of correlated dimensions increased was the perceptual independence of the component stimulus dimensions. They said:

" ... Improvement could only be expected in the compound situation if Ss can judge simultaneously the stimuli on more than one of the component dimensions, and that his judgments along these different dimensions were not completely determined by his judgments of values along any one of them ... "

The authors used this model of independence to predict discrimination performance in both the bidimensional and tridimensional correlated conditions from a knowledge of the subject's identification performance in each of the three unidimensional conditions.

3.11 The Erikson and Hake Model of Perceptual Independence.

Erikson and Hake (1955), couched their notion of perceptual independence in terms of response tendencies in the following way:

3.111 "At any given moment in time the response tendency or rating evoked by a level on one dimension is independent of the response tendency evoked by the levels on the other dimensions."

ERIKSON & HAKE (1955) EXPERIMENT: THE PROBABILITY DISTRIBUTION OF RESPONSES TO THE STIMULUS "SIZE 8" AND THE STIMULUS "HUE 8".

UNDIMENSIONAL PRESENTATION.

TABLE 3.1A "SIZE 8" (STIMULUS)

RESPONSE	
5	.003
6	.007
7	.168
8	.462
9	.240
10	.088
11	.030
12	.002

TABLE 3.1B HUE 8(STIMULUS)

RESPONSE	
5	0.000
6	0.000
7	0.008
8	.496
9	.198
10	.175
11	.095
12	.027

TABLE 3.12 THE PROBABILITY DISTRIBUTION OF OBSERVED RESPONSES TO THE COMPOUND STIMULUS 'HUE 8 - SIZE 8' (THE CORRELATED DIMENSION CONDITION) TOGETHER WITH THE PREDICTED DISTRIBUTION ASSUMING INDEPENDENCE.

SIZE 8 - HUE 8 (STIMULUS)

RESPONSE OR RATING	OBSERVED PROBABILITY	PREDICTED PROBABILITY
5	0.00	0.00
6	0.000	0.00
7	.017	0.021
8	.815	0.728
9	.123	0.183
10	.043	0.056
11	.020	.012
12	.000	0.000

3.112 "The distribution of response frequencies or ratings to a level on a stimulus dimension is a measure of the relative strengths of the response tendencies elicited by that magnitude."

3.113 "In the case of competing response tendencies the stronger tendency will be evoked as a response."

Tables 3.1A and 3.1B illustrate the varying proportions of responses which were given when the two component dimensions (size 8 and hue 8 respectively) were presented to the subjects. The response could be any number from 1 to 20 in both the unidimensional conditions. Table 3.12 shows the proportion of the responses which were elicited by presentation of the bidimensional correlated stimulus 'hue 8 - size 8'. The final column of table 3.12 gives the predicted values of these proportions assuming perceptual independence.

3.20 A characterisation of Perceptual Independence for correlated dimensions in terms of a Response Process and a Sensory Process.

It is quite clear that Erikson and Hake's model involves two distinct components: a sensory and a response process. In what follows, the intuitions outlined in 3.111, 3.112 and 3.113 will be more explicitly characterised in terms of the implied sensory and response processes.

The simple model to be considered postulates that the observable stimulus response relations are a product of two processes: an activation process and a decision (or response) process. The activation process specifies the relation between the external stimulus event and the sensory states of the subject. The decision process specifies the subject's response in terms of his sensory state. To briefly summarise, the correlated stimulus dimensions are fed into the activation process, which converts external energy changes into sensory information giving rise to a sensory state; the decision process then operates on the sensory information to determine the response. We can now specify Erikson and Hake's model in terms of these two independent processes.

3.21 A model of the Sensory Process - State Independence

3.211 In the unidimensional presentation condition each level of any 'independent' stimulus dimension gives rise to a distribution of possible internal perceptual states.

In the experiment, we have been considering, each of the 20 levels of any one of the three component dimensions, has associated with it one of the internal perceptual states D_1, D_2, ... D_{20}

Where D_i is associated with level i, and

$$1 \leq i \leq 20$$

3.212 If level i of any one of the component independent dimensions is presented to the subject, then the most probable internal perceptual state that will be invoked is D_i; but in general the possible internal states that will occur follows a probability distribution, such that the states associated with the levels closest to the presented level will be more likely to occur than those states associated with levels more remote than the level presented.

3.213 In the bidimensional correlated condition, each of the component dimensions give rise to a separate internal perceptual state. The probability distribution of these two separate perceptual internal states which each of the levels of the correlated dimensions give rise to are completely unaffected by the presence of the other dimensions. This, in essence, is the assumption of perceptual independence, but from now on it will be referred to as the state independence assumption for obvious reasons.

3.23 The Decision or Response Model.

3.231 Associated with any internal state D_i, there is a unique response R_i. The responses in the Erikson and Hake experment are any one of the numbers 1 to 20.

3.232 In the bidimensional presentation condition two internal perceptual states occur - one from each of the component dimensions - which have associated with them two responses.

3.233 Associated with each of the two internal states is the probability that level i of each of the stimulus dimensions will elicit that particular internal state. The internal perceptual state with the highest probability of occurrence of the pair is called the dominant state. On a bidimensional trial, since only one response can be made, the response that is made, is the one associated with the most dominant perceptual state for that trial.

This model of state independence merely makes more explicit bot Erikson and Hake's (1955) model and Erikson's [1966] statement. To briefly summarise, the model assumes that when a combination of two perceptual states occur in a correlated bidimensional trial the probability of a given response 'is associated by the state with the highest hit rate associated with it' (Erikson 1966).

The characterisation just presented has some empirical content. For instance, 3.212 can be checked by glancing at table 3.1A. Here 'size 8' has been presented and the most likely perceptual state to be elicited will be D_8. This is so, because the response '8' is given on 46.2% of the unidimensional trials . Moreover, it can be seen that the possible internal states that do occur follow a probability distribution, such that, the states associated with the levels closest to level 8 are more likely to occur than those states associated with levels more remote than the one presented. For instance, it can be seen that it is only on 3% of the trials that the internal state D_{11}

TABLE 3.3

THE COMBINATION OF PERCEPTUAL STATES WHEN THE COMPOUND STIMULUS " HUE 8 - SIZE 8" IS PRESENTED WHICH GIVES RISE TO THE RESPONSE 7. THIS OCCURS WHENEVER THE PERCEPTICAL STATE D7 IS THE MOST DOMINANT OF THE PAIR OF STATES.

STIMULUS DIMENSIONS		PROBABILITY OF THE JOINT OCCURRENCE OF THESE STATES ASSUMING INDEPENDENCE.
HUE 8	SIZE 8	
D_7	D_6	.008 x .007
D_7	D_5	.008 x .003
D_7	D_{12}	.008 x .002
D_{11}	D_7	.095 x .168
D_{12}	D_7	.027 x .168
D_7	D_7	.008 x .168
	$\sum$	= .021 or 2.1% of the trials.

occurs so producing a response 11 whereas the state D_9 occurs on 24.0% of the trials in the unidimensional condition when 'size 8' is presented.

3.24 The Calculation of Erikson & Hake's Predictions of Responses to Multidimensional Correlated Stimuli from the Responses to the Component Stimulus Dimensions.

The more complete characterisation of the Erikson-Hake model presented here makes the task of calculating predictions based on their model relatively easy. Consider the problem of finding the predicted proportion of '7' responses when the correlated stimulus 'size 8 - hue 8' is presented.

Probably the simplest strategy is to list all the possible pairs of perceptual states that can occur with the D_7 state 'dominant'. When this is accomplished it is a trivial matter to calculate the proportion of predicted '7' responses. Table 3.3 shows all the possible pairs that fulfil this requirement. Hence the probability that a '7' is given in the bidimensional condition is 0.021, that is, this response can be expected on 2.1% of the trials.

This model of state independence, in essence, assumes that an increase in identifiability or detection in the correlated condition results from a kind of 'multiple observation' in a single trial, and each of the independent dimensions represents an additional opportunity for a clearer perceptual state (i.e. a more 'dominant' state) to occur. The subject's decision on any trial is determined solely by the observation associated with the clearest perceptual state.

3.25 State Independence, Independent Dimensions, and Psychological Dimensions

In section 2.84 some questions were asked about the nature of psychological dimensions. It was suggested there, that if pitch, loudness and duration are to be urged as candidates for possible psychological dimensions in hearing it was necessary to demonstrate the role played by these variables in perception. One way to do this is to construct a dimensional model and to test any variable to see if it acts like a dimension defined in the model. This was done for the similarity data in experiment 1A and 1B which had been given a dimensional representation. This is also done for similarity judgments in chapter 4 on the same stimuli as these two previous experiments.

This same strategy is adopted here; in identification tasks 'independent' dimensions, it is suggested, correspond to the notion of 'the independent realisation of the components or factors' mentioned in 2.301. Further, two 'physical' dimensions are independent if, and only if, state independence exists between them. If state independence does not exist, then the two proposed dimensions do not have any independent realisation.

3.30 STATE MODELS, State Independence Sensory and Decision Processes (for the case of correlated dimensions presentation).

Most of the theories of signal detection postulate models of both a sensory and a decision process (Swets et al 1961, Norman 1964, Atkinson 1963). Some theories have assumed a continuum of sensory states (Swets et al 1961), others a discrete, but large number of internal perceptual states (McGill 1965), and others also, a set of two or three sensory states (Luce 1963, Krantz 1969). The Erikson-Hake model, although not a model of signal detection, has many formal similarities to one, in particular to those classed as two-state high threshold models.

It was noticed in section 3.20 that a complete characterisation of the state independence hypothesis demands an account of both the decision process and the sensory process. In this section a brief investigation of a model of state independence for the correlated presentation of two independent stimulus dimensions will be undertaken, and in particular an explanation will emerge as to why, in such a task, identification is enhanced compared to when either of the component dimensions is presented singly.

Suppose F and A are two independent stimulus dimensions which have only one value F_o and A_o. Consider the following two experimental conditions:

(a) The control condition. This consists of a unidimensional presentation of each of the component dimensions A_o and F_o. The subject's responses to each of these component dimensions is recorded.

(b) A bidimensional correlated presentation of the stimulus. Here the dimensions A_o, F_o, are presented together as a single stimulus, and detection of only one of the independent dimensions will suffice to identify the stimulus.

3.301 Unidimensional Presentation (Control) Condition

The stimulus-response (S-R) matrices for each of the component independent dimensions are:

$$\text{Response} \quad \begin{array}{c} \\ \text{Yes} \\ \text{No} \end{array} \overset{A_o \text{ (stimulus)}}{\begin{bmatrix} p_a \\ 1 - p_a \end{bmatrix}} \qquad \begin{array}{c} \\ \text{Yes} \\ \text{No} \end{array} \overset{F_o \text{ (stimulus)}}{\begin{bmatrix} p_f \\ 1 - p_f \end{bmatrix}} \qquad 3.301$$

p_a, and p_f, are the probabilities of detection of the A and F dimensions in the control condition.

The Sensory Model

$$\text{(stimulus)}\quad A_o \quad \begin{matrix} D & \bar{D} \\ \begin{bmatrix} p_a & 1 - p_a \end{bmatrix} \end{matrix}$$

3.302

$$\text{(stimulus)}\quad F_o \quad \begin{matrix} D & \bar{D} \\ \begin{bmatrix} p_f & 1 - p_f \end{bmatrix} \end{matrix}$$

These two matrices summarise the sensory process.

For example when stimulus A_o is presented either one of two perceptual states occur:

(i) State D (a detect state) which occurs with a probability of p_a

or

(ii) State $\bar{D}$ (the non-detect state) which occurs with a probability $1 - p_a$

The second matrix, involving the F dimension, similarly summarises the sensory process for this component.

The Response Process.

This can be represented by the following decision matrix:

Sensory States

$$\text{Response}\quad \begin{matrix} & D & \bar{D} \\ \text{Yes} & 1.0 & 0.0 \\ \text{No} & 0.0 & 1.0 \end{matrix}$$

3.303

TABLE 3.4

PERCEPTUAL STATES ARISING FROM THE SEPARATE DIMENSIONS IN THE COMPOUND STIMULUS.		PROBABILITY OF STATE ARISING FROM DIMENSION A	PROBABILITY OF STATE ARISING FROM DIMENSION F	PROBABILITY THAT THIS COMBINATION OF STATES OCCUR.	SUBJECTS RESPONSE.
DIMENSION A	DIMENSION F				
D	D	Pa	Pf	Pa · Pf	YES
$\bar{D}$	$\bar{D}$	1 -Pa	1 - Pf	(1 - Pa) (1 - Pf)	NO
D	$\bar{D}$	Pa	1 - Pf	Pa(1 - Pf)	YES
$\bar{D}$	D	1 - Pa	Pf	(1 - Pa)Pf	YES

N.B. Pa = probability of A state arising

Pf = probability of F state arising

For both the independent dimensions F and A, in the control condition,whenever a D perceptual state occurs the subject reports he has detected or identified the dimension, whereas if a $\bar{D}$ state occurs the subject does not report a detection. This model is a threshold model which allows for no guessing.

Independent Dimensionspresented in a Correlated manner.

When F_oA_o are presented together in a Yes-No detection experiment,table 3.40 gives a listing of all the possible perceptual states which can arise. Columns 3 and 4 show the probability of the indicated perceptual state arising from the given F and A dimensions respectively. The last but one column gives the probability of this combination of perceptual states occurring on any given trial assuming state independence. The final column - the decision column - gives the subject's answers:

"Yes" (a detection)
or
"No" (no detection)

The decision rule is:

If on any trial a particular dimension gives rise to a D state then the subject reports 'yes' (he saw the signal) otherwise he does not (see 3.303).

Assuming state independence, the probability of a detection in the bidimensional correlated condition is:

3.304 $$p_a + p_f - p_a \cdot p_f$$

The probability that there will be no detection is:

3.305 $$1 - p_a - p_f + p_a \cdot p_f$$

If $p_a = p_f = p$

The probability of no detection is:

3.306 $$p^2 - 2p + 1 .$$

The probability that there will be a detection for 2 correlated dimensions in one trial is:

3.307 $$1 - (1 - p)^2$$

Similarly if there are N independent dimensions and these are presented in a correlated manner (that is detection of any one of the dimensions suffices to achieve correct detection of the composite stimulus) the probability of a detection is

$$1 - [1 - p]^N$$

Consider a simple example: a stimulus is composed of four independent but correlated dimensions and the probability of detection of any one of the component dimensions is 0.5. What is the probability of detection of the composite stimulus?

This is clearly equal to:

$$1 - (1 - \tfrac{1}{2})^4$$
$$= .9375$$

Hence the stimulus will be detected on 93.75% of the trials. It is easy to see that each of the independent dimensions affords an 'additional opportunity' for a detect state (D) to occur.

3.40 Independence in Identification Tasks Involving the Orthogonal Presentation of the Stimulus Dimensions.

In the previous section, we have examined independence of the stimulus dimensions in experimental conditions where the values on the component dimensions have been presented in a correlated manner. The main thrust of this section, however, is to investigate independence in experimental paradigms in which there is an orthogonal presentation of the stimulus dimensions. In such a paradigm,it is assumed,at the very least that a number of perceptual dimensions have been isolated and that any combination of values on all dimensions is possible. Moreover it is assumed that it is possible to fix all but a given dimension A (say) and to vary the stimulus along the dimension A, generating the unidimensional values

$$\{A_o, A_1, \ldots\ldots\ldots\ldots A_N\} ,$$

and similarly for F and L we would have the following unidimensional values

$\{F_o, F_1, \dots\dots\dots\dots F_N\}$,

and $\{L_o, L_1, \dots\dots\dots\dots L_N\}$.

In the orthogonal bidimensional situation the following values make up the stimulus sets

(i) The F and A bidimensional stimulus set:

$\{F_oA_o, F_1A_o, F_2A_o, \dots\dots\dots F_NA_N\}$

(ii) The F and L bidimensional stimulus set:

$\{F_oL_o, F_oL_1, F_oL_2, \dots F_1L_o, \dots\dots\dots, F_NA_N\}$

(iii) The A and L bidimensional stimulus set:

$\{A_oL_o, A_oL_1, A_oL_2, \dots A_1L_o, \dots\dots\dots, A_NL_N\}$

In a typical experiment, the set of stimuli is specified, each stimulus is labelled, and then presented for identification a large number of times with the order of presentation randomised. Corcoran's (1967) study provides an example of this mode of presentation. The stimuli were a combination of frequency and rate of interruption, and their 'independence' was under investigation. In the bidimensional presentation condition four stimuli were presented, F_1I_1, F_1I_2, F_2I_1, and F_2I_2, which differed on two dimensions F (frequency) and I (rate of interruption). The F and I dimensions were able to assume one of two values: 1 or 2. In the stimulus presentation each of the four stimuli were equally likely.

The associated responses to each of these four stimuli are labelled: $\overline{F_1I_1}$, $\overline{F_1I_2}$, $\overline{F_2I_1}$, F_2I_2 where the bar over the upper case letters indicates a composite response to the appropriate stimuli. In the actual experiment each of the four tones had the labels $\overline{A}$, $\overline{B}$, $\overline{C}$ and $\overline{D}$ respectively - one composite response to each of the 'two-dimensional stimuli'. However for notational convenience we will use $\overline{F_1I_1}$, $\overline{F_1I_2}$, $\overline{F_2I_1}$, and $\overline{F_2I_2}$. The subjects were trained on the four tones in a no-noise condition until they were able to name each sound without error.

The four stimuli were then presented randomly in noise for identification. Because the subjects had learned the tones, any errors in this condition could be attributed to errors in perception rather than memory.

As Corcoran's experiment is one of the very few which have investigated independence in identification tasks involving an orthogonal presentation of the stimulus dimensions, it will be discussed rather thoroughly. This experiment has been criticised for inaccuracies in arithmetic, (Smith 1972), but the design has many merits. One of the most important of these is the labelling of the stimuli; Corcoran avoided the problem of using a response scheme which depended on the subject identifying each of the levels of the component dimensions. For as Kaufman and Levy (1971) pointed out, to do so, is to assume that the dimensions are perceptually distinct and independent. What will be shown later, is, that Corcoran's response

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Fig 3.41

RECORDED AND PREDICTED VALUES IN THE CORCORAN (1967) EXPERIMENT.

	STIMULUS $F_1 I_1$	$F_1 I_2$	$F_2 I_1$	$F_2 I_2$
$\overline{F_1 I_1}$	0.771 (0.816)	0.063 (0.103)	0.104 (0.107)	0.000 (0.012)
$\overline{F_1 I_2}$	0.177 (0.113)	0.0864 (0.841)	0.031 (0.018)	0.115 (0.112)
$\overline{F_2 I_1}$	0.052 (0.057)	0 000 (0.006)	0.781 (0.735)	0.115 (0.067)
$\overline{F_2 I_2}$	0.000 (0.009)	0.073 (0.056)	0.083 0.146)	0.771 (0.806)

The probabilities obtained in Corcoran's experiment are the non bracketed values. The bracketed values are the probabilities predicted by "independence" considerations.

Fig 3.42

The calculation of the Predicted Values in Corcoran's (1967) Experiment.

(a) $F_1 I_1$ [Stimulus]

	$\overline{F_1}$	$\overline{F_2}$	
$\overline{I_1}$	0.771	0.052	0.823
$\overline{I_2}$	0.177	0.000	0.177
	0.948	0.052	

(b) $F_1 F_2$ [Stimulus]

	$\overline{F_1}$	$\overline{I_2}$	
$\overline{I_1}$	0.063	0.000	0.063
$\overline{I_2}$	0.864	0.073	0.937
	0.927	0.073	

(c) $F_2 I_1$ [Stimulus]

	$\overline{F_1}$	$\overline{F_2}$	
$\overline{I_1}$	0.104	0.781	0.885
$\overline{I_2}$	0.031	0.083	0.114
	0.135	0.864	

(d) $F_2 I_2$ [Stimulus]

	$\overline{F_1}$	$\overline{F_2}$	
I_1	0.000	0.115	0.115
I_2	0.115	0.771	0.886
	0.115	0.886	

The empirical response-response matrix for the stimuli F_1I_1, F_1I_2, F_2I_1, F_2I_2 respectively. Each submatrix has as its entries four column values, for the appropriate stimulus, obtainable from Table 3.41.

Fig 3.43

'Revised' Analysis of Corcoran's (1967) Data. Estimates of Unidimensional Performance from the Empirical Bidimensional Matrix (Fig 3.41).

(a)	F_1	F_2
$\bar{F}_1$	0.948	-
	0.948	-
$\bar{F}_2$	0.052	-
	0.052	-

(a')	I_1	I_2
$\bar{I}_1$	0.813	-
	1.000	-
$\bar{I}_2$	0.187	-
	0.000	-

Estimates obtained from response-response matrix (a) Fig 3.42

(b)	F_1	F_2
$\bar{F}_1$	0.927	-
	0.927	-
$\bar{F}_2$	0.927	-
	0.927	-

(b')	I_1	I_2
$\bar{I}_1$	-	0.068
	-	0.000
$\bar{I}_2$	-	0.932
	-	1.000

Estimates obtained from response-response matrix (b) Fig 3.42

(c)	F_1	F_2
$\bar{F}_1$	-	0.135
	-	0.136
$\bar{F}_2$	-	0.865
	-	0.864

(c')	I_1	I_2
$\bar{I}_1$	0.770	-
	0.903	-
$\bar{I}_2$	0.229	-
	0.096	-

Estimates obtained from response-response matrix (c) Fig 3.42

(d)	F_1	F_2
$\bar{F}_1$	-	0.115
	-	0.114
$\bar{F}_2$	-	0.885
	-	0.886

(d')	I_1	I_2
$\bar{I}_1$	-	0.000
	-	0.129
$\bar{I}_2$	-	1.000
	-	0.870

Estimates obtained from response-response matrix (d) Fig 3.42

Fig 3.44

F dimension

	F_1	F_2
$\bar{F}_1$	0.938	0.125
$\bar{F}_2$	0.062	0.875

I dimension

	I_1	I_2
$\bar{I}_1$	0.872	0.049
$\bar{I}_2$	0.128	0.951

Estimates Unidimensional Performance Using Corcoran's data. Each entry is the mean of 4 estimates of the theoretical probabilities shown in Fig 3.43A.

Fig 3.45

The prediction of a theoretical bidimensional matrix from an estimate of the unidimensional performance assuming state independence.

(a)

- F_1 I_1 stimulus -

	$\bar{F}_1$	$\bar{F}_2$
I_1	.938 x .872 = .818	.062 x .872 = 0.054
I_2	.938 x .128 = .120	.062 x .128 = .008

- F_1 I_1 stimulus -

	$\bar{F}_1$	$\bar{F}_2$
I_1	.938 x .049 = .046	.062 x .049 = .003
I_2	.938 x .951 = .892	.062 x .951 = .059

- F_2 I_1 stimulus -

	$\bar{F}_1$	$\bar{F}_2$
I_1	.125 x .872 = .109	.875 x .872 = .763
I_2	.125 x .128 = .016	.875 x .128 = .112

- F_2 I_2 stimulus -

	$\bar{F}_1$	$\bar{F}_2$
I_1	.125 x .049 = .006	.875 x .049 = .043
I_2	.125 x .951 = .119	.875 x .951 = .832

scheme allows us to infer the identifiability of the different levels of the component stimulus dimensions from a knowledge of the subject's identification response in the bidimensional situation. However, before this is done it would be well to look at Corcoran's experiment in more detail and the assumptions which generated them.

3.41 The Assumptions Generating the Predicted Value of Corcoran's (1967) Experiment

Although Corcoran's (1967) is the study explicitly dealt with here, the same assumptions of that study are those which motivated some other experiments: Corcoran (1966) and Corcoran, Dorfman and Weening (1968).

Table 3.41 gives the recorded and predicted values (on the assumption of independence) for Corcoran's (1967) experiment.

From this table, a set of 4 sub matrices - the empirical response - response submatrices - have been derived. These are shown in Fig 3.42.

Consider Fig 3.42 submatrix (a); it can be seen that each of the 4 entries are the same as those of column 1 of the bidimensional matrix of table 3.41. It was effectively from these four response - response submatrices that Corcoran derived estimates of the identifiability of the unidimensional values of each of the two dimensions.

This point is in fact crucial: his procedure was one of estimating responses to unidimensional from responses to multidimensional stimuli and NOT 'prediction of responses to multidimensional from responses to unidimensional stimuli' as Corcoran (1966) stated.

Fig 3.43 gives these estimates whilst Fig 3.44 shows the mean estimated identifiability of the F and I dimension in the unidimensional situation. From these estimates the theoretical values in the stimulus - response matrix of table 3.41 are derived.

3.42 The Construction of The Theoretical or Predicted Values of The Bidimensional Matrix from the Response-Response Submatrices.

Consider Fig 3.42 which gives the four response-response matrices obtained from Corcoran's empirical bidimensional matrix.

If state independence of the two stimulus dimensions is assumed we have from the response-response submatrix (a) in Fig 3.42:

3.421 $$P(\overline{F_1 I_1} / F_1 I_1)_{2D} = P(\overline{F}_1 / F_1)_{1D} \times P(\overline{I}_1 / I_1)_{1D} = .771$$

The subscripts 2D and 1D indicate whether the response is assumed to arise from the bidimensional experimental condition or the unidimensional situation.

In words 3.421 states:

The probability that the composite response $\overline{F_1 I_1}$ is given to the bidimensional stimulus F_1I_1, is equal to the probability that the response $\bar{F}_1$ is made when the stimulus F_1 is presented in the unidimensional condition times the probability that the response $\bar{I}_1$ is made when the stimulus I_1 is presented in the unidimensional situation.

Similarly:

3.422 $$P(\overline{F_1 I_2} \;/\; F_1 I_1)_{2D} = P(\bar{F}_1 \;/\; F_1)_{1D} \times P(\bar{I}_2 \;/ I_1)_{1D} = .177$$

The two estimates of the hit rate of the stimuli F_1 and I_1 in the unidimensional situation is:

$$P(\bar{F}_1 \;/\; F_1)_{1D} = .948; \quad \text{thus } P(\bar{F}_2 \;/\; F_1)_{1D} = 0.052$$

$$P(\bar{I}_1 \;/\; I_1)_{1D} = .813; \quad \text{thus } P(\bar{I}_2 \;/\; I_1)_{1D} = 0.187$$

From this same submatrix (a) (Fig 3.42) two more estimates of unidimensional identifiability can be obtained:

3.423 $$P(\overline{F_2 I_1} \;/\; F_1 I_1)_{2D} = P(\bar{F}_2 \;/\; F_1)_{1D} \cdot P(\bar{I}_1 \;/\; I_1)_{1D} = 0.052$$

3.424 and $P(\overline{\overline{F}_2 \overline{I}_2} / \overline{F_1 I_1})_{2D} = P(\overline{F}_2 / F_1)_{1D} \cdot P(\overline{I}_2 / I_1)_{1D} = 0.000$

whence: $P(\overline{F}_2 / F_1)_{1D} = 0.052$ and $P(\overline{F}_1 / F_1)_{1D} = .948$

also $P(\overline{I}_2 / I_1)_{1D} = 0.000$ and $P(\overline{I}_1 / I_1)_{1D} = 1.000$

From this response-response submatrix two estimates of

$P(\overline{F}_1 / F_1)_{1D}$

and $P(\overline{F}_2 / F_1)_{1D}$.

Also $P(\overline{I}_1 / I_1)_{1D}$

and $P(\overline{I}_2 / I_1)_{1D}$.

have been obtained. These are shown in Fig 3.43 in submatrices (a) and (a') respectively.

It can be seen now, that each of the response-response submatrices give 2 estimates each of the appropriate unidimensional performance. In Fig 3.43 submatrices a, a', b, b', c, c' and d, d' are the estimates obtained from the corresponding submatrices in Fig 3.42.

Corcoran (1967) and (1971) argued that it is possible to obtain <u>eight</u> estimates of each of the theoretical probabilities. It is argued here, on the contrary, that only FOUR independent estimates of the theoretical probabilities can be obtained. Furthermore, if it is recognised that these estimates are in fact estimates of unidimensional performance it is easy to see why.

3.43 It will be as well to spell out very clearly Corcoran's assemptions.

(a) The method, implicitly involved deriving an estimate (assuming independence) of the identifiability of the component dimensions in a unidimensional situation. See Fig 3.42 3.43 and 3.44.

(b) From these estimates of unidimensional performance, a theoretical bidimensional matrix was generated - assuming state independence. See Fig 3.44 and 3.45. The assumption of state independence guarantees that the identifiability of a level or a value on a component dimension is unaffected by the presence of the other stimulus dimensions.

3.44 Difficulties

Corcoran's experimental procedure is logically unsatisfactory: by initially assuming state independence, he derived from the empirical bidimensional matrix theoretical estimates of unidimensional performance of the component dimensions. He then again assumed state independence; this time of these unidimensional estimates to obtain a theoretical bidimensional matrix. The illogicality of the procedure was further compounded because the theoretical and empirical bidimensional matrices were compared to see how far one deviated from the other.

Such an experimental paradigm is circular and can surely never test independence. The unidimensional performance matrices for the component dimensions should not be estimated - this is an empirical matter and they should be obtained experimentally. It is only these empirically obtained empirical unidimensional performance matrices which should be used to generate - assuming state independence - the theoretical bidimensional matrix.

Another difficulty resides in the 'statistical' definition of independence - a definition which most writers seem to adopt (Garner and Morton 1969, Corcoran 1967, Erikson and Hake 1955, Broadbent 1971). This is, strictly speaking, an operational definition and corresponds to the procedure for generating the theoretical bidimensional matrix from assumed unidimensional performance matrices (see Fig 3.44 and 3.45). However this concept is not rich enough to tell us what to look for, when and if, independence breaks down. In this treatment we have preferred to use the notion of state independence which although operationally corresponds to independence or perceptual independence has a definite psychological content. First it has to do with the independent realisation of the dimensions, and this has many implications for selective dimensional attention (see section 3.58). Second, state independence means that the identifiability of a particular level on one dimension is completely independent of the level of the other dimension. That is, there are no interaction effects between dimensions if state independence holds between them.

In the next section, we will develop some machinery which will help clarify these two notions.

3.50 Uncertainty Analysis and Dimensional Combination

The identification of the values of the two orthogonally presented stimulus dimension in Corcoran's experiment is really a classification problem involving discrete stimuli, and so uncertainty analysis therefore, can provide a natural non-metric measure of correlation. Moreover Garner & McGill (1962) and Hake and Rodwan (1966) have drawn attention to the close parallel that uncertainty analysis has to analysis of variance.

It is advantageous then to analyse the orthogonal stimulus response matrix using uncertainty analysis, because it allows the underlying logic of analysis of variance to be applied to the data from a single subject. This is especially desirable because the pooling of data over many subjects, which is typical of so many studies testing independence in identification tasks,(Corcoran 1967, Miller and Nicely 1955, Conrad 1964, Smith 1973) very possibly swamps out individual subject effects. It is not the individual differences per se, that is of interest, but the possibility that the identifiability of different values of one stimulus dimension varies over different values of the other: in other words there is an interaction effect. However, it is conceivable that different subjects exhibit different patterns of interaction. A dimension, for instance, may have differential identifiability for different subjects due, possibly, to attentional factors. Therefore these very real interactional factors or lack of independence of the dimensions for each subject might very well be masked when data is pooled over many subjects.

Uncertainty analysis enables an experimenter to partial out the different stimulus effects and,like analysis of variance,it can be tailored to a particular experimental design so extracting the maximum amount of information.

Before embarking on a very brief overview of uncertainty analysis a digression as to why the choice of the word "uncertainty" as opposed to "information theory". The word "uncertainty" was finally used - in line with Garner (1962) - because "information" theory carried with it certain echoes and connotations particularly of the information theory industry which burgeoned in

Fig 3.51 Results of an Identification Experiment

CONTROL EXPERIMENT.

(a) F dimension

	F_0	F_1	F_2
$\bar{F}_0$	48	1	0
$\bar{F}_1$	2	44	0
$\bar{F}_2$	0	5	50
			150

$U[F:\bar{F}]_{1D}$ = 1.2986 bits

(b) A dimension

	A_0	A_1	A_2
$\bar{A}_0$	44	5	0
$\bar{A}_1$	6	35	18
$\bar{A}_2$	0	10	32
			150

$U[A:\bar{A}]_{1D}$ = 0.6956 bits

Orthogonal Presentation

(c) THE F A bidimensional matrix

	F_0A_0	F_0A_1	F_0A_2	F_1A_0	F_1A_1	F_1A_2	F_2A_0	F_2A_1	F_2A_2	
$\overline{F_0A_0}$	19	-	2	1	-	-	-	-	-	
$\overline{F_0A_1}$	3	14	1	-	-	-	-	-	-	
$\overline{F_0A_2}$	-	8	20	-	-	-	-	-	-	
$\overline{F_1A_0}$	3	-	-	16	3	1	-	-	-	
$\overline{F_1A_1}$	-	2	-	-	15	5	-	-	-	
$\overline{F_1A_2}$	-	1	2	-	3	18	-	-	-	
$\overline{F_2A_0}$	-	-	-	8	1	1	23	4	2	
$\overline{F_2A_1}$	-	-	-	-	3	1	2	17	11	
$\overline{F_2A_2}$	-	-	-	-	-	-	-	4	12	
	25	25	25	25	25	25	25	25	25	225

$U[FA:\overline{FA}]$ = 1.9405 bits

Fig 3.52. The Derivation of a theoretical bidimensional stimulus-response matrix from the unidimensional performance matrices in the control condition assuming state independence. The one dimensional matrices of Fig 3.51 have been changed into conditional probability matrices for convenience.

<u>Control Condition</u>

(a) F dimension

	F_0	F_1	F_2
$\bar{F}_0$	.96	.02	.00
$\bar{F}_1$	.04	.88	.00
$\bar{F}_2$	.00	.10	1.00

$U[\bar{F}:F]_{1D} = 1.2986$ bits

(b) A dimension

	A_0	A_1	A_2
$\bar{A}_0$	.88	.10	.00
$\bar{A}_1$	.12	.70	.36
$\bar{A}_2$	.00	.20	.64

$U[\bar{A}:A]_{1D} = 0.6956$ bits

Theoretical Response-Response Matrices for each of the dimensional combinations assuming state independence.

(c) $F_0\ A_0$ (stimulus)

	$\bar{F}_0$	$\bar{F}_1$	$\bar{F}_2$
$\bar{A}_0$	.840	.035	.000
$\bar{A}_1$	.120	.005	.000
$\bar{A}_1$	.000	.000	.000

$U_{F_0A_0}[\bar{F}:\bar{A}] = 0.000$bits

(d) $F_0\ A_1$ (stimulus)

	$\bar{F}_0$	$\bar{F}_1$	$\bar{F}_2$
$\bar{A}_0$	.096	.004	.000
$\bar{A}_1$	.670	.028	.000
$\bar{A}_2$	.190	.008	.000

$U_{F_0A_1}[\bar{F}:\bar{A}] = 0.000$ bits

(e) $F_0\ A_2$ (stimulus)

	$\bar{F}_0$	$\bar{F}_1$	$\bar{F}_2$
$\bar{A}_0$	.000	.000	.000
$\bar{A}_1$	.350	.014	.000
$\bar{A}_2$	.610	.026	.000

$U_{F_0A_2}[\bar{F}:\bar{A}] = 0.000$bits

(f) F_1A_0 (stimulus)

	$\bar{F}_0$	$\bar{F}_1$	$\bar{F}_2$
$\bar{A}_0$	.018	.774	.088
$\bar{A}_1$	.002	.106	.012
$\bar{A}_2$	.000	.000	.000

$U_{F_0A_0}[\bar{F}:\bar{A}] = 0.000$bits

(g) F_1A_2 (stimulus)

	$\bar{F}_0$	$\bar{F}_1$	$\bar{F}_2$
$\bar{A}_0$	.002	.088	.010
$\bar{A}_1$	.014	.616	.070
$\bar{A}_2$	.004	.176	.002

$U_{F_0A_1}[\bar{F}:\bar{A}] = 0.000$bits

(h) F_1A_2 (stimulus)

	$\bar{F}_0$	$\bar{F}_1$	$\bar{F}_2$
$\bar{A}_0$	.000	.000	.000
$\bar{A}_1$	.007	.316	.036
$\bar{A}_2$	.013	.564	.064

$U_{F_0A_2}[\bar{F}:\bar{A}] = 0.000$bits

The values in each of these submatrices are obtained by using the same procedure used to generate the submatrices in Fig 3.45

Fig 3.52 (continued)

(i) F_2A_0(stimulus)

	$\bar{F}_0$	$\bar{F}_1$	$\bar{F}_2$
$\bar{A}_0$	.000	.000	.880
$\bar{A}_1$	.000	.000	.120
$\bar{A}_2$	.000	.000	.000

$U_{F_2A_0}[\bar{F}:\bar{A}]$ = 0.000bits

(j) F_2A_1(stimulus)

	$\bar{F}_0$	$\bar{F}_1$	$\bar{F}_2$
$\bar{A}_0$	.000	.000	.100
$\bar{A}_1$	.000	.000	.700
$\bar{A}_2$	.000	.000	.200

$U_{F_2A_1}[\bar{F}:\bar{A}]$ = 0.000bits

(k) F_2A_2 (stimulus)

	$\bar{F}_0$	$\bar{F}_1$	$\bar{F}_2$
$\bar{A}_0$	.000	.000	.000
$\bar{A}_1$	.000	.000	.360
$\bar{A}_2$	.000	.000	.640

$U_{F_2A_2}[\bar{F}:\bar{A}]$ = 0.000bits

Fig 3.53(a) The theoretical conditional probability matrix generated from the 2 unidimensional matrices in the control condition assuming state independence.

	F_0A_0	F_0A_1	F_0A_2	F_1A_0	F_1A_1	F_1A_2	F_2A_0	F_2A_1	F_2A_2
F_0A_0	.840	.096	.000	.018	.002	.000	.000	.000	.000
F_0A_1	.120	.670	.350	.002	.014	.007	.000	.100	.000
F_0A_2	.000	.190	.610	.000	.004	.013	.000	.000	.000
F_1A_0	.035	.004	.000	.774	.083	.000	.000	.000	.000
F_1A_1	.005	.028	.014	.106	.616	.316	.000	.000	.000
F_1A_2	.000	.008	.026	.000	.176	.564	.000	.000	.000
F_2A_0	.000	.000	.000	.088	.010	.000	.880	.100	.000
F_2A_1	.000	.000	.000	.012	.070	.036	.120	.700	.360
F_2A_2	.000	.000	.000	.000	.002	.064	.000	.200	.640
Σ	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

and "Theoretical" $U[\overline{FA}:\overline{FA}]$ = 2.03695 bits

Each column of the theoretical bidimensional matrix is derived one of the theoretical response-response matrices. For example the third column is obtained from submatrix (e) Fig 3.52.

Fig 3.53B. The theoretical performance bidimensional matrix obtained from 3.53A - assuming state independence.

	F_0A_0	F_0A_1	F_0A_2	F_1A_0	F_1A_1	F_1A_2	F_2A_0	F_2A_1	F_2A_2	
$\overline{F_0A_0}$	21	2	-	1	1	-	-	-	-	
$\overline{F_0A_1}$	3	17	9	-	-	2	-	-	-	
$\overline{F_0A_2}$	-	5	15	-	-	-	-	-	-	
$\overline{F_1A_0}$	1	-	-	19	2	-	-	-	-	
$\overline{F_1A_1}$	-	1	-	3	15	8	-	-	-	
$\overline{F_1A_2}$	-	-	1	-	4	14	-	-	-	
$\overline{F_2A_0}$	-	-	-	2	-	-	22	2	-	
$\overline{F_2A_1}$	-	-	-	-	2	-	3	18	9	
$\overline{F_2A_2}$	-	-	-	-	1	1	-	5	16	
	25	25	25	25	25	25	25	25	25	225

Again "theoretical" $U\left[FA:\overline{FA}\right] = 2.03695$ bits

Fig 3.54 Uncertainty Measures obtained from the Empirical Bidimensional Matrix the A and F dimensions derived from the two dimensional matrix in Fig 3.51c.

(a) F dimension

	F_0	F_1	F_2
$\overline{F_0}$	67 (73)	1 (4)	0 (0)
$\overline{F_1}$	8 (2)	62 (65)	0 (0)
$\overline{F_2}$	0 (0)	12 (6)	75 (75)

$U\left[F:\overline{F}\right]_{2D} = 1.1738$ bits

(b) A dimension

	A_0	A_1	A_2
$\overline{A_0}$	70 (66)	7 (7)	5 (0)
$\overline{A_1}$	5 (9)	51 (53)	18 (28)
$\overline{A_2}$	0 (0)	17 (15)	52 (47)

$U\left[A:\overline{A}\right]_{2D} = 0.6255$ bits

The numbers in bracket are the values which would be obtained if the dimensions were independent.

Fig 3.55. Dimensional Interaction. The interaction Submatrices have all been taken from the data presented in the bidimensional matrix in Fig 3.51(c).

(a) Submatrix for A_0 derived from Fig 3.51(c)

	F_0A_0	F_1A_0	F_2A_0	
$\bar{F}_0$	22 (24)	1 (1)	0 (0)	$\bar{F}_0$
$\bar{F}_1$	3 (1)	16 (2?)	0 (0)	$\bar{F}_1$
$\bar{F}_2$	0 (0)	8 (2)	25 (25)	$\bar{F}_2$

$U_{A_0}[F:\bar{F}]$ = .9950 bits

(b) Submatrix for A_1 derived from Fig 3.51(c)

	F_0A_1	F_1A_1	F_2A_1	
$\bar{F}_0$	22 (24)	0 (1)	0 (0)	$\bar{F}_0$
$\bar{F}_1$	3 (1)	22 (22)	0 (0)	$\bar{F}_1$
$\bar{F}_2$	0 (0)	3 (2)	25 (25)	$\bar{F}_2$

$U_{A_1}[F:\bar{F}]$ = 1.2248 bits

(c) Submatrix for A_2 derived from Fig 3.51(c)

	F_0A_2	F_1A_2	F_2A_2
$\bar{F}_0$	23 (24)	0 (1)	0 (0)
$\bar{F}_1$	2 (1)	24 (22)	0 (0)
$\bar{F}_2$	0 (0)	1 (2)	25 (25)

$U_{A_2}[F:\bar{F}]$ = 1.3675 bits

(d) Submatrix for F_0 derived from Fig 3.51(c)

	F_0A_0	F_0A_1	F_0A_2	
$\bar{A}_0$	22 (22)	0 (2)	2 (0)	$\bar{A}_0$
$\bar{A}_1$	3 (3)	16 (18)	1 (9)	$\bar{A}_1$
$\bar{A}_2$	0 (0)	9 (5)	22 (16)	$\bar{A}_2$

$U_{F_0}[A:\bar{A}]$ = 0.8595 bits

(e) Submatrix for F_1 derived from Fig 3.51(c)

	F_1A_0	F_1A_1	F_1A_2	
$\bar{A}_0$	25 (22)	3 (2)	1 (0)	$\bar{A}_0$
$\bar{A}_1$	0 (3)	18 (18)	6 (9)	$\bar{A}_1$
$\bar{A}_2$	0 (0)	4 (5)	18 (16)	$\bar{A}_2$

$U_{F_1}[A:\bar{A}]$ = 0.8583 bits

(f) Submatrix for F_2 derived from Fig 3.51(c)

	F_2A_0	F_2A_1	F_2A_2
$\bar{A}_0$	23 (22)	4 (2)	2 (0)
$\bar{A}_1$	2 (3)	17 (18)	11 (9
$\bar{A}_2$	0 (0)	4 (5)	12 (16)

$U_{F_2}[A:\bar{A}]$ = 0.5524 bits

Fig 3.55. Submatrices a,b, and c show the identifiability of F_0, F_1, F_2 over different values of the A dimension. Similarly submatrices d, e, and f show the identifiability of A_0, A_1 and A_2 over different values of the F dimension. The entries in brackets in each of the cells of these submatrices give the expected value, derived from Fig 3.53B, assuming state independence. Notice, if there is state independence then there is no interaction.

psychology in the early 1960s. Information theory still seems to sustain, at least for me, a definite metaphor or an image of psychological processes, whereas the term uncertainty analysis is relatively neutral.

3.51 Uncertainty Analysis and Perceptual Dimensions

The Experimental Paradigm (Fig 3.51) shows the data and conditions of an actual experiment, reported in the study at the end of the chapter. The two experiments 2A and 2B have exactly the same design, and the data shown in Fig 3.51 is from a single subject (I.S.) in experiment 2A.

There are two experimental conditions

(a) A control condition in which the separate component dimensions Frequency (F), and Loudness (A) are presented in noise. The two dimensions are labelled F and A with values:

(i) A_o, A_1, A_2, on dimension A
(ii) F_o, F_1, F_2, on dimension F.

(b) The bidimensional condition: In this situation, the subject was presented one or other of:

F_oA_o, F_oA_1, F_oA_2, F_1A_o, F_1A_1, F_1A_2,

F_2A_o, F_2A_1, F_2A_2 (orthogonal stimulus presentation).

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F_oA_o, F_oA_1, F_oA_2, F_1A_o, F_1A_1, F_1A_2,

F_2A_o, F_2A_1, F_2A_2 (orthogonal stimulus presentation).

In both conditions, the set of stimuli was specified and each stimulus was labelled and learnt in the no noise trials. When they had been learnt the stimuli in both conditions were presented for identification a large number of times with the order of presentation randomised.

From this experimental set-up 3 sets of stimulus response matrices can be collected.

(i) Two sets of stimulus response (S-R) matrices for the control condition Fig 3.51(a) and (b).

(ii) The bidimensional S-R matrix Fig 3.51(c).

The super posed bar indicates the response to the stimulus. Each S-R matrix has as its entries p(S, R) the probability of identifying the Sth stimulus with the Rth response.

3.52 The Control Condition

When the first dimension, say F, is presented the value of the second dimension (here A) is constant across all values of unidimensional variation.

From the data derived from the control condition the following uncertainty measures can be obtained:

(1) $U\,(F{:}\bar{F})_{1D}$, and $U\,(A{:}\bar{A})_{1D}$. Both are measures of the identifiability of the dimensions in the unidimensional (control)

condition. The subscript "ID" indicates this measure is obtained from the control condition (see Fig 3.51(a) and (b)).

(2) U_{FA} (F:A). This is the quantity which Garner and others refer to as a measure of perceptual independence. It is regarded here as a measure of state independence. See Fig 3.52 a, b, c, d, e, f, g, h.

Uncertainty Measures obtainable from the Control Condition

(1) $U(F:\bar{F})_{1D}$ and $U(A:\bar{A})_{1D}$.

Garner (1962), Attneave (1959) both show that

$$U(F:\bar{F}) = U(F) + U(\bar{F}) - U(F,\bar{F}) \qquad 3.531$$

Both authors demonstrate very clearly how to obtain this measure in an actual instance. Garner (1962) refers to $U(F:\bar{F})$ as a contingent uncertainty term, it is a measure of the amount of correlation rather than degree of correlation.

(2) U_{FA} $(\bar{F}:\bar{A})$ - The quantity usually identified as perceptual independence

Fig 3.52 shows the two control S-R matrices of Fig 3.51 cast in a different form (as conditional probability matrices) for convenience. From these two unidimensional matrices the submatrices c, d, e, f, g, h, i, j, and k are generated on the assumption of state independence. From these submatrices a theoretical bidimensional S-R matrix can be generated (Fig 3.53(a)) and also a performance matrix (Fig 3.53B). It was noted, for instance, that Corcoran obtained estimates of the unidimensional S-R matrices from the empirical bidimensional matrix and then generated a theoretical bidimensional matrix. It was also noted that it is, in fact, a matter of empirical investigation as to whether the "estimated" unidimensional S-R matrices were the same as the actual unidimensional S-R matrices.

Now, U_{FA} $(\bar{F}:\bar{A}) = 0$ if state independence exists. That is, the identifiability of particular values on a stimulus dimension is completely unaffected by the values on the other dimensions. Note also, that U_{FA} $(\bar{F}:\bar{A})$ can only properly be obtained from data obtained from the unidimensional conditions.

3.53 Uncertainty Measures Derived from the Bidimensional Stimulus Response Matrix

From the bidimensional stimulus-response matrix the following uncertainty measures can be derived:

3.531 (i) $U(F:\bar{F})_{2D}$ and $U(A:\bar{A})_{2D}$

3.532 (ii) $U(FA:\overline{FA})$

3.533 (iii) $U_A(F:\bar{F})$ and $U_F(A:\bar{A})$

$U(A:\bar{A})_{2D}$ is a measure of the identifiability of the A dimension ignoring the effects of F. Similarly $U(F:\bar{F})_{2D}$ is a measure of the identifiability of the F dimension ignoring the effects of A. (Kaufman & Levy 1971, Garner 1962).

Fig 3.54(a) and (b) give an example of this, the numbers in the bracket give the expected value of these two submatrices on the assumption of independent dimensions (state independence).

It is instructive to compare:

$U(F:\bar{F})_{2D}$ = 1.1738 bits $U(A:\bar{A})_{2D}$ = 0.6255 bits

$U(F:\bar{F})_{1D}$ = 1.2986 bits $U(A:\bar{A})_{1D}$ = 0.6956 bits

The identifiability of both the F and A dimension is better in the unidimensional context than in the bidimensional context.

$U(FA:\overline{FA})$ This is a measure of the total amount of information transmitted in the bidimensional context, and in our example

$$U(FA:\overline{FA}) = 1.9405 \text{ bits} \quad \text{Fig 3.51(c)}$$

If state independence held we would have

$$U(FA:\overline{FA}) = 2.03695 \text{ bits} \quad \text{Fig 2.53(a)}$$

It does seem that the dimensions are not independent.

3.54 Interaction on the Dimensions

Fig 3.55 illustrates dimensional interaction: the way in which the identifiability of the values of the F (say) dimension varies over different values of the A dimension. Each of the submatrices have an uncertainty measure associated with it, for example from submatrix 3.55b we have:

$$U_{A1}(F:\bar{F}) = 1.2248 \text{ bits}$$

This gives a measure of the identifiability of the F dimension when the A dimension is constant at level A_1. The weighted mean of the measures $U_{A0}(F:\bar{F})$, $U_{A1}(F:\bar{F})$, and $U_{A2}(F:\bar{F})$ signified by $U_A(F:\bar{F})$ reflects the effects of the A values on the overall identifiability of the F component (Kaufman & Levy 1971). If there is state independence there is no dimensional interaction: the identifiability of particular levels of one dimension is the same irrespective of the levels of the other dimension. A similar analysis for ordinal data is made in experiments 3A, 3B and 3C in chapter 4.

3.55 Independence, Selective Dimensional Attention, and the Independent Realisation of the Dimensions

Experiments on selective dimensional attention typically have two conditions:

(1) A control Condition

(2) Selective Attention Condition

(1) The Control Condition: The control experiment is always one which requires the subject to perform some task with a single information source. How well he does in this case provides the base measure by which to compare performance on other tasks. The performance measures are usually either time or accuracy in identification performance (measured for instance by information transmitted). The experiment illustrated in Fig 3.51 has such a control condition. Here the single information sources are the dimension F and A and the baseline measures are provided by quantities $U(F:\overline{F})_{1D}$ and $U(L:\overline{L})_{1D}$.

(2) Selective Attention Condition: Here a set of stimuli is generated from an orthogonal combination of the two or more stimulus dimensions of the control condition. In the experiment just mentioned, the set of stimuli correspond to the nine stimuli presented, randomly, in the bidimensional condition. In the usual dimensional selective attention experiment (Garner & Felfoldy 1970, Egeth and Pachella 1969, Imai and Garner 1965, Morgan and Alluisi, 1967) the subject, in this condition, is required to respond differentially to just the levels on one dimension while ignoring the other dimension. The identifiability of the dimension, or some other performance measure, taken from this condition is then compared with the base line performance for that same dimension in the control condition. This comparison is usually taken as an index of selective attention; of the ability of the subject to treat the two or more dimensions as separate and independent. If baseline performance and experimental condition performance are the same, then perfect selective attention is possible and the two dimensions are independent - i.e. there is an independent realisation of the dimensions.

To clarify the preceding remarks consider a card sorting task reported by Imai & Garner (1965). This experiment corresponded logically to the requirements of a selective attention experiment: a set of stimuli was generated from orthogonal combinations of two or more stimulus dimensions. The subject was required to respond differentially to just the two levels on one dimension by sorting a deck of cards. The dimension by which sorting or responding was to be done was the relevant dimension and the other dimensions were irrelevant. The experimental question usually concerns whether the existence of irrelevant dimension slows performance compared to that when there is no irrelevant dimension (control condition).

Imai & Garner used a card sorting task with stimuli generated from three dichotomous dimensions. The stimuli were two dots on a card and these dots varied on the 3 "dimensions"

(i) horizontal position
(ii) orientation
(iii) and distance between the dots

They found no interference due to the addition of irrelevant dimensions compared to the control condition, that is, these dimensions were independent.

3.56 Composite Responses, Inferring Identifiability of the Component Dimensions, and Selective Dimensional Attention

In the response scheme used by Corcoran (1967) and the one employed in the experiment illustrated in Fig 3.51 a composite response in answer to a stimulus was demanded.

Each of Corcoran's four stimuli were labelled $\overline{A}$, $\overline{B}$, $\overline{C}$ or $\overline{D}$ respectively whilst in the experiments reported here (Experiment 2A and 2B) each stimulus was labelled by a number. The subject was never required to identify a particular level of a dimension as were Imai and Garner's subjects. On the face of it, it appears then, that the experimental paradigm illustrated in Fig 3.51 does not allow us to measure selective attention with respect to the stimulus dimensions.

Fortunately, the uncertainty analysis we have employed gives a procedure whereby the identifiability of both the A and F dimensions in the bidimensional situation can be <u>inferred</u> and is given by the quantities:

(i) $U(F:\overline{F})_{2D}$

(ii) $U(A:\overline{A})_{2D}$

These two quantities, it will be remembered, are measures of the identifiability of the F and A dimension in the bidimensional condition. Also these two measures can be compared with the baseline measures $U(F:\overline{F})_{1D}$, and $U(A:\overline{A})_{1D}$ obtained from the control condition. Hence if

$$\text{inferred } U(F:\overline{F})_{2D} = U(F:\overline{F})_{1D}$$

$$\text{and inferred } U(A:\overline{A})_{2D} = U(A:\overline{A})_{1D}$$

then, as far as these two dimensions are concerned there exists an independent realisation of dimensions, that is perfect selective attention is possible, and as there is no interference due to "irrelevant" dimensions-state independence exists.

3.57 Analysis of Variance and Uncertainty Analysis

The contingent uncertainty term which we have used should be interpreted with some caution. It has for example been suggested as a measure of the amount of information about stimulus events that have been transmitted through the subject (Hake and Garner 1951). The general finding has been, in experiments on absolute judgments, that as the number of categories increases the number of erroneous judgments also increases. The information transmitted (contingent uncertainty) however reaches a maximum level which is maintained as the set of stimulus categories is made larger. MacRae [1970] has pointed out that this is partly an artifact of experimental design: the number of presentations per stimulus did not increase sufficiently rapidly in these experiments with increases in the number of stimuli. The result is an overestimation of $U(X:\bar{X})$ by an amount that increases with $U(X)$. MacRae modified some of the earlier studies by correcting them for bias. This corrected data exhibited a falling off in transmission as the number of stimulus categories increased beyond an optimum.

Garner [1962] suggests an analogy between contingent uncertainty and a main effect variance in analyses of variance. Indeed Garner (1962), ~~Chapt.~~ chapter 5, and Kaufman & Levy (1971) have expressed a term like $U(FA:\overline{FA})_{2D}$ in terms of 'main effects' and interactions where

$$U\left[FA:\overline{FA}\right]_{2D} = U\left[A:\overline{A}\right]_{2D} + U\left[F:\overline{F}\right]_{2D} +$$

(interaction terms of the sort we have discussed)

See Kaufman & Levy (1971), equation 1.

The terms $U\left[A:\overline{A}\right]_{2D}$, and $U\left[F:\overline{F}\right]_{2D}$ correspond to the two main effect terms. The interaction terms referred to above have to do with the amount of $\overline{FA}$ response predictable from unique combinations of F and A stimuli.

3.60 Testing for Independence from Data derived from Master Matrices

Before the experimental work is discussed one final methodological point. Much empirical investigation to do with independence has been carried out on data which derives from experiments having the same logical structure as Conrad's (1964) study. Conrad obtained a confusion matrix for the letters B, C, P, T, V, F, M, S, X, presented auditorily through noise to 300 subjects. A good deal of the work on independence has involved constructing a sub-matrix of confusions from the 'master matrix' (Corcoran et al 1968, Smith 1972). This is achieved by selecting a subset of stimuli and responses from the master matrix and normalising the resulting submatrix to yield estimates of the confusion between the stimulus subset. A submatrix, derived from a master matrix, by normalising is implicitly making use of the constant ratio rule (CRR) first proposed by Clarke 1957 . The CRR when employed on Conrad's data, for example, predicts the acoustic confusions between letters when only a fraction of the original stimulus set is being presented and when the confusions between the letters for the master matrix is known.

In the general case, Clarke's CRR is an empirical law which states that the predicted and observed confusability of any particular submatrix are identical and that both are independent of the other members of the stimulus set. This implies, for example, that if a specific submatrix was embedded in master matrices of different sizes then the derived confusabilities would be invariant with the size of the master matrix.

The CRR is also known as Luce's choice axiom (Luce 1959) and the operation of normalising boils down to the use of one of the several equivalent forms of Luce's axiom - the one known as the law of irrelevent alternatives - which states that preferences between objects do not change when others are added to or subtracted from the overall set of objects.

The propriety of normalising submatrices obtained from a master matrix in order to test for independence is suspect. Conceptually it has the same status as Corcoran's procedure for testing independence. It will be remembered, that independence was assumed in order to obtain estimates of unidimensional performance. Further, these estimates were then used to generate a theoretical bidimensional matrix, on the assumption of independence. This theoretical bidimensional matrix was then employed as a baseline by which to compare the original confusion matrix in order to assess whether the component dimensions were independent.

The use of the CRR to predict confusions in submatrices from the data of a master matrix and then testing these obtained confusions for independence is similarly vulnerable to logical circularity. This is because the CRR is, itself, a statement about independence and its use on a master matrix to produce a submatrix of confusions accordingly makes strong 'independence' assumptions on the very data which is itself being tested for independence. The use of the CRR in such cases then, is quite objectionable; the assumption behind the CRR that a submatrix embedded in master matrices of different sizes would have identical confusabilities is an empirical matter and therefore is the concern for experimental verification. The employment of the CRR and then the subsequent investigation of independence on a subset of the same data serves only at the very least, to confound different forms of independence.

3.70 THE EXPERIMENTS

The two experiments reported here attempt a fairly fine grained investigation of the independence of the dimensions of auditory stimuli. The dimensions are examined two at a time and they are:

Expt 2A : Pitch and loudness (constant duration)

Expt 2B : Pitch and Duration (constant loudness)

The design is fairly similar to Corcoran's (1967) study but there are some differences which were introduced in the light of the criticisms advanced in section 3.40:

(i) In each of the two experiments there is a control condition in which each of the two component dimensions is presented to the subject. The data from the two resulting confusion matrices is used to generate the predicted bidimensional matrix on the assumption of state independence. This enables the predicted value of $U[FA:\overline{FA}]$ to be compared with the 'empirical' value from the subject's actual bidimensional performance.

(ii) Each of the component dimensions vary on three values, instead of only two as in Corcoran's experiment. If there are interaction effects they should be more obvious on dimensions which vary along 3 levels.

(iii) Analysis is performed on the data from individual subjects.

In experiment 2A the values selected for the dimensions of pitch (F) and Loudness (A) are known a priori, to be non-independent (Riesz, 1928, Stevens 1937). It seems probable that there will be interaction effects for individual Ss - the identifiability of pitch will vary, very likely, over different levels of loudness. It is postulated that for individual subjects the deviation between the empirical and theoretical values of $U[FA:\overline{FA}]$ will be larger in most cases than the deviation between the two quantities for the data pooled over all the subjects. This is predicted because the interaction effects are thought to vary from subject to subject and the net effect of pooling is to eliminate interactions and so induce a spurious independence of dimensions which are already known not to be so.

Another and related motivation behind the experiments is to suggest that a reasonable defining property for psychological dimensions in an identification task if that they should exhibit state independence: in particular that the identifiability of a value on one dimension should be independent of the values of the other dimension.

3.71 Experiment 2A

Method: In this experiment the two stimulus dimensions investigated were pitch (F) and loudness (A). There were three levels on each of these components with values F_0, F_1 and F_2 and A_0, A_1, A_2 respectively.

The data for each subject was collected from two conditions:

(i) The two control conditions in which each of the component dimensions F and A were presented randomly in noise for identification.

(ii) The bidimensional condition. In this condition the orthogonal combination of the three values of each of the two component dimensions was presented to the S for identification on noise. Again the presentation was randomised and the 9 stimuli were:

F_oA_o, F_oA_1, F_oA_2, F_1A_o, F_1A_1. F_1A_2,

F_2A_o, F_2A_1, F_2A_2.

3.72 Procedure

The two experimental conditions were divided into two parts:

Control Condition	(a)	Training in identification
	(b)	Noise presentation
Bidimensional Condition	(a)	Training in identification
	(b)	Noise presentation

The data for each subject was collected in the noise presentation situation from both conditions.

The tones were delivered through headphones to the subject who was sitting in a sound proof booth. In front of him was a response panel with 9 'button-light' pairs. Each of the 9 tones was paired to a particular response button with a light just above it. The task was essentially to learn the appropriate button for the tone. As soon as the response button was pressed a light lit up for 500 m secs, above the correct response button giving instant feedback as to the correctness of the response. Each subject was trained to identify the values on each of the component dimensions, until they reached a criterion of 100% correct identification over 60 randomised stimulus presentations.

When this was achieved, the component stimuli was presented in noise, the level of which was adjusted for each S so that in the unidimensional condition they achieved a hit rate of around 70%. The noise level was kept constant for all subsequent conditions for that particular subject. Each of the levels of the component dimensions was presented to the S fifty times in a randomised presentation, i.e. there were 150 stimulus presentations.

In the bidimensional condition exactly the same procedure was followed. First the subject was trained to identify the bidimensional stimulus by pressing the appropriate response button in the no noise presentation. This constituted a composite response to the compound auditory stimulus. When the subject had learned the 9 stimuli (according to the same criteria as before) they were presented each one randomly twenty-five times at the same

TABLE 3.71 Frequency and loudness (EXPT 2A). Identifiability of stimulus dimensions in the control and bidimensional conditions.

	F dimension pitch		A dimension loudness				
SUBJECT	CONTROL $U[F:\bar{F}]_{1D}$ Max=1.5849 bits BITS	$U[F:\bar{F}]_{2D}$ Max:1.5849 bits BITS	CONTROL $U[A:\bar{A}]_{1D}$ Max:1.5849 BITS	$U[A:\bar{A}]_{2D}$ BITS	Empirical $U[FA:\bar{FA}]$ Max:3.1699 BITS	Theoretical $U[FA:\bar{FA}]$ (assuming state independence BITS	Deviation from theoretical value of $U[FA:\bar{FA}]$ BITS
T.McC.	1.3197	0.9091	0.9453	0.3585	2.0062	2.2318	0.2256
I.S.	1.2986	1.1738	0.6956	0.6255	1.9405	2.0369	0.0964
A.J.	1.5846	1.5047	0.6920	0.4538	2.0935	2.3083	0.2148
D.Y.	1.2916	1.3267	0.8685	0.6364	2.3982	2.1806	- 0.2176
L.H.	1.5846	1.5535	0.7939	0.3625	2.2845	2.2099	- 0.0746
N.F.B.	1.5096	1.4456	0.4793	0.4561	1.7428	1.2020	- 0.5408
			POOLED DATA FOR THE SIX SUBJECTS		2.0869	2.0282	0.0587 bits

TABLE 3.72 [Frequency and Loudness] EXPT 2A Dimensional Interaction Effects

	Identifiability of F over dimension A			Identifiability of A over dimension F		
	"A_0" $U_{A0}[F:\bar{F}]$ BITS	"A_1" $U_{A1}[F:\bar{F}]$ BITS	"A_2" $U_{A2}[F:\bar{F}]$ BITS	"F_0" $U_{F0}[A:\bar{A}]$ BITS	"F_1" $U_{F1}[A:\bar{A}]$ BITS	"F_2" $U_{F2}[A:\bar{A}]$ BITS
.McC.	1.0409	1.5040	1.0670	0.6655	0.4733	0.8714
.S.	0.9950	1.2248	1.3675	0.8595	0.8583	0.5524
.J.	1.5834	1.4491	1.5834	0.6708	0.5204	0.5645
.Y.	1.1208	1.4247	1.3670	0.8647	0.7129	0.5614
.H.	1.5834	1.5834	1.5040	0.8741	0.5528	0.7087
.F.B.	0.5636	1.4491	1.5834	0.2880	0.4963	0.5718
"POOLED"	0.9494	1.3670	1.3508	0.6280	0.5299	0.5554

noise level as in the two control conditions, i.e. for 9 stimuli there were 225 stimulus presentations.

Subjects: The subjects were 6 first year psychology students. The subjects volunteered for this experiment in return for ~~dredits~~ credits necessary to fulfil course requirements. Each S required about seven hours to complete the experiment and in view of the time, they were exempted from any further calls from the psychology department subject pool for the whole year. Each of the seven experimental sessions lasted a little over the hour.

Stimuli & Equipment

The stimulus values were

	A_o = 20 db
'Loudness'	A_1 = 30 db
	A_2 = 40 db
	F_o = 300 Hz
'Pitch'	F_1 = 700 Hz
	F_2 = 1000 Hz

The duration of each of the tones was 750 m secs.

3.73 Results

Table 3.71 summarises the data from the six subjects who took part in the experiment. The first four columns

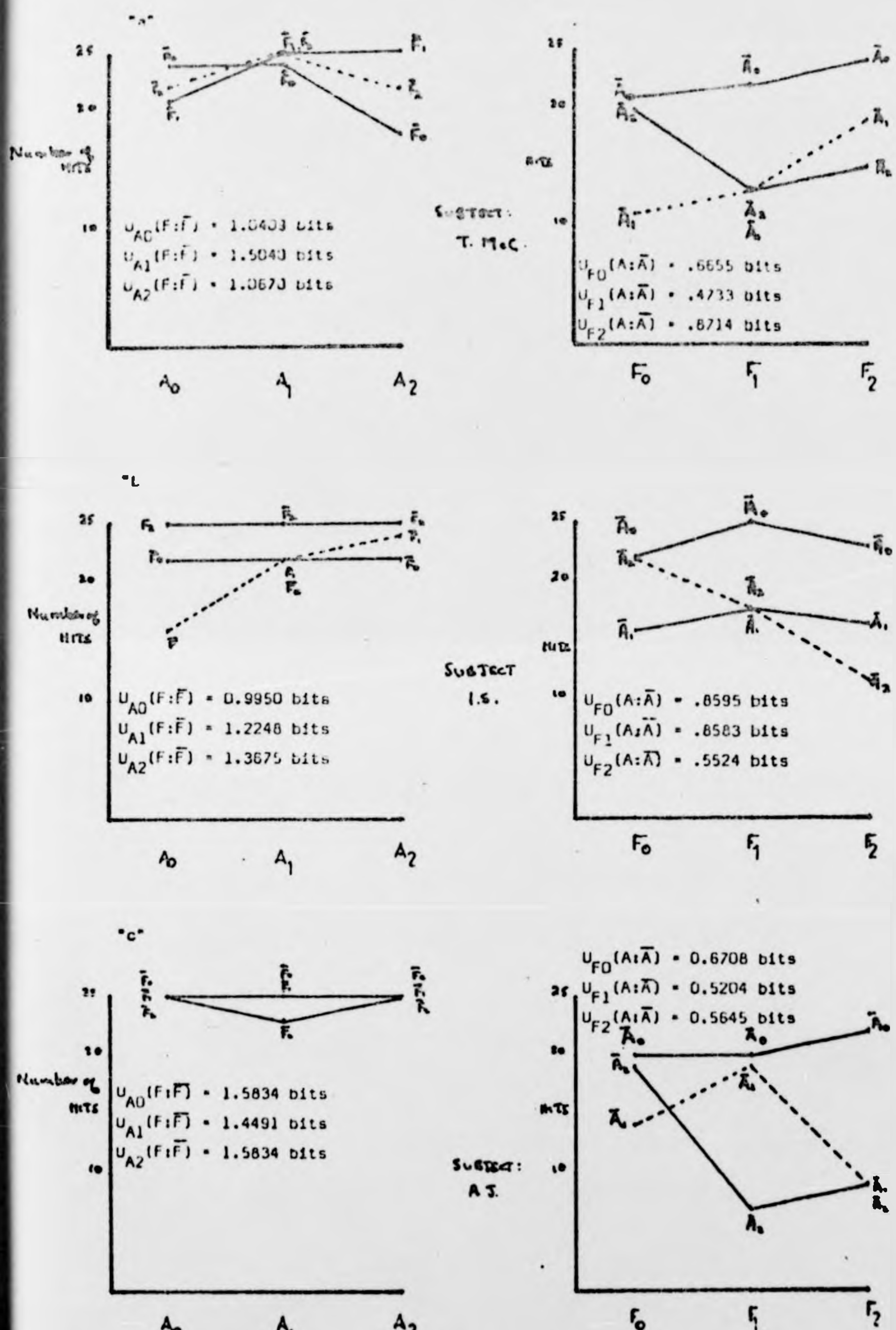


Fig 3.71 EXPERIMENT 2A Identifiability of Pitch (F) over different values of loudness (A) (and vice versa.)

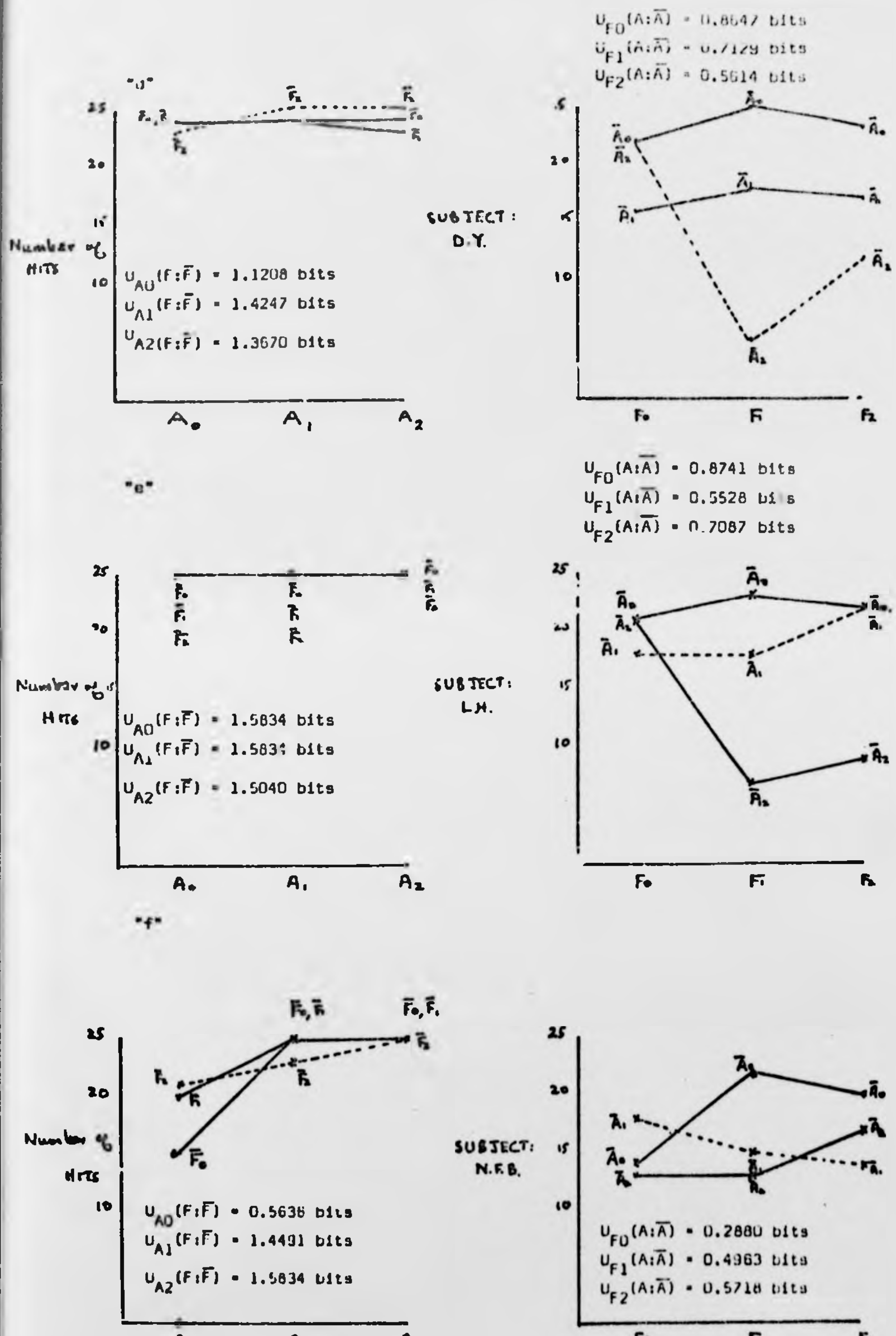


Fig 3.71 (continued). EXPERIMENT 2A. Identifiability of Pitch (F) over different values of Loudness (A) (and vice versa).

compare the identifiability of the F and A dimensions in the control condition with that of the bidimensional condition.

For all subjects (except D.V), $U[F.\bar{F}]_{2D}$ the inferred identifiability of the F dimension in the bidimensional condition - was found to be less than $U[F.\bar{F}]_{1D}$.

Also it was found that $U[A:\bar{A}]_{2D} < U[A:\bar{A}]_{1D}$ for all subjects.

Hence on this criteria the dimensions do not appear to be independent.

The last two columns of table 3.71 compare, for each subject, the theoretical value of $U[FA:\overline{FA}]$ with the empirical value. This theoretical value is obtained by generating the theoretical or predicted bidimensional matrix from the unidimensional stimulus-response matrix on the assumption of state independence. This procedure is illustrated in Fig 3.52. The theoretical and predicted values would be equal if state independence held.

Fig 3.71 shows the value obtained for $U[FA:\overline{FA}]$ - both empirical and theoretical - for the data pooled over six subjects.

The theoretical value was obtained from the two control conditions for the data pooled over the six subjects. The theoretical value was obtained as before. The empirical value of $U[FA:\overline{FA}]$ was obtained by pooling all the bidimensional stimulus response matrices.

The difference between the theoretical value and the empirical value is 0.0587 bits. This is smaller than any of these differences obtained for individual subjects. It seems then, that pooling data, even only over six subjects, has the effect of making the dimensions 'independent'.

Reference to Fig 3.71 and table 3.72 provides a clue as to why on an individual subject basis independence breaks down. The graphs in Fig 3.71 illustrate how the identifiability of the different levels of pitch varies over different levels of loudness (and vice versa). It is obvious that all the subjects were able to identify the value of pitch better than loudness; and moreover, the identifiability of loudness over different pitch values displays more variation for most subjects than the identifiability of pitch over different values of loudness. Table 3.72 shows how the identifiability of one of the dimensions varies with the level of the other.

Fig 3.73 and table 3.7[illegible](a) gives the same dimensional interactional effects pooled over six subjects. Fig 3.73

Fig 3.73 (a) Pooled Data for EXPERIMENT 2A. (Identifiability of Pitch (F) over different values of loudness (A) and vice versa.)

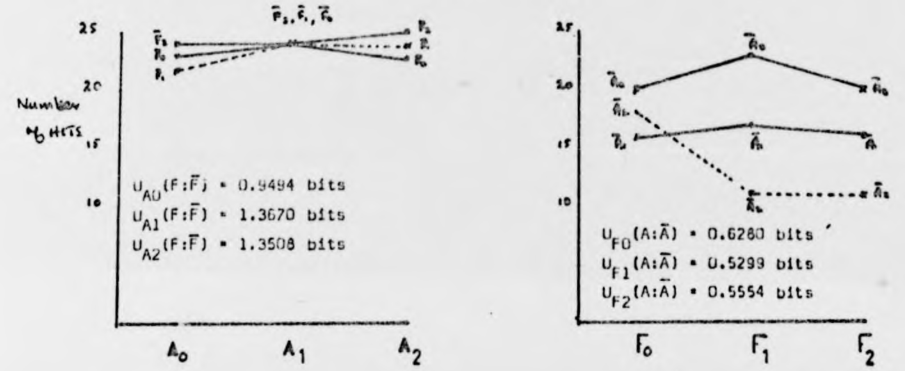
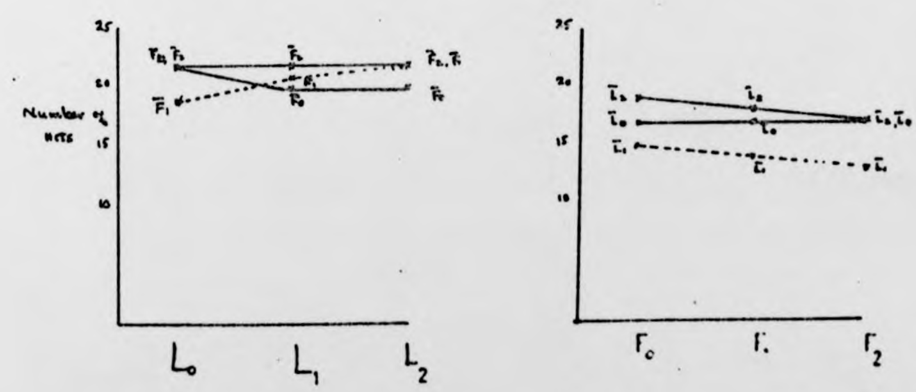


Fig 3.73 (b) Pooled data for EXPERIMENT 2B. (Identifiability of Pitch (F) over different values of Duration (L) and vice versa.)



graphs the 'pooled' dimensional effects and this should be compared with Fig 3.71 which shows shere the interactions occur on an individual basis. It is patently clear that pooling over six subjects acts to 'wipe' out interaction effects. This is seen if, for instance, the identifiability of F over A is considered: the largest variation in identifiability of F over A is for subject N.F.B. where the largest range - the difference between the smallest value and the largest value of identifiability measured in bits - is 1.0194 bits. The smallest variation in identifiability is for subject L.F. and is 0.0000 bits.

The way pooling eliminates interactions is just as dramatic for the identifiability of A over dimension F. For the pooled data the greatest range in identifiability is 0.0981 bits whereas the largest range for an individual subject is 0.3981 bits (T.Mcc).

3.74 Conclusions

The dimension pitch (F) and loudness (A) do not appear to be independent on any of the criteria advanced. When the individual subject is considered:

(a) The theoretical and empirical values of $U[FA:\overline{FA}]$ should be the same if state independence exists (the theoretical bidimensional matrix is obtained from S's confusion matrices in the two control experiments. These confusion matrices are used to generate the theoretical bidimensional matrix assuming state independence).

(b) For independent dimensions

$$U\left[A:\bar{A}\right]_{1D} = U\left[A:\bar{A}\right]_{2D}$$

and $$U\left[F:\bar{F}\right]_{1D} = U\left[A:\bar{A}\right]_{2D}$$

However if the data is <u>pooled</u> the criteria contained in paragraph (a) above, shows that the pooled data gives a better approximation to independence. Moreover this appears to be achieved by the pooled data masking the interaction effect.

This is quite a curious result: a priori it is known that for these particular values of frequency and loudness the dimensions are <u>not</u> independent (Geldard 1953, page 192) yet pooling the data appears to render these dimension 'independent'.

Experiment 2B: Pitch and Duration

The procedure for obtaining the responses was exactly as in experiment 2A except the stimuli were different.

The Control Stimuli were tones which varied only in:

(i) duration L_o, L_1, L_2 (3 levels)
(ii) loudness A_o A_1 A_2 (3 levels)

TABLE 3.74 [Frequency and Duration] EXPT 2B

Dimensional Interaction Effects. Bottom row gives the interaction effects for the data pooled over the six subjects

	Identifiability of F over dimension L			Identifiability of L over dimension F		
	"L_0" U_{L0} [F:$\bar{F}$] BITS	"L_1" U_{L1} [F:$\bar{F}$] BITS	"L_2" U_{L2} [F:$\bar{F}$] BITS	"F_0" U_{F0} [L:$\bar{L}$] BITS	"F_1" U_{F1} [L:$\bar{L}$] BITS	"F_2" U_{L2} [L:$\bar{L}$] BITS
J.McK.	0.4273	0.4614	0.2906	0.3760	0.1806	0.4721
S.M.	1.1683	1.3678	1.3713	0.9111	0.9558	0.8600
I.S.*	1.1384	1.2067	0.9013	0.2312	0.2531	0.1125
M.D.	1.1688	1.2348	1.5039	0.3616	0.6473	0.7076
C.H.	0.8657	0.9475	0.6828	0.5728	0.4246	0.5722
M.R.	1.0442	0.9818	0.8698	0.4898	0.6710	0.5756
POOLED DATA	0.9782	1.0333	0.9389	0.4959	0.5455	0.5574

TABLE 3.74 [Frequency and Duration] EXPT 2B

Dimensional Interaction Effects. Bottom row gives the interaction effects for the data pooled over the six subjects

	Identifiability of F over dimension L			Identifiability of L over dimension F		
	"L_0" U_{L0}[F:F̄] BITS	"L_1" U_{L1}[F:F̄] BITS	"L_2" U_{L2}[F:F̄] BITS	"F_0" U_{F0}[L:L̄] BITS	"F_1" U_{F1}[L:L̄] BITS	"F_2" U_{L2}[L:L̄] BITS
J.McK.	0.4273	0.4614	0.2906	0.3760	0.1806	0.4721
S.M.	1.1683	1.3678	1.3713	0.9111	0.9558	0.8600
I.S.*	1.1384	1.2067	0.9013	0.2312	0.2531	0.1125
M.D.	1.1688	1.2348	1.5039	0.3616	0.6473	0.7076
C.H.	0.8657	0.9475	0.6828	0.5728	0.4246	0.5722
M.R.	1.0442	0.9818	0.8698	0.4898	0.6710	0.5756
POOLED DATA	0.9782	1.0333	0.9389	0.4959	0.5455	0.5574

TABLE 3.73 [Frequency & Duration] EXPERIMENT 2B

Identifiability of the stimulus dimensions in the control and bidimensional conditions

	F dimension pitch		L dimension duration				
	CONTROL $U[\bar{F}:\bar{F}]_{1D}$ BITS	$U[\bar{F}:\bar{F}]_{2D}$ BITS	CONTROL $U[\bar{L}:\bar{L}]_{1D}$ BITS	$U[\bar{L}:\bar{L}]_{2D}$ BITS	Empirical $U[\bar{FA}:\bar{FA}]$ MAX:3.1699 BITS BITS	Theoretical $U[\bar{FA}:\bar{FA}]$ (assuming state in-dependence)	Deviation from theoretical value
J.McK.	0.4999	0.4720	0.2135	0.2817	1.0285	1.8467	0.8182
S.M.	1.4144	1.2697	0.4636	0.8665	2.2379	2.0871	-0.1508
I.S.*	0.8822	0.8731	0.4001	0.1796	1.5125	1.1433	-0.3692
M.D.	1.4196	1.2428	0.2542	0.3872	1.9635	1.7280	-0.2355
C.H.	1.1536	0.8152	0.4573	0.4392	1.8542	1.4473	-0.4069
M.R.	0.8111	0.9175	0.3596	0.5251	1.5827	1.1452	-0.4375
			"POOLED DATE" (For above six subjects)		1.6964	1.5663	-0.1300

Bidiemnsional Presentation 9 tones:

A_oL_o, A_oL_1, A_oL_2, A_1L_o, A_1L_1, A_1L_2, A_2L_o, A_2L_1, A_2L_2

randomised presentations.

Subjects: 6 undergraduate subjects from Stirling University subject pool.

Stimuli:

	A_o	=	20 db
Loudness	A_1	=	30 db
	A_2	=	40 db
	L_o	=	500 m secs
Duration	L_1	=	750 m secs
	L_2	=	1.00 sec

Frequency was constant throughout at 1200 Hz.

Procedure: Exactly as before.

3.81 Results

Table 3.73 summarises the data from the six subjects.

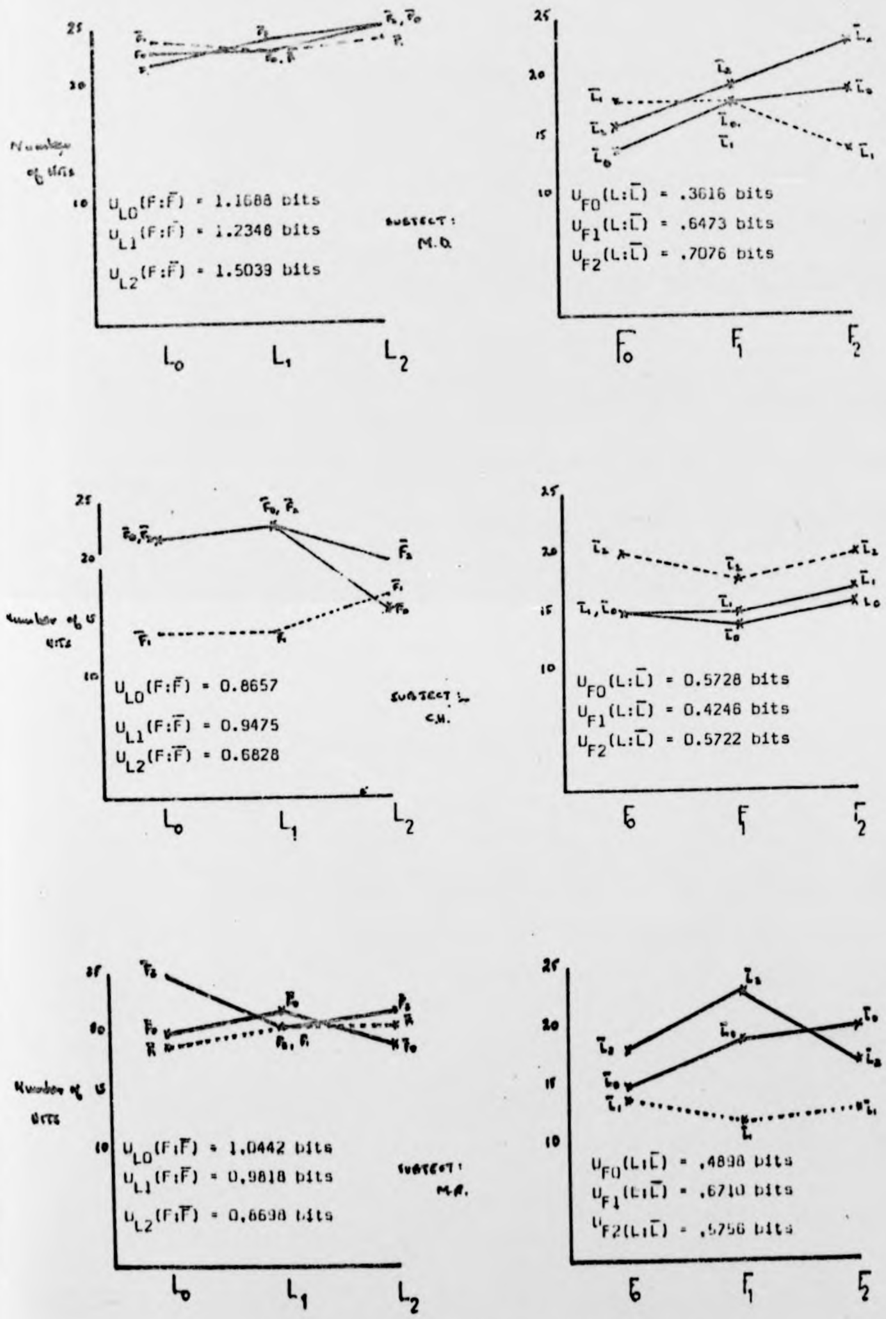


Fig 3.72 EXPERIMENT 2B: Identifiability of Pitch (F) over different Values of Duration (L) (and vice versa).

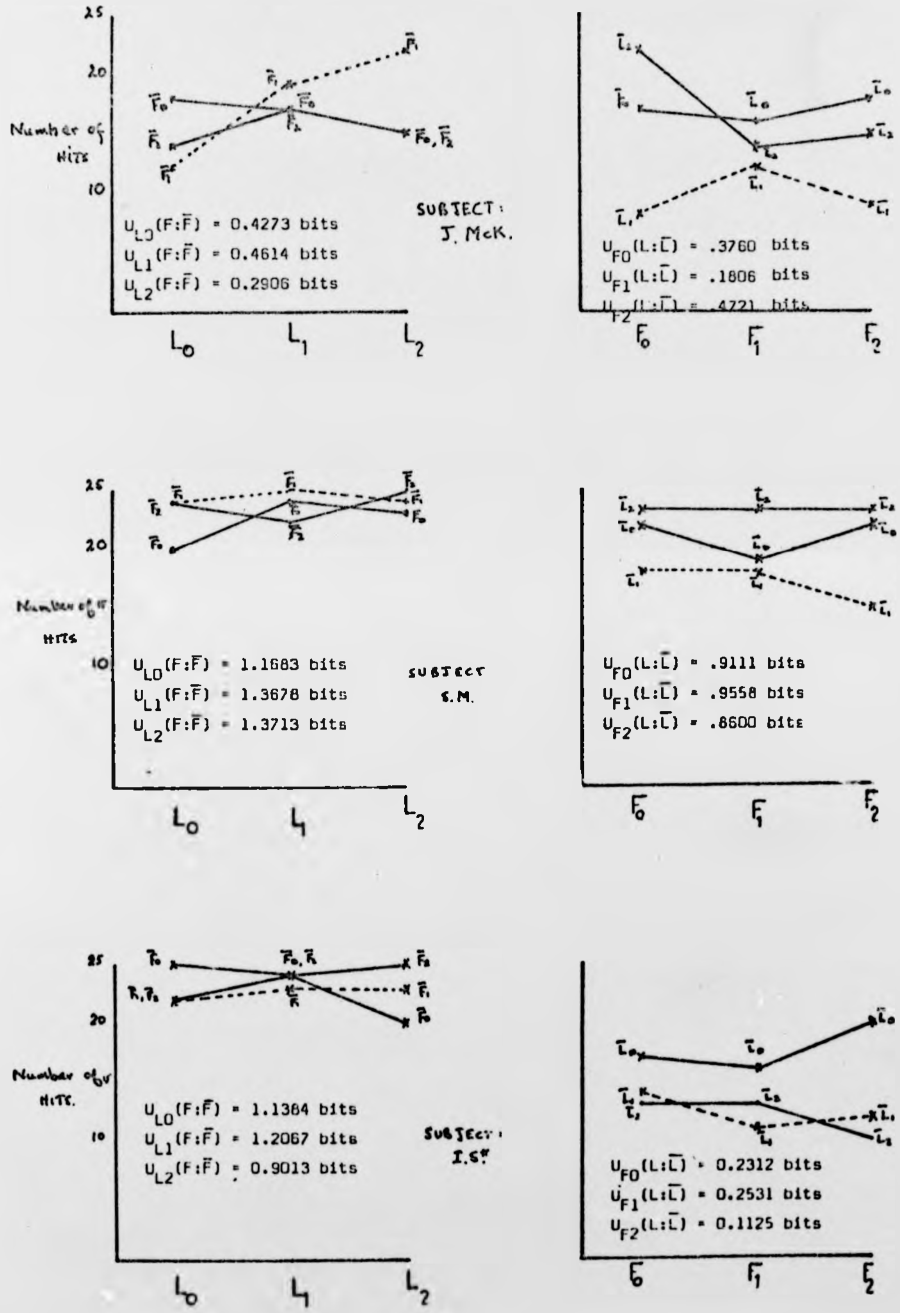


Fig 3.72(Continued) Identifiability of Pitch (F) over different values of Duration (L) (and vice versa).

On average, the inferred identifiability of F in the bidimensional condition, $U[F:\overline{F}]_{2D}$, is nearer to the identifiability of F in the control condition, $U[F:\overline{F}]_{1D}$ than was the case in experiment 2A.

The inferred identifiability of L in the bidimensional condition, $U[L:\overline{L}]_{2D}$ deviated fairly considerably from the identifiability of L in the control condition, $U[L:\overline{L}]_{1D}$.

It does not seem then, that the dimensions F and L are independent on this criterion.

When the theoretical and empirical values of $U[FA:\overline{FA}]$ are compared for each subject they deviate fairly considerably - the smallest deviations being 0.1508 bits above the corresponding predicted value. When the data is pooled the theoretical and empirical values are 1.6964 and 1.5663 bits respectively and the deviation from the theoretical value is .1300 bits. On the basis of pooled data the dimensions are 'independent'.

Fig 3.72 and Table 3.74 show the identifiability of F over L and vice versa.

The identifiability of F is better than L for all subjects.

Secondly if the data for the six subject is pooled and the interactions examined (Fig 3.73b and table 3.74 bottom row), it is again clear that pooling the data acts to eliminate interactions.

3.82 Conclusions

The interaction effects - the variation in identifiability of pitch for different values of duration and vice versa - are not nearly so marked as with the stimuli tested in experiment 2A.

3.90 Discussion

Both experiments were attempts at a 'fine grained' analysis of independence in identification tasks for the case when the stimulus attributes are presented in an orthogonal manner. The principle criticism of the operational definition of perceptual independence which corresponds to the procedure outlined in Fig 3.45 - is that it is devoid of psychological content: if independence does fail it offers no reason why it does so.

The suggestion that two dimensions are independent only when they do not interact in their effects, seems a reasonable one. For if the identifiability of one of the dimensions varied with different values of the other then the notion of a control experiment would be a curious one.

Secondly if the data for the six subject is pooled and the interactions examined (Fig 3.73b and table 3.74 bottom row), it is again clear that pooling the data acts to eliminate interactions.

3.82 Conclusions

The interaction effects - the variation in identifiability of pitch for different values of duration and vice versa - are not nearly so marked as with the stimuli tested in experiment 2A.

3.90 Discussion

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The suggestion that two dimensions are independent only when they do not interact in their effects, seems a reasonable one. For if the identifiability of one of the dimensions varied with different values of the other then the notion of a control experiment would be a curious one.

A control experiment in which for example, we determine the identifiability of the levels of one dimension, must necessarily involve keeping all the other (irrelevant) dimensions of the stimulus constant. If the performance in the control condition depended on the levels of the irrelevant background dimensions then this condition ceases to be a control, in any accepted sense of that word. It seems reasonable then, when embarking on experiments of selective dimensional attention in which the identifiability of the levels of a stimulus are compared with a control condition performance, to check whether there are interaction effects. If there are, we cannot logically talk of a control condition, nor can we compare performance in this spurious control condition with any other. In Chapter 5, an experiment is reported which tends to reinforce this suggestion. Experiments 2A and 2B, in essence, outlined a logically correct procedure to determine whether in fact two dimensions are independent for the case where the stimuli are presented orthogonally. We summarise this below.

(i) Obtain, for each dimension, a unidimensional performance matrix (see for example Fig 3.51 a and b); this is the "control" experiment, where all other irrelevant dimensions are kept constant.

(ii) Obtain the appropriate empirical bidimensional matrix.

(iii) From the two empirical unidimensional performance matrices generate a theoretical bidimensional matrix - on the assumption of state independence. This of course is tantamount to saying that the

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(iv) Compare this theoretical bidimensional matrix with the empirical one, to determine if state independence exists. If not, look at the dimensional interaction effects in the manner suggested in Fig 3.55. Such an examination might establish that not only are the dimensions not independent but might also diagnose why not. The submatrices in Fig 3.55 show how the identifiability of particular values of one dimension vary over different values of the other.

(v) One other indirect measure of independence, which is psychologically meaningful, had to do with selective dimensional attention discussed in Section 3.55. Selective dimensional attention has to do with the ability or inability of treating two physical attributes as separate or independent sources. It was suggested that if, for example:

$$\text{inferred } U(A:\bar{A})_{2D} = U(A:\bar{A})_{1D}$$

then the A dimension is independent of the other dimension (say F).

INDEPENDENCE IN SIMILARITY JUDGEMENT TASKS AND THEIR METRIC REPRESENTATION

4.70 Conclusions for Experiments 3A, 3B, and 3C.

4.80 Experiment 3D: Interdimensional Additivity for Tones varying in Frequency, Loudness, and Duration.

4.00 INDEPENDENCE IN SIMILARITY JUDGEMENTS AND THEIR GEOMETRIC REPRESENTATION

Introduction

In multidimensional scaling (MDS) similarity is assumed to relate to some form of distance defined on psychological dimensions. Similarity is taken as a primitive relation on pairs of stimuli and this relation is assumed to admit of degrees, that is, it is orderable. Thus similarity is defined at least on an ordinal scale.

MDS methods (Shepard,1962, Kruskal, 1964 and Torgerson, 1958) are designed to find the dimensions given the similarities. The typical input consists of an NXN matrix whose cell values indicate the similarity or dissimilarity between pairs of the N stimuli. These similarities or dissimilarities are assumed to measure the psychological distance between the stimuli.

The central idea underlying MDS models for perception is that stimuli are coded internally in terms of continuously varying parameters or dimensions and the task of the MDS techniques is to discover:

(i) The number of dimensions relevant to the perception of the stimuli under consideration.
(ii) The stimulus co-ordinates on each of these dimensions.

The problem of interpretation consists in identifying these physical correlates of the psychological dimensions.

Beals, Krantz, and Tversky(1968) and Tversky and Krantz (1970) have specified the qualitative (non-numerical) properties of ordinal similarity under which certain metric and dimensional representations of stimuli can be made. Both these theoretical papers discussed the metric and dimensional representation problems separately. They showed that each of the ordinal conditions sufficient for either a metric or dimensional representation can be satisfied without the other.

The strategy the authors adopted in both these papers was:

(a) First to describe the ordinal conditions for a very general type of geometric model which embodied the notion of distance.

(b) Independently of the above, to describe two defining properties of psychological dimensions - interdimensional additivity and intradimensional subtractivity together with the associated ordinal constraints.

(c) Given the constraints imposed by (a) and (b), to look for certain distance functions which would simultaneously satisfy both these conditions i.e. the metric and dimensional ones. They claim (Beals et al 1968, Tversky and Krantz 1970) that only one class of distance functions satisfy the constraints imposed by both (a) and (b) and this is one known as the Minkowski r-metric of which the Euclidean, City Block and Dominance metric are special cases.

4.10 METRIC REPRESENTATION OF SIMILARITY DATA

A metric is a scale that assigns to every pair of points x and y a number d(x,y) called their distance.

Consider a set X,and a function assigning a unique number d(x,y) to every pair (x,y) ε X x X (the Cartesian product). This function is called a distance function and the number it assigns, the distance between x and y, if and only if, d(x,y) satisfies the following conditions:

4.101 $d(x,y) \geq 0$ The distance between two points is never negative.

4.102 $d(x,y) = 0$ If and only if x=y. The distance between any two identical points is zero.

4.103 $d(x,y) = d(y,x)$ Distance is symmetric: the distance between x and y is the same as between y and x.

4.104 $d(x,y) + d(y,z) \geq d(x,z)$. The sum of the distances between any point (x and z) and a third point (y) is always greater than the distance between these points. This condition is known as the triangle inequality.

These four conditions are called the metric axioms. Since the similarities are assumed to be measured only ordinally, it seems quite reasonable to assume that among the class of permissable monotonic functions, there is at least one that will transform them into distances (i.e. act as distance functions).

4.12 The Minkowski r-metric.

One class of distance function that not only satisfies all the metric axioms but the other constraints noted in section 4.00 is known as the Minkowski r-metric. This metric is a one parameter class of distance functions defined as follows.

4.121 $$d_r(x,y) = \left[\sum_{i=1}^{N} \left| x_i - y_i \right|^r \right]^{1/r} \quad r \geqslant 1$$

As the parameter, r, varies three special cases are encountered:

(i) When r=1. This is the <u>city block</u> metric, so called because of the analogy of effective distance between points of a city laid out in a rectangular lattice of streets and blocks.

The city block metric takes the simple form

$$d_1(x,y) = \left[\sum_{i=1}^{N} \left| x_i - y_i \right| \right]$$

(ii) When r=2. This corresponds to the usual Euclidean metric:

$$d_2(x,y) = \left[\sum_{i=1}^{N} \left| x_i - y_i \right|^2 \right]^{\frac{1}{2}}$$

(iii) When $r = \infty$ This gives rise to the Dominance metric in which the distance is given by the largest co-ordinate difference.

Coombs, Dawes, and Tversky (1970) rationalise these three different metrics in terms of assumed psychological processes. For instance they follow many other authors (Jones 1962, Butter 1963, Cross 1965) by suggesting that the Dominance metric can be justified psychologically by invoking attentional factors; if the subject can attend to only one dimension at any one time, and if further he attends to that dimension on which the two stimuli are most different the resulting metric is the Minkowski dominance metric so called because the dimension with the largest difference dominates all others. Cross (1965 page 81) argues that the dominance model was implicitly employed by Erikson and Hake (1955) to explain their data. This is a conclusion very difficult to sustain in view of the analysis in section 3.20. Moreover, Erikson and Hake's data arose from an identification experiment, whereas the Minkowski r-metric model and hence the Dominance metric is derived from considerations of similarity data.

4.13 Equidistance Contours

One property of interest about any particular metric is the set of equidistance contours it induces (Beckenback and Bellman, 1961) Equidistance contours

Fig 4.10 Equidistance contours for various values of r.

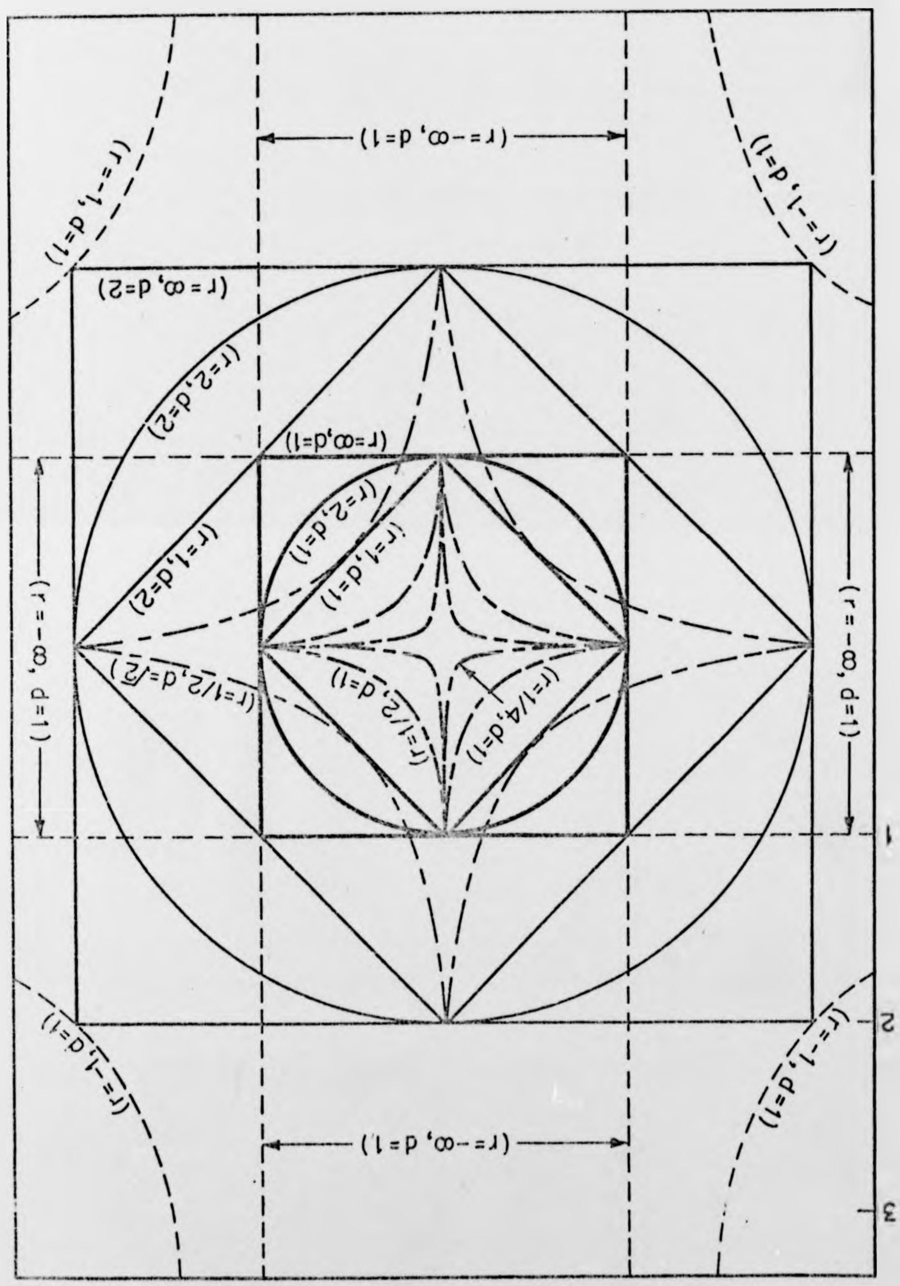


Fig 4.10

are the loci of equal distances from a particular point. Each point in the space has its own set of contours (see Fig 4.10). The Minkowski family of metrics also have the property that members of the family of Equidistance contours for a particular point are mutually similar. This means that contours for two different values of d,(distance) have the same shape although they have different sizes. If the origin is translated to the point whose contours we are considering,the curves for two different values of d are related by a multiplicative constant.

If the two different values of d are d_1 and d_2 the contour for the first will be

$$\frac{d_1}{d_2} \times \text{(the contour of the second curve.)}$$

See Beckenback and Bellman (1961).

4.14 Segmental Additivity

This 'constant ratio' rule for the contours implies that if three points i, j, and k (say) fall in a straight line (with j <u>between</u> i and k) then:

4.141 $$d_{ik} = d_{ij} + d_{jk}$$

This straight line property does not hold for all metrics, Blumenthal (1953), in fact there are some, in which there may be <u>no</u> point 'between' two artibrary points i and k.

The point j is defined as being between i and k if and only if:

$$d_{ik} = d_{ij} + d_{jk}$$

Beals et al use the term metric with additive segments to refer to a metric where there is always a point between every pair of points and they argue that only metric spaces with this segmental additivity property should be considered as candidates for perceptual spaces.

Beals et al continue the discussion of the metric representation of similarity data by describing the ordinal properties required to justify a very general type of geometric model. They showed that if six conditions hold their(P1 to P6) then it is possible to represent similarity measures by distance. In particular they demonstrated that one of these conditions - their P6 - is a necessary condition for similarity to be represented as a distance. This ordinal condition (P6), boils down to a kind of empirical constant ratio rule mentioned in section 4.13. It was noted then, that this property implies segmental additivity of the space (Blumenthal, 1953).

4.20 THE DIMENSIONAL REPRESENTATION OF SIMILARITY DATA

Beals, Krantz and Tversky (1968) and Tversky and Krantz (1970) have isolated two basic properties of subjective dimensions:

(i) interdimensional additivity

(ii) intradimensional subtractivity

To formulate these properties consider x, y, where

$$x = (x_1 \ldots\ldots\ldots\ldots x_N),$$

$$y = (y_1 \ldots\ldots\ldots\ldots Y_N),$$

denote two stimuli which vary along N dimensions, and let $\delta(x,y)$ be an ordinal scale measure of dissimilarity between x and y.

It is assumed that for any $x \neq y$

$$\delta(x,y) = \delta(y,x) > \delta(x,x) = \delta(y,y)$$

Using this index, interdimensional additivity asserts that the contribution of different dimensions to the overall dissimilarity between x and y are combined so that $\delta(x,y)$ is monotonically related to the sum of terms $\Phi_i(x_i,y_i)$ where x_i, y_i are the nominal scale values of x and y on the ith dimension and Φ_i is a symmetric real valued function taking the value 0, when $x_i = y_i$ and positive values otherwise. That is:

4.201 $$\delta(x,y) = F\left[\sum_{i=1}^{N} \Phi_i(x_i y_i)\right]$$

(Where F is a strictly increasing function of one variable).

Intradimensional subtractivity says that the contribution of any one dimension to dissimilarity depends on the absolute difference of the rescaled (interval scale) measures on that dimension. That is:

4.202 $$\delta(x,y) = F\left[\left|f_1(x_1) - f_1(y_1)\right| \ldots \left|f_N(x)_N - f_N(y)_N\right|\right]$$

F is function of N variables, strictly increasing in each variable and the f's are psychophysical functions that rescale the nominal variables $x_1, x_2 \ldots x_N$; to interval scale variables $f_1(x_1), f_2(x_2) \ldots f_N(x)_N$

The above equations describe two 'natural algebraic properties of subjective dimensions;

(i) additivity across dimensions

and

(ii) subtractivity along each dimension

Both properties are assumed in practically all MDS models of similarity judgement. On this basis, therefore, it seems natural to regard them as defining properties for subjective dimensions. This is essentially the position of Tversky and Krantz (1970).

4.21 Joint Factor Independence and Interdimensional Additivity

Equation 4.201 leads to joint factor independence laws: if the dissimilarity produced by some combination of difference on N-1 dimensions exceeds that produced by some other combination on those same dimensions (the difference on the Nth dimension remaining constant) then that same ordering holds as the constant difference on the Nth dimension is varied.

Hence, interdimensional additivity is simply a restatement of joint factor independence in a disguised form and Experiment 1B was not only a test for joint factor independence, but also for inter-dimensional additivity.

4.22 The Additive Difference Model of Similarity Judgements

According to this model (section 2.60 Chapter 2) dissimilarity judgements are described in terms of two sets of scales that apply to each one of the dimensions:

(a) The first set of scales $f_1 \ldots f_N$, applies directly to the physical input and describes its psychological counterpart along each one of the dimensions. These were called psycho-physical functions

(b) The second set of scales $\phi_1 \ldots\ldots \phi_N$, applies to the perceived component-wise differences along the dimensions, and describe their contribution to the overall dissimilarity between the stimuli. These were called similarity functions in Experiment 1A.

In the additive-difference model (a direct test was provided in Experiment 1A for two dimensions) dissimilarity judgements are deemed to be decomposed into two independent processes:

(i) A 'perceptual' process satisfying subtractivity.

(ii) An evaluative process satisfying interdimensional additivity.

4.23 Ordinal Conditions for Dimensional Representation of Similarity Data.

Beals et al (1968) and Tversky and Krantz (1970) in the light of the definitions implied by Equations 4.201 and 4.202, broadened their discussion of the dimensional representation of similarity data by describing ordinal properties (their A1 to A6) sufficient to characterise similarity measures by a dimensional representation. They argued that these six conditions yield additive difference measurement of dissimilarity between multidimensional objects. One of the most important of these - their A3 - implies a kind of context independence property of the similarities and leads directly to interdimensional additivity. This property can be stated verbally as follows:

If 2 stimuli x and x′ have equal values on dimension i and two other stimuli y and y′ also have equal values (not necessarily the same as x and x') on that dimension then the order of $\delta(x,y)$ and $\delta(x' y')$ is dependent only on the value of the stimuli on the N-1 dimensions (in particular this order is not dependent on the pair of values on dimension i)

This says is a rather general way that dimensions do not interact in their effect.

4.24 Psychological Arguments for Interdimensional Additivity and Intradimensional Subtractivity

Interdimensional Additivity was adopted by Tversky and Krantz (1970) as one of the defining properties of a subjective dimension. The kind of non-interaction or independence of the dimensions which it implies, does seem a reasonable condition for a psychological dimension, and certainly this was the view pursued in Chapter 3 when discussing independence in identification experiments. But, it would probably be very difficult to defend it as being a necessary condition for a psychological dimension. However, in this treatment, this 'non-interaction property' is used as a basis for a definition of independent perceptual dimensions (Chapter 3).

Beals's et al's argument for intradimensional subtractivity as a defining property of a psychological dimension does on the face of it seem to be more direct. In the one-dimensional case intradimensional subtractivity is the natural one. Even if the stimuli are multidimensional, if all but one of the dimensions are held constant we are back to the one-dimensional case (implying the need for subtractivity). Also, this one dimensional subtractive 'measure' should, it would seem be independent of the constant values on the other dimensions and in fact must be,if there are no interaction effects in the dimensions. If this is so, then intradimensional subtractivity must hold at least for cases in which only one dimension is varied. While this does not imply that subtractivity must hold overall, it can be shown,however, that it does so if interdimensional additivity is also assumed.

4.30 Distance Functions that Simultaneously satisfy both the Metric and Dimensional Qualitative Conditions.

In the additive different model for similarity judgements, the stimuli are represented in a dimensionally organised space (this fact was exploited in Experiments 1A and 1B and Tversky and Krantz demonstrate this) but the dissimilarity ordering need not coincide with any metric. Tversky and Krantz (1970) showed that the assumption that an additive difference model is compatible with a metric impose strong conditions on the measurement scales. Beals et al (1968) showed that a metric that has both the interdimensional additivity and intradimensional subtractivity properties must be of the form:

4.301 $$d(x,y) = \Phi^{-1}\left[\sum_{i=1}^{N} \Phi\,(x_1 - y_1)\right]$$

with Φ a super additive function

that is, $\Phi(u+v) = \Phi(u) + \Phi(v)$ for all u, and v

The Minkowski r-metric is of this form with:

4.302 $$\Phi(u) = u^r$$

Tversky and Krantz (1970) furthermore, established that of the distance functions of this form, the only one satisfying segmental additivity are the Minkowski r-metrics with $r>1$.

This is a surprising result, for if the premises of Beals et al (1968) and Tversky and Krantz (1970) are accepted; that is, that all three of the properties (interdimensional additivity, intradimensional subtractivity, and segmental additivity) are necessary for a psychological metric, it means that the only permissable metrics are of the Minkowski r-type.

4.31 Decomposability

Tversky and Krantz (1970) analysed a more general model in MDS called decomposability (see section 2.30) which requires that there be no interaction between the dimensions of the subjective space (the dimensions contribute independently to the overall distance.)

The most general equation of psychological distance embodies only decomposability.

4.311 $$d(x,y) = F\left[\Phi_1(x_1 y_1),\ \Phi_2\ (x_2,y_2)\ldots\Phi_N(x_N y_N)\right]$$

where F is increasing function in each of its N arguments and each Φ_i is a symmetric function of two nominal scale arguments.

If subtractivity above is assumed then 4.311 can be written as

4.312 $$d(x,y) = F\left[\left|f_1\ (x_1) - f_1\ (y_1)\right| \ldots\ldots \left|f_N\ (x_N) - f_N\ (y_N)\right|\right]$$

where $\Phi_i\ (x_i,y_i)$ is replaced by $\left| f_i\ (x_i)-f_i\ (y_i)\right|$

N.B. $f_i\ (x_i)$ is the ith co-ordinate of stimulus x in the psychological space.

If Additivity alone is assumed the power metric is generalised to:

4.313 $$d(x,y) = F\left[\sum_{i=1}^{N} \Phi_i\ (x_i y_i)\right]$$

Where F is an increasing function <u>in one</u> argument

If <u>both</u> additivity and subtractivity are assumed we obtain the additive difference model defined by:

4.314 $$d(x,y) = F\left[\sum_{i=1}^{N} \Phi_i\ \left(\left| f_i(x_i) - f_i(y_i)\right|\right)\right]$$

The power metric is a special case of the additive difference model.

If <u>both</u> decomposability and subtractivity are assumed we obtain the following equation for these properties:

4.315 $$d(x,y) = F\left[\Phi_1\left(\left|f_1(x_1) - f_2(x_2)\right|\right)..\Phi_N\left(\left|f_N(x_N)-f_N(Y_N)\right|\right)\right]$$

Experiment 1A (see section 2.702 in particular) constitutes a test of decomposability and subtractivity. The three experiments (3A, 3B and 3C) reported here are also directly concerned with decomposability and subtractivity.

4.40 Introduction to the Experiments Testing Intradimensional Subtractivity and Interdimensional Additivity.

Experiments 3A, 3B and 3C are all concerned with testing intradimensional subtractivity and decomposability for the dimensions, pitch, duration, and loudness of auditory stimuli. Each experiment considers two dimensions at a time:

(i) Pitch and Duration	Experiment 3A
(ii) Pitch and Loudness	Experiment 3B
(iii) Loudness and Duration	Experiment 3C

Experiment 3D, tests the interdimensional additivity condition for these three stimulus dimension taken together.

Tversky and Krantz (1969) have reported a study which tested interdimensional additivity for stimuli which consisted of schematic faces varying on the three dimensions:

(1) Shape of face	(long versus wide)
(2) Eyes	(empty vs filled)
(3) Mouth	(straight vs curved)

The data showed that the overall dissimilarity between faces can be decomposed into three additive components - one for each attribute, and generally their data found strong support for interdimensional additivity.

Wender (1971) tested intradimensional subtractivity for rectangles varying in area and shape. The findings show that subtractivity was violated by most subjects. In particular he reported that the same area interval produces more dissimilarity as the shape level becomes more extreme - this being the case whether the dissimilarity was measured using rating scales or paired comparisons methods. This suggests that decomposability was being violated. Krantz in an as yet unpublished paper, showed in a study of colour similarity, using lightness and chromaticity as dimensions that decomposability (and hence both additivity and subtractivity) is violated.

However, as yet, no published study reports a test of both interdimensional additivity and intradimensional subtractivity for the same stimulus dimensions. It will be recalled that the additive difference model of similarity judgements (section 2.63 and 4.22) postulates a two stage process.

(i) A 'perceptual' process satisfying intradimensional subtractivity.

(ii) An evaluative process satisfying inter-dimensional additivity.

This two stage property should be useful in diagnosing the locus of a breakdown, if one should occur, of either subtractivity or additivity, because subtractivity can be tested for pairs of stimulus dimensions. If intradimensional subtractivity and decomposability is violated for any pair of dimensions, then of course additivity is also violated. Thus, subtractivity for every pair of dimensions is a necessary (but not sufficient) condition for the additive difference model.

The first three experiments then, are concerned with both decomposability and subtractivity of the stimulus dimensions taken two at a time.

EXPERIMENT 3A   Fig 4.41

Physical Values of Stimulus Dimensions:

F_o = 300 Hz L_o = 0.5 secs

F_1 = 700 Hz L_1 = 1.0 sec

F_2 = 1000 Hz L_2 = 1.5 sec

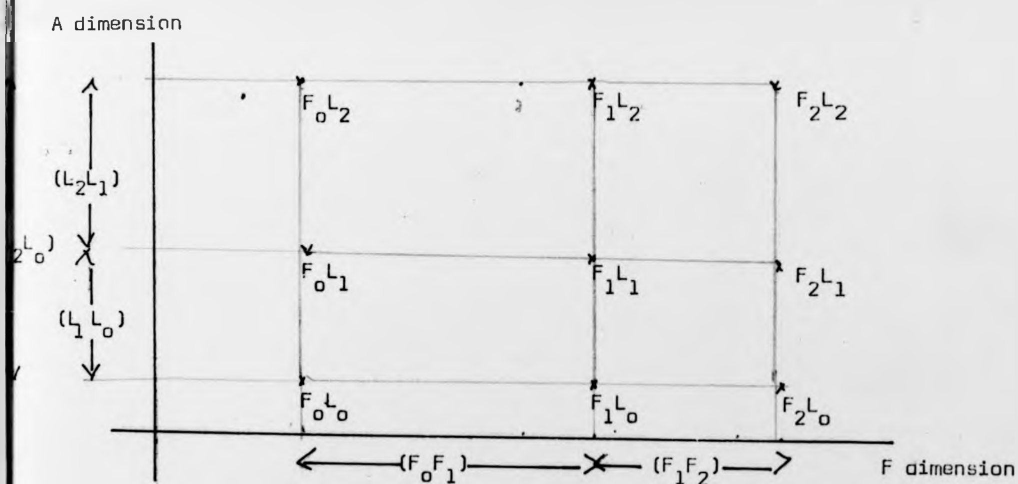
(F_oF_1) denotes physical difference between F_o and F_1 irrespective of value of L

(L_oL_1) denotes physical difference between L_o and L_1 irrespective of value of F

(F_oF_1) = 400 Hz (L_oL_1) = 0.5 sec

(F_1F_2) = 300 Hz (L_1L_2) = 0.5 sec

(F_oF_2) = 700 Hz (L_oL_2) = 1.0 sec



INTRADIMENSIONAL SUBTRACTIVITY: Let 1, 2, 5, and 6 be stimuli such that 1 and 2 have the same value on the F dimension, and 5 and 6 also have equal values on the F dimension (but different from 1 and 2). Also, suppose 2 and 5 have the same value on the A dimension and 6 and 1 also have equal values on the A dimension (but different from 2 and 5) then intradimensional subtractivity asserts: $\delta\ (F_oA_o, F_oA_1) = \delta\ (F_1A_o,\ F_1A_1)$

Decomposability. Since $(F_oF_2) > (F_oF_1) > (F_1F_2)$ then:

$\delta\ (F_oF_2) > \delta(F_oF_1) > \delta\ (F_1F_2)$ over all the three levels of duration.

For example:

$$\delta\ (F_oL_o, F_2L_o) > \delta\ (F_oL_o, F_1L_o) > \delta\ (F_1L_o, F_2L_o)$$

and

EXPERIMENT 3A Fig 4.41

Physical Values of Stimulus Dimensions:

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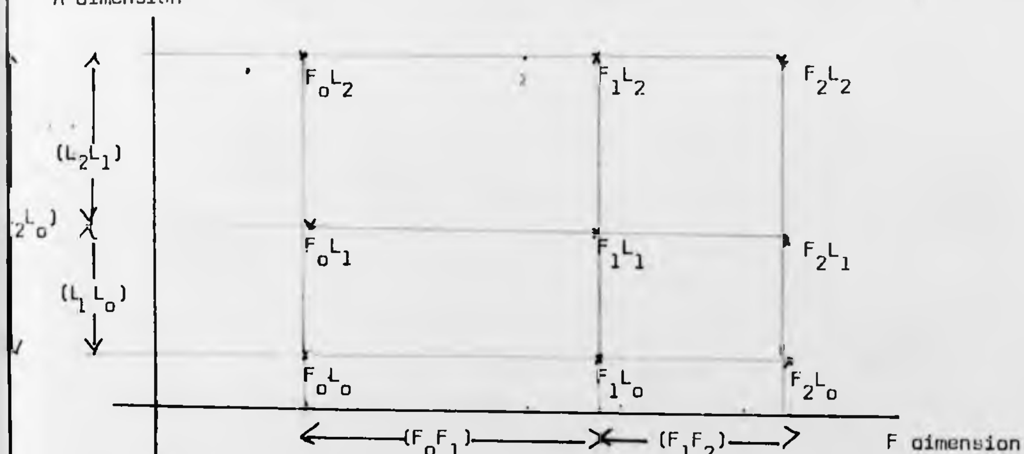
(F_oF_1) denotes physical difference between F_o and F_1 irrespective of value of L

(L_oL_1) denotes physical difference between L_o and L_1 irrespective of value of F

(F_oF_1)	= 400 Hz	(L_oL_1)	= 0.5 sec
(F_1F_2)	= 300 Hz	(L_1L_2)	= 0.5 sec
(F_oF_2)	= 700 Hz	(L_oL_2)	= 1.0 sec



INTRADIMENSIONAL SUBTRACTIVITY: Let 1, 2, 5, and 6 be stimuli such that 1 and 2 have the same value on the F dimension, and 5 and 6 also have equal values on the F dimension (but different from 1 and 2). Also, suppose 2 and 5 have the same value on the A dimension and 6 and 1 also have equal values on the A dimension (but different from 2 and 5) then intradimensional subtractivity asserts: $\delta\ (F_oA_o, F_oA_1) = \delta\ (F_1A_o, F_1A_1)$

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$$\delta\ (F_oF_2) > \delta(F_oF_1) > \delta\ (F_1F_2)$$ over all the three levels of duration.

For example:

$$\delta\ (F_oL_o, F_2L_o) > \delta\ (F_oL_o, F_1L_o) > \delta\ (F_1L_o, F_2L_o)$$

and

Figure 4.41 continued

$$\delta\ (F_0L_1, F_2L_1) > \delta(F_0L_1, F_1L_1)' > \delta\ (F_1L_1, F_2L_1)$$

and $\delta\ (F_0L_2, F_2L_2) > \delta(F_0L_2, F_1L_2) > \delta\ (F_1L_2, F_2L_2)$ ie no interaction between the dimensions.

/

4.41 Experiment 3A (Frequency and Duration)

Introduction

In this experiment 9 tones which varied in pitch (3 levels F_o, F_1, F_2) and duration (3 levels L_o, L_1, L_2) were tested for interdimensional subtractivity and decomposability. The values on the pitch (F) and duration (L) dimensions are shown in Fig 4.41 along with some of the notation used in the three experiments (3A, 3B, and 3C).

Intradimensional subtractivity and decomposability is also depicted Fig 4.41, but the general case follows from application of Equation 4.315 : If x,y, x', y' are stimuli such that:

(i) $x_i = y_i$

(ii) $x'_i = y'_i$

for some i

and with

(iii) $x_j = x_j'$

(iv) $y_j = x_j'$

for all $j \neq i$ then $\delta(x,y) = \delta(x',y')$

Intradimensional subtractivity leads to the following equalities in ordering between pairs of tones with respect to dissimilarity (see Fig 4.41 and table 4.40).

4.42 (a) $\delta(F_oL_o, F_1L_o)=\delta(F_oL_1, F_1L_1)=\delta(F_oL_2, F_1L_2)$

(b) $\delta(F_1L_o, F_2L_o)=\delta(F_1L_1, F_2L_1)=\delta(F_1L_2, F_2L_2)$

(c) $\delta(F_oL_o, F_2L_o)=\delta(F_oL_1, F_2L_1)=\delta(F_oL_2, F_2L_2)$

further

(d) $\delta(F_oL_o, F_oL_1)=\delta(F_1L_o, F_1L_1)=\delta(F_2L_o, F_2L_1)$

(e) $\delta(F_oL_1, F_oL_2)=\delta(F_1L_1, F_1L_2)=\delta(F_2L_1, F_2L_2)$

(f) $\delta(F_oL_o, F_oL_2)=\delta(F_1L_o, F_1L_2)=\delta(F_2L_o, F_2L_2)$

One test for subtractivity then, is that the dissimilarity scores for the single pair of tones within the triples(a),(b),(c), etc., should be equal.

4.43 Ordinal Interactions in Similarity Data (Decomposability).

This is not to be confused with ordinal interaction mentioned in section 2.40, but there are of course very close conceptual similarities.

There is another consequence of subtractivity and decomposability which needs to be exploited - and indeed was tested in a restricted way in experiment 1A - and it concerns the ordering of 'differences'.

From Fig 4.41 is can be seen that:

$$(F_oF_2) > (F_oF_1) > (F_1F_2)$$

In words this means that the physical difference in frequency between F_o and F_2 is greater than that between F_o and F_1 which in turn is greater than that between F_1 and F_2 (Fig 4.41 explains this). This implies providing there are no interactions (i.e. we have decomposability)

4.431 $$\delta(F_oL_o, F_2L_o) > \delta(F_oL_o, F_1L_o) > \delta(F_1L_o, F_2L_o)$$

$$\delta(F_oL_1, F_2L_1) > \delta(F_oL_1, F_1L_1) > \delta(F_1L_1, F_2L_1)$$

$$\delta(F_oL_2, F_2L_2) > \delta(F_oL_2, F_1L_2) > \delta(F_1L_2, F_2L_2)$$

or $\delta(F_oF_2) > \delta(F_oF_1) > \delta(F_1F_2)$ for all values of L.

Where the notation $\delta(F_oF_2)$ etc denotes the dissimilarity between two tones with a common duration but with frequencies F_o and F_2. This ordering, then, must be maintained over all values of the L dimension for decomposability.

If the ordering differed (or is not consistent) over different values of L we would get a 'cross over' effect as sometimes occurs in analysis of variance, see Section 2.43 Chapter 2. Moreover, if there are interactions of this nature it would be impossible to obtain an additive representation.

In this experiment (as also with experiments 3B and 3C) the diagnosis of subtractivity and decomposability is two fold:

1. A direct test of subtractivity; this means that the dissimilarity of pairs of stimuli within (a), (b), (c)... etc (4.42) should be equal.

2. A test for decomposability; this means thatthe predicted orderings in 4.431 should be maintained over all values of the other dimension concerned.

In the experiments (3A, 3B, and 3C) both these tests are made on the data.

4.44 Method

Since exactly the same procedure is followed in the first three experiments the description of the method will suffice for all the studies.

Dissimilarity judgements were obtained by presenting a single pair of tones to a subject who rated their dissimilarity by marking a point on a scale from 1 (minimal dissimilarity) to 20 (very dissimilar). Since there were 9 different tones the number of pairs of stimuli was 36. Each of these 36 pairs was presented four times in one experimental session. So as to minimise order effects the tones of a single pair was reversed in order of presentation twice, but of the 36 pairs of stimuli presented to the subject only 18 pairs were necessary for the analysis;these are indicated in Table 4.40.

Each subject took part in 5 experimental sessions and the analysis was carried out on the data from the final two sessions. This means, for instance, that the dissimilarity scores for the 18 pairs of stimuli in Table 4.40 are the means of 8 dissimilarity judgements. Ss were tested two at a time. Each of the five sessions lasted just over the hour.

SUBJECTS 10 undergraduate students, all of whom were paid for their services.

4.45 Apparatus and Stimuli

The stimuli consisted of 9 pure tones varying in duration and pitch at a level of loudness of 98.5 db. The values of the parameters are indicated in Fig 4.41.

The Ss were run two at a time and were sat in a sound proof room. The tones which were prerecorded were delivered via a tape recorder over a loudspeaker. First one tone was presented, followed 2 seconds later by another. Ss simply rated the dissimilarity between the tones.

4.46 Direct Test of Subtractivity

TABLE 4.40: EXPERIMENT 3A (RESULTS) Pitch and Duration

Mean dissimilarity Ratings between Pairs of Tones

(F_oF_1) = 400Hz	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
(F_oL_o, F_1L_o)	11.0	8.8	5.2	10.9	12.3	12.8	16.3	13.0	12.0	19.4
(F_oL_1, F_1L_1)	11.8	7.6	5.6	12.2	12.1	14.2	12.8	12.8	13.7	18.0
(F_oL_2, F_1L_2)	13.0	8.6	5.9	12.7	12.5	15.0	14.3	15.1	13.5	18.3
(F_1F_2) = 300Hz										
(F_1L_o, F_2L_o)	8.7	8.4	5.8	12.0	9.8	8.4	13.0	11.3	12.5	7.3
(F_1L_1, F_2L_1)	7.9	8.0	7.0	12.2	10.3	9.3	12.5	9.9	13.5	6.3
(F_1L_2, F_2L_2)	8.9	7.7	6.5	12.6	9.9	11.9	15.8	9.6	12.5	7.9
(F_oF_2) = 700Hz										
$(F_oL_o\ F_2L_o)$	12.8	14.4	7.7	18.0	15.9	14.5	17.0	15.0	14.3	18.1
$(F_oL_1\ F_2L_1)$	13.6	13.4	7.5	18.3	14.6	15.0	16.6	13.4	14.0	17.5
$(F_oL_2\ F_2L_2)$	12.8	14.5	8.0	18.3	16.9	14.4	17.5	15.9	14.8	18.8
(L_oL_1) = 0.5 sec										
(F_oL_o, F_oL_1)	1.4	3.4	1.8	1.9	1.6	3.3	3.0	2.8	1.3	2.5
(F_1L_o, F_1L_1)	1.8	5.9	1.5	2. 0	1.9	4.1	3.6	2.8	3.8	2.3
(F_2L_o, F_2L_1)	2.4	3.0	1.6	2.0	1.8	3.4	3.3	2.7	4.5	2.0
(L_1L_2) = 0.5 sec										
(F_oL_1, F_oL_2)	1.3	2.7	1.8	1.7	2.5	3.1	2.9	2.0	1.8	2.1
(F_1L_1, F_1L_2)	1.9	1.6	1.8	1.8	1.9	3.1	3.4	2.0	3.5	1.6
(F_2L_1, F_2L_2)	2.0	1.8	1.8	1.4	2.1	2.5	2.8	1.9	4.0	1.6
(L_oL_2) = 1.0 sec										
(F_oL_o, F_oL_2)	1.6	4.3	2.6	2.9	3.0	3.9	4.0	2.6	1.3	2.9
(F_1L_o, F_1L_2)	1.8	4.2	2.9	2.8	2.9	4.6	4.6	3.1	4.0	4.1
(F_2L_o, F_2L_2)	2.2	3.6	2.5	2.8	2.6	3.1	4.1	2.9	4.0	3.6

EXPERIMENT 3A ORDERING OF DISSIMILARITIES (Ordinal Interaction)

	L_o constant (0.5 secs)			F_o constant (300 Hz)		
L_o constant	$\delta\ (F_oF_2)$	$\delta\ (F_oF_1)$	$\delta\ (F_1F_2)$	$\delta (L_oL_2)$	$\delta\ (L_oL_1)$	$\delta\ (L_1L_2)$
S1	1 (12.8)	2 (11.0)	3 (8.7)	1 (1.6)	2 (1.4)	3 (1.3)
S2	1 (14.4)	3 (8.8)	2 (8.4)	1 (4.3)	2 (3.4)	3 (2.7)
S3	1 (7.7)	3 (5.2)	2 (5.8)	1 (2.6)	2.5 (1.8)	2.5 (1.8)
S4	1 (18.0)	3 (10.9)	2 (12.0)	1 (2.9)	2 (1.9)	3 (1.7)
S5	1 (15.9)	2 (12.3)	3 (9.8)	1 (3.0)	3 (1.6)	2 (2.5)
S6	1 (14.5)	3 (12.8)	2 (8.4)	1 (3.9)	2 (3.3)	3 (3.1)
S7	1 (17.0)	3 (16.3)	2 (13.0)	1 (4.0)	2 (3.0)	3 (2.9)
S8	1 (15.0)	3 (13.0)	2 (11.3)	2 (2.6)	1 (2.8)	3 (2.0)
S9	1 (14.3)	2 (12.0)	3 (12.5)	2.5 (1.3)	2.5 (1.3)	1 (1.8)
S10	1 (18.1)	3 (18.4)	2 (7.3)	1 (2.9)	2 (2.5)	3 (2.1)

Table 4.41 RESULTS. Ranking of Dissimilarity Scores for pairs of tones.

The bracketed numbers refer to the dissimilarity between pairs of tones and the unbracketed refer to the ranking.

The above rankings of the dissimilarity scores was obtained from Table 4.40. The notation (F_oF_2) refers to the dissimilarity between the tones F_oL_o, and F_2L_o.

Thus for S1 (above) the two tones which are the most dissimilar are ranked 1, the next most dissimilar, 2, and so on. The ordering, for S1 with respect to dissimilarity is $\delta(F_oL_o,F_2L_o) > \delta\ (F_oL_o,F_1L_o) > \delta\ (F_1L_o,F_2L_o)$ or: $\delta\ (F_oF_2) > \delta(F_oF_1) > \delta\ (F_1F_2)$. This notation is used in all the tables.

EXPERIMENT 3A ORDERING OF DISSIMILARITIES

	L_1 constant (1.0 secs)			F_1 constant (700 secs)		
L_1 constant	(F_0F_2)	(F_0F_1)	(F_1F_2)	(L_0L_2)	(L_0L_1)	(L_1L_2)
S1	1	2	3	2.5	2.5	1
S2	1	3	2	2	1	3
S3	1	3	2	1	3	2
S4	1	2	3	1	2	3
S5	1	2	3	1	2.5	2.5
S6	1	2	3	1	2	3
S7	1	2	3	1	2	3
S8	1	2	3	1	2	3
S9	1	2	3	1	2	3
S10	1	2	3	1	[illegible]	3

TABLE 4.42 RESULTS. Ranking of Dissimilarity scores for pairs of tones.

The above rankings of the dissimilarity scores was obtained from table 4.40. The notation (F_0F_2) refers to the dissimilarity between the tones F_0L_1, and F_2L_1. The hypothesis was that the dissimilarity rankings between (F_0F_2), (F_0F_1) and (F_1F_2) with L_1 held constant would be:

$\delta(F_0F_2) > \delta(F_0F_1) > \delta(F_1F_2)$ because $(F_0F_2) > (F_0F_1) > (F_1F_2)$ (Fig 4.41).

Also $(L_2L_0) > (L_1L_0) = (L_2L_1)$ Hence the predicted ordering of these differences (assuming decomposability) is $\delta\ (L_2L_o) > \delta(L_1L_0)$ and $\delta\ (L_2L_0) > \delta(L_2L_1)$ and $\delta(L_1L_0) = \delta > L_2L_1)$. These predictions were upheld.

EXPERIMENT 3A ORDERING OF DISSIMILARITIES

	L_2 constant (1.0 secs)			F_2 constant (700 secs)		
L_2 constant	(F_0F_2)	(F_0F_1)	(F_1F_2)	(L_0L_2)	(L_0L_1)	(L_1L_2)
S1	1	2	3	2	1	3
S2	1	2	3	1	2	3
S3	1	3	2	1	3	2
S4	1	2	3	1	2	3
S5	1	2	3	1	3	2
S6	1	2	3	2	1	3
S7	1	3	2	1	2	3
S8	1	2	3	1	2	3
S9	1	2	3	2	1	3
S10	1	2	3	1	2	3

TABLE 4.43 RESULTS. Ranking of Dissimilarity scores for pairs of tones.

These rankings were obtained from Table 4.40. The experimental hypothesis was that the dissimilarity ranking between (F_0F_2), (F_0F_1) and (F_1F_2) with L_2 held constant would be: $\delta(F_0F_2) > \delta(F_0F_1) > \delta(F_1F_2)$.

Likewise the dissimilarity ranking of (L_0L_2), (L_0L_1) and (L_1L_2) with F_2 held constant would be since $\delta(L_0L_2) > \delta(L_oL_1)$ and $\delta(L_0L_2) > \delta(L_1L_2)$ and also since $(L_0L_1) = (L_1L_2)$ then $\delta(L_0L_2) > (L_0L_1)$ and $\delta(L_1L_2)$.

This prediction of the ranking was upheld, and thus constituted a test of decomposability (no interactions between the dimensions).

Table 4.40 gives the dissimilarity scores for each of the 10 Ss for the 18 pairs of stimuli listed in the subclasses (a), (b), (c), (d), (e), and (f) (section 4.42).

For each set of 3 pairs within the six subclasses of Table 4.40 the dissimilarity scores for a particular S should be equal. The data strongly suggests that even for rating data these scores are very consistent for each S, within each of the subclasses.

As a further test the Friedman two-way analysis of variance by ranks for related samples was carried out on the 6 sets of subclasses. Again, none of the scores within subclasses showed significant differences in dissimilarity ($p < 0.005$) for any one subject.

4.45 Testing predicted orderings for Decomposability

Tables 4.41, 4.42 and 4.43 are all obtained from Table 4.40. The numbers in brackets refer to the dissimilarity rating given by the S to a pair of tones. To see this, consult Table 4.41. S1, rated (F_oF_2) - that is (F_oL_o, F_2L_o) as 12.8,tones(F_oF_1)as 11.0 and so on. The rank ordering for S1; with respect to dissimilarity for these intervals is as shown and is in the direction predicted.

Table 4.42 gives the same information as the last table but this time L_1 is constant for the first three columns, and F_1 is constant for the last three columns. Likewise table 4.42 gives the same information as to ordering, but with L_2 and F_2 constant respectively.

These 3 tables then, give the ordering of the differences

$$(F_oF_2),\ (F_oF_1),\ \text{and}\ (F_1F_2)$$

$$(L_oL_2),\ (L_oL_1),\ \text{and}\ (L_1L_2)$$

for different values of the other dimension (either L or F).

4.46 Ordering of Pitch differences.

The hypothesis is that since:

$$(F_oF_2) > (F_oF_1) > (F_1F_2) \quad \text{see Fig 4.41}$$

then $\delta(F_oF_2) > \delta(F_oF_1) > \delta(F_1F_2)$

This prediction of the rank ordering for the 10 subjects for each of L_o, L_1, and L_2 was tested by using Page's test (Page, 1963), which is a trend test for related samples.

It was found that the predicted ranking occurred for all values of L and the prediction was upheld beyond the .001 level of significance.

4.46 Ordering of Duration Differences

Since $(L_oL_2) = 1.0$ secs

and $(L_oL_1) = (L_1L_2) = 0.5$ secs

The prediction is that:

$$\delta(L_oL_2) > \delta(L_oL_1)$$

and $\delta(L_oL_2) > \delta(L_1L_2)$

for all levels of F

Also $\delta(L_oL_1) = \delta(L_1L_2)$ over all levels of F.

All these predictions were upheld beyond the .05 level of significance.

Conclusions

Intradimensional subtractivity, for the dimensions pitch and duration, seems to be upheld and no S seemed to seriously violate it. Moreover for no S was decomposability violated, because the ordering of differences in pitch between tones for all levels of duration were the same.

EXPERIMENT 3B   Fig 4.51

Physical Values of Stimulus Dimensions

F_o	=	300 Hz	A_o	=	92 dbs
F_1	=	700 Hz	A_1	=	95 dbs
F_2	=	1000 Hz	A_2	=	98 dbs

(F_oF_1)	=	400 Hz	(A_oA_1)	=	3 dbs	
(F_1F_2)	=	300 Hz	(A_1A_2)	=	3 dbs	Stimulus Differences
(F_oF_2)	=	700 Hz	(A_oA_2)	=	6 dbs	

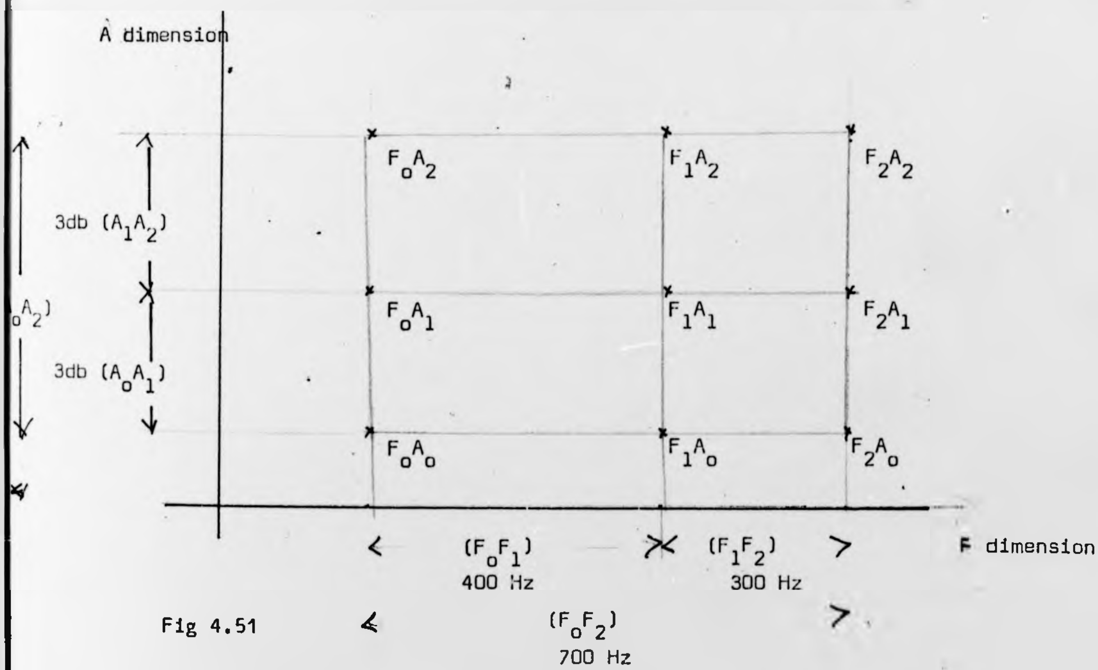


Fig 4.51

If there is no interaction between the dimensions (decomposability)

Since $(F_oF_2) > (F_oF_1) > (F_1F_2)$

then $\delta(F_oF_2) > \delta(F_oF_1) > \delta(F_1F_2)$ irrespective of the level of A

Also since $(A_oA_2) > (A_oA_1)$

and $(A_1A_2) = (A_oA_1)$

then $\delta(A_oA_2) > \delta(A_oA_1)$

and $\delta(A_oA_2) > \delta(A_oA_2)$.

4.50 Experiment 3B Pitch and Loudness

Direct Test for Subtractivity

If the interdimensional subtractivity condition holds for pitch and loudness then the following equalities hold between pairs of tones with respect to dissimilarity (see also Fig 4.51 and Table 4.50).

(a) $\delta(F_oA_o, F_oA_1) = \delta(F_1A_o, F_1A_1) = \delta(F_2A_o, F_2A_1)$

(b) $\delta(F_oA_1, F_oA_2) = \delta(F_1A_1, F_1A_2) = \delta(F_2A_1, F_2A_2)$

(c) $\delta(F_oA_o, F_oA_2) = \delta(F_1A_o, F_1A_2) = \delta(F_2A_o, F_2A_2)$

(d) $\delta(F_oA_o, F_1A_o) = \delta(F_oA_1, F_1A_1) = \delta(F_oA_2, F_1A_2)$

(e) $\delta(F_1A_o, F_2A_o) = \delta(F_1A_1, F_2A_1) = \delta(F_1A_2, F_2A_2)$

(f) $\delta(F_oA_o, F_2A_o) = \delta(F_oA_1, F_2A_1) = \delta(F_oA_2, F_2A_2)$

The results of these tests are shown in Table 4.50, where again each of the 18 scores from each S are grouped in subclasses of three shown by (a), (b), (c), etc. above.

Ordering of Differences (Testing decomposability).

Again referring to Fig 4.51 we have since:

$(F_oF_2) > (F_oF_1) > (F_1F_2)$ [Where again (F_oF_2) for example

4.50 Experiment 3B Pitch and Loudness

Direct Test for Subtractivity

If the interdimensional subtractivity condition holds for pitch and loudness then the following equalities hold between pairs of tones with respect to dissimilarity (see also Fig 4.51 and Table 4.50).

(a) $\delta(F_oA_o, F_oA_1) = \delta(F_1A_o, F_1A_1) = \delta(F_2A_o, F_2A_1)$

(b) $\delta(F_oA_1, F_oA_2) = \delta(F_1A_1, F_1A_2) = \delta(F_2A_1, F_2A_2)$

(c) $\delta(F_oA_o, F_oA_2) = \delta(F_1A_o, F_1A_2) = \delta(F_2A_o, F_2A_2)$

(d) $\delta(F_oA_o, F_1A_o) = \delta(F_oA_1, F_1A_1) = \delta(F_oA_2, F_1A_2)$

(e) $\delta(F_1A_o, F_2A_o) = \delta(F_1A_1, F_2A_1) = \delta(F_1A_2, F_2A_2)$

(f) $\delta(F_oA_o, F_2A_o) = \delta(F_oA_1, F_2A_1) = \delta(F_oA_2, F_2A_2)$

The results of these tests are shown in Table 4.50, where again each of the 18 scores from each S are grouped in subclasses of three shown by (a), (b), (c), etc. above.

Ordering of Differences (Testing decomposability).

Again referring to Fig 4.51 we have since:

$(F_oF_2) > (F_oF_1) > (F_1F_2)$ [Where again (F_oF_2) for example

TABLE 4.50: EXPERIMENT 3B (Results) Pitch and Loudness

Mean Similarity Ratings between pairs of Tones

$(A_oA_1) = 3$ db	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
(F_oA_o, F_oA_1)	1.0	1.0	1.0	2.5	1.0	1.0	1.0	1.0	1.0	1.0
(F_1A_o, F_1A_1)	1.0	1.3	1.0	2.0	1.0	1.0	1.0	1.0	1.0	1.0
(F_2A_o, F_2A_1)	1.0	1.5	1.0	2.0	1.0	1.0	1.3	1.0	1.0	1.0
$(A_1A_2) = 3$ db										
(F_oA_1, F_oA_2)	1.0	1.1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
(F_1A_1, F_1A_2)	1.0	1.5	1.0	2.0	1.0	1.0	1.0	1.0	1.0	1.0
(F_2A_1, F_2A_2)	1.0	1.0	1.0	1.5	1.0	1.0	1.1	1.0	1.0	1.0
$(A_oA_2) = 6$ dbs										
(F_oA_o, F_oA_2)	3.1	4.1	3.4	3.7	3.3	4.4	4.4	4.3	5.2	2.1
(F_1A_o, F_1A_2)	3.2	4.7	3.5	3.4	3.2	4.7	4.3	4.9	5.0	2.3
(F_2A_o, F_2A_2)	3.1	4.4	3.3	3.9	3.7	4.8	4.5	3.9	5.4	2.4
$(F_oF_1) = 400$ Hz										
(F_oA_o, F_1A_o)	15.3	14.1	14.0	18.0	9.5	9.3	12.1	11.9	13.3	12.4
(F A , F A)	15.3	14.1	14.0	15.8	10.9	9.5	13.8	15.0	15.0	13.1
(F_oA_2, F_1A_2)	15.8	14.6	14.0	15.9	9.0	10.3	12.8	12.5	15.4	12.9
$(F_1F_2) = 300$ Hz										
(F_1A_o, F_2A_o)	5.4	4.3	9.0	13.0	9.0	4.4	10.6	6.9	10.9	8.0
(F_1A_1, F_2A_1)	5.9	6.0	9.0	15.0	9.5	4.8	10.6	9.4	12.0	7.9
(F_1A_2, F_2A_2)	6.0	6.0	10.0	18.0	9.0	4.3	13.0	9.8	11.1	8.0
$(F_oF_2) = 700$ Hz										
(F_oA_o, F_2A_o)	17.1	15.9	15.1	16.9	13.0	10.6	13.9	11.9	16.1	13.9
(F_oA_1, F_2A_1)	16.8	15.7	15.1	16.3	13.3	10.8	13.9	12.7	16.1	14.9
(F_oA_2, F_2A_2)	16.7	15.3	15.4	16.4	12.4	10.7	12.9	12.8	15.9	14.5

EXPERIMENT 3B <u>Ordering of Dissimilarities obtained from Table 4.50</u>

	F_0 Constant (300 Hz)			A_0 Constant (95db)		
	(A_0A_2)	(A_1A_2)	(A_0A_1)	(F_0F_2)	(F_0F_1)	(F_1F_2)
S1	(3.1) 1	(1.0) 2.5	(1.0) 2.5	(17.1) 1	(15.3) 2	(5.4) 3
S2	(4.1) 1	(1.1) 2.5	(1.0) 2.5	(15.4) 1	(14.1) 2	(4.3) 3
S3	(3.4) 1	(1.0) 2.5	(1.0) 2.5	(15.1) 1	(14.0) 2	(9.0) 3
S4	(3.7) 1	(1.0) 3	(2.5) 2	(16.9) 2	(16.0) 1	(13.0) 3
S5	(3.3) 1	(1.0) 2.5	(1.0) 2.5	(13.0) 1	(9.5) 2	(9.0) 3
S6	(4.4) 1	(1.0) 2.5	(1.0) 2.5	(10.6) 1	(9.3) 2	(4.4) 3
S7	(4.4) 1	(1.0) 2.5	(1.0) 2.5	(13.9) 1	(12.1) 2	(10.6) 3
S8	(4.3) 1	(1.0) 2.5	(1.0) 2.5	[illegible] 1.5	[illegible] 1.5	[illegible] 2
S9	(5.2) 1	(1.0) 2.5	(1.0) 2.5	(16.1) 1	(13.3) 2	(10.9) 3
S10	(5.1) 1	(1.0) 2.5	(1.0) 2.5	(13.9) 1	(12.4) 2	(8.0) 3

Table 4.51. The bracketed numbers in each cell of the last three columns refer to the dissimilarity between (F_0F_2), (F_0F_1), (F_1F_2) with A_0 constant. The unbracketed numbers are the rank orders of these dissimilarities for each S. The bracketed numbers in the first three columns refer to the dissimilarieits between (A_0A_2), (A_1A_2) and (A_0A_1). The unbracketed numbers refer to the rank orders of the dissimilarities of these three tones for each S. Fig 4.51 give the predicted ranking of these differences assuming decomposability.

EXPERIMENT 3B Ordering of Dissimilarities Obtained from table 4.50

	F_1 Constant (700 Hz)			A_1 Constant (97.5db)		
	(A_0A_2)	(A_1A_2)	(A_0A_1)	(F_0F_2)	(F_0F_1)	(F_1F_2)
S1	(3.2) 1	(1.0) 2.5	(1.0) 2.5	(17.1) 1	(15.3) 2	(5.9) 3
S2	(4.7) 1	(1.5) 2	(1.3) 3	(15.9) 1	(14.1) 2	(6.0) 3
S3	(3.5) 1	(1.0) 2.5	(1.0) 2.5	(15.1) 1	(14.0) 2	(9.0) 3
S4	(3.4) 1	(2.0) 2.5	(2.0) 2.5	(16.9) 1	(15.8) 2	(15.0) 3
S5	(3.2) 1	(1.0) 2.5	(1.0) 2.5	(13.0) 1	(10.9) 2	(9.5) 3
S6	(4.7) 1	(1.0) 2.5	(1.0) 2.5	(10.6) 1	(9.5) 2	(4.8) 3
S7	(4.3) 1	(1.0) 2.5	(1.0) 2.5	(13.9) 1	(13.8) 2	(10.6) 3
S8	(4.9) 1	(1.0) 2.5	(1.0) 2.5	(11.9) 2	(15.0) 1	(1.4) 3
S9	(5.6) 1	(1.0) 2.5	(1.0) 2.5	(16.1) 1	(15.0) 2	(12.0) 3
S10	(2.3) 1	(1.0) 2.5	(1.0) 2.5	(13.9) 1	(13.1) 2	(7.9) 3

Table 4.52. The bracketed numbers in each cell of the last three column refer to the dissimilarity between (F_0F_2),(F_0F_1),(F_1F_2) with A_1 constant. The unbracketed numbers are the rank orders of these dissimilarities for each S. The bracketed numbers in the first three columns refer to the dissimilarities between (A_0A_2),(A_1A_2), and (A_0A_1). The unbracketed numbers refer to the rank order of the dissimilarities of these three tones for each S. Fig 4.51 gives the predicted rankings of these differences if there are no interactions between the dimensions.

EXPERIMENT 3 B. <u>Ordering of dissimilarities obtained from table 4.50</u>

	F_2 constant (1000 Hz)			A_2 Constant (98.5db)		
	(A_1A_2)	(A_1A_2)	(A_0A_1)	(F_0F_2)	(F_0F_1)	(F_1F_2)
S1	(3.1) 1	(1.10) 2.5	(1.0) 2.5	(16.7) 1	(15.8) 2	(6.0) 3
S2	(4.4) 1	(1.0) 3	(1.5) 2	(15.3) 1	(14.6) 2	(6.0) 3
S3	(3.3) 1	(1.0) 2.5	(1.0) 2.5	(15.4) 1	(14.0) 2	(10.0) 3
S4	(3.4) 1	(1.5) 3	(2.0) 2	(16.4) 2	(15.4) 3	(18.0) 1
S5	(3.7) 1	(1.0) 2.5	(1.0) 2.5	(12.4) 1	(9.0) 2.5	(9.0) 2.5
S6	(4.8) 1	(1.0) 2.5	(1.0) 2.5	(10.7) 1	(10.3) 2	(4.9) 3
S7	(4.5) 1	(1.1) 3	(1.3) 2	(12.4) 2	(12.8) 3	(13.0) 1
S8	(3.9) 1	(1.0) 2.5	(1.0) 2.5	(12.8) 1	(12.5) 2	(9.8) 3
S9	(5.4) 1	(1.0) 2.5	(1.0) 2.5	(15.9) 1	(15.4) 2	(11.1) 3
S10	(2.4) 1	(1.0) 2.5	(1.0) 2.5	(14.3) 1	(12.9) 2	(8.0) 3

Table 4.53 The bracketed numbers in each cell of the last three columns refer to the dissimilarity between (F_0F_2), (F_0F_1), (F_1F_2) with A_2constant. The unbracketed numbers are the rank orderings of thse dissimilarities for each S. The bracketed numbers in the first three columns refer to the dissimilarities between (A_0A_2),(A_1A_2) and (A_0A_1) with F_2 constant.

The unbracketed numbers are the rank orders of the dissimilarities of these three tones for each S.

refers to the physical difference between two tones of frequency F_o, and F_2 but with a common duration between them.]

then if decomposability holds:

$\delta(F_oF_2)>\delta(F_oF_1)>\delta(F_1F_2)$ irrespective of the level of duration (L_o, L_1, L_2)

Again referring to Fig 4.51 we have that

$$(A_oA_1) = (A_1A_2)$$

this means that for any constant level of duration between tones if there is no interaction

$$\delta(A_oA_1) = \delta(A_1A_2)$$

Similarly since:

$$(A_oA_2) > (A_1A_2)$$

and $(A_oA_2) > (A_oA_1)$

then $\delta(A_oA_2) > \delta(A_1A_2)$

and $\delta(A_oA_2) > \delta(A_oA_1)$

The results of these tests for 'non-ordinal' interaction appear in tables 4.51, 4.52, and 4.53.

RESULTS:

Direct Test for Subtractivity. From table 4.50 a direct test of subtractivity revealed that the hypothesis of intradimensional subtractivity was upheld (beyond the .05 level of significance).

Test for Decomposability for the predicted rankings of differences: all the predicted orderings were upheld beyond the .001 level of significance. Again these orderings were all consistent for different levels of the other dimension, implying no interaction between the dimensions.

CONCLUSIONS

The hypothesis of intradimensional subtractivity and decomposability for pitch and loudness was upheld for the values tested.

EXPERIMENT 3C   Fig 4.61

Physical Values of Stimulus Dimensions:

L_0 = 0.5 secs A_0 = 87 db

L_1 = 1.5 secs A_1 = 102 db   Constant F_0 throughout the experiment (700 Hz).

L_2 = 2.0 secs A_2 = 106 db

(L_0L_1) = 1.0 secs (A_0A_2) = 19 dbs

(L_1L_2) = 0.5 secs (A_0A_1) = 15 dbs

(L_0L_2) = 1.5 secs (A_1A_2) = 4 dbs

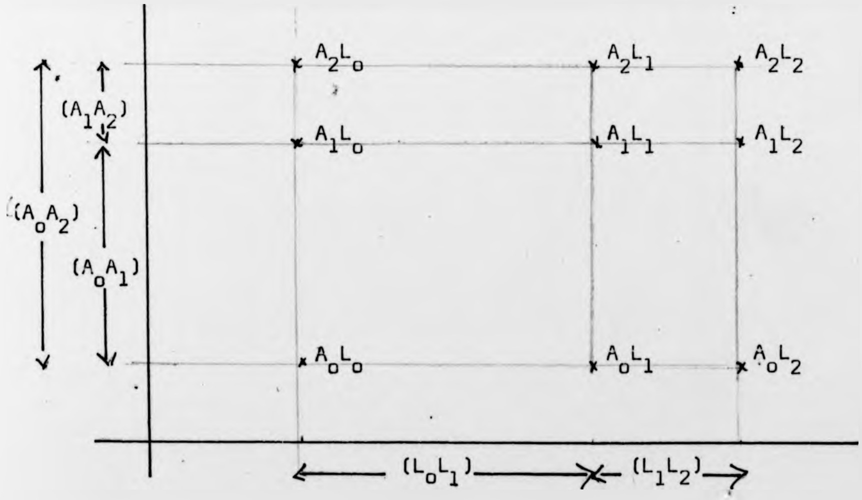


Fig 4.61 For decomposability:

Since $(A_0A_2) > (A_0A_1) > (A_1A_2)$

then $\delta(A_0A_2) > \delta(A_0A_1) > \delta(A_1A_2)$ irrespective of the level of L

also since $(L_0L_2) > (L_0L_1) > (L_1L_2)$

then $\delta(L_0L_2) > \delta(L_0L_1) > \delta(L_1L_2)$ This is irrespective of the level of A

This tests decomposability.

4.60 Experiment 3C (Duration and Loudness)

The dimensions are the same as those used in Experiment 1A. Indeed this experiment was carried out before experiment 1A and it is readily seen that if subtractivity and decomposability were violated (i.e. ordinal interaction occurred with respect to the ordering of differences) then the results of Experiment 1A would not have been possible.

(1) If interdimensional Subtractivity holds for duration and loudness then the following equalities hold between pairs of tones with respect to dissimilarity (see also Fig 4.61 and Table 4.60).

(a) $\delta(A_oL_o,\ A_1L_o) = \delta(A_oL_1,\ A_1L_1) = \delta(A_oL_2,\ A_1L_2)$

(b) $\delta(A_1L_o,\ A_2L_o) = \delta(A_1O_1,\ A_2L_1) = \delta(A_1L_2,\ A_2L_2)$

(c) $\delta(A_oL_o,\ A_2L_o) = \delta(A_oL_1,\ A_2L_1) = \delta(A_oL_2,\ A_2L_2)$

Also

(d) $\delta(A_oL_o,\ A_oL_1) = \delta(A_1L_o,\ A_1L_1) = \delta(A_2L_o,\ A_2L_1)$

(e) $\delta(A_oL_1,\ A_oL_2) = \delta(A_1O_1,\ A_1L_2) = \delta(A_2L_1,\ A_2L_2)$

(f) $\delta(A_oL_o,\ A_oL_2) = \delta(A_1L_o,\ A_1L_2) = \delta(A_2L_o,\ A_2L_2)$

Table 4.60 RESULTS EXPERIMENT 3C

(A_0A_1) = 15 db	S1	S2	S3	S4	S5	S6	S7	S8
(A_0L_0, A_1L_0)	3.3	4.3	8.0	4.0	7.8	7.4	6.4	8.3
(A_0L_1, A_1L_1)	3.3	4.8	8.3	4.5	9.5	8.6	7.3	10.8
(A_0L_2, A_1L_2)	2.9	4.3	7.3	3.6	10.4	7.8	7.3	10.4
(A_1A_2) = 4 db								
(A_1L_0, A_2L_0)	2.2	3.7	6.4	2.0	7.4	6.8	5.1	11.5
(A_1L_1, A_2L_1)	2.8	5.0	6.9	3.5	8.3	7.6	3.9	7.3
(A_1L_2, A_2L_2)	1.7	6.4	6.1	2.9	9.8	7.2	4.0	8.5
(A_0A_2) = 19 db								
(A_0L_0, A_2L_0)	2.3	5.5	8.5	5.1	10.0	10.1	9.4	16.6
(A_0L_1, A_2L_1)	2.3	6.0	7.6	7.3	12.4	10.4	8.8	15.9
(A_0L_2, A_2L_2)	4.2	4.0	10.7	8.6	16.0	11.1	11.1	16.5
(L_0L_1) = 1.0 sec.								
(A_0L_0, A_0L_1)	4.8	6.9	6.3	7.6	5.0	8.0	7.9	8.0
(A_1L_0, A_1L_1)	5.9	7.5	5.6	7.1	7.1	8.3	9.5	11.9
(A_2L_0, A_2L_1)	5.3	6.3	5.4	7.4	6.9	7.6	10.1	11.4
(L_1L_2) = 0.5 sec.								
(A_0L_1, A_0L_2)	4.6	7.3	3.5	5.6	4.3	3.8	2.5	4.8
(A_1L_1, A_1L_2)	2.6	6.9	6.0	2.4	5.8	4.3	3.6	5.8
(A_2L_1, A_2L_2)	4.6	7.2	5.1	6.3	2.9	7.5	4.4	7.4
(L_0L_2) (1.5 sec)								
(A_0L_0, A_0L_2)	8.6	13.6	4.4	8.0	4.5	9.0	10.6	11.3
(A_1L_0, A_1L_2)	8.6	12.0	5.9	10.0	8.3	10.8	11.0	15.8
(A_2L_0, A_2L_2)	8.8	12.3	6.5	9.8	10.5	10.5	11.4	15.4

Experiment 3C Ordering of Dissimilarities

	L_0 constant (0.5 secs)			A_0 constant (87db)		
	(A_0A_2)	(A_0A_1)	(A_1A_2)	(L_0L_2)	(L_0L_1)	(L_1L_2)
S1	2 (2.3)	1 (3.3)	3 (2.2)	1 (8.6)	2 (4.8)	3 (4.6)
S2	1 (5.5)	2 (4.3)	3 (3.7)	1 (13.6)	3 (6.9)	2 (7.3)
S3	1 (8.5)	2 (8.0)	3 (6.4)	2 (4.4)	1 (6.3)	3 (3.5)
S4	1 (5.1)	2 (4.0)	3 (2.0)	1 (8.0)	2 (7.6)	3 (5.6)
S5	1 (10.0)	2 (7.8)	3 (7.4)	2 (4.5)	1 (5.0)	3 (4.3)
S6	1 (10.1)	2 (7.4)	3 (6.8)	1 (9.0)	2 (8.0)	3 (3.8)
S7	1 (9.4)	2 (6.4)	3 (5.1)	1 (10.6)	2 (7.9)	3 (2.5)
S8	1 (16.6)	3 (8.3)	2 (11.5)	1 (11.3)	2 (8.0)	3 (4.8)

Table 4.61

The bracketed numbers in each cell of the first three columns refer to the dissimilarity between (A_0A_2), (A_0A_1), and (A_1A_2) with L_0 constant. The unbracketed numbers are the rank orders of these dissimilarities for each S. The last three columns refers to the dissimilarity between (L_0L_2), (L_0L_1), (L_1L_2) with A_0 constant. The prediction is that with respect to the rank ordering of dissimilarities: $\delta(A_0A_2) > \delta(A_0A_1) > \delta(A_1A_2)$ and $\delta(L_0L_2) > \delta(L_0L_1) > \delta(L_1L_2)$.
This prediction was upheld in both cases. Hence the lack of interaction between these dimensions implied decomposability. The results of this experiment could have been anticipated from those of Experiment 1A.

Experiment 3C   Ordering of Dissimilarities obtained from Table 4.60

	L_1 constant (1.5 secs)			A_1 constant (102 db)		
	(A_0A_2)	(A_0A_1)	(A_1A_2)	(L_0L_2)	(L_0L_1)	(L_1L_2)
S1	3 (2.3)	1 (3.3)	2 (2.8)	1 (8.6)	2 (5.9)	3 (2.6)
S2	1 (6.0)	3 (4.8)	2 (5.0)	1 (12.0)	2 (7.5)	3 (6.9)
S3	2 (7.6)	1 (8.3)	3 (6.9)	2 (5.9)	3 (5.6)	1 (6.0)
S4	1 (7.3)	2 (4.5)	3 (3.5)	1 (10.0)	2 (7.1)	3 (2.4)
S5	1 (12.4)	2 (9.5)	3 (8.3)	1 (8.3)	2 (7.1)	3 (5.8)
S6	1 (10.4)	2 (8.6)	3 (7.6)	1 (10.8)	2 (8.3)	3 (4.3)
S7	1 (8.8)	2 (7.3)	3 (3.9)	1 11.0	2 (9.5)	3 (3.6)
S8	1 (15.9)	2 (10.8)	3 (7.3)	1 (15.8)	2 (11.9)	3 (5.8)

Table 4.62. The bracketed numbers in each cell of the first three columns refer to the dissimilarity between (A_0A_2), (A_0A_1) and (A_1A_2) with L_1 constant. The unbracketed numbers are the rank orders of these dissimilarities for each S. The last three columns refer to the dissimilarity between (L_0L_2), (L_0L_1) and (L_1L_2) with A_1 constant. The prediction is, that with respect to the rank ordering of dissimilarities,: $\delta(A_0A_2)>\delta(A_0A_1)>\delta(A_1A_2)$ and $\delta(L_0L_2)>\delta(L_0L_1)>\delta(L_1L_2)$. This prediction was upheld in both cases, implying decomposability or lack of interaction between these dimensions.

Experiment 3C   Ordering of Dissimilarities obtained from
Table 4.60

	L_2 constant (2.0 secs)			A_2 constant (106 db)		
	(A_0A_2)	(A_0A_1)	(A_1A_2)	(L_0L_2)	(L_0L_1)	(L_1L_2)
S1	1 (4.2)	2 (2.)	3 (1.7)	1 (8.8)	2 (5.3)	3 (4.6)
S2	3 (4.0)	2 (4.3)	1 (5.4)	1 (12.3)	3 (6.3)	2 (7.2)
S3	1 (10.7)	2 (7.3)	3 (6.1)	1 (6.5)	2 (5.4)	3 (5.1)
S4	1 (8.6)	2 (8.6)	3 (2.9)	1 (9.8)	2 (7.4)	3 (6.3)
S5	1 (16.0)	2 (10.4)	3 (9.8)	1 (10.5)	2 (6.9)	3 (2.9)
S6	1 (11.1)	2 (7.8)	3 (7.2)	1 (10.5)	2 (7.6)	3 (7.5)
S7	1 (11.1)	2 (7.3)	3 (4.0)	1 (11.4)	2 (10.1)	3 (4.4)
S8	1 (16.5)	2 (10.4)	3 (8.5)	1 (15.4)	2 (11.4)	3 (7.4)

Table 4.63. The bracketed numbers in each cell of the first three columns refer to the dissimilarity between (A_0A_2), (A_0A_1), (A_1A_2) with L_2 constant. The unbracketed numbers are the rank orders of these dissimilarities for each S. The last three columns refer to the dissimilarity between (L_0L_2), (L_0L_1) and (L_1L_2) with A_2 constant. The prediction is that with respect to the rank ordering of dissimilarities. $\delta(A_0A_2) > \delta(A_0A_1) > \delta(A_1A_2)$ and $\delta(L_0L_2) > \delta(L_0L_1) > \delta(L_1L_2)$. This prediction was upheld in both cases implying decomposability or lack of interaction between the dimensions.

The results of this test are shown in Table 4.60

(2) Ordering of Differences (Decomposability Test)

From Fig 4.61

Since $(A_oA_2) > (A_oA_1) > (A_1A_2)$

Then $\delta(A_oA_2) > \delta(A_oA_1) > \delta(A_1A_2)$

Further, Since

$$(L_oL_2) > (L_oL_1) > (L_1L_2)$$

Then $\delta(L_oL_2) > \delta(L_oL_1) > \delta(L_1L_2)$

The results of this test appear in tables 4.61, 4.62, and 4.63.

RESULTS

Subtractivity (from Table 4.60) - a direct test of subtractivity revealed that the hypothesis of intradimensional subtractivity was upheld beyond the .001% level of significance.

DECOMPOSABILITY: for the predicted difference orderings all the predictions were upheld beyond the .001% level of significance. Moreover these orderings were consistent for different levels of the other dimension: thus, these two dimensions exhibited lack of interactions and the hypothesis of decomposability was upheld.

CONCLUSIONS:

The hypothesis of intradimensional subtractivity and decomposability for these two dimensions loudness, and duration, was strong upheld.

4.70 Conclusions for Experiments 3A, 3B and 3C.

Each of the three experiments constituted tests of intradimensional subtractivity and decomposability for the dimensions; pitch, duration and loudness taken two at a time. The diagnoses was two fold

(i) A test for decomposability in which the predicted orderings in differences along a dimension was compared with the actual ordering over different levels of the second dimension. This was really a test for ordinal interaction effects.

(ii) A direct test for subtractivity.

In a very real sense the first mentioned test was fundamental: if this had failed - that is the ordering of the difference depended on the level of the other dimension - then both subtractivity and the additive difference model would have been violated. So if subtractivity had failed in this experiment it may well have been due to failure of decomposability.

For all three experiments, for the dimensions tested, both decomposability and subtractivity were strongly confirmed. In particular Experiment 3C was of interest because it was for these two dimensions (see Experiment 1A) that it was discovered that dissimilarity judgements could be represented by an additive process.

4.70 Experiment 3D Interdimensional Additivity

This experiment constitutes in part, a replication, of Krantz and Tversky's (1969) study which examined interdimensional additivity for schematic faces varying on three binary attributes. This experiment was discussed in section 4.40. In the experiment reported here, there are again three attributes:

pitch (F), loudness (A) and duration (L) all of which vary on two levels so as to make up 8 different tone combinations varying in these 3 parameters.

4.71 Notation and values

The two levels of pitch are denoted by F_o and F_1 where:

F_o = 300 Hz (F_oF_1) = 400 Hz

F_1 = 700 Hz

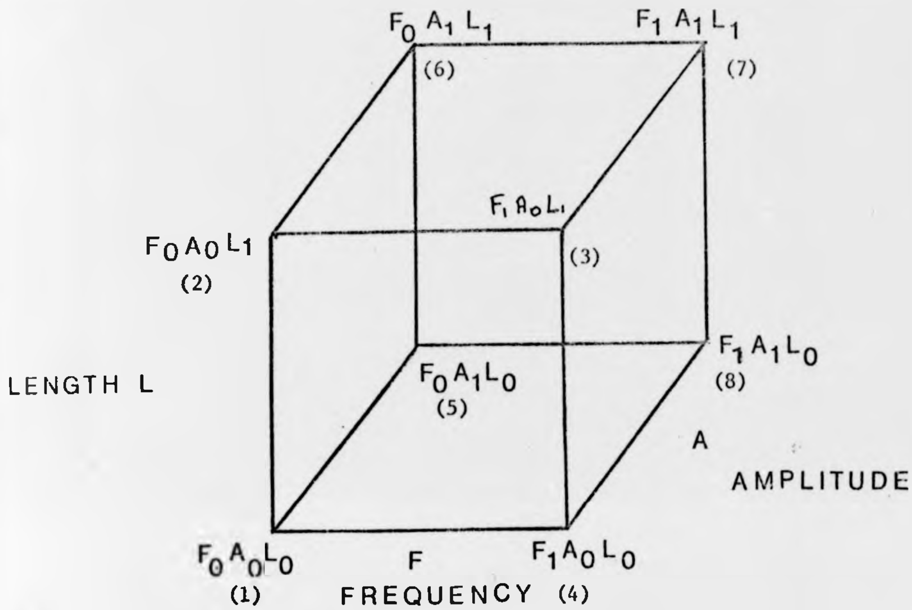
Where (F_oF_1) denotes the difference in the physical values of frequency between the tones with a common value of all other attributes.

The two levels of duration L_o, and L_1 are :

L_o = 0.5 sec and (L_oL_1) = 0.5 secs

L_1 = 1.0 sec

Fig 4.71



REPRESENTATION OF 8 SIGNALS EACH COMPOSED OF THREE PARAMETERS F, A, AND L WHICH VARY ON TWO LEVELS AS VERTICES OF A RECTANGULAR SOLID.

The two levels of loudness A_o and A_1 are:

$$A_o = 95.0 \text{ db} \quad \text{and } (A_oA_1) = 2.5 \text{ db}$$

$$A_1 = 97.5 \text{ db}$$

The 8 tones are shown in Fig 4.71, as vertices of a rectangular solid.

4.72 Using the same notation as before and applying Equation 4.201 which summarises the interdimensional model we have:

$$\delta(x,y) = F\left[\sum_{i=1}^{N} \Phi_i (x_i, y_i)\right]$$

N = number of dimensions.

δ (x,y) is a measure of the dissimilarity between x and y.

F a strictly increasing function of one variable.

Φ_i a symmetric function of two (nominal scale) variables.

$\Phi_i = 0$ if two variables are equal

$\Phi_i > 0$ otherwise

x_i is the value of stimulus x on the ith dimension.

For our case, where N=3 we can rewrite the above quotation as

4.721 $$\delta(x,y) = F\left[\Phi_1 (x_1, y_1) + \Phi_2 (x_2, y_2) + \Phi_3(x_3, y_3)\right]$$

TABLE 4.71 A

MEAN SIMILARITY RATINGS N = 10, FOR 7 SUBJECTS OF PAIRS OF TONES WHICH VARIED ON TWO LEVELS OF FREQUENCY, AMPLITUDE AND DURATION (F_0, F_1; A_0A_1; L_0, L_1;)

Numbers in brackets identify pair of tones in Fig

A ONE ATTRIBUTE VARYING BETWEEN PAIRS		[f'al]	RATINGS FOR SUBJECTS S1	S2	S3	S4	S5	S6	S7
(1) Frequency Varying									
$(F_0A_0L_0)$	$(F_1A_0L_0)$	[1,4]	4.2	8.3	5.0	7.0	10.3	16.3	9.1
$(F_0A_0L_1)$	$(F_1A_0L_1)$	[2,3]	4.7	8.6	5.0	7.4	10.1	17.0	8.7
$(F_0A_1L_0)$	$(F_1A_1L_0)$	[5,8]	4.2	8.7	5.1	7.4	10.4	17.1	8.7
$(F_0A_1L_1)$	$(F_1A_1L_1)$	[6,7]	4.8	8.4	5.0	7.0	10.3	16.6	9.1
(2) Amplitude Varying		[fa'l]							
$(F_0A_0L_0)$	$(F_0A_1L_0)$	[1,5]	1.1	2.3	1.0	2.3	1.2	2.0	5.7
$(F_0A_0L_1)$	$(F_0A_1L_1)$	[2,6]	1.2	2.1	1.0	2.2	1.5	1.9	5.1
$(F_1A_0L_0)$	$(F_1A_1L_0)$	[4,8]	1.7	2.1	1.0	2.6	1.7	2.3	5.4
$(F_1A_0L_1)$	$(F_1A_1L_1)$	[3,7]	1.3	2.1	1.0	2.5	1.5	2.6	6.2
(3) Duration Varying		[fal']							
$(F_0A_0L_0)$	$(F_0A_0L_1)$	[1,2]	2.3	3.1	3.2	2.5	3.7	4.0	5.8
$(F_0A_1L_0)$	$(F_0A_1L_1)$	[5,6]	2.4	3.2	3.0	2.9	3.4	3.9	5.4
$(F_1A_0L_0)$	$(F_1A_0L_1)$	[4,3]	2.4	3.4	3.0	3.0	3.8	3.9	5.6
$(F_1A_1L_0)$	$(F_1A_1L_1)$	[8,7]	2.5	3.1	3.0	2.9	3.4	3.8	6.2

Table 4.7.1 A (Cont). RESULTS EXPERIMENT 3D (Cont)

				S1	S2	S3	S4	S5	S6	S7
B	TWO ATTRIBUTES VARYING BETWEEN PAIRS									
(4)	Frequency and Amplitude		[f'a'l]							
	$(F_0A_0L_0)$	$(F_1A_1L_0)$	[1,8]	6.2	10.3	5.0	8.1	11.1	17.7	11.5
	$(F_0A_0L_1)$	$(F_1A_1L_1)$	[2,7]	6.3	9.7	9.0	7.4	11.2	17.0	13.2
	$(F_1A_0L_0)$	$(F_0A_1L_0)$	[4,5]	6.1	9.8	5.0	7.9	11.3	16.5	13.3
	$(F_1A_0L_1)$	$(F_0A_1L_1)$	[3,6]	6.1	9.7	12.5	7.9	10.9	16.9	12.0
(5)	Duration and Amplitude		[fa'l']							
	$(F_0A_0L_0)$	$(F_0A_1L_1)$	[1,6]	5.1	6.3	3.8	3.3	4.1	4.3	10.6
	$(F_1A_0L_0)$	$(F_1A_1L_1)$	[4,7]	5.1	6.2	9.4	3.7	3.9	4.7	8.1
	$(F_0A_1L_0)$	$(F_0A_0L_1)$	[5,2]	4.9	6.7	13.4	3.1	4.1	4.6	10.2
	$(F_1A_1L_0)$	$(F_1A_0L_1)$	[8,3]	4.9	6.4	8.0	3.3	4.1	4.5	9.5
(6)	Frequency and Length		[f'al']							
	$(F_0A_0L_0)$	$(F_1A_0L_1)$	[1,3]	7.2	12.3	8.0	8.8	10.7	17.7	11.3
	$(F_0A_1L_0)$	$(F_1A_1L_1)$	[5,7]	7.3	12.1	8.0	9.1	10.3	18.0	11.2
	$(F_1A_0L_0)$	$(F_0A_0L_1)$	[4,2]	7.4	11.9	8.0	8.4	10.4	17.6	11.1
	$(F_1A_1L_0)$	$(F_0A_1L_1)$	[8,6]	7.6	12.3	8.1	9.0	10.5	17.3	12.4
C	ALL THREE ATTRIBUTES VARYING BETWEEN PAIRS		[f'a'l']							
(7)	$(F_0A_0L_0)$	$(F_1A_1L_1)$	[1,7]	10.3	14.3	8.3	9.4	13.3	18.3	16.7
	$(F_0A_0L_1)$	$(F_1A_1L_0)$	[2,8]	10.7	13.6	8.0	9.5	12.9	18.2	15.6
	$(F_1A_0L_0)$	$(F_0A_1L_1)$	[4,6]	9.8	13.1	8.3	9.7	12.7	18.9	17.2
	$(F_1A_0L_1)$	$(F_0A_1L_0)$	[3,5]	9.7	14.3	8.2	9.1	11.3	19.0	16.5

TABLE 4.71B RESULTS (CONTINUED) OF EXPERIMENT 3D.

D(8) No Attributes Varying Between Pairs [f a l]

		S1	S2	S3	S4	S5	S6	S7
$(F_oA_oL_o)$, $(F_oA_oL_o)$	[1,1]	1.0	1.0	1.0	1.0	1.0	1.0	1.0
$(F_1A_oL_o)$, $(F_1A_oL_o)$	[4,4]	1.0	1.0	1.0	1.0	1.0	1.0	1.0
$(F_1A_1L_1)$, $(F_1A_1L_1)$	[7,7]	1.0	1.0	1.0	1.0	1.0	1.0	1.0
$(F_oA_1L_1)$, $(F_oA_1L_1)$	[6,6]	1.1	1.0	1.0	1.1	1.0	1.0	1.0

TABLE 4.71B RESULTS (CONTINUED) OF EXPERIMENT 3D.

D(8) No Attributes Varying Between Pairs [f a l]

		S1	S2	S3	S4	S5	S6	S7
$(F_oA_oL_o)$, $(F_oA_oL_o)$	[1,1]	1.0	1.0	1.0	1.0	1.0	1.0	1.0
$(F_1A_oL_o)$, $(F_1A_oL_o)$	[4,4]	1.0	1.0	1.0	1.0	1.0	1.0	1.0
$(F_1A_1L_1)$, $(F_1A_1L_1)$	[7,7]	1.0	1.0	1.0	1.0	1.0	1.0	1.0
$(F_oA_1L_1)$, $(F_oA_1L_1)$	[6,6]	1.1	1.0	1.0	1.1	1.0	1.0	1.0

In order to demonstrate how to use this equation consider the following example:

$$x = (F_o A_o L_o) \quad \text{tone 1 (Fig 4.71)}$$

$$y = (F_1 A_o L_o) \quad \text{tone 4 (Fig 4.71)}$$

The subscripts indicate the nominal scale values on the three attributes (F, A, and L).

$$\delta(x,y) = F\left[\Phi_1(F_1F_1) + \Phi_2(A_oA_o) + \Phi_3(L_oL_o)\right]$$

$$\text{i.e. } \delta(x,y) = F\left[\Phi_1(F_oF_1)\right]$$

Again consider tones 5 and 8 (Fig 4.71) which we will denote by x' and y'

$$\text{i.e.} \quad x' = F_oA_1L_o \quad \text{tone 5}$$
$$y' = F_1A_1L_o \quad \text{tone 8}$$

$$\delta(x'y') = F\left[\Phi_1(F_oF_1) + \Phi_2(A_1A_1) + \Phi_3(L_oL_o)\right]$$

$$\text{i.e.} \quad \delta(x'y') = F\,\Phi_1(F_oF_1)$$

and therefore $\delta(1,4) = \delta(5,8)$

If this equation is applied systematically to all possible stimulus pairs the following predictions can be made which are adumbrated in Table 4.71. In this table there are 7 sets of four stimulus pairs shown on the left hand side and labelled A(1), A(2)...B(1)...etc. Within each of these sets the 4 dissimilarities should be the same for each subject.

4.722 The first prediction then, is that the dissimilarities between pairs within any of these 7 subclasses should be the same. This was one of the predictions tested and upheld by Krantz and Tversky with their stimuli.

4.723 Another prediction which Krantz and Tversky derived from equation 4.201 was that the dissimilarity between stimuli increased as the number of attributes in which they differed increased. This is easily appreciated at an intuitive level. For example according to Equation 4.201 the following statement must be true for any one subject:

$$\delta(F_oA_oL_o,\ F_1A_oL_o) > \delta(F_oA_oL_o,\ F_oA_1L_1) > \delta(F_oA_oL_o,\ F_1A_1L_1)$$

The first pair only vary with respect to one attribute; the second pair varies on two attributes, and the third pair varies on three attributes.

4.724 A third prediction which follows from the Equation of Interdimensional Additivity is that joint factor independence should hold - this was discussed in Section 4.21. This was not a prediction made by Krantz and Tversky (1969) and their design in fact did not allow it to be tested. Joint factor independence has already been applied to dissimilarity data in

Experiment 1B, where it was tested for a small subset of stimuli. Briefly, for dissimilarity orderings, joint factor independence asserts: if the dissimilarity produced by some combination of difference on N-1 dimensions exceeds that produced by some other combination on those same dimensions the difference on the Nth dimension remaining constant, then that same ordering holds as the constant difference on the Nth dimension is varied. See Fig 4.72, which should be also compared to Fig 2.52A and B in Chapter 2.

METHOD

A single pair of tones was presented successively to the subject who rated the dissimilarity between them by marking a point on a rating scale from 1 (identical) to 20 (extremely dissimilar).

The stimulus set consisted of:

(i) 28 non identical pairs of tones. These are shown in Table 4.71A, with their numbers (see Fig 4.71).

(ii) 4 identical pairs of tones:

$(F_oA_oL_o, F_oA_oL_o)$ (1,1)
$(F_1A_oL_o, F_1A_oL_o)$ (4,4)
$(F_1A_1L_1, F_1A_1L_1)$ (7,7)
$(F_oA_1L_1, F_oA_1L_1)$ (6,6)

Fig 4·70 EXPERIMENT 3D (Interdimensional Additivity)

Dissimilarity matrix derived for S7 in a similarity rating experiment between pairs of tones presented successively. Each of the tones is composed of 3 physical dimensions: Frequency, Amplitude, and Length which vary on 2 levels.

	$F_0A_0L_0$	$F_0A_1L_0$	$F_1A_1L_0$	$F_1A_0L_0$	$F_0A_0L_1$	$F_0A_1L_1$	$F_1A_1L_1$	$F_1A_0L_1$
$F_0A_0L_0$	-	5.7	11.5	9.1	5.8	10.6	16.7	11.3
$F_0A_1L_0$		-	8.7	13.3	10.2	5.4	11.2	16.5
$F_1A_1L_0$			-	5.4	15.6	12.4	6.2	9.5
$F_1A_0L_0$				-	11.1	17.2	8.1	5.6
$F_0A_0L_1$					-	5.1	13.2	8.7
$F_0A_1L_1$						-	9.1	12.0
$F_1A_1L_1$							-	6.2
$F_1A_0L_1$								-

The complete stimulus set, consisted of 32 stimulus pairs; 28 non identical and 4 identical pairs, all of which are generated from the 8 tones shown in Fig 4.71.

Each experimental session consisted of four blocks of these 32 stimulus pairs, so each stimulus pair was presented four times in one session for a dissimilarity rating. On two of these presentations the order was reversed to counteract order effects. Altogether each subject took part in 7 sessions. All subsequent analysis was performed on the data from the last 2½ sessions.

SUBJECTS

7 undergraduate students, all from the Stirling University Subject Pool. Each session of 128 pair presentations took about an hour to complete. Most of the sessions had two (at most three) subjects present.

EQUIPMENT

Recorded and presented in exactly the same manner as the three previous experiments. (See Appendix 3)

4.74 Results

The data from the last 2½ sessions are summarised in Tables 4.71 A and B. Fig 4.7Q shows a typical dissimilarity matrix derived from Subject 7. This

This can be checked with S7's performance in the table of results (Table 4.71A). Each of the entries in Table 4.71 A and B are the means of the ratings (N=10) for each of 28 pairs of non-identical tones, and 4 pairs of identical tones respectively. Table 4.71 A illustrates how each of the 28 pairs of non-identical tones can be grouped into 3 classes A, B, and C. Moreover A and B both contain three subclasses, containing 4 pairs of tones.

Class A: consists of all pairs of tones in which there is only one attribute varying between pairs. These pairs can be further subdivided into the following subclasses.

A(1) A subclass of four pairs in which only frequency varies between tones - this is labelled

[f'a l]

A(2) A subclass in which only loudness varies between tones

labelled [f a' l]

A(3) A subclass in which only duration varies between tones

labelled [f a l']

Class B: consists of all pairs of non-identical tones wuch that there are two attributes varying between

Fig 4.72

ΔA: a

ΔF	ΔL	
f'	f'a l	f' a l'
f	f a l	f a l'

ΔA: a'

ΔF	ΔL	
f'	f' a' l	f'a'l'
f	f a' l	f a'l'

Interdimensional Additivity as a Joint Factor Independence Condition (Experiment 3D).

In this design, there are three factors, which correspond to the differences between tones in frequency duration, and loudness denoted by ΔF, Δ L, and ΔA respectively: f and f' correspond to two levels of difference in frequency between tones of a pair (zero and 400 Hz respectively). Also l and l' correspond to two levels of duration differences between tones (zero seconds and 0.5 seconds). Likewise a and a' correspond to two levels of loudness difference between tones (zero and 2.5 db). Fig 4.73 shows the same factorial structure in the context of Experiment 3D.

For joint independence: If f' a l > f a l' then f'a'l > f a'l'. In words: If the dissimilarity produced by a combination of a difference in frequency (400 Hz) and non zero difference in duration is greater than the dissimilarity produced by zero difference in frequency and a non zero (0.5 seconds) difference in duration then this same ordering holds as the constant difference in loudness is varied.

Fig 4.73

$\Delta A \rightarrow$ a

	1	1'
f'	(1,4) (2,3) A(1) (5,8) (6,7) f'a 1	(1,3) (5,7) B(6) (4,2) (8,6) f'a 1
f	(1,1) (4,4) D(8) (7,7) (6,6) f a 1	(1,2) (5,6) A(3) (4,3) (8,7) f a 1'

↑ ΔF $\Delta L \rightarrow$

$\Delta A \rightarrow$ a'

	1	1'
f'	(1,8) (2,7) B(4) (4,5) (3,6) f'a'1	(1,7) (2,8) C(7) (4,6) (3,5) f'a'1'
f	(1,5) (2,6) A(2) (4,8) (3,7) f a'1	(1,6) (4,7) B(5) (5,2) (8,3) f a'1'

↑ ΔF $\Delta L \rightarrow$

All possible pairs of stimuli in Experiment 3D can be assigned to one of the 8 cells of a 2x2x2 factorial structure in which each cell corresponds to a particular combination of differences in frequency duration, and loudness between tones of a pair. The factors are these 'differences' between tones and are denoted by ΔF, ΔL, and ΔA respectively. Since this is a conjoint structure we may test for joint factor independence which is one of the consequences of inter-dimensional additivity. For instance if f'a 1 > f a 1' then f'a'l > f a'l'. This means, that if

$$\delta(1,4) > \delta(1,2)$$

$$\text{then } \delta(1,8) > \delta(1,6)$$

N.B. The labels A(1), B(6), etc identify the subclasses in Table 4.71 to which each of the groups of 4 pairs belong.

tones. These pairs may be further subdivided into the following subclasses:

B(4) A subclass in which only frequency and loudness (amplitude) vary between tones:

labelled [f'a' l]

B(5) A subclass in which only duration and loudness vary between tones:

labelled [f a'l']

B(6) A subclass in which frequency and duration vary:

labelled [f'a l']

Class C: In this class are all non-identical pairs which vary in three attributes. There is only one such subset, labelled

[f'a'l']

Class D: Table 4.71 B gives a subset of 4 identical pairs, labelled

[f a l]

It should be noted that each of these 8 subsets contain only four pairs of tones and more over, each of these subsets can be represented by a cell in the 2x2x2 factoral structure shown in Fig 4.73.(Fig 4.72 clarifies the above also)

Testing the Three Predictions

The first prediction then, (see 4.722) is that the dissimilarities for any one subject, should be constant within any one of the subclasses A(1), A(2), A(3), B(4), B(5), B(6), C(7), and D(8).

The differences among the four means within any one class, for any one S, are so small as to be insignificant. This first prediction then, is strongly upheld.

The second prediction(see 4.723)is also strongly confirmed, there is hardly any overlap between the pairs, differing in one attribute, two attributes and three attributes for any one subject. So the prediction that dissimilarity increases if new differences between stimuli are added is strongly confirmed. The only exception to this was S3. The third prediction(see 4.724) concerns joint factor independence. Fig 4.72 and Fig 4.73 show how each of the 8 subclasses of four stimulus pairs - covering all the possible stimulus pairs in the experiment can be assigned to one or other of the eight cells of the 2x2x2 factorial structure, where the factors are differences in frequency, duration, and loudness denoted by ΔF, ΔL, and ΔA respectively. Each of these factors have two levels denoted by:

(i) f and f' - the two levels of ΔF

(ii) l and l' - the two levels of ΔL

Fig 4.74 Experiment 3D

(a) Joint Factor Independence of ΔA over ΔF and ΔL

a

	l	l'
f'	A(1) f'a l	B(6) f'a l'
f	D(8) f a l	A(3) f a l'

a'

	l	l'
f'	B(4) f'a'l	C(7) f'a'l'
f	A(2) f a'l	B(5) f a'l'

For any one S, the ordering of dissimilarities between any of the four pairs in subclass A(1) and A(3) must be the same as that between any of the four pairs in B(4) and B(5).

Example: Consider S1 (Table 4.71). The dissimilarity between pairs in A(1) is greater than the dissimilarities between pairs in A(3). This same ordering must hold between B(4) and B(5). It does, because the dissimilarity between any of the pairs in B(4) is greater than for any of the dissimilarities between pairs in B(5).

Likewise the ordering of dissimilarities between any of the four pairs in B(6) and D(8) must be the same as that between C(7) and A(2).

(b) Joint Factor Independence of ΔL over ΔF and ΔA.

l

	a	a'
f'	A(1) f'a l	B(4) f'a'l
f	D(8) f a l	A(2) f a'l

l'

	a	a'
f'	B(6) f'a l'	C(7) f'a'l'
f	A(3) f a l'	B(5) f a'l'

(Fig 4.74 Continued)

For any one S, the ordering between any one of the four pairs in subclass A(1) and A(2) must be the same as the ordering between any of the four pairs in subclass B(6) and B(5). Likewise the ordering of dissimilarities between any of the four pairs in B(4) and D(8) must be the same as that between C(7) and A(3).

(c) Joint Factor Independence of ΔF, over ΔL and ΔF.

f

	1	1'
a'	A(2) f a'l	B(5) f a' l'
a	D(8) f a l	A(3) f a l'

f'

	1	1'
a'	B(4) f'a'l	C(7) f'a'l'
a	A(1) f'a l	B(6) f'a l'

For any one S, the ordering between any one of the four pairs in A(2) and A(3) must be the same as the ordering between any of the four pairs in subclasses B(4) and B(6).

Likewise the ordering between any of the four pairs in B(5) and D(8) must be the same as that between C(7) and A(3).

(iii) a and a' - the two levels of ΔA

Fig 4.72 and 4.73 give further details of this empirical conjoint structure.

Figure 4.74 explains in detail the requirements of joint factor independence and suggests how joint factor independence may be tested. This we do for each subject, in conjunction with Table 4.71.

(a) Joint Factor Independence of ΔA over ΔF and ΔL. From Fig 4.74(a) it can be seen that this means that the ordering of dissimilarities between any of the four pairs in subclass A(1) and A(3) must be the same as that between B(4) and B(5). There is only one violation and that is for S3.

Likewise the ordering between B(6) and D(8) must be the same as that between C(7) and A(2). This was found to be so for all Ss.

(b) Joint Factor Independence of ΔL over ΔF and ΔA. From Fig 4.74 (b) this amounts to saying that the ordering of dissimilarities between any of the four pairs in subclass A(1) and subclass A(2) must be the same as that between B(6) and B(5). Both S3 and S2 violated this restriction. The ordering between all pairs in B(4) and D(8) msut be the same as those between C(7) and A(3). This was found to be true for all Ss.

(c) Joint Factor Independence of ΔF over ΔL and ΔF. From Fig 4.74 (c) joint factor independence of ΔF over ΔL and ΔF means that the ordering between A(2) and A(3) is the same as that between B(4) and B(6). This restriction was violated by S7 only. Likewise the ordering between B(5) and D(8) must be the same as that between C(7) and A(1). This was found to be true for all S.

4.75 Conclusions

The three predictions of the interdimensional additivity equation (4.201) have all been fairly strongly confirmed. The last prediction,that of joint factor independence which is really the crux of interdimensional additivity, is really a very strong condition, and its consequence have been spelled out in detail in section 2.55. Joint factor independence was confirmed (except for some 'minor' violations) for all subjects except for S3 who displayed some very strong and idiosyncratic violations.

Equations 4.201 includes as special cases the Euclidean, City block, and Dominance metric and rejection of 4.201 for these dimensions would have ruled out the applicability of any of these three distance models for similarity judgements.

Experiment 3D in a sense worked in reverse; from a given model to a test of the predictions. Experiment 1B, however, was an exploratory foray into the applicability of the additive difference model for similarity judgements assuming a dimensional representation only, and bereft of any metric constraints. Experiment 3D although it tested both the metric and dimensional requirements was set up in such a way as to exploit its latent conjoint structure. There are indeed other ways of testing the implied ordering predictions of Equation 4.201 - which Krantz and Tversky (1969) attempted - but it can involve (as it did with those authors) some truly hair-raising and messy non-parametric statistical work. The advantages of testing the ordering predictions in the manner attempted are two fold: First, the design pointed to the measurement theoretic basis of Beal's et al's original speculations. Second, it urges, that if possible, experiments should be designed so as to lay as bare and as transparent as possible the underlying psychological theory.

4.80 Discussion

It now seems worthwhile to consider experiments 1A and 1B together with 3D. Experiments 1A and 1B investigated various independence conditions for similarity data. The data indeed showed that some fairly stringent independence conditions were satisfied for the dissimilarity judgement tasks, in fact, stringent enough, to assert that the dissimilarity judgements could be represented by some type of additive independence model. It was shown in 1A, that this form of additive independence satisfied all the requirements of the additive difference model for similarity judgements for the two attribute case, N=2. This is an interesting result because Krantz et al (1971) and Tversky (1967) have shown that additive conjoint measurement can be applied to both the case when N≥3 and N=2. However the case N=2 presents problems, because the ordinal requirements of additive conjoint measurement are slightly different than for the case when N≥3. This finding is reflected in Beal's et al (1968) and Tversky and Krantz (1970) discussions on the metric requirements of the additive difference model and they give an axiomitisation only for N≥3. However in experiment 1A, where N=2, the additive nature of dissimilarity judgements was handled by using the double cancellation condition (Luce and Tukey, 1964).

Experiment 1B investigated both single factor and joint factor independence for a three factor dissimilarity judgement experiment in which the factors were differences of varying magnitudes along the three dimensions, pitch (ΔF), loudness (ΔA) and duration (ΔL) between tones. Because both single factor and joint factor independence was satisfied it again suggested that these three dimensions combine additivity, (and hence there are no interactions between them) to determine dissimilarity.

Both experiments 1A and 1B demonstrated the plausibility then, of the additive difference model for similarity judgements for the case N=2, and N=3 respectively.

Tversky (1965) was the first to recognise that additive conjoint measurement could appropriately be applied to metric representations of dissimilarity orderings. He showed that the distance equation (4.121):

$$d_r(x,y) = \left[\sum_{i=1}^{N} |x_i - y_i|^r\right]^{1/r} \qquad r \geqslant 1$$

could be generalised in two ways:

4.313 $$d(x,y) = F\left[\Phi_i(x_i, y_i)\right]$$

4.312 $$d(x,y) = F\left[|f_1(x_1)-f_2(y_2)| \ldots\ldots |f_N(x_N)-f_N(y_N)|\right]$$

(The numbers 4.313 and 4.312 refer to the sections where these were first mentioned).

Equation 4.313 says that the dimensions combine additivity to determine dissimilarity and 4.312 specifies that the contribution of any one dimension can be represented by the absolute difference on the scale values (intradimensional subtractivity). If d(x,y) is not required to be a metric then 4.313 is simply the equation for additive conjoint measurement, in N variables (see section 2.31 Chapter 2). It was this realisation then,that animated Experiments 1A and 1B.

Experiment 3D constituted a test for interdimensional additivity, a condition, which has embedded in it, both dimensional and metric requirements. The dimensional requirements include the twin conditions of the additive combination of the dimensions and lack of interaction between them. The most important metric requirement is segmental additivity. Experiments 1A and 1B merely tested the dimensional requirements of the additive difference model. One last methodological point to do with the strategy of model testing adopted here. All the experiments involved the same dimensions. Thus the following pattern of results could have emerged:

Either (1) For all subjects, there was widespread violations of the various independence conditions for Experiment 1B i.e. dimensional requirements not satisfied. If this had occurred, interdimensional additivity would have failed anyhow since it has the dimensional requirements built into it.

or (2) No systematic violations in Experiment 1B, i.e. dimensional requirements are intact but widespread violations for interdimensional additivity. This would mean that whereas the dimensional requirements are met the metric ones are not.

or (3) No violations in Experiment 1B or 3D, both the metric and dimensional requirements are intact, as was in fact the case.

CHAPTER 5

INFORMATION PROCESSING AND THE STIMULUS METRIC : AN ALTERNATIVE EXPLANATION

5.00 Introduction

5.10 Metric Properties of the Stimulus in Information Processing. Attneave's [1950] experiment. The Euclidean and City Block Metric. Shepard's study. Hyman and Well. Lockhead's distinction between integral and non-integral stimuli.

5.20 Redundancy Gains and Interference Effects in Absolute Judgement Tasks.

5.30 Speeded Classification Tasks. Garner and Felfoldy's Experiment.

5.40 Explanations of Redundancy Gains in Absolute Judgement and Speeded Classification Tasks. The Independent Dimensions Model. The Psychological Distance Hypothesis. Difficulties with 'Metric' explanations of Redundancy Gains. An Alternative Dimensional Account for Redundancy Gains based on the Independent Dimensions Model.

5.00 Experiment 4A. Testing the Independent Dimensions Model.

5.00 Introduction

In Chapter 4 it was seen that the distance function which embodied both dimensional and metric properties involved the following four assumptions:

(i) Decomposability: The distance between objects can be decomposed into contributions from each of the dimensions or components of the space.

(ii) Intradimensional Subtractivity: Each contribution to the distance measure between two points is based upon the absolute difference in scale values within any one dimension between the points.

(iii) Interdimensional Additivity: The distance measure combines additively from each dimension.

(iv) Metric: All the differences in (ii) are transformed by the same power function or Minkowski metric.

Recently, there have been experimental psychologists,(Garner, 1974, Lockhead, 1972, Hyman and Well, 1967, 1968, Handel, 1968, Shepard, 1964, Rabbitt, 1971), who have suggested that the type of information processing which takes place within an organism depends upon particular properties of the stimulus, in particular, on its metric properties.

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5.10 Metric Properties of the Stimulus and Information Processing

It was Attneave (1950) who first suggested that attentional phenomena could interact with the metric of the stimulus space. His study of similarity judgements of geometrical stimuli (parallelograms) gave some empirical support for his belief that the City Block or Minkowski 1-metric might provide a better explanation of judgemental behaviour than the usual Euclidean distance, or Minkowski 2-metric The important feature of his stimuli seemed to be that the dimensions could easily be isolated perceptually, and attended to independently of one another, Attneave 's suggestion was that in the case of such dimensions the sum of absolute differences formula of the City Block metric was more appropriate than the square-root of the sum of squared differences Euclidean formula. The City Block metric, it was suggested is the one which would be used if subjects judged the interstimulus distances for each dimension independently, and then combined them additively to derive an overall subjective distance. However for stimuli such as colour where the dimensions could not be easily isolated Attneave proposed that the more wholistic Euclidean metric was still appropriate.

5.12 Shepard (1964) followed Attneave by making a similar distinction between analysable and unitary stimuli, and he wrote of these two types:

'..... there are those that are related as homogenous unitary wholes, and those that tend to be analysed into perceptually distinct components or properties ... (Shepard, 1964 page 80).

The first sort of stimulus he called unitary and the second analysable. Moreover he suggested that a metric somewhere between the Euclidean and City Block was required to account for similarity judgements involving analysable stimuli and the Euclidean metric for unitary stimuli.

In one experiment, Shepard used highly analysable geometric stimuli. These were a series of circles of different sizes with inscribed radii varying in angle of inclination. This series was so constructed that one of the stimuli exactly matched a standard stimulus on just one of the two dimensions (diameter and inclination of the radius). Another circle in the same series matched the standard on the second dimension. All the remaining stimuli in the series were compromises on these two dimensions. The task for each subject was to select the stimulus in the series that was closest to the standard.

Shepard's argument was, that if the psychological space had the same structure as the physical space and if its metric was Euclidean then neither of these two special stimuli (the two that matched the standard on one or other of their dimensions) would be picked as closest to the standard, but rather a compromise stimulus which was moderately close to the standard on both dimensions. The date showed, on the contrary, that subjects did not choose the compromise nor any of the intermediate stimuli. Instead, the distribution of choices (across subjects) was bimodel with the two modes corresponding to the two special stimuli which differed from the standard on only one of the two relevant physical dimensions. This tendency of the subjects to match the standard stimulus on just one of the two dimensions was interpreted by Shepard as implying that the perceptual metric was non-Euclidean and in particular it deviated toward the City Block metric.

5.13 Hyman & Well (1967, 1968) collected the complete similarity data on three types of stimuli and employed multidimensional scaling to obtain the best fit in a Euclidean space. They used the Euclidean metric as the baseline by which to study the pattern of similarity judgements of the following sets of stimuli:

(i) Munsell Colour patches, (9 different ones varying in Chroma and Value)

(ii) Geometrical Figures, (7 parallelograms varying in size and tilt)

(iii) Circles with radii (8, varying in diameter and angle of radius)

The same Munsell Colour patches were used by Torgerson (1952), the parallelograms were those studied by Attneave (1950) and the circles, of course, were the same as those considered by Shepard (1964).

Hyman and Well, then, used the Euclidean metric as a yard-stick to study the pattern of judgements for all three types of stimulus material. They reasoned that if subjects made their judgements according to the City Block metric then relative to their one dimensional judgements (Hyman and Well 1967, page 234), they would judge bidimensional differences as being larger than would be predicted from the Euclidean distance model.

Thus, when Hyman and Well examined the pattern of deviations from the Euclidean model, they expected to find in the case of the City Block model, that the average deviation of the bidimensional comparisons to be relatively positive, and the average deviation of the unidimensional comparisons to be relatively negative. So if subjects were truly 'City Block' the difference between the average deviation of the bidimensional comparison should be positive. Conversely if subjects deviated from the Euclidean model toward the Dominance model then the difference (average deviation from bidimensional minus average deviation from unidimensional comparison) should be negative. Hyman and Well concluded that their results confirmed that the Euclidean metric was appropriate for judgement of colour patches but that the City Block was more appropriate for describing similarity judgements of geometrical stimuli.

The Hyman and Well study is vulnerable on logical grounds. Their MDS procedure assumed a Euclidean metric in order to investigate systematic deviations. Usually MDS is employed to 'discover' the minimum number of dimensions necessary to describe the similarity data. This, Hyman and Well did not do; they assumed the identity and number of the component dimensions a priori. Their procedure had embedded in it, strong dimensional assumptions which they used to study the deviations from a Euclidean baseline. Before physical attributes - such as value and chroma for colour - can be offered as candidates for psychological dimensions, they must fulfil certain rigorous qualitative criteria: decomposability, intradimensional subtractivity, and interdimensional additivity. In fact, it is known that for colour dissimilarity judgements (involving hue and saturation) decomposability breaks down thereby ruling out any of the Minkowski r-metrics, (Krantz; personal communication).

5.14 Lockhead (1966, 1970, 1972) has also made a similar distinction between stimuli:

> '.... the similarity space which results from the combination of some dimensions produces a Euclidean metric while others produce a different perhaps a City Block metric.' (Lockhead 1970).

However Lockhead used the term <u>integral</u> to describe those stimuli which give rise to a Euclidean metric and non-integral for those that give rise to a City Block metric in direct distance scaling.

5.20 Redundancy Gains and Interference Effects in Absolute Judgement and Speeded Classification Tasks

Lockhead (1966), Garner and Felfoldy (1970), and Garner (1974) have all used the notion of integral and non-integral stimuli to explain results arising from two types of information processing experiments: absolute judgement and speeded classification tasks. Indeed Garner and Felfoldy (1970) on the basis of their experiments and from the work of Hyman and Wells (1968) and Torgerson (1958) offer the following operational definition of an integral stimulus:

(i) An integral stimulus gives rise to a Euclidean metric in direct distance scaling

(ii) An integral stimulus gives rise to a redundancy gain when the dimensions are correlated for both absolute judgement and speeded classification tasks

(iii) An integral stimulus gives rise to interference effects when the dimensions are added orthogonally in both absolute judgement and speeded classification tasks.

Non-Integral Stimuli were likewise given an operational definition by Garner and Felfoldy:

(i) Non-integral stimuli give rise to a City Block metric in direct distance scaling

(ii) There is no redundancy gain when the dimensions are correlated for either absolute judgement or speeded classification tasks

(iii) Non-integral stimuli do not give rise to interference effects when the dimensions are added orthogonally in either absolute judgement or speeded classification task.

We shall refer to the above definitions as the Garner - Felfoldy hypothesis.

The Logical Structure of Absolute Judgement and Speeded Classification Tasks for testing the Garner - Felfoldy Hypothesi

5.21 Absolute Judgement Tasks

Suppose we consider two stimulus dimensions A and L with two values on each dimension A_0, A_1; L_0 and L_1. In practice we need not think of only two values, for each dimension but we do so here, just to make the logical structure of an experiment testing the Garner - Felfoldy hypothesis apparent.

In such an experiment there are three modes of stimulus presentation

(i) Unidimensional Presentation of each of the component dimensions for identification

Fig 5.21 The Logical Structure of an Absolute Judgement Experiment to Test the Garner-Felfoldy Hypothesis

Stimulus Values to be identified	Mode of Stimulus Presentation		
	Unidimensional (Control)	Correlated	Orthogonal
A_o and A_1 Measure Used: information transmitted	Stimuli Presented A_oL_o, A_1L_o (Identify A_o and A_1)	Stimuli Presented A_oL_o, A_1L_1 (Identify A_o and A_1)	Stimuli Presented $A_oL_o, A_oL_1, A_2, L_o, A_1L_1$ (Identify A_o and A_1)
L_o and L_1 Measure Used: information transmitted	Stimuli Presented A_oL_o, A_oL_1 (Identify L_o and L_1)	Stimuli Presented A_oL_o, A_1L_1 (Identify L_o and L_1)	Stimuli Presented $A_oL_o, A_oL_1, A_1L_o, A_1L_1$ (Identify L_o and L_1)

Fig. 5.22 Outcome Structure of the Experiment shown in Fig. 5.21

Stimulus Values to be identified	Mode of Stimulus Presentation		
	Unidimensional	Correlated	Orthogonal
A_o and A_1	Baseline	(i) better than baseline (ii) Equal to baseline (iii) Worse than baseline	(i) better than baseline (ii) Equal to baseline (iii) Worse than baseline
L_o and L_1	Baseline	(i) better than baseline (ii) Equal to baseline (iii) Worse than baseline	(i) better than baseline (ii) Equal to baseline (iii) Worse than baseline

(ii) Correlated Dimension Presentation. Here a redundant dimension is added. For example A_oL_o, A_1L_1 always appear together, identification of one dimension suffices to identify the complete stimulus. The S is asked to attend to one of the dimensions and identify its value.

(iii) Orthogonal Stimulus Presentation. Here each of the two values of each of the dimensions occur with both values of the other dimension, with a resultant four different stimuli. Ss are required to identify each of the dimensional values and suppress the irrelevant second dimension. In our example the orthogonal stimulus set would be:

$$A_oL_o,\ A_oL_1,\ A_1L_o, A_1\ L_1$$

The subject would be required to identify A_o and A_1, ignoring different values of L and then in another condition to identify L_o and L_1 and ignore different values of A.

Each subject appears in six experimental conditions see (Fig. 5.21). There are two control conditions, two correleated conditions and two orthogonal conditions for each subject. Fig. 5.22 illustrates the possible outcome for each S (or for the averaged results of many subjects taking part in all six conditions). The measure is the amount of information transmitted in each of the six conditions.

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The Garner - Felfoldy hypothesis states that in absolute judgement experiments, stimuli which have integral dimensions would result in:

(a) Increase in information transmission in both the correlated presentation conditions compared with either base line performance : redundancy gain.

(b) In both the orthogonal presentation conditions a decrease in identifiability of either of the component dimensions compared with base line performance. That is to say there is interference: and selective dimensional attention breaks down.

For non-integral stimuli however the Garner - Felfoldy hypothesis asserts :

(c) No increase in information transmission in the correlated presentation condition; i.e. no redundancy gain.

(d) The identifiability of the component dimensions in the orthogonal presentation conditions are the same as the base line identifiability, that is 'perfect' selective dimensional attention is possible, and there are interference effects.

Fig. 5.21 and Fig. 5.22 summarises the above remarks for an absolute judgement task.

Erikson and Hake's (1955) study did not coincide with the above structure in all details - there were two control conditions, but not two 'correlated' conditions and of course there was no orthogonal presentation. They did report a redunency gain however. Lockhead (1966) used the dimensions of hue and value, and reported no redundancy gain, when the dimensions were separated but again there was only one correlated condition. Garner (1962) reports a few studies where dimensions were added orthogonally and judgements of all dimensions required - that is there were two orthogonal conditions for two dimensional stimuli - the results seemed to indicate that there was some improvement.

It is the case however, that there are no studies yet on absolute judgements which fit the logical requirements of a test for the Garner - Felfoldy hypothesis.

5.30 Speeded Classification Tasks

The structure of an experiment on speeded classification in order to test the Garner - Felfoldy hypothesis is exactly the same as the one for absolute judgements. The subject is required to classify stimuli by one of two dimensions represented in a set of stimuli and speed of classification is measured. Suppose we consider two stimulus dimensions A and L with two values on each dimension A_0, A_1 and L_0, L_1 respectively. Again there are three methods of stimulus presentation.

Fig. 5.31 The Logical Structure of a Speeded Classification Task Testing the Garner-Felfoldy Hypothesis

Stimulus Values to be Identified	Mode of Stimulus Presentation		
	Unidimensional (Control)	Correlated	Orthogonal
A_o and A_1 Measure used: sorting time	Stimuli Presented A_oL_o, A_1L_o (Identify A_o and A_1)	Stimuli Presented A_oL_o, A_1L_1 (Identify A_o and A_1)	Stimuli Presented $A_oL_o, A_oL_1, A_1L_o, A_1L_1$ (Identify A_o and A_1)
L_o and L_1 Measure used: sorting time	Stimuli Presented A_oL_o, A_oL_1 (Identify L_o and L_1)	Stimuli Presented A_oL_o, A_1L_1 (Identify L_o and L_1)	Stimuli Presented $A_oL_o, A_oL_1, A_1L_o, A_1L_1$ (Identify L_o and L_1)

Fig. 5.32 Outcome Structure of the Experiment shown in Fig. 5.31

Stimulus Values to be identified	Mode of Stimulus Presentation		
	Unidimensional (Control)	Correlated	Orthogonal
A_o and A_1 Measure used: sorting time	Baseline	(i) better than baseline (ii) Equal to baseline (iii) Worse than baseline	(i) better than baseline (ii) Equal to baseline (iii) Worse than baseline
L_o and L_1 Measure used: sorting time	Baseline	(i) better than baseline (ii) Equal to baseline (iii) Worse than baseline	(i) better than baseline (ii) Equal to baseline (iii) Worse than baseline

(i) Unidimensional Presentation. Here S is required to sort (say) a deck of cards on the basis of just one of the two dimensions into two piles.

(ii) Correlated Presentation. Here the two dimensions appear in a perfectly correlated way. Again the subject is required to sort by each of two dimensions, by sorting into two piles.

(iii) Orthogonal Presentation. Each of the two values of each of the dimensions occur with both values of the other dimensions with a resultant four different stimuli. S is required to identify each of the dimensional values and suppress the irrelevant second dimension.

Again each subject appears in six experimental conditions (Fig. 5.31) with two correlated and two orthogonal conditions. Fig. 5.32 shows the possible outcome structure of such an experiment.

The Garner - Felfoldy Hypothesis states that in speeded classification tasks, stimuli which have integral dimensions would result in

(a) An increase in sorting time in both the correlated presentation conditions compared with either base line performance i.e. there is a redundancy gain

(b) A decrease in sorting time of either component dimension in the orthogonal presentation conditions compared with baseline performance.

For non-integral stimuli however the Garner - Felfoldy hypothesis asserts:

(c) No increase in sorting time in the correlated presentation conditions: no redundancy gain

(d) No decrease in sorting time in the orthogonal presentation conditions.

Garner and Felfoldy (1970) themselves provided a study, which in all experimental details fulfils the requirements of a test for their hypothesis. Their experiment involved four different types of stimulus but each involved the same experimental arrangement.

In each of the four experiments, subjects were required to sort a deck of 32 cards into two piles as rapidly as possible. Sorting time was the measure. Each experiment involved two stimulus dimensions with two levels or values per dimension. Thus four different stimulus values could exist. There were three different methods of presentation;

control, correlated and orthogonal. For each type of stimulus presentation the subjects were required to sort by each of two dimensions, so for each subject there were six different edperimental conditions. These were:

(i) Two unidimensional presentations, there were only two stimuli varying on a single dimension and subjects were required to sort by that dimension. This condition provided a baseline.

(ii) Two correlated presentations. Here the two dimensions occurred in a perfectly correlated or redundant manner. There were still only two different stimuli but subjects were required to sort by each of two different dimensions separately.

(iii) Two orthogonal stimulus presentations. Here each of the two values of each of the dimensions occurred with both values of the other dimension. The subject was required to classify or sort the stimuli separately by each dimension.

The four stimulus materials were:

1. Munsell Colours. The two dimensions employed were value (or brightness) and chroma (saturation) of single Munsell chips. These were the same stimulus dimensions that Hyman and Well (1967) and Torgerson (1952) argued gave rise to a Euclidean metric in

direct distance scaling. According to the Garner - Felfoldy hypothesis this is an integral stimulus so there would be facilitation for correlated dimensions and interference for orthogonal dimensions.

2. Separated Colours. The two dimension value (brightness) and chroma (saturation) were made separable by having one colour chip varying in value and another colour chip on the <u>same</u> card varying in chroma. Hyman and Well (1968) had in this case obtained a City Block metric. Lockhead (1966) using hue and value found no redundancy gain for this type of separable dimension. The Garner - Felfoldy hypothesis would predict for this non-integral stimulus that there would be no redundancy gain and no interference effects.

3. Dot location. The two dimensions were horizontal and vertical location (two values on each). This stimulus was thought to be integral and so the prediction was redundency gain and interference when dimensions are presented orthogonally.

4. Diameter of a Circle and Angle of Radius. These two dimensions were found by Shepard (1964) and Hyman and Well (1969) to be non-integral and to yield a City Block Metric. Again the prediction was no redundancy gain and no interference effects in the orthogonal presentation conditions.

The predictions of the Garner - Felfoldy hypothesis were upheld for the first two stimulus materials. However for dot location they reported a barely significant redundancy gain and a barely significant interference effect (for Munsell colours it was five times as great!)

For the circles and diameters, they reported no redundancy gain when judging the size of the circle (and the diameter was redundant). However when classifying by diameter they reported some redundancy gain. For neither of the orthogonal presentation conditions did they find any interference.

5.40 Explanations of Redundancy Gains in Absolute Judgement and Speeded Classification Tasks

There are two approaches, one dimensional in its orientation, and the other metric which is not really an explanation but an operational definition, to account for redundancy gains in these two types of task. They will be referred to as the Independent Dimensions Model and the Metric Hypothesis respectively.

5.41 Independent Dimensions Model

This has already been extensively discussed in Chapter 3 Section 3.10. This model assumes that the dimensions are psychologically independent and that errors in the identi-

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5.41 Independent Dimensions Model

This has already been extensively discussed in Chapter 3 Section 3.10. This model assumes that the dimensions are psychologically independent and that errors in the identification of the values of one of the dimension are independent of errors in identifying the particular value of a second dimension: there are no interaction effects. The Erikson Hake (1955) model postulated that independent dimensions,presented in a correlated manner, provide an additional opportunity for a clearer perceptual state and this provides an increase in identifiability. A method of examining independence effects using a method of analysis inspired by Kaufman and Levy (1971) was demonstrated in Experiments 2A and 2B.

5.42 The Psychological Distance Hypothesis

This has already been spelled out in Section 5.20 and as it is essentially an operational definition it provides no explanation for redundancy gains. Garner (1970) has gone a little way, however to explain what he means by an integral stimulus when he suggests:

> '... that two dimensions are "integral" if in order for a level on one dimension to be realised there must be a dimensional level specified on the other. For example: a visual stimulus must have a brightness and a hue and a saturation and a size and a form, that fact makes any pair of these dimensions integral.'

5.43 Difficulties with 'Metric' Explanations of Redundancy Gains

There are two major difficulties met when employing metric explanations to account for redundancy gains in absolute judgement and speeded classification tasks. The first has to do with the fact that before a particular stimulus can be said to give rise to a Euclidean or a City Block metric certain rigorous non qualitative conditions must be met: interdimensional additivity, intradimensional subtractivity and decomposability. None of the stimuli used by Garner and Felfoldy have been investigated in a measurement theoretic way. An unpublished study by Krantz of the colour dimensions value (brightness) and chroma (saturation) goes against the suggestion that these dimensions are Euclidean. Krantz showed, however, that for these dimensions decomposability (and hence both additivity and subtractivity) is violated in a very systematic way and hence cannot possibly produce a Euclidean metric in direct distance scaling.

The second difficulty has to do with those circumstances in which Euclidean and City Block metrics can be legitimately invoked. The notions of metric and dimensional representations can properly only be applied to similarity data, and their use to explain results in identification experiments, whether absolute judgement, or speeded classification tasks, is entirely unwarranted. In the experiments reported in the second and fourth chapters on similarity data, it was confirmed that various ordinal independence conditions held. In the identification experiments reported in chapter three on the same stimuli it was seen that there were interaction effects; and independence in this idenficiation task and for these stimuli did not hold. Identification tasks and similarly Judgement tasks therefore seem to demand entirely different types of psychological processing; and concepts like Euclidean or City Block metrics while appropriate for similarity data in some circumstances cannot be transplanted as an explanatory construct for identification experiments. It seems better then, to dispense with any 'metric' explanations of redundancy gains in absolute judgement and speeded classification tasks and to look for others.

5.44 It is suggested instead, that we abandon the notions of integral and non-integral stimuli with all its metric connotations and concentrate instead on the dimensional aspects of the stimulus. It is hypothesised then, that in identification experiments such as absolute judgement and speeded classification tasks that

(i) Dimensions which are perceptually independent give rise to redundancy gains (Erikson & Hake 1955). This is because the correlated conditions (for independent dimensions) results in a type of multiple observation on a single trial, where each of the independent dimensions represent an additional opportunity for a clearer perceptual state to occur.

(ii) Dimensions which are perceptually independent do not result in interference effects when the stimulus dimensions are presented orthogonally

(iii) Dimensions which are not independent can give rise to either redundancy gains or not depending on the interaction effects. In particular, whether the identifiability of one or other of the dimensions at a particular level is enhanced or not at a particular level of the other dimension.

(iv) For non independent dimensions. In the orthogonal presentation condition sometimes interference or lack of it arises depending on the specific interaction effects.

Results (iii) and (iv) are more likely to occur for stimuli like colour or sound which are "integral" except of course they probably do not give rise to a Euclidean metric. It is further argued that for 'attributes' like hue, or saturation it is not meaningful to describe them as psychological dimensions in the sense that they can be abstracted. To adopt a metaphor it is best to think of a 'raw plug' being removed. Some raw plugs when removed bring out some other parts of the wall with it. This could happen when attempting to selectively abstract an 'integral' dimension like hue from a stimulus. This dimension can be abstracted but with more or less of the other attributes. However with independent dimensions - dimensions which do not form an intrinsic stimulus like

Shepard's wheels and radii,)it is certainly possible to 'abstract' the angle of radius from the size of the circle. However it is very difficult to visualise selectively abstracting a dimension like hue without at the same time extracting brightness. Neither brightness, nor hue can exist above in the same way that the size of a circle and the angle of one of its radii. Therefore to talk of colour dimensions like hue and saturation in the same way that we talk about dimension like dot location (the two dimensions being horizontal versus vertical) is clearly not comparable.

EXPERIMENT 4A

5.50 Introduction

This experiment is similar in design and conception to the Garner and Felfoldy (1970) study and the details of this present one are summarised in Fig 5.51.

There were 10 experimental conditions, each one corresponding to a particular method of stimulus presentation and to a particular pair of dimensions for identification. The conditions comprise for each dimension

(i) two control presentations
(ii) two correlated presentations
(iii) and one orthogonal presentation.

The task was an identification one, and the dimensions used were those of loudness (A) and duration (L) of auditory tones, both of which varied on two levels of the respective dimensions. The subject was required to identify either A_o and A_1 or L_o and L_1. The measure was the time taken in milliseconds to identify each of the tones.

The aim of the experiment was two-fold. First to replicate the Garner-Felfoldy experiment; but using both auditory stimuli and more strictly controlled time measurements. (Their measure was the time taken, in seconds, to sort 32 cards into two piles. In the present experiment, however, the measure used was the time taken, in milliseconds, to categorise one of two tones.) The other aim was to study baseline and correlated condition performance for each of the dimensions; so for each dimension there were two control conditions and two correlated conditions. Baseline performances were compared to determine whether they varied significantly for different background values of the other dimensions. If they did so, then the use of a control or baseline condition by which to compare performance in all other conditions becomes problematical. For, supposing the time taken to categorise a tone as either A_o or A_1 in the control condition depended on the duration of the tone. This would mean, there would be no one baseline value by which to compare the latencies of A_o and A_1 in either of the correlated or orthogonal presentations. That is, the presence of an interaction effect so that the identifiability of A_o and A_1 depends on a particular value of L renders the notion of a control or baseline condition meaningless. In experiments 2A and 2B, reported in chapter 3, it was established that the identifiability of a particular level of loudness depended on the particular level of pitch. Also, it was found, that the identifiability of a particular value of pitch depended on the duration of the tone. Moreover Scott (1974) in an unpublished undergraduate dissertation showed that the identifiability of a particular level of loudness (A) depended on the duration of the tone.

It can be seen then, that to compare the two 'control' conditions for each stimulus is not at all trivial if there exists the possibility of interaction effects. Moreover, if for each stimulus the performance in the two correlated conditions differ both between themselves and with respect to either of the control conditions then an explanation might more fruitfully be sought, by looking at possible interaction effects between the stimulus dimensions rather than by invoking the mediation of Euclidean or City Block metrics.

5.51 Method

The tones were delivered to the subject (in a sound proof booth) via earphones. In front of the subject were two buttons labelled 'soft' (for responses to the A_o tone) and 'loud' (for responses to the A_1 tone). In the 'A_o A_1'stimulus conditions the subject was told to respond only to the loudness or softness of the tone and ignore all other attributes. In the'L_oL_1'stimulus condition the subject was seated in front of two similar buttons labelled 'long' and 'short', and again he was told to respond only to the duration and ignore all other aspects.

The identity of the delivered tone, the response to it, and the time taken to respond, were all recorded on a tape via a data transfer unit.

The 10 conditions were subdivided into two experimental blocks which were labelled

(i) The A_oA_1 identification block, on which the subject was required to identify A_o or A_1 under different conditions of stimulus presentation (two control, two correlated, and one orthogonal).

(ii) The L_oL_1 identification block on which there were again five conditions of stimulus presentation.

Each subject served in all 10 conditions. Before going on to a different block, a subject was required to complete all the conditions within a block with the order being randomised within that block. Eight subjects served in each of the blocks three times. Four of them served in the A_oA_1 block first and four of them served in the L_oL_1 block first.

For each block and within each of the five conditions 100 randomised tones were delivered for identification. Only the data from performance in a block, the third time round, was eventually analysed, and within each of the five conditions only the results of the last 50 stimulus presentations were considered.

It was found that errors in each of these last fifty presentations were practically non existent. Indeed six of the eight subjects made no mistakes at all while the other two made only 9 between them (an error rate of less than 2% over all 10 conditions). Hence these errors were ignored in the final data analysis.

Table 5.54 Experiment 4A Results

Stimulus to be Identified	Mode of Stimulus Presentation				
	Unidimensional Conditions		Correlated Conditions		Orthogonal
	L_o constant	L_1 constant	A_oL_o, A_1L_1	A_oL_1, A_1L_o	
A_o and A_1	422 msecs	317 msecs	337 msecs	354 msecs	431 msecs
L_o and L_1	416 msecs	409 msecs	406 msecs	414 msecs	526 msecs

N.B. The data was analysed using a Repeated Measures Design with two factors (mode of presentation and stimulus to be identified) and 8 people. Each subject served in all conditions. The Mode of Presentation x subjects, the stimulus to be Identified x subjects, and the Mode of Presentation x stimulus to be Identified x subjects interaction terms were all non-significant. $(p < .005)$.

5.52 Stimuli

The apparatus used to generate the tones and record responses was the same as that used in Experiment 2A and 2B and is described in the appendix.

Each tone was of constant frequency (1000 Hz and

$$A_o = 87 \text{ db}$$
$$A_1 = 102 \text{ db}$$
$$L_o = 500 \text{ milliseconds}$$
$$L_1 = 750 \text{ milliseconds}$$

Fig 5.51 shows which stimuli were presented to the subject in each condition, and in all conditions the number of different stimuli presented to the subject in a randomised manner were the same.

5.53 SUBJECTS

The eight subjects were all sixth year pupils from Stirling High School and their ages ranged from about 16.5 years to 18.5 years. The subjects were all paid for their services (30 pence per hour) and each subject took about six hours over three days to complete the experiment.

5.54 RESULTS AND DISCUSSION

The results are shown in Table 5.54. Each of the entries in each of the conditions represents the mean latency for the eight subjects. Moreover each of the '8' scores

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which make up this mean is, itself, the mean of 50 latencies. These fifty latencies were each subjected to a logarithmic transform to stabilise the variances. It was found even after extensive practice in each of the conditions that there was a 'Mean' and 'Variance' drift: in a particular condition the mean latency seemed to increase with an increase in variance that is, the latency and variance estimates were correlated. Converting the latencies to logarithms removed this.

The means for each subject and for each of the 10 conditions were subjected to an analysis of variance (a repeated measures design). The two factors were

(i) Stimulus to be identified A_o and A_1 and L_o and L_1.

(ii) Mode of stimulus presentation

a_1 a_2 two control conditions

b_1 b_2 two correlated conditions

c_1 orthogonal condition

A planned comparison was carried out (Hays page 459). It was found in the A_oA_1 identification block:

(a) There was a significant difference in latencies between the two control conditions. That is, the time required to identify A_o or A_1 in the control condition depended on the background duration of the tones ($p = 0.05$).

(b) No significant difference in identification times between the two correlated presentation conditions was detected.

(c) For the A_oL_o, A_1L_1 (Correlated) versus A_oL_o, A_1L_o (control) conditions the differences in latencies were <u>just</u> significant ($p = .005$).

(d) For the orthogonal presentation condition versus the A_oL_o, A_1L_o control condition there is again a significant difference between the latencies. However there is no significant difference in latencies between the orthogonal and the other control condition.

For the A_oA_1 block, then, whether there is interference or not in the orthogonal presentation condition depends on <u>which</u> control condition is being used as the baseline.

Further whether or not there is facilitation in the correlated condition compared with the control condition depends on which of the correlated presentations is considered and which correlated condition.

It seems clear then, that what is happening is some fairly complex interaction process between the stimulus dimensions. Within the A_oA_1 block there is no one value of duration (L_o or L_1) which provides a typical estimate of the identifiability of A_1 or A_o: that is, there is an interaction effect which makes the employment of a baseline condition from which to compare all other conditions within the block as not very helpful.

For the L_oL_1 block a planned comparison revealed

(e) No significant difference between the two control conditions.

(f) No significant differences between the two correlated conditions, or any difference between these conditions or any of the controls.

(g) A significant difference between the orthogonal and both unidimensional conditions.

Result (g) is not very satisfactory for the Garner-Felfoldy hypothesis which proposes that facilitation in the correlated condition should lead to interference in the orthogonal conditi and vice versa.

It seems clear, then,on the basis of the experiments reported in chapter 3 on stimulus independence and the theoretical work reported in chapter 4, that a more parsimonious account for the results of Garner and his colleagues can be found in studies investigating the presence or absence of interaction effects of the stimulus dimensions rather than by invoking metric concepts like Euclidean, or City Block dimensions.

CONCLUSIONS

Mighty is the charm
Of those abstractions to a mind beset
With images and haunted by himself,
And specially delighted unto me
Was that clear synthesis built up aloft
So gracefully; even then when it appeared
Not more than a mere plaything, or a toy
To sense embodied: not the thing it is
In verity, an independent world,
Created out of pure intelligence.

William Wordsworth
The Prelude, (Book 6)

The attempt in this thesis has been to mount a sustained experimental and conceptual analysis of independence in similarity and identification data.

This investigation has led to an examination of psychological dimensions and their independence. In the first two experiments, various types of independence were tested on similarity data. It was found that a dimensional representation of a similarity judgement task was possible and that it accorded with the predictions of the Additive Difference Model. It was also seen that the measurement theoretic analysis employed, helped make the underlying psychological processes more apparent.

Chapter 3 examined the notion of independence in identification tasks. The main point of this Chapter was to stress that perceptual independence if it is to be meaningful psychologically must be tied to the notion of a lack of interaction between the stimulus dimensions. This type of independence, - perceptual independence - was then scrutinised experimentally. It was found that for auditory stimuli delivered through noise there were indeed interaction effects, and that the identifiability of a particular level on one dimension varied over different levels of the other.

Chapter 4 constituted a generalisation of the work in Chapter I inasmuch as it reviewed and examined the theoretical literature on the foundations of multidimensional scaling. In this discussion, the plausability of a dimensional representation of the similarity data in Experiments 1A and 1B was joined to a metric representation. With the auditory stimuli investigated, it was found that both a dimensional and metric representation was possible. Moreover the class of distance functions consistent with the metric representation were of the Minkowski r-type. This in turn meant that any one of the Euclidean, City Block and Dominance metrics was appropriate for describing the similarity judgement tasks.

Chapter 5 reviewed some of the studies which have suggested that the type of information processing possible depended on the metric properties of the stimulus. It was pointed out that none of these published studies had investigated in a measurement theoretic manner the dimensional or metric requirements. In particular it was noted that explaining results in <u>identification</u> experiments by invoking Euclidean or City Block metrics was entirely unwarrented. These types of metrics certainly have some meaning when applied to similarity data, and then, only when very stringent ordinal conditions are satisfied. The final experiment suggested that a better explanation of some of the identification results in the published literature might be due to interactions on the dimensions rather than to their alleged metric properties.

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APPENDIX 2A

Dissimilarity Data Matrices for each of the 2 conditions and 4 replications of Experiment 1A.

Experiment 1A

Dissimilarity Data Matrices (Scores ranked) SUBJECT 3

CONDITION I

Replication 1

12	14	15	16
7	9	11	13
5	6	8	10
1	2	3	4

Replication 2

12	14	15	16
8	10.5	10.5	13
5.5	5.5	7	9
1.5	1.5	3.5	3.5

Replication 3

12	14	15	16
8	10	11	13
5	6	7	9
1	2	3	4

Replication 4

12	14	15	16
8	10.5	10.5	13
5	6	7	9
1	3	3	3

CONDITION 2

Replication 1

11	14	15	16
8	8	12	13
4	5	8	8
1	2	3	8

EXPERIMENT 1A

Dissimilarity Data Matrices (Scores ranked) SUBJECT 1

CONDITION 1

Replication 1

5	9	13	16
4	8	12	15
2	6	10	14
1	3	7	11

Replication 2

11	14	15	16
9	10	12	13
4	5	7	8
1	2	3	6

Replication 3

12	15	15	15
8	10	12	12
4	5	8	8
2	2	2	6

Replication 4

11	14½	14½	14½
9	11	11	14½
5	5	7	8
1	2	3	5

CONDITION 2

Replication 1

13	15	14	16
9	11	10	12
5	7	6	8
1	3	2	4

Replication 2

4	8	12	16
3	7	11	15
2	6	10	14
1	5	9	13

Replication 3

3	8	12	16
3	7	11	14.5
3	6	10	14.5
1	5	9	12

Replication 4

5	8	12	16
3	7	12	15
2	5	12	12
1	5	9	12

EXPERIMENT 1A

DISSIMILARITY DATA MATRICES (SCORES, RANKED)

SUBJECT 2

CONDITION 1

Replication 1

4	11	11	16
3	7	11	15
2	6	11	11
1	5	11	11

Replication 2

11	14	15	16
9	10	12	13
4	4	7	8
1	2	4	6

Replication 3

11	13.5	15.5	15.5
9	10	12	13.5
4	5	7	8
2	2	2	6

Replication 4

11	14	15	16
9.5	9.5	12.5	12.5
3.5	5	7	8
1	2	3.5	6

Replication 1

11	13	15.5	15.5
9.5	9.5	12	14
3	5	7.5	7.5
1.5	1.5	4	6

Replication 2

11	13	15.5	15.5
9.5	9.5	12	14
3	6	6	8
1	2	4	6

Replication 3

11	13	15.5	15.5
9.5	9.5	12	14
3	5	7	8
1	2	4	6

Replication 4

11	13	15	16
9	10	12	14
3	5	7.5	7.5
1	2	4	6

APPENDIX 2B

Ranking of the stimulus pairs with respect to dissimilarity for four subjects.

APPENDIX 1 B1

TABLE 1B1 (S1) RANKING OF THE 12 PAIRS OF TONES OBTAINED BY THE METHOD OF PAIRED COMPARISONS. THE PAIR (9,7) IS THE MOST DISSIMILAR AND THE PAIR (1,1) THE LEAST DISSIMILAR. S(x.y) REFERS TO THE DISSIMILARITY SCORE AND INDICATES THE NUMBER OF TIMES THE ROW PAIR IS JUDGED TO BE MORE DISSIMILAR THAN THE COLUMN PAIR.

	(9,7)	(9,6)	(9,8)	(9,5)	(1,7)	(1,6)	(1,8)	(1,5)	(1,3)	(1,2)	(1,4)	(1,1)	S(xy)	Ranking
(9,7)	/	4	4	4	4	4	4	4	4	4	4	4	44	12
(9,6)	0	/	4	4	4	4	4	4	4	4	4	4	40	11
(9,8)	0	1	/	4	4	4	4	4	3	4	4	4	36	10
(9,5)	0	0	0	/	4	4	4	4	4	4	4	4	32	9
(1,7)	0	0	0	0	/	4	4	4	4	4	4	4	28	8
(1,6)	0	0	0	1	1	/	4	3	3	4	4	4	24	7
(1,8)	0	0	0	1	1	2	/	3	4	4	4	4	23	6
(1,5)	0	0	0	0	1	1	2	/	3	4	4	4	19	5
(1,3)	0	0	0	0	0	0	2	3	/	4	4	4	17	4
(1,2)	0	0	0	0	0	0	0	0	3	/	4	4	11	3
(1,4)	0	0	0	0	0	0	0	0	0	0	/	4	4	2
(1,1)	0	0	0	0	0	0	0	0	0	1	2	/	3	1

	(9,7)	(1,7)	(9,6)	(1,6)	(9.8)	(1,8)	(9,5)	(1,5)	(1,3)	(1,2)	(1,4)	(1,1)	S(x,y)	RANK
(9,7)	/	4	4	4	4	4	4	4	4	4	4	4	44	12
(1,7)	0	/	4	4	4	4	4	4	4	4	4	4	40	11
(9,6)	0	3	/	1	2	2	4	4	4	4	4	4	32	10
(1,6)	0	1	1	/	2	3	4	4	4	4	4	4	31	8.5
(9,3)	0	0	1	2	/	4	4	4	4	4	4	4	31	8.5
1,8	0	0	0	0	2	/	1	4	3	4	4	4	22	7
9,5	0	0	0	0	0	1	/	4	4	4	4	4	21	6
1,5	0	0	0	0	0	0	1	/	4	3	4	4	16	5
1,3	0	0	0	0	0	0	1	2	/	4	4	4	15	4
1,2	0	0	0	0	0	0	0	1	1	/	4	4	10	3
1,4	0	0	0	0	0	0	0	1	1	1	/	4	7	2
1,1	0	0	0	0	0	0	0	0	0	1	1	/	2	1

TABLE 1 B2(S2) RANKING OF THE 12 PAIRS OF TONES BY THE METHOD OF PAIRED COMPARISONS FOR ONE SUBJECT. S(x,y) IS THE DISSIMILARITY SCORE AND INDICATES THE NUMBER OF TIMES THE ROW PAIRS IS JUDGED TO BE MORE SIMILAR THAN THE COLUMN PAIR.

	(9,7)	(9,8)	(9.6)	(9,5)	(1,7)	(1,8)	(1.6)	(1.5)	(1,3)	(1.4)	(1,2)	(1,1)	S(x,y)	RANK
(9,7)	/	4	4	4	4	4	4	4	4	4	4	4	44	12
(9,8)	0	/	4	4	4	4	4	4	4	4	4	4	40	11
(9,6)	0	1	/	2	3	4	4	4	4	4	4	4	34	10
(9.5)	0	0	1	/	3	3	4	3	4	4	4	4	30	9
(1,7)	0	0	1	1	/	3	4	4	4	4	4	4	29	8
(1,8)	0	0	0	1	1	/	2	2	3	4	4	4	21	7
(1,6)	0	0	0	0	0	0	/	3	4	4	4	4	19	6
(1.5)	0	0	0	0	0	0	0	/	4	4	4	4	11	5
(1,5)	0	0	0	0	1	1	0	1	/	4	4	4	15	4
(1,4)	0	0	0	0	0	0	0	0	1	/	4	4	9	3
(1,2)	0	0	0	0	0	0	0	0	1	1	/	4	6	2
(1,1)	0	0	0	0	0	0	0	0	0	0	2	/	2	1

TABLE 1.B3 (S3) RANKING OF THE 12 PAIRS OF TONES OBTAINED BY THE METHOD OF PAIRED COMPARISONS FOR ONE SUBJECT S(x,y) IS THE DISSIMILARITY SCORE AND INDICATES THE NUMBER OF TIMES THE ROW PAIR IS JUDGED TO BE MORE SIMILAR THAN THE COLUMN PAIR.

	9,7	9,6	1,7	(9,8)	(1,6)	9,5	1,8	1,5	1,3	1,2	1,4	1,1	S(x,y)	RANKING
(9,7)	/	2	3	4	4	4	4	4	4	4	4	4	41	12
(9,6)	0	/	4	4	4	4	4	4	4	4	4	4	40	11
(1,7)	0	1	/	3	4	4	3	4	4	4	4	4	35	10
(9,8)	0	1	2	/	3	3	3	4	4	4	4	4	32	9
(1,6)	0	0	2	3	/	3	4	4	4	3	4	4	31	8
(9,5)	0	0	1	3	2	/	4	4	4	4	4	4	30	7
(1,8)	0	0	0	1	1	3	/	4	4	3	4	4	24	6
(1,5)	0	0	0	0	0	0	1	/	4	4	4	4	17	5
(1,3)	0	0	0	0	1	0	0	1	/	4	4	4	14	4
(1,2)	0	0	0	0	0	1		0	1	/	3	4	9	3
(1,4)	0	0	0	0	0	0	2	0	1	1	/	3	7	2
(1,1)	0	0	0	0	0	0	0	0	1	1	1	/	3	1

TABLE 1B.4(S4) RANKING OF THE 12 PAIRS OF TONES OBTAINED BY THE METHOD OF PAIRED COMPARISONS FOR ONE SUBJECT. S(x.y) IS THE DISSIMILARITY SCORE AND INDICATES THE NUMBER OF TIMES THE ROW PAIR IS JUDGED TO BE MORE SIMILAR THAN THE COLUMN PAIR.