A MODEL FOR THE SELECTION OF INVESTMENT PROJECTS UNDER INFLATIONARY CONDITIONS

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ABSTRACT

Current methods of investment appraisal under inflation are shown to have deficiencies that may result in the wrong selection of investment opportunities. A more efficient alternative is proposed here, that looks into two aspects of the problem: (1) the analysis of independent projects, and (2) the analysis of corporate investment decisions over the long term, involving several interdependent projects.

For the first case, a variant of common discounting techniques is used. This method, called "Terminal Value", is based on the projection of cash flows to obtain an estimate of the net incremental wealth that would be attained if the project in question was adopted. In doing this, the method takes account of the effect of inflation on the fixed investment, and the different treatment given to the amounts of debt and equity used to finance the project. This results in a more realistic appraisal of opportunities. The method can deal with minor interdependencies between small sets of projects. For the case of several interdependent projects over a certain planning horizon, where decisions are linked to problems of liquidity, limited availability of funds and subject to much higher risk, a Mixed Integer Programming simulation model is proposed. This model maximises "Terminal Value" subject to constraints on cash balance, liquidity, gearing and taxation. The choice of this particular set of constraints responds to the need to check those factors on which the effects of inflation are more damaging. The model is based on the premise that inflation forecasts are very imprecise. Thus, to account for the inherent risk, the project selection criterion is such that the chosen portfolio is the one with the highest probability of being optimal for the inflation scenario that materialises. An estimate of the portfolio's sensitivity to inflation is also produced by the model.

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In the absence of inflation, methods of investment appraisal such as Net Present Value, Discounted Cash Flow Rate of Return and Payback Period are commonly used. These technhiques require information about the inflows and outflows generated by the investment projects, tax rate, depreciation method allowed and cost of capital. Under conditions of inflation these factors are affected in different ways; few of them are fully responsive to inflation, and the magnitude and direction of the response could greatly vary from one element to another, or from one project to another.

Under the high inflation rates that are now common to many countries, most firms still use the traditional project evaluation techniques with some implicit or explicit way of accounting for inflation. Whether these techniques can help management reach robust investment decisions is doubtful; the cash crisis faced by many companies in recent years, reinforces the belief of many, that project evaluation techniques especially developed for inflationary conditions are necessary. "Inflation, tight money, the international liquidity crunch, high interest rates and cash flow problems represent a set of interdependent problems which very few firms throughout the world have managed to escape" (Naylor, 1977). The only way the manager can cope with the situation is through "an adequate early warning system to enable him to anticipate liquidity problems before they become acute, especially while he still has a number of viable options available to him" (ibid).

Several works have been reported in the literature that attempt to adapt traditional project appraisal methods to inflationary conditions. Unfortunately, even at a time when the inflation rate was very low, the existing evaluation techniques were strongly criticised (Adelson, 1970; Fawthrop, 1971; Bromwich, 1970; Salkin et al, 1973). The main reason for criticism is that these methods are based on assumptions about the reality of the economic environment that very seldom hold. Assumptions such as a perfect capital market, independence between projects, and the divisibility of any sum of investment capital between projects are characteristic of Discounted Cash Flow techniques. It is felt that high inflation rates with their intricate effects on the economy, have changed the investment environment to such an extent that the use of rules of thumb is no longer safe.

In adapting traditional evaluation techniques to inflationary conditions, very little effort has been made to try to relax any of the strong assumptions on which they are based. As a result, methods are produced that are only able to deal with the effects inflation would have on a non-existent perfect capital market. This superimposition of simplifying assumptions in our complex economic environment can only increase the risk of taking wrong decisions.

In order to improve decision making a more realistic analysis is needed. From the viewpoint of inflation, there are several factors that must be carefully considered: liquidity, leverage, taxation, depreciation of fixed investment and working capital amongst them. These factors are affected by inflation both directly and through interactions between each other. Interdependencies between projects are introduced by the effects of inflation on taxable profit, liquidity and gearing. Thus, as the number of investment opportunities increases, so does the difficulty in selecting the best portfolio of projects. An efficient way to handle all these complications is Mixed Integer Programming. The use of this optimisation technique makes it possible to take account of all the intervening factors and reach a decision that maximises the chosen measure of profit. While a company could have more than one measure of profit and they could be different from those used by other companies, it is possible to find one long term objective common to most profit making corporations. Such an objective could be the maximisation of assets value at the planning horizon, which is referred to in this work as the Terminal Value of the corresponding investment decision. Its use in this model stems from the need to tackle particular difficulties brought about by inflation; this will be shown in detail in Chapter 2.

Most of the economic variables used in the analysis, however, have to be obtained by means of forecasts subject to high uncertainty. Amongst the sources of such uncertainty, the inflation rate is probably the most important one, affecting each variable.

To account for the uncertainty on the forecast of the inflation rate, Monte Carlo simulation can be used. This requires the estimation of probabilities of occurrence for the inflation rate in every year up to and including the horizon. Because these values are interdependent, estimates of the correlations between them must be obtained and introduced by means of conditional probability distributions.

In a sufficient number of runs a frequency of portfolio selection can be obtained. Further to this, the effect on the portfolio terminal value of other possible inflation rates must be analysed. This can be done by constructing a portfolio terminal value histogram by random sampling of annual inflation rates.

The combined use of MIP and a statistical technique like Monte Carlo simulation can prove to be an invaluable tool for the decision maker. There seems to be, however, a growing disregard for mathematical approaches to decision making among industrialists. It is argued that it is useless to have a sophisticated method of analysis if it is to be fed with unreliable data. This argument, although valid in itself, ignores the fact that a well devised mathematical method would be intended to make the best possible use of the information available, taking account of the uncertainty involved. Finally, the main contribution of a model like the one described in this work, is only to reduce the decision region to a more manageable size, allowing the decision maker to use his experience and intuition more effectively.

It is the purpose of this work to present an alternative approach to the appraisal of investment opportunities under inflationary conditions. Existing methods are analysed and compared with each other in order to identify their shortcomings and select the most adequate one. This method is then used as the basis for the development of an alternative approach, that attempts to overcome some of the shortcomings of the existing techniques. The strongest limitations on the proposed alternative, are imposed by the assumptions of a perfect capital market and independency between investment opportunities. This leads to the next stage of this work, which deals with those more complex cases, for which it is too risky to expect these assumptions to hold. The method developed in the previous stage is then adapted for use in this more realistic situation. The proposed methodology produces a more robust ranking of investment opportunities affected by different inflation rates; a proper evaluation of the benefits derived from the use of debt; and a detailed analysis of the interdependencies introduced by the interactions between inflation, the taxation system, liquidity requirements and the different sources of finance.

In the first chapter, methods of economic evaluation under conditions of inflation reported in the literature are analysed. In the second chapter, a more detailed method of evaluation for the simplest cases is proposed; this refers to the analysis of investment opportunities for which it is safe to assume a perfect capital market. The third chapter deals with the capital budgeting problem; this is the analysis and selection of investment projects arising within a certain planning period. For this more complicated case, the assumption of a perfect capital market is dropped and the effects of taxation, liquidity and gearing constraints on the selection process are realistically taken into account. The fourth chapter presents a case study using simulated data; the refusal on the part of private industrial corporations to disclose the relevant financial information, has prevented me from conducting a proper case study. The best attempt is made, however, to keep the simulated data as close to reality as possible. In the fifth and final chapter, the ultimate conclusions are drawn and topics for further study are proposed.

CHAPTER I

ANALYSIS OF EXISTING TECHNIQUES FOR PROJECT APPRAISAL UNDER INFLATIONARY CONDITIONS

1.1 INTRODUCTION

The purpose of this chapter is to identify the most adequate of current approaches to the problem of investment appraisal under inflation. Existing methods are analysed in relation to each other and their relative advantages and disadvantages illustrated with numerical examples. Because most of these techniques are based on corrections to the same method, Net Present Value (NPV), it will be seen that, once the most adequate method is identified, the others have very little to contribute to further improvements. The selected method is used in subsequent chapters as the starting point for the development of a more rigorous analysis. For the purpose of this chapter, most of the general limitations usually attributed to the traditional NPV method are ignored; they will be dealt with in the second and third chapters. A perfect capital market is assumed for the purposes of this chapter.

The review of the literature covers the most representative works on the subject. Some related topics are also briefly discussed. Finally the results of the analysis are contrasted with the actual practice in industry.

1.2 REVIEW OF THE LITERATURE

Four closely inter-related topics are covered in this review: finance, costing, accounting and investment appraisal. Only the last of these four topics is further discussed in this chapter. The importance of the other three and their inter-relation with investment appraisal are highlighted in later chapters. The views of different authors on each of these topics are briefly discussed.

Finance

Works on this topic address the problem of the negative effects of inflation on the financial performance of a firm (Williams, 1976; Hull et al, 1976; Buffet, 1977). This is reflected through overstated profits which increase the tax actually paid; higher dividends in real terms which by reducing the amount of funds reinvested impair the growth of the company; increase in working capital requirements. Such a situation may result in a cash crisis. Adequate strategies, however, can offset most of these disadvantageous effects; Hull et al (1976) propose the use of a computer model which produces balance sheet-like output with which different strategies could be tested.

Also concerned with this topic is the question of corporate financial viability and share valuation under inflation (Merrett, 1975; Kennedy, 1976; Hguyen, 1976). Merrett (1975) proposes a method of share valuation whereby the opportunity cost of retentions is deducted from all later profits. By comparison with this method, all other techniques grossly overstate the rate of increase of profitability. In a related paper by Kennedy (1976), it is argued that proper use of Merrett's method applied to historic cost profits of British industrial and commercial companies, for the period from May 1972 to January 1975, would indicate that share prices should have been rising over the period, rather than falling, as was the case.

Accounting

Two aspects of the problem of accounting under inflation are covered:

- Validity of the current cost accountancy (CCA) method supported by Sandiland's report (1975) against the approach using current purchasing power (CCP) (Myddelton, 1976; Scapen, 1976; Reuben, 1976; Buckley, 1976).

- Accountancy methods in countries with very high inflation rates (Buckley, 1976; Eaves, 1976; Thorne, 1978).

Costing

Papers reviewed deal with the change in costs for the different commodities used in the chemical industry, cost control, and construction of cost indices (Forrest, 1975; Cran, 1976; Eaves, 1976).

Investment Appraisal

Papers on this topic can be grouped according to their proposed way to cope with inflation.

Method 1:

Papers under this heading base the analysis solely on corrections to the discount rate (Fleischer et al, 1967; Reisman et al, 1970; Reisman et al, 1972; Rose, 1976; Carsberg et al, 1976; Jelen et al, 1974).

Method 2:

Projecting the cash flows and correcting the discount rate (Bromwick, 1969; Cooley et al, 1975; Davidson, 1975; Holland et al, February 1977; Holland et al, March 1977; Mitchell, 1977; Waters et al, 1976; Watson et al, 1977; Thorne, 1978; Allen B, 1977; Hanke et al, 1975; Styh & Patersen, 1977; Sullivan et al, 1977).

Method 3:
Cash flows are projected and the observed market rate of interest (or its forecast) is used as discount rate (Wilkes, 1972; Van Horne, 1971).

Other papers deal mainly with the effects of different taxation and depreciation policies on profitability (Benford, 1977; Landers et al, 1973). The depreciation method allowed and the taxation system in operation are very important factors affecting the viability of a project. Depreciation allowances are based on historic costs. If inflation occurs over the life of a project, there is a loss in purchasing power because that portion of revenue designated as a recovery of capital has less purchasing power than was originally invested. In real terms, the taxable income is overstated (Landers et al, 1973). In the UK however, it is presently permissible to write-off the whole of the capital investment on plant and equipment in the first year of operation, if the other profits of the company are large enough to permit this (Watson et al, 1977).

A paper by Westwick et al, 1976, reports the results of a survey of methods of investment appraisal under inflation in 81 British companies. Unfortunately, because of the way in which the results are presented, it is not possible to know some important details about the methods used by those companies. Different ways of calculating the minimum rate of return are reported; it is also reported that some companies make projections of their cash flows, and some others do not. It would be interesting to know, for example, how the minimum rate of return is calculated by a company whose cash flow estimates are based simply on existing prices.

The literature on the subject can be very confusing mainly because of a lack of convention in the usage of terms, and poor definition of values and assumptions. The approach proposed by Fleischer et al (1967), for example, has been classified here as being of the type of Method 1; Davidson (1975) agrees with this classification. Sullivan (1977) however, considers the approach by Fleischer et al to be of the type of Method 2. In the same paper, Sullivan makes reference to a method in which one has to "estimate outcomes in terms of constant-worth dollars and use only the cost of capital for comparing alternatives". If this method is conceptually correct, the cost of capital is implicitly assumed to be expressed in real terms (net of inflation). This is opposite to the usage in most papers in which "cost of capital" is referred to in terms of a nominal rate (incorporates an inflationary element) (Wilkes, 1972; Van Horne, 1971). Stuart et al (1975) escalate future costs according to the expected inflation rate. These costs are discounted using an uncorrected interest rate which, if the method is conceptually correct, has to be a nominal rate. Watson et al (1977) project cash flows on the basis of future costs and prices; cash flows are then expressed in terms of purchasing power in year zero by discounting them using the general rate of inflation; finally cash flows are discounted once again using the "cost of capital". This cost of capital is presumably expressed in real terms. However its value is calculated assuming that "... the best available risk free use of the money were at a nominal 10 per cent per year ...".

In all these cases the papers are assumed to be conceptually correct. Inconsistencies are attributed, as stated above, to a lack of convention in the usage of terms.

In general repetition abounds, and a tendency towards newer developments does not seem to have emerged.

1.3 COMPARATIVE ANALYSIS OF EXISTING METHODS

Before proceeding to explain these methods in more detail, it is necessary to define some of the terms that will be used throughout this chapter.

Current money units refer to cash amounts expressed in terms of costs and prices which, it is forecast, will prevail at a specified future date. Cash amounts based on existing prices are projected into the future to express them in terms of current money units.

Constant purchasing power units apply to future cash amounts expressed in terms of money units prevailing at the time the analysis is made.

The real rate of interest is the inflation-free discount rate.

The money rate of interest is the nominal or market rate of interest, eg the bank lending rate.

Method 1

This method is based on the formula given in (1.1) below:

$$NPV = \sum_{j=o}^{n} \frac{C_j}{(1+\alpha)^j(1+i)^j} \qquad (1.1)$$

where C_j is the after tax cash flow for year j. This amount is not projected into the future.

α is the annual after tax return required on invested capital, expressed in real terms

i is the annual inflation rate

n is the duration of the project (in years).

This method can be criticised from two different viewpoints:

(a) It fails to project the cash flows. The effect of this is to introduce a distortion which in the majority of cases would understate the current money value of cash flows. In cases where costs escalate much faster than prices, the effect of this practice would be to exaggerate the current money value of cash flows.

(b) The correction to the discount rate assumes that the cost of capital keeps pace with inflation.

Detailed criticism to the double-discounting formula on which this method is based, is given later in this chapter.

This method will, in general, produce a very pessimistic forecast of the profitability of a project (unescalated cash flows discounted at a high rate of interest). In the case of negative cash flows the method would understate the loss.

<u>Method 2</u>

This method is based on the formula given in (1.2) below:

$$NPV = \sum_{j=o}^{n} \frac{C_j}{(1+\alpha)^j (1+i)^j} \qquad (1.1)$$

where C_j is the after tax cash flow for year j. This amount is not projected into the future.

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Method 2

This method is based on the formula given in (1.2) below:

$$NPV = \sum_{j=o}^{n} \frac{C'_j}{(1+\alpha)^j (1+i)^j} \qquad (1.2)$$

where C'_j is the after tax cash flow for year j projected into the future.

All other terms are as defined before.

For the projection of the cash flows, two cases can be considered:

(a) General inflation. This assumes that all cash flow components are equally responsive to inflation. The before-tax cash flows are projected according to the expected inflation rate, as shown below:

$$C'_{j\ BT} = C_{j\ BT} (1 + i)^j \qquad (1.3)$$

Where $C'_{j\ BT}$ is the before tax cash flow for year j expressed in current money units.

$C_{j\ BT}$ is the before tax cash flow for year j in terms of existing prices.

(b) Differential inflation. In this more realistic case, prices and costs have a different response to inflation. Each cash flow component has to be projected in the way described above, and then net cash flows are calculated in the usual manner.

The correction to the discount rate is based on equation 1.4 given below:

$$1 + m = (1 + \sigma)(1 + i) \qquad (1.4)$$

where m is the market or nominal rate of interest.

All other terms are as defined before.

The reason for this correction is, according to advocates of the method, that the cash flows obtained by projecting costs and prices do not have a common purchasing power. Therefore it is necessary to express all cash flows in terms of purchasing power at year zero before discounting using a real rate of interest. Alternatively, cash flows expressed in current money units are discounted using the market rate of interest. Equation (1.4) can also be expressed as:

$$m = \alpha + i + \alpha i \qquad (1.4a)$$

For reasonably low inflation rates, the last term in equation (1.4a) can be ignored. This is a formal justification of the practice of some authors who assume that the rates of interest and inflation are additive.

A general aggregate such as the Retail Price Index (RPI) is commonly used as i, in equation 1.4. One other possible value for i, is the particular package of goods and services required by the project (Davidson, 1975; Cooley et al, 1975). It is argued that use of a general aggregate can produce misleading results if it escalates at a different rate than the particular package of goods and services required by the project. If the desired package escalates faster than the general aggregate, use of the general aggregate can result in the selection of poor projects. On the other hand, if the desired package escalates more slowly than the general aggregate, the opposite occurs; suitable projects are needlessly rejected. This approach has certain relevance, particularly if the fixed investment is to be replaced in the physical sense. When this is not the case, a general aggregate such as the RPI should be used: The NPV represents the maximum amount which could be distributed to shareholders without reducing the capital value of the firm in the present period. Hence the appropriate rate is that which reflects the consumption pattern of shareholders.

The main defect of this method stems from the correction to the discount rate. According to Wilkes, 1972, it is erroneous to start with a supposed real rate which remains constant over the life of the project, and to evaluate the nominal rate m from (1.4). In actual fact, the values of m and i are data. It has been proved by several authors (Mundell, 1963; Kessel et al, 1962) that nominal interest rates rise by less than the rate of inflation, and therefore the real rate of interest will fall. The value of m, as calculated from (1.4) is overstated. As a result this method will produce a rather pessimistic forecast of the profitability of a project.

Method 3

This method is based on the formula given in (1.5) below:

$$NPV = \sum_{j=o}^{n} \frac{C_j'}{(1+m)^j} \qquad (1.5)$$

where all terms are as defined before.

The difference between Methods 3 and 2 lies on the way in which m is estimated. It has been argued above that, because nominal interest rates do not fully adjust for inflation, equation (1.4) should not be used. Wilkes, 1972, contends that "national and international economic policies may be thought to be more important factors than inflation in determining nominal interest rates".

Although the difficulties associated with forecasting can be equally large, the direct estimation of nominal interest rates from past behaviour and other factors such as those mentioned above, is likely to involve less error than the estimates obtained from equation (1.4).

This method is conceptually superior to Methods 1 and 2. Because nominal interest rates embody an element attributable to anticipated inflation, and cash flows are estimated on the basis of future prices, this method avoids the type of distortion of profitability produced by Method 1 (see Van Horne, 1971, pp 653); by using a more realistic estimate of the market rate of interest, the bias produced by Method 2 is also avoided.

Having described the methods to incorporate inflation in investment appraisal, it is possible to illustrate the differences between alternative approaches with a numerical example.

The details of the example can be summarised as follows:

Depreciable fixed capital assets amount to £5,000,000. The working capital required to fund the inventories and secure operating expenses until the plant starts generating profit is £2,000,000. It will be assumed that the tax debt of the company is sufficient to permit the whole of the fixed capital to be written off in the first year of operation and that tax is due at the end of the year of earning. Tax rate on marginal income is taken as 50 per cent. It is further assumed that working capital is recovered in the final year of operation at its historic cost. The market rate of interest is assumed to remain constant at 14% over the entire life of the project. The after tax market rate of interest is 7%. The company uses an after tax real discount rate of 5% to evaluate projects, which is also assumed to be constant. For simplicity the risk inherent in the project is here ignored.

Three cases will be considered:

(a) General inflation, where all cash flow items are equally affected by inflation.

(b) Differential inflation, where prices and costs have a different response to inflation.

(c) A particular case, where different tax situations are analysed.

(a) The cash flow situation for the case of general inflation is shown in Table 1.1(a). Sales income calculated on the basis of year zero prices is £7,000,000; operating costs, on the same basis, amount to £5,000,000.

Table 1.1(a)

Year	0	1	2	3	4	5
Annual Inflation Rate (%)		16	15	14	13	12
Sales Income, £M/year		8.1200	9.3380	10.6453	12.0292	13.4727
Operating Costs £M/year		5.8000	6.6700	7.6038	8.5923	9.6234
Cash Flow (B.Tax) £M/year		2.3200	2.6680	3.0415	3.4369	3.8493
Tax Payable £M/year		-1.3400	1.3340	1.5208	1.7185	1.9247
Cash Flow (A.Tax) £M/year		3.6600	1.3340	1.5208	1.7185	5.7740

Table 1.1(b)

Method	NPV
1	- 747,567
2	1,719,212
3	4,254,922

On the basis that Method 3 offers the most appropriate way of accounting for inflation, it can be seen from Table 1.1(b) that, in this example, Methods 1 and 2 understate profitability. Use of Method 1 would lead to the rejection of the project. In the case of negative cash flows, Methods 1 and 2 would understate the loss.

(b) For the case of differential inflation, sensitivity factors will be used. These factors indicate the extent to which different cash flow elements adapt to changes in the inflation rate. It is assumed in this example that sales price escalates at 0.8 times the annual inflation rate. Therefore we use a value of (0.8)* (Annual Inflation Rate)/100 to obtain the annual sales income. Operating costs are assumed to escalate at a rate of (1.2)* (Annual Inflation Rate)/100. Details about the cash flows are shown in Table 1.2(a).

Table 1.2(a)

Year	0	1	2	3	4	5
Annual Inflation Rate (%)		16	15	14	13	12
Sales Income £M/ year		7.8960	8.8435	9.8340	10.8567	11.8990
Operating Costs £M/year		5.9600	7.0328	8.2143	9.4957	10.8631
Cash Flow (B.Tax) £M/year		1.9360	1.8107	1.6197	1.3610	1.0358
Tax Payable, £M/ year		-1.5320	0.9054	0.8098	0.6805	0.5179
Cash Flow (A.Tax) £M/year		3.4680	0.9054	0.8098	0.6805	4.3673

Table 1.2(b)

Method	NPV
1	- 747,567
2	- 461,003
3	1,325,919

The relative performance of the three methods of evaluation, as shown in Table 1.2(b), is similar to that of the previous example. Method 2, however, predicts a negative NPV which would lead to the rejection of the project. Because Method 1 ignores escalation of the cash flows, the NPV is identical with that of Table I. It can be seen from these examples that, under certain circumstances Methods 1 and 2 can lead to the wrong accept/reject decision.

(c) So far in these examples, it has been assumed that capital allowances can be taken immediately they become available. Although this treatment is possible in the UK, it is not always applicable, in which case it exaggerates profitability and can lead to the erroneous acceptance of a project.

A pessimistic case on the other end of the scale would assume that project's tax allowances can only be set against net revenue cash flows from the same project (Wilkes, 1977). This implies that the project's something separate from the firm to which it is incremental, or else, that the firm is at a break-even situation and the only taxable income is generated by the project. This case is exemplified in Table 1.3. Most real situations would probably fall somewhere in between the cases in Tables 1.2 and 1.3 . In Table 1.4, fixed capital is depreciated using a straight line calculation.

Table 1.3 (a)

Year	0	1	2	3	4	5
Annual Inflation Rate (%)		16	15	14	13	12
Sales Income £M/year		7.8968	8.8435	9.8340	10.8567	11.8990
Operating Costs £M/year		5.9600	7.0328	8.2143	9.4957	10.8631
Cash Flow (B.Tax) £M/year		1.9360	1.8107	1.6197	1.3610	1.0358
Tax Payable £M/year		0.	0.	0.1832	0.6805	0.5179
Cash Flow (A.Tax) £M/year		1.9360	1.8107	1.4365	0.6805	4.3673

Table 1.3 (b)

Method	NPV
1	- 952,038
2	- 680.004
3	1,196,444

Table 1.4(a)

Year	0	1	2	3	4	5
Annual Inflation Rate (%)		16	15	14	13	12
Sales Income £M/year		7.8960	8.8435	9.8340	10.8567	11.8990
Operating Costs £M/year		5.9600	7.0328	8.2143	9.4957	10.8631
Cash Flow (B.Tax) £M/year		1.9360	1.8107	1.6197	1.3610	1.0358
Tax Payable £M/year		0.4680	0.4054	0.3098	0.1805	0.0179
Cash Flow (A.Tax) £M/year		1.4680	1.4054	1.3098	1.1805	4.8673

Table 1.4(b)

Method	NPV
1	-1,194,995
2	908,431
3	1,039,569

The before tax cash flows in these two examples are identical with those of Table 1.2(a), and the difference in the after tax cash flows is solely due to the less favourable depreciation methods. The corresponding NPVs, shown in Tables 1.3 (b) and 1.4(b), can be seen to be less optimistic than those in Table 1.2(b). The straight line depreciation method produces the least favourable effect on profitability. The fact that inflation overstates taxable income, makes an accelerated depreciation method particularly important. Tax allowances, however, can only be taken if the company generates enough profit. For the example in Table 1.3, only net revenue from the same project is considered. In a more realistic case, the net revenue from all the activities of the firm has to be considered for tax purposes. This can create interdependencies between investment projects, as will be seen in a later chapter.

The Program used for the calculations of the above examples is shown in Appendix A.

1.4 ACTUAL PRACTICE

Although a comprehensuve survey of the actual practice is not within the scope of this work's coverage, ten interviews with managers and consultants were carried out. It was found that half of the people interviewed ignored inflation in their investment analysis. Amongst those in the other half of the group, three used Method 2, and one used Method 3. The one left, is a management consultant firm. According to these consultants, the majority of their clients request that cash flows be projected into the future and discounted using the firm's cut off rate. In many cases, this cut off rate has remained unchanged for many years, and bears little or no relation to the fluctuating rate of inflation or the market rate of interest.

However, in a survey conducted by Westwick et al (1976), 62 out of 81 companies interviewed, did consider inflation in the appraisal of investment projects. The most popular method used was the projection of cash flows assuming differential inflation. Thirteen companies used this method in isolation; no indication was given about the discount rate used in these cases. The results are shown in Table 1.5, which has been taken from the original paper. The column headed "only" indicates that they used only that method; the column headed "total" indicates they used that method with one or more other methods. "Although a company may use a combination of methods ...", say the authors, "... they do not necessarily use more than one method on any one project, but may use a combination of methods on the totality of their projects".

Table 1.5

		Only	Total
1.	Increasing the cut off rate of inflation	6	26
2.	Applying different rates of inflation	13	31
3.	Applying the same rate of inflation	3	14
4.	Considering inflation at the risk analysis/sensitivity stage	7	22
5.	Other	8	13
		37	106

Two of the "other" ways of accounting for inflation are: assuming that cost inflation will be passed on in prices; and, allowing for inflation on the fixed capital cost during period of construction.

Another section of this survey deals with the selection of the minimum rate of return for use in investment appraisal decisions. It was found that the most popular method was to use the company's bank overdraft rate. A few companies used a variation on the gilt rate, equity rate or overdraft rate. However, all these companies put together represent only one-third of the surveyed sample; at least ten other methods were reported.

Even if we assume that most companies consider inflation in their investment appraisal, the evidence suggests a relatively poor acceptance of the methods reported in the literature. This could be a reflection of their low credibility amongst industrialists.

It is obvious that using the wrong methodology to account for inflation can be just as misleading as ignoring it. The consequences of this state of affairs can be underinvestment on the one hand, or the selection of poor projects on the other. A strong company with a good share of the market would probably not be too badly affected in the short term. In the long term, however, underinvestment could cause the loss of its share of the market. The damage to small companies could be fatal. The financial difficulties faced by many companies during the last few years could well be the result of this situation.

The evidence suggests that a more realistic method of appraisal is necessary.

CHAPTER II

PROPOSED METHODOLOGY FOR THE APPRAISAL OF INVESTMENT PROJECTS UNDER INFLATIONARY CONDITIONS - THE CASE OF A PERFECT CAPITAL MARKET

2.1 INTRODUCTION

The applicability of existing methods of evaluation such as those described in Chapter I, is limited to cases where it is safe to assume the existence of a perfect capital market, independence between projects and the divisibility of the investment capital. Only one kind of capital market imperfection will be considered here, the inequality of borrowing and lending rates of interest. It is shown, however, that even in this case, inflation can create complications which cannot be adequately treated by the existing methods. Nevertheless, uncomplicated methods of project appraisal are necessary in those cases where the amount of money at risk does not justify the cost of highly sophisticated and more realistic techniques. An alternative method of evaluation for this type of situation is here proposed. This method, called Terminal Value (TV), is similar to Method 3. The more detailed analysis it entails however, is shown to result in better decision making. The weaknesses of Method 3 are highlighted by contrasting its performance with the decisions that could be reached using TV.

The analysis of the results obtained with the TV method for simple cases, suggests the need for a Mathematical Programming approach to the analysis of the more complex situations, involving a large number of projects. This case is dealt with in Chapter III.

2.2 DEFINITION OF THE TERMINAL VALUE METHOD

This method is designed to estimate the profit or loss generated by a project at the end of its life after replacing the initial investment plus the extra capital needed to make up for the loss in purchasing power caused by inflation. Two possible cases are considered: (a) physical replacement of the productive equipment; and (b) replacement of the investment when the project is not to be continued for another period. This Terminal Value is net of all expenditures, interest on loan included. The necessary steps for the calculation of the TV are as follows:

1. Projection of costs and prices according to the expected inflation rate:

- In the most general case, differential inflation is assumed and sensitivity factors as those defined in example (b) in Chapter I are used. Although sensitivity factors for every cash flow component could be estimated, it seems likely that there will be diminishing returns to progressively greater numbers of individual components being projected, yielding smaller and smaller improvements in forecast accuracy (Gee, 1977, page 17). Depending on the particular project, broad categories, such as labour, raw materials and sales price could be defined and projected using collective sensitivity factors.

2. Estimation of annual cash flows net of all expenditures, interest on loan included:

- Sales revenue and total operating expenses are first calculated on an accruals basis. From these, the taxable profit from operations is calculated. Total taxable profit is then obtained by deducting capital allowances and annual interest on loan from the taxable profit from operations. The before tax cash flows are then calculated taking account of the lag in payments for sales and operating expenses. Finally, net after tax cash flows are obtained by deducting the total tax payable and the interest on loan from the before tax cash flows.

Following Watson et al (1977) and Gee (1977), the "cash flows to equity" procedure is used. This treats debt finance flows in the same way as all other cash flows. This is explained by Merrett and Sykes (1973) as follows:

"In analysing a capital project it should be apparent that we are attempting primarily to ascertain its advantages to the equity shareholders. From this it would appear that, in strict logic, we should in every case set out the net cash flows from and to the equity shareholders. In this analysis, the debt which could be raised on the assets of a project would be regarded as an inflow reducing the total capital required from the equity shareholders, and in the subsequent years of the project's life the interest and debt repayments should be regarded as normal cash outflows."

3. Assume reinvestment of the annual net inflows at the market rate of interest:

- The interest rate at which the company can lend money is used for this purpose. By treating the interest on loan as part of the cash flows and assuming reinvestment at the lending rate, the common assumption of equal lending and borrowing rates of interest can be dropped.

4. Estimation of the cumulative reinvested cash flows at the end of the project's life:

- The selection of a point in the time horizon is required. The length of this planning period can be determined by at least one of the following factors: the duration of the longest project under analysis, or that point in time at which forecasts are deemed to be too unreliable to make any further contribution to the analysis. If a project's termination date does not coincide with the horizon, its TV is either compounded or discounted back to the horizon at the market lending rate of interest.

5. Projection of the initial investment at the expected inflation rate:

- If the fixed investment is to be replaced in the physical sense at the end of the project's life, the inflation rate expected to affect the relevant plant and equipment is used. If the project is not going to be replaced, the general rate of inflation should be used instead.

6. Subtraction of the amount in 5 from the amount in 4.

The calculation of the TV for the alternative use of capital is not included in these six steps. The standard alternative is simply another investment project (if we ignore risk), and as such its TV is evaluated and compared with the other alternatives.

The concept of a future as opposed to a present value is not new. The main difference between this Terminal Value and other methods based on a similar concept lies in the way in which the initial outlay is projected into the future. This is done, as stated in step 5 above, using the forecast of the escalation rate for the initial investment, ignoring the opportunity cost of capital. In terms of the method being described here, the opportunity cost of capital is, simply, the TV of the alternative use of the investment capital. The opportunity cost is, therefore, actually considered when making comparisons between the different alternatives. The advantages of this approach will be better appreciated later in this chapter, in the analysis of projects affected by different inflation rates, or projects competing for limited funds.

Equation (2.1) shows the formula for the TV in its simplest form:

$$TV = \sum_{j=1}^{n} C_j'(1+m)^{n-j} - \sum_{k=o}^{n} C_k(1+i)^{n-j} \qquad (2.1)$$

where all terms are as defined in Chapter 1. The form of the second term in the RHS of (2.1) is intended to allow for the possibility that the investment will extend beyond year zero.

As it stands, equation (2.1) is only applicable to projects requiring the same initial investment and fully financed with equity funds. Because the opportunity cost of the investment capital is not considered in equation (2.1), explicit allowance must be made for differences in the initial investment in alternative projects.

2.3 THE EVALUATION OF INVESTMENT PROJECTS USING THE TV METHOD

Different cases of increasing complexity are analysed using both TV and NPV (in what follows Method 3 is referred to as NPV). Equation (2.1) is modified, as required in each case.

Appraisal of Individual Investment Projects

a. General Inflation:

For this simple case Equation (2.1) can be used without alteration. The TV for the project, however, has to be compared with the TV of the standard alternative. Thus,

$$TV-TV_B = \sum_{j=1}^{n} C'_j(1+m)^{n-j} - \sum_{k=o}^{n} C_k(1+i)^{n-j}$$

$$-\left(\sum_{k=o}^{n} C_k(1+m)^{n-j} - \sum_{k=o}^{n} C_k(1+i)^{n-j}\right)$$

$$= \sum_{j=1}^{n} C'_j(1+m)^{n-j} - \sum_{k=o}^{n} C_k(1+m)^{n-j}) \qquad (2.2)$$

where TV_B represents the TV of the standard alternative.

Dividing every term in (2.2) by $(1+m)^n$ and rearranging:

$$\sum_{j=o}^{n} \frac{C'_j}{(1+m)^j} = NPV$$

where the terms with k subscripts in (2.2) are included in C'_j with negative values.

This shows that a simple NPV would produce just the same result. However, the assumption of general inflation very seldom holds. A more realistic situation is illustrated below.

b. Differential Inflation:

The case where the fixed investment escalates at a rate different from the general inflation rate, is of special interest. Comparing the project's TV with the standard alternative:

$$TV - TV_B = \sum_{j=1}^{n} C_j(1+m)^{n-j} - \sum_{k=o}^{n} C_k(1+i')^{n-j}$$

$$- \left(\sum_{k=o}^{n} C_k(1+m)^{n-j} - \sum_{k=o}^{n} C_k\ (1+i)^{n-j} \right) \qquad (2.2)$$

where i' is the particular inflation rate affecting the initial investment. Rearranging this equation and grouping similar terms, we get:

$$TV - TV_B = \sum_{j=o}^{n} C_j(1+m)^{n-j} - \left[\sum_{k=o}^{n} C_k(1+i')^{n-j} - \sum_{k=o}^{n} C_k(1+i)^{n-j} \right] \qquad (2.3)$$

The corresponding NPV would be:

$$NPV = \sum_{j=o}^{n} C_j\ (1+m)^{-j}$$

Depending on the relative values of i' and i, the quantity between square brackets in (2.3) will be positive or negative. If this amount has a sufficiently large positive value, the TV of the project will be negative. This indicates that the project does not generate enough money to replace plant and equipment. Still, if the first two terms in the RHS of (2.3) add up to a positive number (positive NPV), the project generates more cash than the standard alternative. The decision in this case depends on whether or not the project has to be replaced. If physical replacement is necessary, it would be desirable to know in advance whether injection of capital will be required. Analysis of the project using TV, predicts the necessity to inject capital to renew the project. A positive NPV indicates that the project is more desirable than the standard alternative, however, it gives no indication as to whether injection of capital will be necessary to replace the project.

Treatment of borrowed capital under inflation

Because of long term fixed interest contracts, holding debt during inflationary periods can improve the profitability of an investment project. This gain, however, has to be carefully weighed against the opportunity cost of the equity capital saved by the use of borrowed capital (Baxter, 1975). If the firm is a net borrower over the planning period, a real gain can be realised because the loan principal is repaid with currency of less purchasing power than it was originally borrowed. Fixed interest rates over a period of growing inflation are advantageous to the borrower, because the real interest paid on debt decreases as inflation increases. If the firm is a net lender, however, the opposite occurs and a holding or lending loss is incurred. Investment projects are often appraised in isolation and this makes it more difficult to estimate the gain or loss derived from borrowing. The problem is better appreciated by using the "cash flows to equity" approach, mentioned earlier. The use of debt will reduce the total amount of equity capital required to finance the project. If the equity capital so saved is lent in the market, it will be subject to the same eroding effect inflation has on borrowed money. It is possible to estimate the highest interest on loan which, under these circumstances, would result in increased profitability. To this purpose, the TV for the case of a mixture of debt and equity is compared with the corresponding all equity TV. Risk considerations are here ignored:

$$\sum_{j}^{n}((C_j - Pgr) - (T_j - Pgr)t)(1+e)^{n-j} - gP - (1-g)(1+i)^n P +$$

$$gP\ (1+e)^n - gP(1+i)^n \geqslant \sum_{j}^{n} (C_j - T_j t)(1+e)^{n-j} - P\ (1+i)^n$$

Cancelling equal terms and rearranging,

$$r \leqslant \frac{(1+e)^n - 1}{(1-t)\sum_{j}^{n}(1+e)^{n-j}} \qquad (2.4)$$

Where C_j is the net cash flow from operations in year j

P is the fixed investment

g is the proportion of debt used to finance the project

T_j is the taxable profit from operations in year j

r is the annual interest rate on loan before tax

t is the tax rate (fractional)

e is the market lending rate of interest after tax

i is the annual inflation rate

n is the duration of the project

$\{(C_j\text{-}Pgr) - (T_j\text{-}Pgr)t\}$ is the net annual cash flow after subtracting the interest paid on borrowed money (Pgr) and the total amount of tax payable after deduction of the allowances due to debt interest payments ($(T_j - Pgr)t$). This amount is compounded (as opposed to discounted) to year n. gP represents the loan principal which is repaid at the end of the project's life. $(1-g)P(1+i)^n$ represents the amount required at the end of the project's life to pay for shareholders' contribution to the initial investment without loss of purchasing power. $gP(1+e)^n - gP(1+i)^n$ represents the TV of the equity capital saved by borrowing, which is lent in the market. It has been assumed that interest and inflation rates remain constant over the project's life and that the interest on debt is independent from the amount of debt used. On this basis, inequality (2.4) gives the highest interest on loan which would result in increased profitability if the firm is a net lender over the planning period concerned. Observe that the maximum value of r in (2.4) is not a function of the inflation rate. It is, however, indirectly affected by inflation through the effects of this on the lending rate (which is here the opportunity cost of equity capital).

Selecting between alternative investment projects

Three different situations will be analysed:

(a) Mutually exclusive projects of equal duration and initial investments, but affected by different escalation rates. Projects are fully financed with equity funds:

When Method 3 is used, the selection is based on the highest NPV. This procedure, however, does not take account of the fact that the initial outlays are affected by different escalation rates, and that when the project has to be replaced, the difference in replacement value can change the ranking of the projects. If TV is used, the difference in replacement value is actually taken into account. Because the initial investments are the same, the opportunity cost of the investment capital is only relative to each alternative and not to the option of lending the money in the market (eg deposit in a bank). It is, of course, necessary that at least the selected project be better than the bank investment. Equation (2.1) is therefore used without any correction. Example 2.3.1 below illustrates this case:

Example 2.3.1

Two mutually exclusive investment projects are analysed. The general assumptions are the same as stated in (a) above. The cash flows details are given in Table 2.1:

Table 2.1

	Year	0	1	2	3	4	5
Project A	Escalation Rate (%)		12	13	14	15	16
	Cash Flows ($x10^6$)	-4	1.0	1.5	3.5	4.0	6.0
Project B	Escalation Rate (%)		8	8	8	8	8
	Cash Flows ($x10^6$)	-4	1.0	1.5	3.0	3.5	5.5

A bank interest rate of 12% per year is assumed. The projects' NPVs are:

$$NPV_A = 10.5 - 4 = 6.5 \times 10^6$$

$$NPV_B = 9.6 - 4 = 5.6 \times 10^6$$

This would lead to the selection of project A. If the projects are analysed using TV, however, the decision would be in favour of project B.

$$TV_A = 18.5 - 7.7 = 10.8 \times 10^6$$

$$TV_B = 16.9 - 5.9 = 11.0 \times 10^6$$

Project B generates a larger profit after replacement because the particular production equipment needed is affected by a smaller escalation rate than in the case of project A.

(b) Mutually exclusive projects with different initial investments.

I. With common escalation rates:

Assume projects A and B require initial outlays of C_o^A and C_o^B respectively, where $C_o^A < C_o^B$ and both escalate keeping pace with the general inflation rate. To simplify the notation in what follows, it will be assumed that the whole initial investment takes place in year zero, only. Their TVs would be evaluated as follows:

$$TV_A = \sum_j^n C_j^A(1+m)^{n-j} - C_o^A(1+i)^n + (C_o^B - C_o^A)(1+m)^n - (C_o^B - C_o^A)(1+i)^n$$

where $(C_o^B - C_o^A)(1+m)^n - (C_o^B - C_o^A)(1+i)^n$ is the TV of the difference in initial investments between projects B and A.

$$TV_B = \sum_j^n C_j^B(1+m)^{n-j} - C_o^B(1+i)^n$$

To decide between these two projects, their TVs have to be compared. Subtracting TV_B from TV_A:

$$TV_A - TV_B = \sum_j^n C_j^A(1+m)^{n-j} - C_o^A(1+m)^n - \left(\sum_j^n C_j(1+m)^{n-j} - C_o^B(1+m)^n\right)$$

dividing every term by $(1+m)^n$ we get:

$$\sum_j^n \frac{C_j^A}{(1+m)^j} - C_o^A - \left(\sum_j^n \frac{C_j^A}{(1+m)^j} - C_o^B\right) = NPV_A - NPV_B$$

which shows that TV and NPV would lead to exactly the same result, in which case TV has no additional advantages over NPV.

II. With different escalation rates:

If the fixed investments escalate at a rate different from the general index, however, the terms containing i in the derivation above can not be cancelled against each other, because their values are not the same in every case. Thus, the difference in TV would be:

$$TV_A - TV_B = \sum_j^n C_j^A(1+m)^{n-j} - C_o^A(1+i_A)^n + (C_o^B - C_o^A)(1+m)^n$$
$$- (C_o^B - C_o^A)(1+i)^n - (\sum_j^n C_j^B(1+m)^{n-j} - C_o^B(1+i_B)^n)$$

where i_A and i_B are the escalation rates applying to projects A and B. Under these circumstances Method 3 could lead to the wrong selection of projects.

(c) Example 2.3.2 below illustrates the type of interactions between projects, generated by the use of debt in an inflationary environment.

<u>Example 2.3.2</u>

It is assumed that the firm has £2.5 million available for investment, and that a further £2.0 million could be borrowed. The alternative investment opportunities are:

Project A $2.0 \times 10^6 \xrightarrow{T} 4 \times 10^6$)
Project B $2.5 \times 10^6 \xrightarrow{T} 5 \times 10^6$) Gross Profit

A bank or similar investment over the same period of time would yield an overall growth of 50%:

$$2.5 \times 10^6 \xrightarrow{T} 3.75 \times 10^6 \quad (m = 0.5)$$

Inflation rate over the period is forecast at 75%:

$$2.5 \times 10^6 \xrightarrow{T} 4.375 \times 10^6$$

this represents the amount in current money units with purchasing power of £2.5 million in terms of year zero money.

The interest rate on loan is 60% over the whole period.

Using Method 3, the analysis of alternatives would be as follows:

$$NPV_A = \frac{4 \times 10^6}{1.5} - 2.0 \times 10^6 = 0.67 \times 10^6$$

$$NPV_B = \frac{5 \times 10^6}{1.5} - 2.5 \times 10^6 = 0.83 \times 10^6$$

Project B would obviously be preferred. It is still possible to raise the extra £2.0 million necessary to finance both projects. To analyse this possibility the "cash flows to equity" approach is used. For illustration purposes, it is assumed that the loan is equally distributed between the projects, however, a different assumption in this respect would not affect the analysis.

$$NPV_{A'} = \frac{4 \times 10^6 - 0.6 \times 10^6}{1.5} - 2.0 \times 10^6 = 0.27 \times 10^6$$

$$NPV_{B'} = \frac{5 \times 10^6 - 0.6 \times 10^6}{1.5} - 2.5 \times 10^6 = 0.43 \times 10^6$$

The joint NPV is

$$NPV_{A'+B'} = 0.27 \times 10^6 + 0.43 \times 10^6 = 0.7 \times 10^6$$

This option is not better than project B alone and therefore it is rejected.

If the problem is analysed using a TV approach, the results are as follows:

$$TV_A = 4x10^6 - 2x10^6\ (1.75) + \left[.75x10^6 - .5x10^6\ (1.75)\right]$$
$$= .5x10^6 + (-.125x10^6)$$
$$= .375x10^6$$

where the amount in square brackets represents the TV of the £0.5 million not invested in the project which are lent in the market.

$$TV_B = 5x10^6 - 2.5x10^6\ (1.75) = .625x10^6$$

If £2.0 million loan is taken.

$$TV_{A'} = (4x10^6 - .6x10^6) - 1x10^6 - 1x10^6\ (1.75) = 0.65x10^6$$

$$TV_{B'} = (5x10^6 - .6x10^6) - 1x10^6 - 1.5x10^6\ (1.75) = .775x10^6$$

$$\text{Joint } TV_{A'+B'} = 1.425 \times 10^6$$

The joint venture is more attractive than the individual activities, when properly analysed using TV. This method allows us to take account of the real effect of inflation in the project. Only that part of the initial investment contributed by the firm has to be protected from inflation; the loan principal is repaid at its nominal value. By taking the loan and investing in both projects, the amount that could be distributed to shareholders without reducing the capital value of the firm, is maximised. As the number of investment opportunities increases, so does the difficulty to analyse all the possible combinations of projects and a more efficient method of analysis becomes necessary. This situation can be further complicated by the interactions introduced by the taxation system (see Berry et al, 1979). A method to deal with these complications is described in Chapter III. Notice that, because in the example above the positive NPVs for the two projects already indicate that the bank investment would not be a better alternative, the corresponding TVs need not be calculated.

2.4 SUMMARY OF THE ADVANTAGES OF THE TV METHOD OVER NPV

It was mentioned above that, the main difference between TV and NPV lies in the way in which the initial outlay is projected into the future. A feature common to the examples given above is that the effect of inflation on the initial outlay is not the same for all the alternatives. This introduces changes in their relative opportunity costs which can only be appreciated if the effect of inflation on the initial investment is explicitly considered. As a result, an analysis using TV will produce a more appropriate ranking of investment opportunities.

By treating debt finance flows in the same way as all other cash flows, the common assumption about equal lending and borrowing rates can be dropped. This feature avoids some of the difficulties involved in calculating a weighted average cost of capital, and makes it possible to perform a more realistic assessment of the advantages of the use of borrowed capital under inflation.

The TV method evolved as a result of an analysis of the factors which were thought likely to introduce most of the complications in the appraisal of investment projects under inflation, using NPV. TV is, therefore, no more than an extension of NPV broken these complications is described in Chapter III. Notice that, because in the example above the positive NPVs for the two projects already indicate that the bank investment would not be a better alternative, the corresponding TVs need not be calculated.

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The TV method evolved as a result of an analysis of the factors which were thought likely to introduce most of the complications in the appraisal of investment projects under inflation, using NPV. TV is, therefore, no more than an extension of NPV broken down into a somewhat more detailed analysis. It can be seen that the TV method is not uniquely defined by one rigid formula; instead equation (2.1) can be modified depending on the particular case under analysis. Because the effects of inflation are different for each project, flexibility is a very important feature of any method of investment appraisal under inflation. Accordingly, TV could be thought of as a simple set of rules intended to account for the likely effects inflation could have on investment projects under different circumstances. Still, TV is only another simplistic method whose applicability is limited by the assumptions on which it is based. In this respect, it does not differ from any other method, as stated by Allen (Allen, D H, 1980): "There is no single all-embracing method of carrying out the economic evaluation of a project. The method chosen should take account of the specific purposes for which the evaluation is required, the availability of information and the effort which can be justified."

2.5 A NOTE ON THE TREATMENT OF UNCERTAINTY

The increased uncertainty introduced by inflation makes it too risky to base decisions on point estimates, such as the values predicted by a TV or other similar method. No special technique to account for this uncertainty is here proposed. Several methods of allowing for risk and uncertainty in economic evaluations are reported in the literature (see for example, Allen, D H, 1980). The selection of any particular method is dependent upon the level of confidence required and the information available.

CHAPTER III

CAPITAL BUDGETING IN AN INFLATIONARY ENVIRONMENT. CORPORATE INVESTMENT DECISIONS OVER THE LONG - TERM

3.1 INTRODUCTION

This chapter is concerned with the overall analysis of the investment and financing activities of the firm. In order to bring more realism into the analysis, the assumption of a perfect capital market is relaxed, and factors such as liquidity and leverage are taken into account. Many other interacting factors are involved in the analysis. Investment opportunities arising at different times within the planning horizon are likely to be interdependent due to technological factors and financial limitations. As in the simpler cases discussed in Chapter II, the presence of inflation increases the complexity of the problem: interdependencies are introduced by the interactions between the taxation system and the diverse inflation rates that apply to different projects; interdependencies between projects can also be introduced when a mixture of debt and equity is used for financing; under conditions of inflation a gain on owing can be realised if the firm is a net borrower, otherwise a holding or lending loss has to be set against the gain on borrowing. To deal with these complications a Mixed Integer Programming (MIP) model is used. The model maximises assets value at the planning horizon, subject to constraints on cash balance, liquidity, gearing and taxation. The objective function evolved as an extension of the TV approach discussed in Chapter II; the selection of this particular set of constraints is intended to help anticipate some of the effects of inflation on corporate financial performance, such as increases on working capital requirements and overstatement of profits. The judicious selection of the investment and financing opportunities will minimise these harmful effects. To facilitate the understanding of this chapter, a brief description of the theory and properties of the general mathematical programming type of problem is given in Appendix B.

3.2 EXISTING MODELS FOR CAPITAL BUDGETING USING MATHEMATICAL PROGRAMMING

A large number of models have been proposed in the literature since 1955. In his annotated bibliography, Larson (1970) comments on 116 works on the subject, published between May 1955 and June 1969. Many of them draw heavily on the work of Weingartner (1963). Because of the importance of Weingartner's work in this field, it is convenient at this point to give a brief description of his Basic Horizon model:

> The model is based on the maximisation of assets value at some time in the future, called the horizon. This objective function is subject to cash balance constraints, one for each year before, and including the horizon. Lending and borrowing without limit take place at some stated rate of interest r.

> The model is expressed as a linear programme in the following way:

$$\text{Max.} \sum_j \hat{a}_j x_j + V_T - W_T$$

Subject to:

(a) $\sum_j a_{1j} x_j + V_1 - W_1 \leq D_1$

(b) $\sum_j a_{tj} x_j - (1+r)V_{t-1} + V_t + (1+r)W_{t-1} - W_t \leq D_t,$

$t = 2, \ldots, T$

(c) $0 \leq x_j \leq 1, \; j = 1, \ldots, n$

(d) $V_t, W_t \geq 0, \; t = 1, \ldots, T$

where

$\hat{a}_j$ is the value of all flows subsequent to the horizon, if any, discounted to the horizon at the market rate of interest.

x_j represents the fraction of project j adopted

V_t represents the amount available for lending in year t

W_t denotes the amount borrowed in year t

a_{tj} is the flow in year t resulting from acceptance of project j. a_{tj} is positive when it represents an expenditure and negative when it represents revenue

D_t denotes the funds anticipated to be generated by the firm from operations in year t and which may be attributed to the resources the firm currently controls

T is the horizon year

r is the rate of interest for lending and borrowing

> Lending and borrowing are accomplished by means of "renewable" one year contracts, where all interest is payable at the end of the year. (Weingartner, 1967).

The maximisation of the Net Terminal Value has been used by other authors after Weingartner (Byrne et al, 1969; Struve, 1966; Bernhard, 1967; Byrne et al, 1967; Chambers, 1967; Harvey, 1967; Robertson, 1967; Salazar et al, 1968; Arzac, 1968). Other typical objective functions reported in the literature include the maximisation of some function of the dividends of the firm (Unger, 1969; Weingartner, 1966; Baumol et al, 1965; Bernhard, 1969) and the maximisation of NPV (Wilson, 1969; Hillier, 1969; Alexander, 1968; Thompson et al, 1968; France, 1968; Lesso, 1967; Seppälä, 1967; Cohen et al, 1967). Constraints on cash balance, availability of funds and mutually exclusive projects are some of the most common ones.

Bernhard (1969) criticises Weingartner's Net Terminal Value objective function. His argument is that such an objective function ignores flows (which Bernhard seems to equate to dividend payments) prior to the horizon. The fact is that the term (V_T - W_T) in Weingartner's objective function (the net amount of financial assets accumulated at the horizon) accounts for flows prior to the horizon reinvested at the rate of interest. The objective function used in the model to be described in this chapter is similar to that of Weingartner, although it originated from the TV method of Chapter II.

Some of the most interesting works on the subject, published in more recent years, include articles by Chambers (1971), Unger

(1974), Ashton et al (1976 and 1979), Bhaskar (1978), and Berry et al (1979). Chambers (1971) analyses the use of multiple methods of new financing including the option of issuing new equity. The objective function used in this model is the maximisation of the cash flows generated at and after the horizon discounted to a value at the horizon. For each alternative specification of a programme of investments prior to the horizon, there is a stream of post-horizon cash flows. In this way the objective function reflects the effect at the horizon of pre-horizon activities. The objective function is subject to constraints on gearing and cash balance. The gearing constraints allow for the effect of different projects on the gearing position of the firm to be taken into account. The cash balance constraints differ from those of Weingartner, in that lending and borrowing activities are included in the analysis in the form of alternative "projects". Variables of the type of V_t and W_t used in the Basic Horizon model are, thus, unnecessary. Unger (1974) illustrates the use of duality theory for integer programming problems (see Appendix B), to obtain economic interpretations of the models. He applies his results to Weingartner's Basic Horizon model, but contends that similar results could be obtained for other discrete capital budgeting problems. Ashton et al (1976 and 1979) show that "rules of thumb", such as NPV, can give tolerably close approximations to the linear programming solution. The analysis is based on the approximations that rules of thumb imply to the dual of MP formulations. It is argued that the feasible decision space is extremely small, making the use of more complex analytical techniques unnecessary. They do, nontheless, acknowledge the need for programming models in situations of greater complexity. They illustrate their findings using models proposed by Weingartner (1963) and Chambers (1971), with constraints on cash balance and debt capacity. Bhaskar (1978) maximises the discounted stream of dividends to be paid during the planning period. He uses a "soft" type of capital rationing, where leverage can be increased at an increased cost of capital. Bhaskar contends that this type of capital rationing makes the feasible decision space (ie feasible region) much larger than in the models developed by Weingartner (1963) and Chambers (1971), therefore the conclusions of Ashton et al (1976) do not apply to his model. Berry et al (1979) propose a mathematical programming approach to deal with taxation induced interdependencies in investment appraisal. Their model maximises the sum of the discounted pre tax NPVs minus the discounted sum of tax payments. Only one type of constraint is used: that necessary to determine the total taxable income and the total unrelieved balance of capital allowances in each year. Their work is particularly relevant in an inflationary environment characterised by reduced profitability, partly due to overstatement of taxable income. A judicious selection of investment opportunities can help reduce the inflationary burden, by making the best possible use of tax allowances.

3.3 A CAPITAL BUDGETING MODEL USING MIP WHICH INCORPORATES INFLATION IN THE ANALYSIS

The advantages of a mathematical programming approach to capital budgeting have been discussed at length by several authors (Weingartner, 1963; Fawthrop, 1971; Bernhard, 1969; Bhaskar, 1978; and others). Amongst these advantages we could mention:

(1) The possibility to analyse the whole set of investment alternatives, even when projects are not independent.

(2) The joint analysis of investment and financing activities.

(3) Market imperfections, such as different lending and borrowing rates and a limited availability of investment capital, can be incorporated in the analysis. Other unrealistic assumptions, such as "the divisibility of any sum of investment capital between projects", become unnecessary in the context of an MP model.

(4) The application of MP to a particular case provides more information and a better understanding of the problem, than would otherwise be achieved.

The presence of inflation increases both the importance of an analysis of this type and the difficulties involved. Berry et al (1979) have demonstrated that taxation can induce interdependencies in investment appraisal, which can be handled by the use of an MIP formulation. One of the causes of these interdependencies is that tax allowances can be taken immediately only if profits are large enough. This might appear not to be the case when projects are analysed in isolation, but a mixture of projects can complement each other to take better advantage of the capital allowances. This issue attains much greater importance under inflationary conditions, as the value of tax allowances decreases with time, not only because of opportunity costs but also because of increasing replacement value of assets. The case is further complicated when the replacement value of assets for different projects is affected by different escalation rates, altering the rankings of alternative combinations of projects. As illustrated in example 2.3.2, the use of debt and equity in an inflationary environment, can also create interdependencies between projects. The possible advantages of holding debt under conditions of inflation can be determined if an overall analysis of the lending and borrowing operations of the firm is carried out. All this suggests an analysis of the investing and financing activities of the firm, such as the one that can be made using MP techniques. The selection of a mixed integer algorithm stems from the need to decide between investment opportunities which can not be fractionally accepted or rejected, mixed with other activities, such as lending and borrowing which can, theoretically, take any continuous value. The relative advantages of MIP over LP need further clarification. The main disadvantage of MIP is its much higher cost. An LP approach, however, can produce some undesirable results:

- The selection of fractional projects by LP, is accompanied by the assumption of a linear relationship between amount of benefit and level of resources used for a given project (Bell et al, 1970). This is, precisely, one of the unrealistic assumptions, implicit in traditional project selection techniques (Adelson, 1970), which we are trying to avoid by making use of mathematical programming. It is argued that its effect can be minimised by the presence of alternative project versions, but these are not always available.

- LP yields a higher objective function value because the resources are better utilised (Bell et al, 1970). This better utilisation of resources, however, is based on the assumption of linearity mentioned above. Moreover, the most frequent case in capital budgeting is that capital is not sufficient to undertake all the viable opportunities. The most important resource in this problem (money) is then, fully utilised in the best options expected to be available to the firm.

- It is claimed that "the classification of projects by linear programming model into fully accepted, partially accepted and rejected conveys more information than a fully integer solution" (Bell and Read, 1970). This extra information is obtained through sensitivity analysis, which shows the interactions between the fractions selected and the constraints, allowing the analyst "to make an intuitive judgment on which fractional projects should be rejected" (Ibid). Sensitivity studies, however, can also be performed on an MIP model, as will be seen in a later section of this chapter. The analysis can produce the same type of information as in the continuous case.

- The LP solution to the problem will have to be converted into an integer form. Either rejecting or accepting fractional projects will often fail to lead to the optimum integer solution which, after all, is the objective of the analysis.

Inflation is incorporated in the analysis in two ways: indirectly, by defining the constraint sets which measure and control the effects of inflation on financial performance; and directly, by projecting all the parameters on which inflation has an immediate effect.

Because the forecast of the inflation rate is so uncertain, it is very important to quantify the effects of the whole spectrum of possible values of the inflation rate on the selection of investment projects. Postoptimality analysis of the type that can be performed with MP packages is not very useful in this case, because the inflation rate does not appear explicitly in the MIP model. Instead, Monte Carlo simulation is used. This provides an adequate form of risk analysis which will be discussed at length, later on in this chapter.

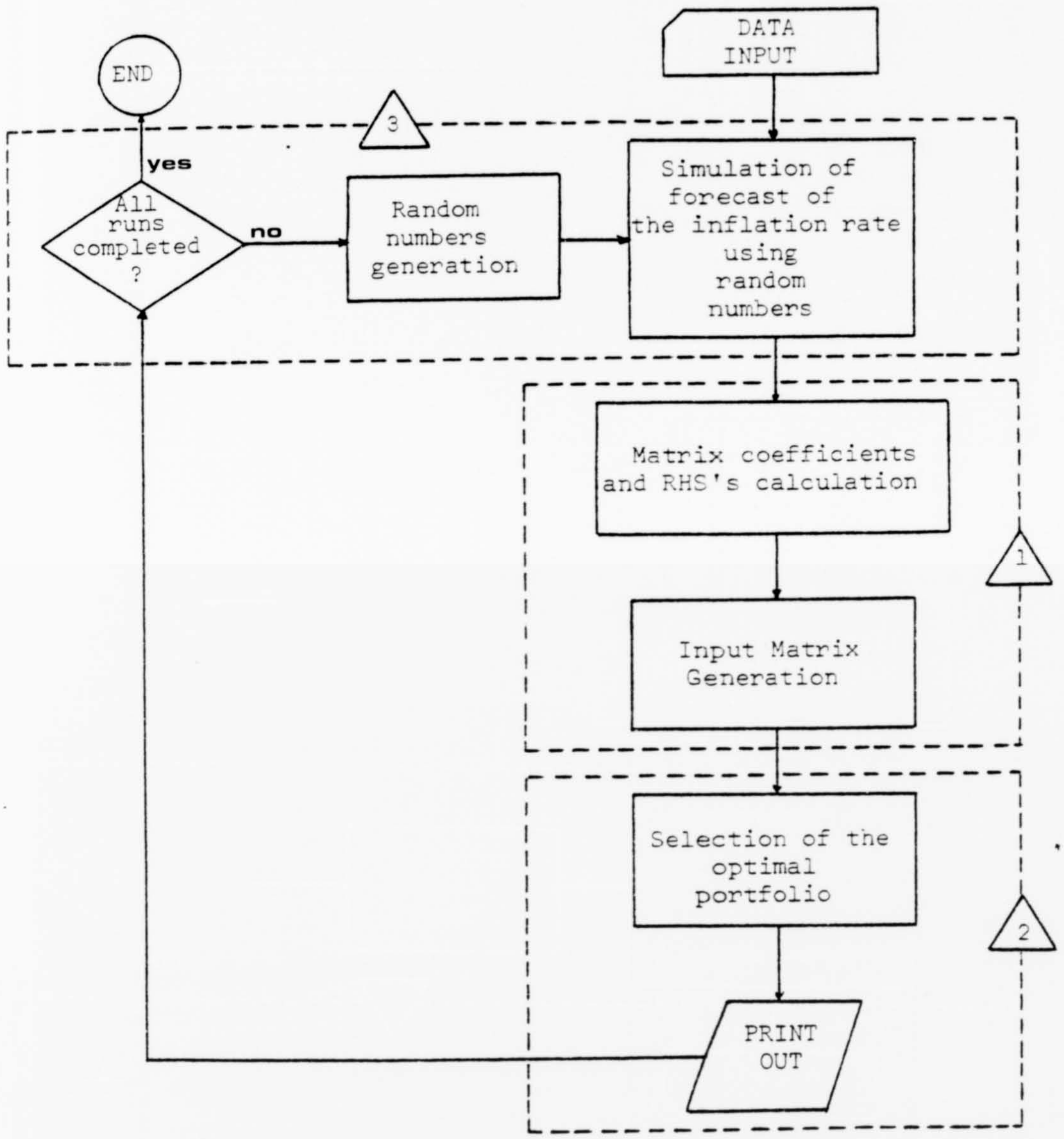
The model comprises two parts:

(1) a program in Fortran where primary information about the cash flows, inflation and interest rates is processed to define basic relationships, and

(2) a Mixed Integer Programming sub-model which uses the processed information to select the investment projects and simulate the lending and borrowing operations subject to the required constraints.

The processing of information carried out in part (1) of the model depends on the particular set of investment projects and the economic forecasts prevailing at the time the evaluation is made. A detailed description of it will, therefore be given for the case study in Chapter IV. This part of the model will also generate the input matrix necessary for the MIP sub-model. Figure 3.1 illustrates the interconnections between the different parts of the model.

Figure 3.1



△1 Part (1): Process of information and Matrix generation

△2 Part (2): Mixed Integer Programming sub-model

△3 Risk Analysis: Monte Carlo Simulation

3.3.1 The MIP sub-model

The MIP sub-model maximises assets value at the planning horizon. This is an extension of the TV methodology discussed in Chapter II, applied to the more general case of an imperfect capital market. A few simplifying assumptions are necessary:

- The capital not used in projects is invested in short term securities or bank deposits.
- All cash balances are lent in the market.
- No capital is borrowed beyond the horizon. If any money is borrowed at the horizon year, the full amount plus the after tax interest are subtracted from the objective function. Because the loan is repaid a year later, it has to be discounted back to the horizon at the after tax lending rate of interest.
- Tax is paid one year in arrears from the date it becomes due.

Two types of projects are allowed:

(1) projects with a fixed starting date; and
(2) projects which can be started at any date within the planning period, provided there is enough capital.

This second type of project is of particular interest under inflationary conditions. While these projects can be postponed to allow investment in other projects, their profitability and effect on the company will change with time. Fixed investment and production costs will probably rise with inflation; however, deferment can allow for a more convenient scheduling of investments resulting in increased overall profitability. An example of a project of this type is the investment to increase plant capacity.

Four different types of constraints are used: cash balance, liquidity, gearing and taxation. The selection of this particular set of constraints is intended to help anticipate some of the harmful effects of inflation on corporate financial performance. Cash balance constraints are used in most MP models to ensure that all cash available is used, either in investment activities, or to cover liabilities. Increases on working capital requirements caused by inflation have, in turn, caused liquidity problems to many companies. These companies must find an optimum level of investment, high enough to minimise the erosive effect of inflation, but not so high that it would endanger the position of the firm if inflation fluctuated in an unexpected manner. Liquidity constraints help finding that optimal level of investment. A similar role, but from a different viewpoint, is played by the gearing constraints. To finance increases in working capital due to inflation, the company may have to resort to debt finance. Holding debt during a period of rising inflation could be advantageous to the firm, but on the other hand, increased corporate risks would call for lower gearings in a period of fluctuating inflation. A careful control of the total level of debt in the company is then, of special importance. The interplay between the liquidity and gearing constraints will be made clearer at a later stage on this section. The overstatement of taxable income is one of the damaging effects of inflation on investment projects. 100% first year capital allowances can help reduce the burden, but they can only be taken if the company generates enough profit. The selection of the right mixture of investment opportunities allows the decision maker to take better advantage of the capital allowances offered within the taxation system. The set of taxation constraints in this model (based on the work of Berry et al (1979) calculates the capital allowances and taxable profit for each set of possible combinations of projects.

The MIP sub-model can be stated as follows:

(3.1.0) $$\max_{x} \sum_{j=1}^{m+n} \hat{a}_j x_j + L_T - \frac{1+b'_T}{1+l'_T} B_T - \frac{h}{1+l'_T} S_T$$

s.t.

(3.1.1) $$\sum_{j=1}^{m+n} a_{oj} x_j + L_o - B_o \leq P_o$$

$$\sum_{j=1}^{m+n} a_{tj} x_j + L_t - (1+l_{t-1})L_{t-1} - B_t + (1 + b_{t-1})B_{t-1}$$

$$+ h S_{t-1} \leq P_t, \; t = 1, \ldots, T \qquad (\rho_t)$$

(3.1.2) $$L_o - rB_o \geq 0$$

$$L_t - rB_t - rh S_{t-1} \geq 0, \; t = 1, \ldots, T \qquad (\beta_t)$$

(3.1.3) $$B_t - gL_t - g \sum_{k=o}^{t} \sum_{j=1}^{m+n} (C_{kj} - D_{kj}) x_j \leq 0,$$

$$t = 0, \ldots, T \qquad (\gamma_t)$$

(3.1.4) $$\sum_{j=1}^{m+n} a_{oj} x_j - A_o + S_o = P_o'$$

$$\sum_{j=1}^{m+n} a_{tj} x_j + b_{t-1}B_{t-1} - l_{t-1}L_{t-1} - A_t + A_{t-1}$$

$$+ S_t = P'_t, \; t = 1, \ldots, T \qquad (\lambda_t)$$

(3.1.5) $$\sum_{j=m+1}^{n} x_j \leq 1 \qquad (\delta)$$

$$x_j = 0 \text{ or } 1, \; j = 1, \ldots, m, \ldots, n \qquad (\mu_j)$$

$$L_t, B_t, A_t, T_t \geq 0, \; t = 0, \ldots, T$$

The following notation is used:

$\hat{a}_j$ denotes the flows arising after the horizon discounted back to the horizon at the after tax lending rate of interest.

x_j is an integer variable which takes a value of zero if the project is rejected, or 1 if it is accepted. The subscripts $j = 1, m$ refer to projects with a fixed starting date; $j = m+1, n$ refer to projects with a flexible starting date.

L_t denotes the amount lent in year t at a rate of interest l_t. L_T contains the terminal value of cash flows prior to the horizon.

B_t denotes the amount borrowed in year t at a rate of interest b_t.

b'_T and l'_T are after tax borrowing and lending interest rates respectively.

b_t and l_t are before tax interest rates.

S_t is the total taxable income in year t, after allowances. A one year lag between tax becoming payable and date of payment is assumed. Tax owed on income at the horizon year is, therefore, paid one year after. This value has to be discounted back to the horizon at the after tax lending interest rate, and subtracted from the objective function.

h is the corporate tax rate.

$a_{t,j}$ is the flow in year t resulting from acceptance of project j. This value is positive when it represents an expenditure and negative when it represents revenue.

P_t represents the projection of the cash that will be available for investment in year t from old activities, minus debt commitments already existing at the outset, minus the planned payment of dividends. Advanced Corporation Tax is here ignored.

r is the current (liquidity) ratio, defined as the quotient of current assets and current liabilities.

g is the gearing ratio, defined as the proportion of debt in the total net assets of the firm.

$C_{k,j}$ denotes the fixed investment on project j made in year k.

$D_{k,j}$ is the depreciation on plant and equipment due to project j in year k.

A_t is the total unrelieved balance of capital allowances up to and including year t.

P'_t is the projection of the cash available in year t from old activities.

t and k are time subscripts, j is the project number subscript, and T is the horizon year.

It should be noticed that interest rates within the planning period are on a before tax basis, and those beyond the horizon are after tax interest rates. The reason for this is that, within the planning period, the total tax payable by the firm is calculated every year and subtracted from total annual earnings. After the horizon calculations are less detailed, and cash flows are estimated on an after tax basis.

Objective Function

The objective function is based on the same principle as the TV model discussed before. However, because all the available opportunities are being considered and all the capital is invested, the initial layout need not be subtracted from the amount accumulated at the horizon (ie, it is a constant and consequently does not affect the optimisation process).

$$\text{Maximise} \sum_{j=1}^{m+n} \hat{a}_j \, x_j + L_T - \frac{1+b'_T}{1+l'_T} B_T - \frac{h}{1+l'_T} S_T$$

The first two terms represent the after tax net income accumulated through the planning period, plus an estimate of the after tax net income after the horizon generated by activities started within the planning period. The last two terms represent the payment at year T + 1 of debt and tax incurred at the horizon. The horizon year is defined as that point in time beyond which forecasts are deemed to be too unreliable to make any further contribution to the analysis. Events taking place after that point in time are too uncertain to be worthy of detailed analysis: For this reason the objective function is designed to achieve the highest profits at the horizon, making sure that the firm will be able to cover all liabilities incurred before or at that year. This will leave the firm in a strong position to deal with events taking place after the horizon. No borrowing takes place after year T, therefore flows after then are discounted at the after tax lending interest rate.

Cash Balance Constraints

This set of constraints ensures that all the cash available in year t will be given some use, eg investment in projects, bank, payment of debt, dividends, etc.

$$\sum_{j=1}^{m+n} a_{o,j}\, x_j + L_o - B_o \leqslant P_o$$

$$\sum_{j=1}^{m+n} a_{t,j}\, x_j + L_t - (1+l_{t-1})\, L_{t-1} - B_t$$

$$+ (1+b_{t-1})\, B_{t-1} + hS_{t-1} \leqslant P_t, \quad t = 1, \ldots, T$$

The cash left after investment in projects, payment of tax and debt is lent in the market, ie bank deposit, one year securities, or similar short term investment. It is assumed that both lending and borrowing are carried out by means of renewable one year contracts.

Liquidity Constraints

$$L_o - rB_o \geqslant 0$$

$$L_t - rB_t - rhS_{t-1} \geqslant 0, \quad t = 1, \ldots, T$$

This set is used to keep the amount of cash in each year at the required level. The liquidity ratio used is defined as the quotient of current assets and current liabilities. In this model, total current assets are equated with cash balances, and total current liabilities are equated with outstanding debt plus tax payable in the current year. The frequency with which firms face liquidity problems under inflationary conditions, makes it very important to take account of this factor in the selection of investment projects.

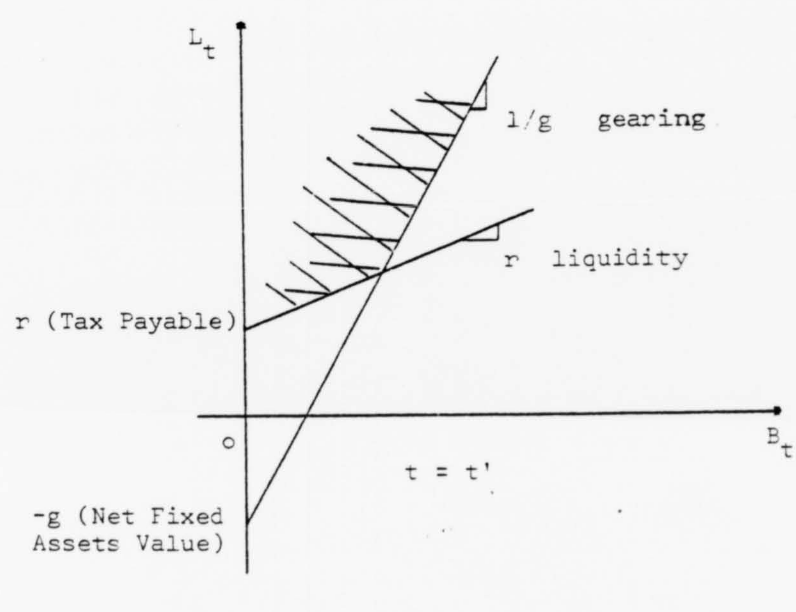
Gearing Constraints

$$B_t - gL_t - g \sum_{k=o}^{t} \sum_{j=1}^{m+n} (C_{k,j} - D_{k,j})\, x_j \leq 0, \quad t = 0, \ldots, T$$

This set of constraints keeps the level of debt within the allowed gearing ratio, defined as the proportion of debt in the total net assets of the firm. The effect of inflation on gearing is mainly felt through the change in the replacement value of plant and equipment. This should be taken into account in the calculation of depreciation, as suggested by Baxter (1975) and Merrett and Sykes (1973), in order to obtain the real value of the assets of the firm. Debt financing can be very advantageous, but the limits imposed by management policy and interest rates, must be observed to achieve an optimum selection of investment opportunities.

The interactions between the liquidity and gearing constraints are illustrated in Figure 3.2:

Figure 3.2



The shaded area represents a simplified view of the feasible region for a given year t', and a given set of selected projects. Tax payable and the value of net fixed assets in year t depend mostly on the projects selected and the way in which the inflation rate affects them. For relatively low values of L_t the total amount available for investment, $L_t + B_t$ will be limited by the liquidity constraint. As the investment generates profit, the value of L_t will increase, and debt will now be limited by the gearing constraint. Liquidity and gearing constraints could be binding in the same year. This could occur if the optimum value of L and B in that year, was at the intersection of the two constraints. Liquidity and gearing constraints do not remain static over the planning period. During investment, the liquidity constraints will move towards the origin, while the gearing constraints will move away from it (on the negative side of the L axis). Investment on new fixed assets generates tax allowances, thus reducing the total tax payable. Inflation will also cause these constraints to move, by altering taxable profit and the value of fixed assets.

Taxation Constraints

$$\sum_{j=1}^{m+n} a_{oj}\, x_j - A_o + S_o = P_o'$$

$$\sum_{j=1}^{m+n} a_{oj}\, x_j + b_{t-1}B_{t-1} - l_{t-1}L_{t-1} - A_t + A_{t-1} + S_t$$

$$= P'_t\, ,\ t = 1, \ldots, T$$

These constraints include all those terms which, in a given year contribute to increase or reduce taxable profit. If the net balance on the LHS is positive (an expenditure), the variable - A_t

(allowances) will absorb it to maintain the equality. If the net balance is negative (inflow), it will be absorbed by the variable S_t (taxable profit) in order to maintain the equality. Because the variables S_t appear in all the cash balance constraints reducing the total amount available for investment, and consequently reducing the potential value of the objective function, S_t will be made zero whenever possible. Therefore A_t and S_t will not simultaneously achieve positive values. The combination $- \quad A_{t-1}$ allows the unrelieved balance from previous years to be taken into account.

3.3.2 The Dual of the MIP sub-model

The dual analysis of a capital budgeting model helps identifying interesting features of the optimal solution, and provides an economic interpretation of the model. Analysis of the dual problem, however, is often based on the assumption of divisibility of projects. Some of the important results obtained in this way are, therefore, not directly applicable to the discrete case.

In this section, Balas' duality theory, as reported by Unger (1974), is used to obtain the dual of the MIP sub-model. Based on this dual formulation, results similar to those obtained by Weingartner (1963) are applied to this model.

The dual to problem (3.1) may be stated as:

(3.2.0) $\underset{x}{\text{Max.}}\ \underset{\rho\lambda\mu\delta}{\text{Min.}}\ \sum_{t=o}^{T} P_t\rho_t + \sum_{t=o}^{T} P'_t\lambda_t + \delta - \sum_{j=1}^{m+n} \mu_j\, x_j$

s.t.

(3.2.1) $\sum_{t=o}^{T} a_{tj}\rho_t - g \sum_{t=o}^{T}\sum_{k=o}^{t} (C_{kj} - D_{kj})\gamma_t + \sum_{t=o}^{T} a_{tj}\lambda_t$

$- \mu_j = \hat{a}_j,\ j = 1, \ldots, m$ (x_j)

(3.2.2) $\sum_{t=o}^{T} a_{tj}\rho_t - g \sum_{t=o}^{T}\sum_{k=o}^{t} (C_{kj} - D_{kj})\gamma_t + \sum_{t=o}^{T} a_{tj}\lambda_t$

$+ \delta - \mu_j = \hat{a}_j,\ j = m + 1, \ldots, n$ (x_j)

(3.2.3) $\rho_t - (1+l_t)\rho_{t+1} - \beta_t - g\gamma_t - l_t\lambda_{t+1}$

$\geqslant 0,\ t = 0, \ldots, T - 1$ (L_t)

(3.2.4) $\rho_T - \beta_T - g\gamma_T \geqslant 1$ (L_T)

(3.2.5) $-\rho_t + (1+b_t)\rho_{t+1} + r\beta_t + \gamma_t + b_t\lambda_{t+1}$

$\geqslant 0,\ t = 1, \ldots, T - 1$ (B_t)

(3.2.6) $- \rho_T + r\beta_T + \gamma_T \geqslant - \frac{1+b'_T}{1+l'_T}$ (B_T)

(3.2.7) $h\rho_{t+1} + rh\beta_{t+1} + \lambda_t \geqslant 0,$

$t = 0, \ldots, T - 1$ (S_t)

(3.2.8) $\lambda_T \geqslant - \frac{h}{1+l'_T}$ (S_T)

(3.2.9) $- \lambda_t + \lambda_{t+1} \geqslant 0,\ t = 0, \ldots, T - 1$ (A_t)

(3.2.10) $- \lambda_T \geqslant 0$ (A_T)

$\rho_t, \beta_t, \gamma_t \geqslant 0,\ t = 0, \ldots, T;\ \delta \geqslant 0,\ \lambda_t$ unrestricted, $t = 0, \ldots, T.$ μ_j unrestricted, $j = 1, \ldots n,\ x_j = 0 \text{ or } 1,\ j = 1, \ldots, n$

The original variable associated with each constraint is indicated in brackets at the right of that constraint. The following complementary slackness conditions can be obtained:

$$\bar{\rho}_t \, \bar{F}_t = 0, \; t = 0, \ldots, T \qquad (3.3.1)$$

$$\bar{\beta}_t \, \bar{Q}_t = 0, \; t = 0, \ldots, T \qquad (3.3.2)$$

$$\bar{\gamma}_t \, \bar{G}_t = 0, \; t = 0, \ldots, T \qquad (3.3.3)$$

$$\bar{\delta} \, \bar{R} = 0 \qquad (3.3.4)$$

$$(\bar{\rho}_t - (1+l_t)\bar{\rho}_{t+1} - \bar{\beta}_t - g\,\bar{\gamma}_t - l_t\bar{\lambda}_{t+1})\,\bar{L}_t = 0, \quad t = 0, \ldots, T-1 \qquad (3.3.5)$$

$$(-\bar{\rho}_t + (1+b_t)\bar{\rho}_{t+1} + r\bar{\beta}_t + \bar{\gamma}_t + b_t\bar{\lambda}_{t+1})\,\bar{B}_t = 0, \quad t = 0, \ldots, T-1 \qquad (3.3.6)$$

$$(h\bar{\rho}_{t+1} + rh\bar{\beta}_{t+1} + \bar{\lambda}_t)\, S_t = 0, \; t = 0, \ldots, T-1 \qquad (3.3.7)$$

$$(-\bar{\lambda}_t + \bar{\lambda}_{t+1})\,\bar{A}_t = 0, \; t = 0, \ldots, T-1 \qquad (3.3.8)$$

$$(-\bar{\lambda}_T)\,\bar{A}_T = 0 \qquad (3.3.9)$$

where F_t, Q_t, G_t and R are the slack variables associated with constraints (3.1.1), (3.1.2), (3.1.3) and (3.1.5) respectively and the superscript bar denotes a variable's optimal value.

3.3.3 Properties of the optimal solution

Some important properties of the optimal solution to the primal and dual problem can be derived:

Property 1. $\overline{\rho}_t \geqslant 1$, $t = 0, \ldots, T$

Proof: From (3.2.4) we obtain that

$$\overline{\rho}_T \geqslant 1 \tag{3.4.1}$$

By substituting (3.4.1) in (3.2.3) and making $t = T - 1$ we get:

$$\overline{\rho}_{T-1} \geqslant (1+l_{T-1})\overline{\rho}_T + \overline{\beta}_{T-1} + g\overline{\gamma}_{T-1} + l_{T-1}\overline{\lambda}_T \tag{3.4.2}$$

In general, it can be assumed that $0 < l_t < 1$, $t = 0, \ldots, T$; from the dual problem (3.2) we know that $\beta_t, \gamma_t \geqslant 0$, $t = 0, \ldots, T$, and λ_t is unrestricted in sign for all t.

From (3.2.8) $\lambda_T \geqslant -\frac{h}{1+l'_T}$, where $h < 1$ and $l'_T > 0$, by definition.

Hence, $\frac{h}{1+l'_T} < 1$, and from this $|l_{T-1}\overline{\lambda}_T| < l_{T-1}\overline{\rho}_T$. Substituting this result in (3.4.2) we obtain $\rho_{T-1} \geqslant 1$. By successive substitution in (3.2.3) and using (3.2.9) we obtain:

$$\overline{\rho}_t \geqslant 1, \; t = 0, \ldots, T$$

and also

$$\overline{\rho}_t \geqslant \overline{\rho}_{t+1}, \; t = 0, \ldots, T$$

Property 2. $\overline{F}_t = 0$, $t = 0, \ldots, T$

Proof: This follows immediately from Property 1 and complementary slackness condition (3.3.1).

These two properties have important economic interpretations. Following Weingartner (1963), $\overline{\rho}_t$ represents the yield at the horizon of an additional pound in year t. Given that $l_t > 0$ for all t, cash in the earlier years is expected to have a higher yield at the horizon than cash becoming available at a later time in the planning period. If money at any time within the planning period has a positive yield at the horizon, it makes economic sense to invest all the cash remaining after paying debt, dividends and tax. This is the meaning of Property 2.

From (3.2.1) above we obtain:

$$\mu_j = -(\hat{a}_j - \sum_{t=0}^{T} a_{t,j}\rho_t + g\sum_{t=o}^{T}\gamma_t \sum_{k=o}^{t} (C_{k,j} - D_{k,j})$$
$$- \sum_{t=o}^{T} a_{tj}\lambda_t - \delta),\ j = 1, \ldots, m + n$$

The RHS of this equality is the negative of the horizon value of project j. It is useful to give an economic interpretation of the components of this horizon value.

$\hat{a}_j$, as defined before, is the value at time T of post horizon cash flows from project j.

$- \sum_{t=o}^{T} a_{t,j}\rho_t$ is the value at time T of pre-horizon cash flows; this quantity appears here with a negative sign, because of the sign convention adopted for the $a_{t,j}$.

$-g \sum_{t=o}^{T} \gamma_t \sum_{k=o}^{t} (C_{k,j} - D_{k,j})$ represents the contribution of project j to tightening or loosening the leverage position through its effect on total net assets. $\sum_{k=o}^{t} (C_{k,j} - D_{k,j})$ is the value at year t of the fixed investment on project j, after deducing depreciation charges. The effect of inflation on this quantity depends on the escalation rate of the fixed assets and the way this is taken into account in calculating depreciation. It can be seen that, a high escalation rate on fixed assets would make this quantity negative; the net effect would be to reduce the horizon value of the project.

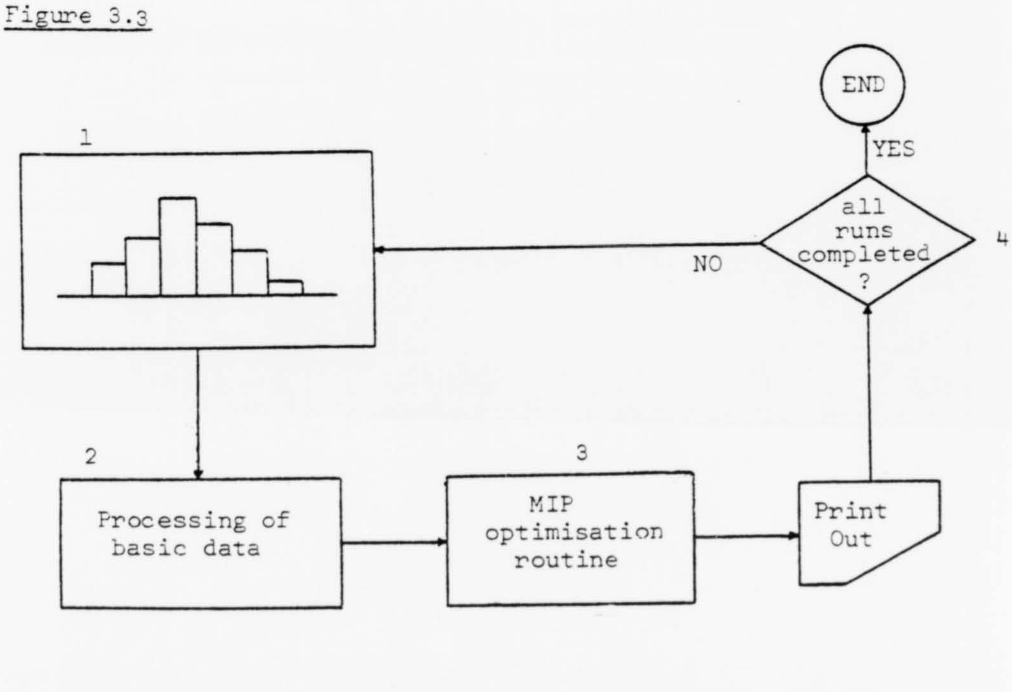
$-\sum_{t=o}^{T} a_{t,j}\lambda_t$ represents the contribution of the project to taxable profit and tax allowances over the whole planning period. The dual problem (3.2) indicates that λ_t is unrestricted in sign. However, it can be shown that $\lambda_t \leqslant 0$, $t = 0, \ldots, T$ for any set of values of A_t and S_t, and furthermore, that $\lambda_t < 0$, $t = 0, \ldots, T$ provided $S_T > 0$. This is shown in Appendix C. Thus, for negative values of λ_t, $- a_{t,j}\lambda_t$ will have a positive contribution to the project's horizon value if $a_{t,j} > 0$ (an outflow), and a negative contribution if $a_{t,j} < 0$ (an inflow). The project's horizon value receives a bonus when it contributes to reduce the tax burden of the company: If expenditure on the project takes place when $|\lambda_t|$ attains its largest values, and income arises when $\lambda_t = 0$ or $|\lambda_t|$ is very small.

$-\delta$ is the penalty imposed on projects x_j $j = m+1, \ldots, n$ for being restricted to only one version out of the set of n alternatives. Notice that, if none of the versions is chosen, $\delta = 0$ (see complementary slackness condition, 3.3.4). If constraint (3.1.5) in the primal problem were relaxed by one unit (ie two versions of the problem set were allowed), the objective function would be increased δ units.

3.4 TAKING ACCOUNT OF UNCERTAINTY IN THE FORECAST OF THE INFLATION RATE

The results produced by one run of the MIP sub-model are optimal with respect to point estimates of the data involved. Because of the high uncertainty in the information required, it is necessary to quantify the effects of the whole range of possible values of the parameters on the optimal solution. It is useful to know, for example, whether a different portfolio of projects would produce better results, if the inflation rate decreased at a faster pace than originally predicted. It is important to know, too, how sensitive the program value for a chosen portfolio of projects is: would the liquidity conditions still be satisfied, if the inflation rate suddenly increased due to a change in the Government economic policies? Monte Carlo simulation provides an adequate way of conducting this type of analysis. It is particularly suitable to account for the uncertainty in the forecast of the inflation rate, as used in this model. The inflation rate does not explicitly appear in the MIP sub-model (ie, as a coefficient, RHS or a variable); instead, the effect of inflation on coefficients and RHSs is worked out in part one of the model, where primary information is processed. Figure 3.3 illustrates the interconnections between the simulation routine and the two other parts of the model.

Figure 3.3



to know, for example, whether a different portfolio of projects would produce better results, if the inflation rate decreased at a faster pace than originally predicted. It is important to know, too, how sensitive the program value for a chosen portfolio of projects is: would the liquidity conditions still be satisfied, if the inflation rate suddenly increased due to a change in the Government economic policies? Monte Carlo simulation provides an adequate way of conducting this type of analysis. It is particularly suitable to account for the uncertainty in the forecast of the inflation rate, as used in this model. The inflation rate does not explicitly appear in the MIP sub-model (ie, as a coefficient, RHS or a variable); instead, the effect of inflation on coefficients and RHSs is worked out in part one of the model, where primary information is processed. Figure 3.3 illustrates the interconnections between the simulation routine and the two other parts of the model.

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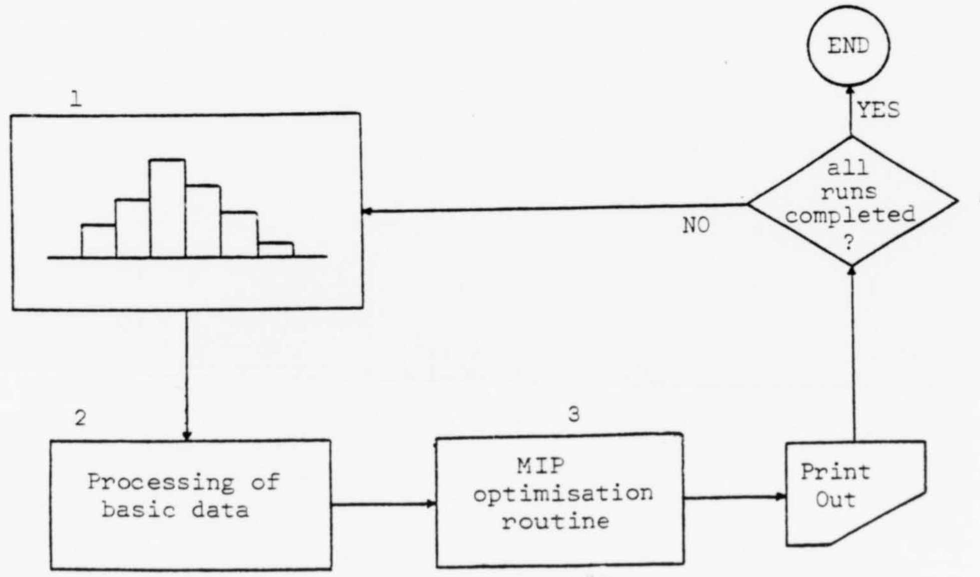


(1) The simulation routine randomly selects a value of the inflation rate from a histogram, and inputs it to the part of the program that computes coefficients and RHSs.

(2) The value obtained in (1) is used to calculate cash flows, interest rates, depreciation, etc. These parameters are used for the creation of the matrix to be input to the MIP sub-model.

(3) The optimum portfolio is obtained, and the result is printed out.

(4) If the maximum number of iterations has been reached, the program stops. If not, a new value of the inflation rate is selected, and the entire process is repeated.

The most frequently selected portfolio is then tested for sensitivity. For this purpose, the set of x_j variables corresponding to the selected portfolio are declared as constants in the MIP program, and the process described in Figure 3.3 is then repeated.

3.4.1 Monte Carlo Simulation

This technique will be illustrated with a numerical example. A generalisation of the theory behind it will be offered afterwards.

Let us assume that the forecast of the inflation rate in a given year can be described by a discrete random variable. The probability density function (pdf) can then be represented by a table relating likely values of the annual inflation rate to their respective probabilities:

x	18	19	20	21	22
p(x)	.10	.25	.30	.25	.10

The program then works out the cumulative density function (CDF) as

x	18	19	20	21	22
F(x)	.10	.35	.65	.90	1.00

The random numbers generated by the computer are then categorised to define the random deviates x uniquely as follows:

$0 \leq R \leq .10$	$x = 18$
$.10 < R \leq .35$	$x = 19$
$.35 < R \leq .65$	$x = 20$
$.65 < R \leq .90$	$x = 21$
$.90 < R \leq 1.00$	$x = 22$

Generally, this means that, for any discrete random variable x, the random deviate assumes the value j if the random number is such that:

$$F(j-1) < R \leq F(j)$$

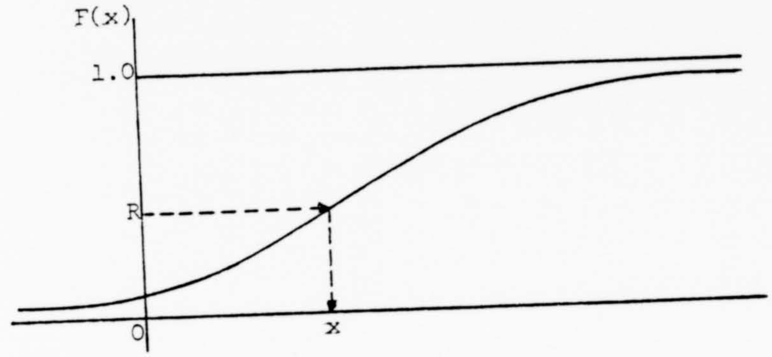
This process is repeated for each year within the planning horizon.

The method is based on the fact that "given any CDF F(x) of the random variable x, the new random variable y = F(x), is uniformly distributed over the (0,1) interval" (Taha (1971) provides a proof of this statement). This applies to any probability distribution of x. If a random number, R, on the (0,1) interval is generated, then the corresponding random deviate x for any probability density function (pdf) f(x) will be given by:

$$x = F^{-1}(R)$$

This is illustrated in Figure 3.4.

Figure 3.4



It is not always possible to represent the pdf in a closed form. In this case the so-called tabular method can be used provided that the random variables are discrete. This is the approach followed in the example above, and in the model here described.

Because annual forecasts of the inflation rate are interdependent, estimates of the correlations between them must be obtained. Several studies have been reported in the literature, that deal with the problem of forecasting inflation (Lawson, 1980; Patterson, 1976; Carlson et al, 1975; Lahiri, 1976). Their methods include time series analysis, random walk processes and Bayesian techniques. They are based on historic data and expectations of the future. All these methods are of a probabilistic nature and could be used for the Monte Carlo simulation routine. Some of them, however, are essentially short-term forecasting methods (Petterson, 1976) and therefore not suitable for use here. The important task of devising forecasting techniques for the medium and long-term is beyond the scope of this work. For the purposes of this research a very simple heuristic method to introduce correlations between consecutive values is used. The following example illustrates this method.

Let us assume that the pdf's for two consecutive years are:

Year 1	x_1	18	19	20	21	22
	$p(x_1)$	.10	.25	.30	.25	.10

Year 2	x_2	16	17	18	19	20
	$p(x_2)$	.15	.22	.26	.22	.15

Our subjective estimate of the most likely value the rate of inflation can have in year 2 (18%), is partly based on the assumption that our estimate of the most likely value in year 1 (20%) is correct. If the inflation rate in year 1 turned out to be 18% instead of 20%, we would revise the probabilities originally attached to the inflation rate in year 2: more weight would be given to lower inflation rates, and the values of $p(x_2)$ would be shifted to the left. The relation between the magnitude of the "shift" and the value obtained for x_1 in the simulation run represents the estimate of the correlation between pdf in year 1 and pdf in year 2.

Because of the probabilistic nature of the method, the whole process described above has to be repeated a large number of times in order to obtain a reasonable estimate of the effect of likely future inflation on the project selection process. No precise figure for the number of iterations required can be given. However, it could be said that the number of iterations required will increase with the number of possible alternative solutions, which in turn could depend on the variance of the stochastic variables involved. In any case, the larger the number of iterations, the greater the accuracy obtained. The analyst will have to base his decision on his understanding of the problem, his subjective estimate of the risk involved, and weigh them against the cost of a large number of iterations.

This methodology can be applied to all uncertain parameters in the model for which a subjective probability forecast may be estimated. In this model, only uncertainty in the forecast of the inflation rate is considered. However, because the inflation rate is a factor common to most of the economic variables used in the model, this simplification can help give an insight into the effects of uncertainty in the entire model.

CHAPTER IV

A CASE STUDY USING SIMULATED DATA

4.1 INTRODUCTION

A much better understanding of the model, its advantages and limitations can be gained by the analysis of its use in a particular case. While the MIP sub-model is flexible enough to be applied to a wide variety of situations without changes, its flexibility greatly depends on the processing of the data carried out in part (1) of the model and the subsequent generation of the input matrix. Assumptions concerning the way in which inflation affects different cash flow elements depend on the particular cases being analysed. These and other assumptions determine the final form of the cash flows used in the MIP sub-model.

The refusal on the part of private industrial corporations to disclose the relevant financial information, has forced me to use hypothetical data for this case study. Financial data such as lending, borrowing and inflation rates at year zero were taken from the Financial Times at the time the analysis was made (February 1981); the rest of the information is imaginary, although wherever possible, it resembles the values they could have in reality.

Part (1) of the model is described, for this particular example, in Section 4.2.1.

4.2 DEFINITION OF THE PROBLEM

A certain British company is willing to ascertain its best investment strategy over the next eleven years (the planning period), based on the information about prospective opportunities available at the time. It is foreseen that seven investment projects will become available within the next three years, and that three other projects will be available a few years hence. An eleventh project, the investment on increased plant capacity, could be made at any time within the planning period. Decisions on these projects cannot be made independently because of three main reasons:

(1) It is desirable to invest in projects whose combined effect can provide a hedge against fluctuating inflation. A set of projects showing low sensitivity to inflation would be advantageous.

(2) The projects are competing for the limited funds available.

(3) In selecting the projects, the company wishes to make the best possible use of the tax allowances available within the taxation system. The projects are likely to be interdependent in this respect.

Tables 4.1, 4.2 and 4.3 show the forecasts of fixed investment, total expenses and sales revenue, for each one of the eleven projects, in terms of costs and prices prevailing at the present time. Initial outlays on Table 4.1 are inclusive of fixed investment and working capital. It can be seen that some projects' lives extend beyond the end of the planning period.

The capital available for investment at the outset is £500,000 and the pre-tax income expected from the ongoing activities is as follows:

Year	1	2	3	4	5	6	7	8	9	10
£000	100	110	120	130	140	150	160	170	180	190

Table 4.1 Fixed Investment (£000s)

Project:	1	2	3	4	5	6	7	8	9	10	11
Year:											
0	140	130								300	120
1	120					200	280	330			
2			140				60				
3											
4											
5											
6									110		
7											
8											
9				160	40						

Table 4.2 Total Expenses (£000s)

Project:	1	2	3	4	5	6	7	8	9	10	11
Year:											
0											
1		30								60	54
2		30				68		78		68	60
3	93	30	28			45	149	90		68	54
4	96	29	28			34	130	103		60	68
5	85	29	34			170	118	90		52	67
6	72	27	40			14	90	71		50	63
7	51	25	34			14	68	51	26	38	
8	39	21	28			57	40	32	26	30	
9	18	16	22			45	28	19	26	22	
10		9	17	64	19	34	17	6.5	25	7	
11		3	12	58	17	23	12		22	52	
12			5.5	45	12	12			18	45	
13				32	9				13	38	
14				13	6.5				8	30	
15				15	2.7				2.7	22	
16				58						15	
17				64						7	

Table 4.3 Sales Revenue (£000s)

Project:	1	2	3	4	5	6	7	8	9	10	11
Year:											
0											
1		53								107	80
2		53				135		146		120	97
3	194	53	56			90	281	170		120	110
4	180	51	56			68	259	194		107	100
5	170	51	68			34	236	170		93	90
6	136	48	79			20	180	134		80	80
7	107	43	68			20	135	97	49	67	
8	73	37	56			113	79	61	49	53	
9	34	29	45			90	56	36	49	40	
10		16	34	121	36	68	34	12	47	10	
11		3	23	101	32	45	23		43	93	
12			11	85	22	23			37	80	
13				61	17				28	67	
14				24	12				19	53	
15				14	5				9	40	
16				111						27	
17				121						13	

The company's dividend policy is such that distributions grow at a rate equal to that of the inflation. The dividend distribution in the current year will be £50,000. The liquidity ratio, defined as the ratio of current assets (less stocks) to current liabilities is 0.75; the gearing ratio, defined as the proportion of debt in total net assets is 0.4; the corporation tax rate is 50%.

4.2.1 Processing of the data and matrix generation Part (1) of the model

The calculations that follow and the input matrix generation are performed by Part (1) of the model. Although these are particular to the case analysed here, they exemplify the procedure to follow in this type of analysis.

Modelling on current money units

The information in Tables 4.1, 4.2 and 4.3 has to be calculated in terms of the current costs and prices of the years to which they refer. The conversion into current money units is carried out using inflation sensitivity factors. This requires the forecast of the general inflation rate (ie Retail Price Index, RPI), from which the forecasts for specific cash flow components are derived. Forecasts of inflation, both generally and for specific commodities, are provided by several sources, such as Phillips and Drew, London Business School Index, and Central Statistical Office HMSO (Westwick et al, 1976). The forecasting of inflation sensitivity factors, suggests Gee (1977), may be carried out by extrapolating the past behaviour of cost components then modifying the results in the light of managerial expectations concerning future inflation. In this simplified example only three different sensitivity factors are used. They relate to the broad categories of fixed investment, sales price and total expenses, and are assumed to remain constant over the planning period. A greater level of disaggregation, with sensitivity factors relating to individual cash flow components (eg raw material, labour) would probably improve the forecast accuracy.

For the case of total expenses, the conversion into current money units is carried out by means of the next equation:

$$E_{j,m} = \prod_{k=1}^{j} (1 + f_m i_k)^k P_{jm}$$

where E_{jm} is the total expenditure in current money units for project m in year j

f_m is the inflation sensitivity factor for the total expenses of project m

i_k is the inflation rate (RPI) in year k

P_{jm} is total expenditure in terms of present time costs and prices, for project m in year j.

Similar equations are used for fixed investment and sales revenue.

Inflation sensitivity factors used for the eleven projects are shown in Table 4.4 below:

Table 4.4

Projects:	1	2	3	4	5	6	7	8	9	10	11
Fixed Investment	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.2
Sales Revenue	.85	.9	.85	.95	1.0	.8	.9	1.0	.8	.9	.85
Total Expenses	1.0	1.0	1.05	1.1	1.1	1.0	.95	1.2	1.0	1.05	1.1

Forecasting of lending and borrowing interest rates

It is assumed that short-term interest rates will not be fully responsive to changes in the inflation rate. A sensitivity factor of 0.6 is used over the planning period. This means that if the inflation rate increases (say) 10% in one particular year, interest rates will increase by only 6%. The next equation is used:

$$r_j = r_{j-1} \left(1 + .6 \left(i_j - i_{j-1}\right) / i_{j-1}\right)$$

where r_j is the (lending or borrowing) interest rate in year j, and

i_j is the inflation rate in year j.

It is further assumed that lending and borrowing interest rates will not fall below 6% and 8% respectively during the planning period, and that, beyond the horizon, they will remain constant at the value attained at the horizon year.

Inflation is not necessarily the most important factor in the forecasting of short-term interest rates. Other factors, such as Government economic policies, can be of more relevance (Wilkes, 1972). A more rigorous forecasting scheme should therefore, take account of those factors.

Increases in Working Capital

After start-up, working capital is assumed to grow in line with the value of annual sales. The next relationship is used:

$$CCT_{i,j} = (CCT_{i-1,j}) \, (V_{i,j} \, / \, V_{i-1,j})$$

where $CCT_{i,j}$ is the working capital required for project j in year i, and

$V_{i,j}$ is the value of sales due to project j in year i (expressed in current money units)

This presupposes that the level of stock is proportional to expected sales.

Taxable Profit from Operations and Tax Allowances

Net annual taxable profit is calculated using the values in current money units obtained above.

$$TCF_i = \sum_{j=1}^{m} (V_{ij} - E_{ij} - C_{ij})$$

where TCF_i is total taxable profit minus allowances in year i

E_{ij} is the total expenditure due to project j in year i

C_{ij} is the fixed investment on project j in year i

V_{ij} is defined as before.

The net value of $(V_{ij} - E_{ij})$ is restricted to non-negative values, to avoid losses to be taken as tax allowances by the MIP sub-model. It is assumed that working capital remaining at the end of a project's life is recovered free from taxation, unless its value exceeds its historic cost, in which case, the difference is taxed at the corporation rate. For simplicity, stock relief and clawback are ignored.

Cash Flows

The procedure suggested by Gee (1977) is followed. It is assumed that three months' credit is taken on sales and two months' credit on the payment of total expenses. From this, a forecast of operating cash flows can be obtained as follows:

$$F_{ij} = vV_{i-1,j} + (1 - v)V_{i,j} - eE_{i-1,j} + (1 - e)E_{i,j}$$

where $F_{i,j}$ is the operating cash flow from project j in year i

v is the delay in the payment of sales (in years)

e is the delay in the payment of total expenses (in years),

all other terms are as previously defined.

Capital investment cash flows are calculated assuming a six months' delay in payments, as follows:

$$I_{i,j} = cC_{i-1,j} + (1 - c)C_{i,j}$$

Net annual cash flows are obtained subtracting capital cash flows and working capital increases from operating cash flows. Working capital remaining at the end of a project's life is added on to the net cash flow of the corresponding year. The following equation is used:

$$CF_{i,j} = F_{i,j} - I_{i,j} - (CCT_{i,j} - CCT_{i-1,j})$$

where $CF_{i,j}$ is the net annual cash flow from project j in year i, all other terms are as previously defined.

Depreciation of Fixed Assets

In order to calculate the total value of the firm's assets in any one year, straight line depreciation over the life of the projects, and negligible disposal values are assumed. The next formula is used:

$$D_{k,j} = (A_j / d_j) \prod_{l=s_j}^{k} (1 + a_j i_l)$$

where $D_{k,j}$ is the amount fixed assets from project j depreciate in year k

A_j is the value of the fixed investment on project j in the first year of operation

d_j is the duration of project j (in years)

a_j is the inflation sensitivity factor for the fixed assets of project j

s_j is the start-up year for project j

i_l is the inflation rate in year l

$D_{k,j}$ is used in the calculation of the gearing ratio in every year within the planning period. The way the inflation rate affects the gearing ratio depends on how the company calculates depreciation, and on whether assets are revalued. In this example, the value of assets is maintained constant, but depreciation keeps pace with the escalation rate of fixed assets (as in Baxter, 1975). This has a negative effect on gearing. A more optimistic approach, suggested by Merrett and Sykes (1973), is to revalue assets in line with their escalation rate. This increases the debt raising potential of the firm.

A probabilistic forecast of the inflation rate

For the purposes of this example, probabilistic forecasts of the inflation rate are produced. The subjective probability distributions, shown in Table 4.5, predict falling inflation rates over the planning period. For the first three years these distributions are assymetrical with higher probabilities given to lower inflation rates; for all other years, the distributions are symmetrical.

Table 4.5

Year														
1	P(i)					.15	.28	.30	.22	.05				
	i					.13	.14	.15	.16	.17				
2	P(i)				.07	.12	.25	.27	.20	.06	.03			
	i				.11	.12	.13	.14	.15	.16	.17			
3	P(i)			.04	.06	.10	.24	.26	.20	.06	.02	.02		
	i			.09	.10	.11	.12	.13	.14	.15	.16	.17		
4	P(i)		.025	.025	.035	.075	.22	.24	.22	.075	.035	.025	.025	
	i		.07	.08	.09	.10	.11	.12	.13	.14	.15	.16	.17	
5	P(i)	.02	.02	.03	.04	.08	.20	.22	.20	.08	.04	.03	.02	.02
	i	.05	.06	.07	.08	.09	.10	.11	.12	.13	.14	.15	.16	.17
6	P(i)	.03	.03	.04	.05	.10	.16	.18	.16	.10	.05	.04	.03	.03
	i	.03	.04	.05	.06	.07	.08	.09	.10	.11	.12	.13	.14	.15
7	P(i)	.03	.03	.05	.06	.11	.14	.16	.14	.11	.06	.05	.03	.03
	i	.03	.04	.05	.06	.07	.08	.09	.10	.11	.12	.13	.14	.15
8	P(i)	.03	.04	.06	.08	.10	.12	.14	.12	.10	.08	.06	.04	.03
	i	.03	.04	.05	.06	.07	.08	.09	.10	.11	.12	.13	.14	.15
9	P(i)	.05	.05	.07	.08	.09	.10	.12	.10	.09	.08	.07	.05	.05
	i	.03	.04	.05	.06	.07	.08	.09	.10	.11	.12	.13	.14	.15
10	P(i)	.06	.06	.07	.08	.09	.09	.10	.09	.09	.08	.07	.06	.06
	i	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10	.11	.12	.13

i = fractional value of the inflation;
P(i) = probability of i.

The correlations between consecutive values are calculated according to the procedure described in Section 3.4.1. The "shift" is assumed to follow the next equation:

$$\text{Shift}_j = (.7)(\phi_{j-1} - \psi_{j-1}) + (.3)(\phi_{j-2} - \psi_{j-2})$$

where ϕ_{j-1} is the forecast of the inflation rate obtained for year j-1 .

ψ_{j-1} is the expected value (or the mode, if the distribution is not symmetrical) of the inflation rate in year j-1.

This means that forecasts in year j depend on the values forecast for years j-1 and j-2, according to the weights .7 and .3. The magnitude of the shift represents the change in the values of i_j in Table 4.5 (subscript j = 1, ..., 10).

Some large corporations produce their own probabilistic forecasts of the inflation rate. Forecasts produced by independent bodies, such as The Bank of England, Phillips and Drew, the St James Group, can also be used. If these forecasts are not presented in terms of probability distributions, the transformation can be achieved by taking the point estimates as the most likely values, and attaching lower subjective probabilities to other possible values.

Matrix Generator

Having calculated the quantities above, the program proceeds to generate the input matrix in the format required by the particular MP package being used (Honeywell MPS is used in this example). The generated matrix is printed on a computer file from where it is read by the MP package. The flexibility of this matrix generator allows the analyst to make changes to the model with relative facility. This attains great importance if the model is to be adapted to particular situations.

The program used for the above calculations and matrix generation is shown in Appendix D. A sample input matrix is shown in Appendix G.

4.3 USE OF THE MODEL TO SELECT THE OPTIMUM PORTFOLIO OF PROJECTS

The simulation is carried out in two stages. In the first stage, one hundred forecasts of the inflation rate during the planning period are produced. For each inflation scenario, the MIP sub-model selects the most profitable set of projects. This generates one or more portfolios selected with different frequencies. The most frequently selected portfolio in a sufficiently large number of iterations stands the best chance of being the optimal one for the inflation scenario that materialises (see Allen et al, 1970). However, an inflation scenario may occur for which this portfolio is not optimal. It is important then to determine its sensitivity to inflation. This is measured in terms of the variability of the portfolio's Terminal Value (the objective function) to changes in the forecast of the inflation rate. This is the purpose of the second stage of the analysis, which is carried out by producing another one hundred inflation scenarios, while keeping the project selection variables (the x_j's) fixed to the values corresponding to the selected portfolio. The advantage of finally choosing this particular portfolio of projects is that it is the most robust in the face of uncertainty in the forecast of the inflation rate.

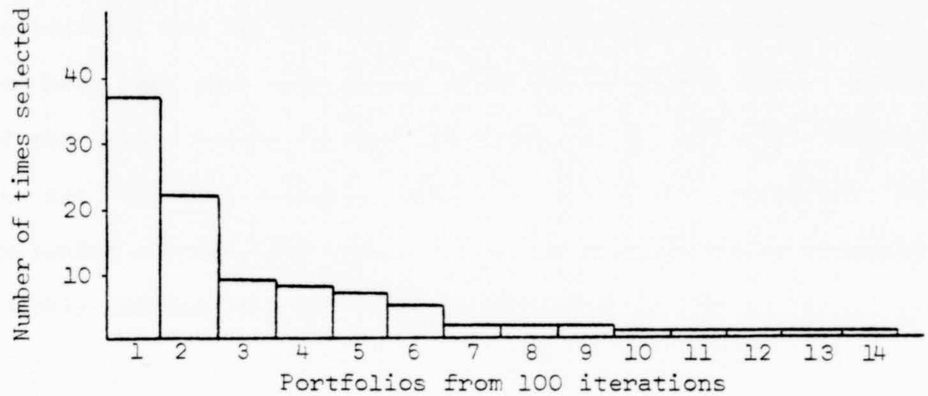
4.3.1 Analysis of the Results

The results of the first stage of the simulation are summarised in Table 4.6 and a diagram of selection frequencies is shown in Figure 4.1. Project 21 in Table 4.6 corresponds to the alternative of starting project 11 at the horizon. All other project numbers correspond to those indicated in Tables 4.1 to 4.3.

Table 4.6

Portfolio No.	Projects Selected	Frequency of Selection
1	2, 3, 5, 6, 7, 21	37
2	2, 3, 4, 5, 6, 7, 21	22
3	2, 3, 6, 7, 21	9
4	1, 3, 5, 7, 21	8
5	2, 3, 4, 5, 6, 7, 9, 21	7
6	3, 5, 7, 10, 21	5
7	2, 3, 4, 5, 6, 7	2
8	2, 3, 5, 6, 7, 9	2
9	3, 5, 7, 9, 10, 21	2
10	3, 4, 7, 10, 21	1
11	1, 3, 7, 21	1
12	3, 4, 5, 7, 10, 21	1
13	1, 3, 4, 7, 21	1
14	1, 3, 4, 5, 7, 21	1
		99

Figure 4.1 Portfolio Selection Frequencies



The results show that fourteen different portfolios are generated in 100 iterations. It will be observed that the sum of the frequencies of selection adds up to only 99. This is due to the fact that not all of the 100 iterations were successful. The rather simplistic forecasting scheme used for interest rates, can create situations with relatively low inflation (less than 10%) and very high interest rates (of the order of 30% or more). These cases are assumed unrealistic, and, therefore, excluded from the analysis. The differences in frequency of selection between portfolio 1 and portfolios 2 and 3, seem to be large enough to assume that the selection of portfolio 1 as optimal is robust. To reinforce this assumption, further 110 iterations were run. This second simulation generated 14 portfolios, of which, the top eight coincide with portfolios selected in the first run, although not in the same ranking order. The bottom six portfolios do not appear in the first run. The top three portfolios are the same ones as in the first run. Some changes in ranking order can be observed in the other portfolios, these changes being more radical as we approach the less frequently selected ones. This indicates that 100 iterations are just sufficient to identify the top three portfolios, although some doubt has now been cast upon the ranking order of the other ones. Because of the random nature of each iteration, it is valid to combine the two runs into a single simulation run of 210 iterations (203 excluding unsuccessful ones). The top six portfolios obtained in this combined run are shown in Table 4.7.

Table 4.7

Portfolio No.	Projects Selected	Frequency of Selection
1	2, 3, 5, 6, 7, 21	79
2	2, 3, 4, 5, 6, 7, 21	40
3	2, 3, 6, 7, 21	20
4	2, 3, 4, 5, 6, 7, 9, 21	15
5	1, 3, 5, 7, 21	10
6	3, 5, 7, 10, 21	9

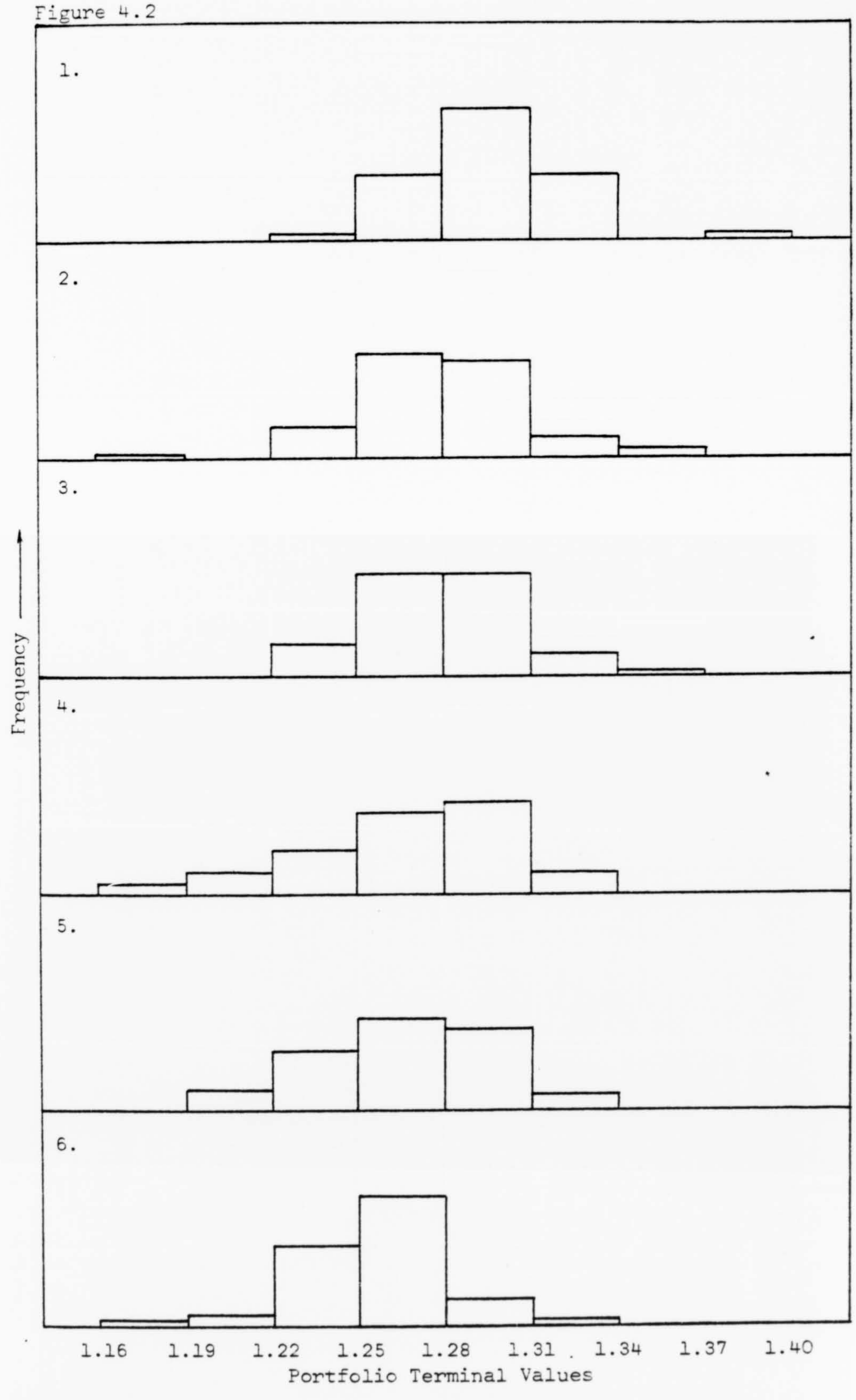
It can be seen that portfolios 1, 2, 3 and 6 are the same ones obtained in the first run. Portfolios 4 and 5 in the combined run appear in reversed order. The combined results provide satisfactory evidence of the optimality of portfolio 1. This type of result, however, does not always occur. A situation could arise, in which, the differences in frequency of selection are not large enough to warrant that the choice of any portfolio as optimal, is robust. In this case the analyst could decide to increase the number of iterations in order to obtain a more clearly defined top portfolio. There is, however, no guarantee that this approach will resolve the situation: similar results could occur again. This would indicate that those portfolios stand similar chances of being optimal under a wide range of different inflation scenarios. A second option open to the analyst, is to base the selection of an optimal portfolio on its sensitivity to inflation. If at the end of this second stage of the analysis, no clear favourite has been identified, the manager will have to base his decision on other considerations. But even in this case, the model would have largely reduced the decision space to only a few different portfolios.

It was decided, for this example, to subject the top 6 portfolios to inflation sensitivity analyses using 50 iterations in each case. The results are shown in Figure 4.2. The figures on the left refer to portfolio numbers as defined in Table 4.7, and the values along the bottom of the Table are the portfolio Terminal Values in millions of pounds. The sensitivity analysis reinforces the assumption of optimality of portfolio 1, made earlier. The histograms show that the probabilities of achieving values on the top half of the scale (greater than 1.28) are higher for portfolio 1 than for any other portfolio. A tendency towards an increased risk of Terminal Values on the lower half of the scale and flatter frequency distribution, can be observed as we move down Figure 4.2.

A sample output for the two stages of the analysis is shown in Appendix E.

A closer study of the selected portfolio can be made by examining the values of the other variables involved in the analysis. Because of the assumptions implicit in the formulation of the model and the uncertainty present in most of the variables, especial attention must be given to liquidity and gearing constraints which are binding or have a small slack at

Figure 4.2


1.
2.
3.
4.
5.
6.
Frequency
1.16
1.19
1.22
1.25
1.28
1.31
1.34
1.37
1.40
Portfolio Terminal Values

the optimum. Cash difficulties are more likely to arise in those years in which the constraints are binding. Slack values for the liquidity and gearing constraints of the selected portfolio are given in Table 4.8 below.

Table 4.8

Year	0	1	2	3	4	5	6	7	8	9	10
Liquidity (x 10^6)	.385	.152	.0	.154	.376	.533	.685	.797	.954	1.087	
Gearing (x 10^6)	.206	.329	.049	.379	.470	.476	.467	.434	.470	.539	.643

Only the liquidity constraint in year 2 is binding, and careful planning will be necessary to prevent cash problems in that year. The amounts lent, borrowed and the tax payable are shown in Table 4.9.

Table 4.9

Year	0	1	2	3	4	5	6	7	8	9	10
Lending (x 10^6)	.385	.152	.332	.174	.541	.715	.862	.958	1.160	1.240	1.153
Borrowing (x 10^6)	-	-	.442	-	-	-	-	-	-	-	-
Tax Payable (x 10^6)	-.130	-.508	-.533	.054	.441	.484	.473	.431	.551	.408	.108

Tax allowances are expressed as a negative tax. The high interest rates and the fact that inflation has a negative effect on all the projects (expenses escalate much faster than sales revenue), maintains borrowing at a zero-level, except in year 2. The rejected projects do not generate enough profit to justify raising the extra capital needed. This situation, although somewhat pessimistic, is not uncommon in an inflationary environment. Appendix F illustrates a situation in which one of the projects is less severely affected by inflation, ie there is a smaller difference between the escalation rates of expenses and sales revenue. The resulting increase in project profitability makes borrowing worthwhile and a different set of projects is selected. The same situation is further analysed using an improved version of the model, in which two types of borrowing, short- and medium-term, are possible. This analysis illustrates the trade-off between a "cheap", but difficult to renew short-term debt, and a more expensive medium-term debt, with a fixed interest rate in a period of falling inflation, but with repayments spread over a period of several years.

4.3.2 Analysis of the Dual problem

Table 4.10 below, shows the values of $\bar{\rho}_t$ (the yield at the horizon of an additional pound in year t), compared with the compounded values of the market lending rates, calculated by the program in the way described in Section 4.2.1.

Table 4.10

Year	ρ	Annual Market Lending Rate (After Tax) $(1 + r_t)$	Cumulative Lending Rate $\prod_t (1 + r_t)$	Inflation Rate i_t	Cumulative Inflation $\prod_t (1 + i_t)$
0	1.874	1.065	1.737		
1	1.738	1.063	1.631	.140	1.14
2	1.622	1.061	1.534	.133	1.29
3	1.471	1.056	1.446	.117	1.44
4	1.388	1.058	1.369	.123	1.62
5	1.308	1.061	1.294	.134	1.84
6	1.230	1.040	1.220	.057	1.94
7	1.180	1.067	1.173	.121	2.18
8	1.102	1.056	1.099	.087	2.37
9	1.042	1.041	1.041	.047	2.48
10	1.000	1.033	-	.033	2.56

It can be seen from this table, that the ρ_t values are higher than the compounded market lending rates for every year of the planning period. The chosen investment strategy generates higher returns than lending in the market. If investment in the market were more profitable than the internal projects, all the capital would be lent and no projects would be selected. This does not necessarily mean that the investment strategy maintains the purchasing power of the capital. The last column of Table 4.10 shows that costs and prices increase at a much faster pace than capital. The real value of the firm's assets is decreasing over the years; yet, rejecting all the projects would put the firm in an even more disadvantageous position. Notice that $\overline{\rho}_t \geq 1$ and $\overline{\rho}_t \geq \overline{\rho}_{t+1}$, $t = 0, \ldots, T$. as proved in Section 3.3.3.

The dual values for the project selection variables are shown in Table 4.11.

According to the properties of the dual problem defined in Section 3.3.3, μ_j is the negative of the horizon value of project j. Therefore, positive μ_j's have a negative effect on Terminal Value. None of the projects with positive μ_j-value has been selected. Three projects with negative μ_j's (1, 8 and 10) have been rejected because of budget limitations. Although these three projects have larger negative μ-values than most of the selected projects, their large initial investments would absorb the capital which could be invested in smaller projects (2, 3 and 6), whose combined effect on total Terminal Value is larger than that of projects 1, 8 or 10 taken individually. As explained in Section 4.3.1, high interest

Table 4.11

Project	x-Value	μ_j (x 10^6)
1	0	- 0.188
2	1	- 0.051
3	1	- 0.073
4	0	+ 0.001
5	1	- 0.001
6	1	- 0.126
7	1	- 0.342
8	0	- 0.120
9	0	+ 0.007
10	0	- 0.159
11	0	(BASIC)
12	0	+ 0.016
13	0	+ 0.024
14	0	+ 0.030
15	0	+ 0.053
16	0	+ 0.075
17	0	+ 0.085
18	0	+ 0.113
19	0	+ 0.120
20	0	+ 0.123
21	1	- 0.056

rates and relatively low project profitability make borrowing not viable. Only the selected version of the investment in increased plant capacity, 21, has a negative μ_j- value. Notice that all the x_j variables, excepting x_{11}, have dual values different from zero, whether they are selected or rejected. The reason is that none of these variables is basic. Their positive values are assigned by the Branch and Bound routine (see Appendix C). On the other hand, x_{11} has a value of zero in spite of being a basic variable. This "degenerate solution" indicates that the constraint limiting the investment in increased plant capacity to only one version, is redundant in this particular case. The interplay between other constraints in the model and the Branch and Bound algorithm keeps the number of selected versions to only one. However, this is not always the case, and that constraint can become active under slightly different circumstances. The complete MIP output and the input matrix are shown in Appendix G.

4.3.3 Computing Requirements

The model, with a total of 45 rows and 65 columns, was implemented using an MPS package in a Honeywell 66/80. The 100 iterations needed in the first stage of the analysis required a total of 6.6 minutes. Each of the sensitivity analyses (50 iterations) took approximately 1.2 minutes. These figures include compilation, generation of input matrices, processing of the MIP sub-model and report generation.

4.4 CONCLUSIONS

An example has been given of the use of the model and the data processing required for the generation of the input matrix.

Although the model is able to select one optimal portfolio in the light of the best quantitative information available, no final recommendation is made. Instead, several alternative options and an estimate of their inherent risks are given. The final decision is left to the manager who, based on other qualitative factors, experience and intuition, can select the best investment strategy for the company from the largely reduced decision space produced by the model.

CHAPTER V

CONCLUSIONS AND FURTHER WORK

CONCLUSIONS AND FURTHER WORK

An alternative approach to the analysis of investment opportunities under inflationary conditions has been proposed. This alternative method attempts to overcome some of the deficiencies of the existing techniques, bringing more realism into the analysis. The proposed alternative comprises two parts:

(a) the appraisal of individual projects; and

(b) the joint analysis of the investment and financing activities of the firm involving several interdependent projects.

Existing investment appraisal techniques were examined and their shortcomings identified. Current methods were found to be deficient on several counts:

(1) Inadequate treatment of the effects of external debt on the viability of a project under inflationary conditions. Even in simple cases, interdependencies between projects introduced by the use of debt, can lead to the wrong selection of investment alternatives.

(2) In cases where the fixed investment escalates at a rate different from the general inflation rate, existing methods give no indication as to whether the returns cover the replacement cost of the project.

(3) Current methods can lead to the wrong choice amongst mutually exclusive projects whose initial investments are affected by different escalation rates.

These three shortcomings can, in fact, be reduced to only one: current methods of investment appraisal do not properly consider the effect of inflation on the initial investment. To overcome this deficiency, a Terminal Value method was proposed. This is a variation on the existing methods, incorporating some of their salient features:

- The observed market rate of interest (or its forecast) is used as reinvestment rate; and

- The "cash flows to equity" procedure is followed.

The more detailed analysis this method entails, was shown to result in a more adequate appraisal of investment opportunities, which effectively overcomes the deficiencies mentioned above. Its only disadvantage in relation to other methods lies on the more elaborate calculations it requires. The Terminal Value method also suffers from several of the limitations common to most other techniques of investment appraisal. These methods can only be applied in cases where it is safe to assume the existence of a perfect capital market, independence between projects and divisibility of the investment capital. These limitations make it unsuitable for the joint analysis of the investment and financing activities of the firm. Thus, a Mixed Integer Programming model based on the maximisation of the Terminal Value was developed. The model is incorporated in a Monte Carlo simulation routine to account for uncertainty in the forecast of the inflation rate. This type of model does not rely on any of the assumptions mentioned above, and therefore, provides us with a more realistic form of analysis.

The proposed model helps to minimise the erosive effect of inflation in several ways:

(1) By making the best possible use of tax allowances in the project selection process.

(2) Maximising the level of investment without incurring too big a risk.

(3) It determines the optimal amount and the type of debt to be used.

(4) Handling the interdependencies introduced by the taxation system and the combined use of debt and equity.

(5) By giving an indication of the risk inherent in the selected portfolio.

These features enable the model to identify the portfolio of projects standing the best chance of being the optimal one for the inflation scenario that materialises.

The model is practical from two viewpoints:

(a) The information it requires is usually available within the company; and

(b) Its usefulness is not limited by the availability of highly precise data. Based on the premise that forecasts are imprecise, the model estimates the effects of uncertain inflation on the selected portfolio.

The use of this model as a corporate planning tool would provide the manager with a means to conduct periodical reappraisals of the selected investment strategy as better information is obtained.

This would, in turn, ensure that corrective measures are taken while viable options are still available. In the long term, the company's sensitivity to inflation and the risk of incurring liquidity problems would be minimised. Increased corporate profitability would be the end result from the more sound investment strategy that a model of this type makes possible.

The main disadvantages of the model can be summarised as follows:

- It assumes that interest rates depend only on the inflation rate.
- Only uncertainty in the forecast of the inflation rate is considered in the risk analysis.
- Advanced Corporation Tax, stock relief and clawback are ignored. These are important factors in a period of inflation.
- The model does not consider long-term debt and the issue of new stock as sources of external finance.
- It is more complex and expensive than traditional methods.

The last disadvantage, however, is justified by the much greater risk involved in the cases for which the model is intended. The other disadvantages can be overcome, although at the expense of increasing the complexity and cost of the model. Improvements should be weighed against the increased computer time required.

As it stands, the final output produced by the model is as shown in Appendix E. A more practical report generator could be created to have tables of portfolio selection frequencies and histograms of portfolio sensitivity to inflation (Table 4.6 and Figure 4.2) directly produced by the model.

Practical improvements can only be achieved after a comprehensive analysis of the model's behaviour under a wide variety of different situations simulated in the computer. Ideally, the model should be adapted to the needs of a particular company, and the simulation runs should be a reflection of the firm's past experience and expectations of the future. This would help give managers a better understanding of the model and greater confidence in its usefulness and potential.

APPENDIX A

Programs used for the calculation of the examples in Chapter 1

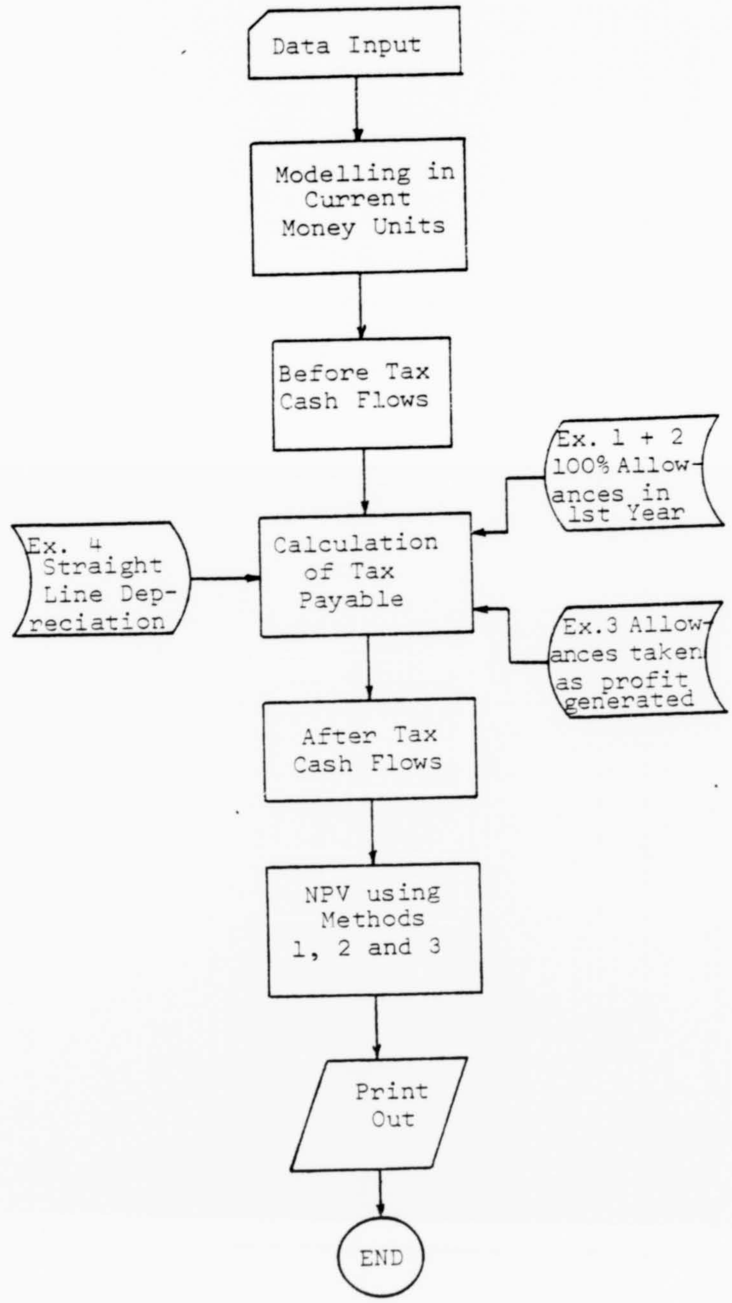
The examples summarised in Tables 1.1, 1.2, 1.3 and 1.4 were calculated using the three programs shown in this Appendix.

As illustrated in the flow chart in Figure A1, the programs only differ in the way depreciation is calculated for tax purposes:

- Program 1 assumes that project's tax allowances can only be set against net revenue cash flows from the same project;
- Program 2 assumes that the whole of the fixed capital can be written off in the first year of operations;
- Program 3 assumes that fixed capital is depreciated using a straight line method over the life of the project.

The computer print out is shown at the end of this Appendix.

Figure A1


Data Input
Modelling in Current Money Units
Before Tax Cash Flows
Ex. 1 + 2 100% Allowances in 1st Year
Ex. 4 Straight Line Depreciation
Calculation of Tax Payable
Ex.3 Allowances taken as profit generated
After Tax Cash Flows
NPV using Methods 1, 2 and 3
Print Out
END

```
*  PROGRAM 1
*LIST

10$EA,J
20$      IDENT    SGR24
30$      OPTION   FORTRAN
40$      USE      .GTLIT
50$      FORTRAN ASCII
60      DIMENSION GIR(10),SI(10),OP(10),CFB(10),TP(10),CFA(10)
70      DIMENSION CI(10),CDR(10),CNPV(10),UNPV(10),PNPV(10)
80C  ***DATA INPUT***
90      READ,N,FC,RC,TAX,DR,RM
100      READ,(GIR(I),I=2,N)
110      READ,SI(1),OP(1)
120      READ,FS,FO,FW
130      CI(1)=1.0
140      CDR(1)=1+DR
150C  ***MODELLING IN CURRENT MONEY UNITS***
160      DO 15 J=2,N
170      CI(J)=CI(J-1)*(1+GIR(J)*FW)
180      SI(J)=SI(J-1)*(1+GIR(J)*FS)
190      OP(J)=OP(J-1)*(1+GIR(J)*FO)
200      CDR(J)=CDR(J-1)*(1+GIR(J))
210  15  CONTINUE
220      RCC=RC*CI(N)
230C  ***BEFORE TAX CASH FLOWS***
240      DO 25 I=2,N
250      CFB(I)=SI(I)-OP(I)
260      IF(CFB(I).LE.0.0)GO TO 130
270      IF(I.GT.2)GO TO 35
280      IF(CFB(I)-FC.LE.0.0)GO TO 65
290      TP(I)=TAX*(CFB(I)-FC)
300      DIF=FC-CFB(I)
310      GO TO 55
320  65  TP(I)=0.0
330      DIF=FC-CFB(I)
340      GO TO 55
350  35  CONTINUE
360      IF(DIF.LE.0.0)GO TO 75
370      IF(CFB(I)-DIF.LE.0.0)GO TO 100
380C  ***TAX PAYABLE***
390      TP(I)=TAX*(CFB(I)-DIF)
400      GO TO 110
410 100  TP(I)=0.0
420 110  DIF=DIF-CFB(I)
430      GO TO 55
```

```
75   TP(I)=TAX*CFB(I)
     GO TO 55
130  TP(I)=0.0
C   ***AFTER TAX CASH FLOWS***
55   CFA(I)=CFB(I)-TP(I)
25   CONTINUE
     CFA(N)=CFA(N)+WC
C   ***NPV USING METHODS 1,2 AND 3***
     CNPV(1)=-FC-WC
     UNPV(1)=-FC-WC
     PTP=SI(1)-OP(1)
     IF(PTP-FC.LE.0.0)GO TO 105
     PNPV(2)=-FC-WC+(PTP-(PTP-FC)*TAX)/CDR(2)
     KEY=1
     GO TO 115
105  PNPV(2)=-FC-WC+PTP/CDR(2)
     RES=FC-PTP
     KEY=0
115  CONTINUE
     DO 45 K=2,N
     CNPV(K)=CNPV(K-1)+CFA(K)/CDR(K)
     UNPV(K)=UNPV(K-1)+CFA(K)/(1+RN)**(K-1)
     IF(K.LE.2)GO TO 45
     IF(PTP.LE.0.0)GO TO 120
     IF(KEY.EQ.1)GO TO 125
     IF(RES.LE.0.0)GO TO 125
     IF(PTP-RES.LE.0.0)GO TO 135
     PNPV(K)=PNPV(K-1)+(PTP*(1-TAX)+RES*TAX)/CDR(K)
     RES=RES-PTP
     GO TO 45
135  PNPV(K)=PNPV(K-1)+PTP/CDR(K)
     RES=RES-PTP
     GO TO 45
125  PNPV(K)=PNPV(K-1)+PTP*(1-TAX)/CDR(K)
     GO TO 45
120  PNPV(K)=PNPV(K-1)+PTP/CDR(K)
45   CONTINUE
     PNPV(N)=PNPV(N)+WC/CDR(N)
C   ****RESULTS ARE PRINTED OUT***
     WRITE(06,50)(DIR(I),I=2,N)
50   FORMAT(1H ,'ANNUAL INFLATION RATE(%)',5F10.2)
     WRITE(06,60)(SI(I),I=2,N)
60   FORMAT(1H ,'SALES INCOME                ',5F10.0)
     WRITE(06,70)(OP(I),I=2,N)
70   FORMAT(1H ,'OPERATING COSTS             ',5F10.0)
     WRITE(06,80)(CFB(I),I=2,N)
80   FORMAT(1H ,'CASH FLOW(B. TAX)           ',5F10.0)
     WRITE(06,90)(TP(I),I=2,N)
90   FORMAT(1H ,'TAX PAYABLE                 ',5F10.0)
     WRITE(06,95)(CFA(I),I=2,N)
95   FORMAT(1H ,'CASH FLOW(A. TAX)           ',5F10.0)
     WRITE(06,65)CNPV(N),UNPV(N),PNPV(N)
65   FORMAT(1H ,'NPV2=',F10.0,'NPV3=',F10.0,'NPV1=',F10.0)
     STOP
     END
$      EXECUTE
$        LIMITS   .002,30K,,10K
$        DATA     I*
5,5000000.,2000000.,.5,.05,.7
.16,.15,.14,.13,.12
7000000.,5000000.
```

```
1050$0.8,1.2,1.0
1060$      ENDCOPY
1070$      ENDJOB
```

* PROGRAM 2

```
10EEA,J
20$      IDENT    SG824
30$      OPTION   FORTRAN
40$      USE      .GTLIT
50$      FORTRAN ASCII
60C  *****WHOLE OF THE FIXED CAPITAL WRITTEN DOWN ON FIRST YEAR*****
70      DIMENSION OIR(10),SI(10),OP(10),CFB(10),TP(10),CFA(10)
80      DIMENSION CI(10),COR(10),CNPV(10),ONPV(10),PNPV(10)
90C  ***DATA INPUT***
100      READ,N,FC,AC,TAX,OR,RN
110      READ,(OIR(I),I=2,N)
120      READ,SI(1),OP(1)
130      READ,FS,FO,FW
140      CI(1)=1.0
150      COR(1)=1+OR
160C  ***MODELLING IN CURRENT MONEY UNITS***
170      DO 15 J=2,N
180      CI(J)=CI(J-1)*(1+OIR(J)*FW)
190      SI(J)=SI(J-1)*(1+OIR(J)*FS)
200      OP(J)=OP(J-1)*(1+OIR(J)*FO)
210      COR(J)=COR(J-1)*(1+OIR(J))
220  15  CONTINUE
230      ACC=AC*CI(N)
240C  ***CASH FLOWS CALCULATION***
250      DO 25 I=2,N
260      CFB(I)=SI(I)-OP(I)
270      IF(I.GT.2)GO TO 35
280      TP(I)=TAX*CFB(I)-TAX*FC
290      GO TO 55
300  35  TP(I)=TAX*CFB(I)
310  55  CFA(I)=CFB(I)-TP(I)
320  25  CONTINUE
330      CFA(N)=CFA(N)+ACC
340C  ***NPV USING METHODS 1,2 AND 3***
350      CNPV(1)=-FC-AC
360      ONPV(1)=-FC-AC
370      PNPV(2)=-FC-AC+((SI(1)-OP(1))*TAX+FC*TAX)/COR(2)
380      DO 45 K=2,N
390      CNPV(K)=CNPV(K-1)+CFA(K)/COR(K)
400      ONPV(K)=ONPV(K-1)+CFA(K)/(1+RN)**(K-1)
410      IF(K.LE.2.OR.K.GE.N)GO TO 45
420      PNPV(K)=PNPV(K-1)+((SI(1)-OP(1))*TAX)/COR(K)
430  45  CONTINUE
440      PNPV(N)=PNPV(N-1)+((SI(1)-OP(1))*TAX+AC)/COR(N)
450C  ***RESULTS PRINT OUT***
460      WRITE(06,50)(OIR(I),I=2,N)
470  50  FORMAT(1H ,'ANNUAL INFLATION RATE(%)',5F10.2)
480      WRITE(06,60)(SI(I),I=2,N)
490  60  FORMAT(1H ,'SALES INCOME                 ',5F10.0)
500      WRITE(06,70)(OP(I),I=2,N)
510  70  FORMAT(1H ,'OPERATING COSTS              ',5F10.0)
520      WRITE(06,80)(CFB(I),I=2,N)
530  80  FORMAT(1H ,'CASH FLOW(B. TAX)            ',5F10.0)
540      WRITE(06,90)(TP(I),I=2,N)
```

```
  90  FORMAT(1H ,'TAX PAYABLE                    ',5F10.0)
      WRITE(06,95)(CFA(I),I=2,N)
  95  FORMAT(1H ,'CASH FLOW(A. TAX)              ',5F10.0)
      WRITE(06,85)CNPV(N),UNPV(N),PNPV(N)
  85  FORMAT(1H ,'NPV2=',F10.0,'NPV3=',F10.0,'NPV1=',F10.0)
      STOP
      END
$      EXECUTE
$      LIMITS  .001,20K,,10K
$      DATA    1*
£6,5000000.,2000000.,.5,.05,.07
.16,.15,.14,.13,.12
£7000000.,5000000.
£1.0,1.0,1.0
$      ENDCOPY
$      ENDJOB
```

* PROGRAM 3

```
££A,J
$      IDENT   SG624
$      OPTION  FORTRAN
$      USE     .GTLIT
$      FORTRAN ASCII
C  *****STRAIGHT LINE DEPRECIATION******
     DIMENSION DIR(10),SI(10),OP(10),CFB(10),TP(10),CFA(10)
     DIMENSION CI(10),COR(10),CNPV(10),UNPV(10),PNPV(10)
C  ***DATA INPUT***
      READ,N,FC,SC,TAX,OR,RM
      READ,(DIR(I),I=2,N)
      READ,SI(1),OP(1)
      READ,FS,FO,FN
      CI(1)=1.0
      COR(1)=1+OR
C  ***MODELLING IN CURRENT MONEY UNITS***
      DO 15 J=2,N
      CI(J)=CI(J-1)*(1+DIR(J)*FN)
      SI(J)=SI(J-1)*(1+DIR(J)*FS)
      OP(J)=OP(J-1)*(1+DIR(J)*FO)
      COR(J)=COR(J-1)*(1+DIR(J))
      DEP=FC/(N-1)
  15  CONTINUE
      SCC= C*CI(N)
C  ***CASH FLOWS CALCULATION***
      DO 25 I=2,N
      CFB(I)=SI(I)-OP(I)
      TP(I)=TAX*CFB(I)-TAX*DEP
      IF(TP(I).LT.0.0)TP(I)=0.0
      CFA(I)=CFB(I)-TP(I)
  25  CONTINUE
      CFA(N)=CFA(N)+SCC
C  ***NPV USING METHODS 1,2 AND 3***
      CNPV(1)=-FC-SC
      UNPV(1)=-FC-SC
      PNPV(1)=-FC-SC
      TPT=(SI(1)-OP(1))*TAX-TAX*DEP
      IF(TPT.LT.0.0)TPT=0.0
      DO 45 K=2,N
      CNPV(K)=CNPV(K-1)+CFA(K)/COR(K)
      UNPV(K)=UNPV(K-1)+CFA(K)/(1+RM)**(K-1)
```

```
      PNPV(K)=PNPV(K-1)+(SI(1)-OP(1)-TPT)/CDR(K)
45    CONTINUE
      PNPV(N)=PNPV(N)+WC/CDR(N)
      WRITE(06,50)(UIR(I),I=2,N)
50    FORMAT(1H ,'ANNUAL INFLATION RATE(%)',5F10.2)
      WRITE(06,60)(SI(I),I=2,N)
60    FORMAT(1H ,'SALES INCOME                    ',5F10.0)
      WRITE(06,70)(OP(I),I=2,N)
70    FORMAT(1H ,'OPERATING COSTS                 ',5F10.0)
      WRITE(06,80)(CFB(I),I=2,N)
80    FORMAT(1H ,'CASH FLOW(B. TAX)               ',5F10.0)
      WRITE(06,90)(TP(I),I=2,N)
90    FORMAT(1H ,'TAX PAYABLE                     ',5F10.0)
      WRITE(06,95)(CFA(I),I=2,N)
95    FORMAT(1H ,'CASH FLOW(A. TAX)               ',5F10.0)
      WRITE(06,85)CNPV(N),UNPV(N),PNPV(N)
85    FORMAT(1H ,'NPV2=',F10.0,'NPV3=',F10.0,'NPV1=',F10.0)
      STOP
      END
$      EXECUTE
$      LIMITS   .001,20K,,10K
$      DATA     I*
E6,5000000.,2000000.,.5,.05,.07
.16,.15,.14,.13,.12
E7000000.,5000000.
E0.8,1.2,1.0
$      ENDCOPY
$      ENDJOB
```

```
*OFF
**COST:  $    0.26 TO DATE:  $ 9627.35= 10%
**ON AT 12.823 - OFF AT 12.840 ON  04/15/81
```

LINE TERMINATED - CP

```
FUNCTION?ACTI 2
FUNCTION?EFFI 06
 ANNUAL INFLATION RATE(%)         0.16        0.15        0.14        0.13        0.12
 SALES INCOME                 -120000.    4356000.   10045320.   12024212.   13472717.
 OPERATING COSTS              5600000.    6570000.    7003600.    8542294.    9623369.
 CASH FLOW(B. TAX)            2320000.    2000000.    3041520.    3430910.    3849346.
 TAX PAYABLE                 -1340000.    1334000.    1520760.    1715454.    1424074.
 CASH FLOW(A. TAX)            3660000.    1334000.    1520760.    1715454.    5774422.
 NPV2=  1714212.NPV3=  -254422.NPV1=   -747567.

END OF  06
FUNCTION?HOLD

*JOB  6525T

16:02:42 6525T OUTPUT WAITING    ID=J0.----FINI----

FUNCTION?ACTI 2
FUNCTION?EFFI 06
 ANNUAL INFLATION RATE(%)         0.16        0.15        0.14        0.13        0.12
 SALES INCOME                 7696000.    8843520.    9633994.   10650730.   11644470.
 OPERATING COSTS              5960000.    7032800.    8214310.    9445743.   10683130.
 CASH FLOW(B. TAX)            1936000.    1810720.    1019684.    1360467.    1035640.
 TAX PAYABLE                 -1532000.     905360.     809842.     680493.     517423.
 CASH FLOW(A. TAX)            3468000.     905360.     609642.     680493.    4587271.
 NPV2=  -461003.NPV3=  1325914.NPV1=   -747567.

END OF  06
FUNCTION?HOLD

*JOB  6345T

16:03:15 6345T OUTPUT WAITING    ID=J0.----FINI----

FUNCTION?ACTI 2
FUNCTION?EFFI 06
 ANNUAL INFLATION RATE(%)         0.16        0.15        0.14        0.13        0.12
 SALES INCOME                 7696000.    8843520.    9633994.   10650730.   11644470.
 OPERATING COSTS              5960000.    7032800.    8214310.    9445743.   10683130.
 CASH FLOW(B. TAX)            1936000.    1810720.    1019684.    1360467.    1035640.
 TAX PAYABLE                        0.          0.     183202.     680493.     517423.
 CASH FLOW(A. TAX)            1936000.    1810720.    1436482.     680493.    4387271.
 NPV2=  -660004.NPV3=  1196444.NPV1=   -952036.

END OF  06
FUNCTION?HOLD

*JOB  6404T

16:04:22 6404T OUTPUT WAITING    ID=J0.----FINI----

FUNCTION?ACTI 2
FUNCTION?EFFI 06
 ANNUAL INFLATION RATE(%)         0.16        0.15        0.14        0.13        0.12
 SALES INCOME                 7696000.    8843520.    6833994.   10650730.   11644470.
 OPERATING COSTS              5960000.    7032800.    8214310.    9445743.   10663130.
 CASH FLOW(B. TAX)            1936000.    1810720.    1619684.    1360467.    1035640.
 TAX PAYABLE                   468000.     405360.     309642.     180493.      17423.
 CASH FLOW(A. TAX)            1468000.    1405360.    1309642.    1180493.    4867271.
 NPV2=  -908431.NPV3=  1039569.NPV1= -1194995.

END OF  06
FUNCTION?HOLD
```

APPENDIX B

B.1 Mathematical Programming - A brief description of this technique

Mathematical programming (MP) is concerned with the optimisation of a linear function, called the objective function, subject to a set of linear equalities and/or inequalities, called the constraints.

Under the generic term *Mathematical Programming* we are including three different types of problems:

(1) Linear Programming (L), in which the variables can take any non-negative value that does not violate the constraints.

(2) Integer Programming (IP), in which the variables are constrained to take only non-negative integer values that do not violate the constraints.

(3) Mixed Integer Programming (MIP), which combines the two types of variables allowed in (1) and (2).

Linear Programming

For illustration purposes, a numerical example of an LP type of problem will be given.

Let us assume the manager of a factory is interested in finding the daily production levels of articles x_1 and x_2 that maximise profitability. Product x_1 yields a net benefit of £4/unit, and x_2 yields £5/unit. These products must undergo two different processes, C1 and C2; C1 can operate for a maximum of 12 hours/day and C2 for 15 hours/day. Product x_1 requires 4 hours/day of process C1 and 3 hours/day of process C2. Product x_2 requires 3 hours/day of process C1 and 5 hours/day of process C2.

With this information we can set up the following linear problem:

Objective Function (OF) is the maximisation of the net benefit:

$$x_o = 4x, + 5x_2$$

Subject to the constraints:

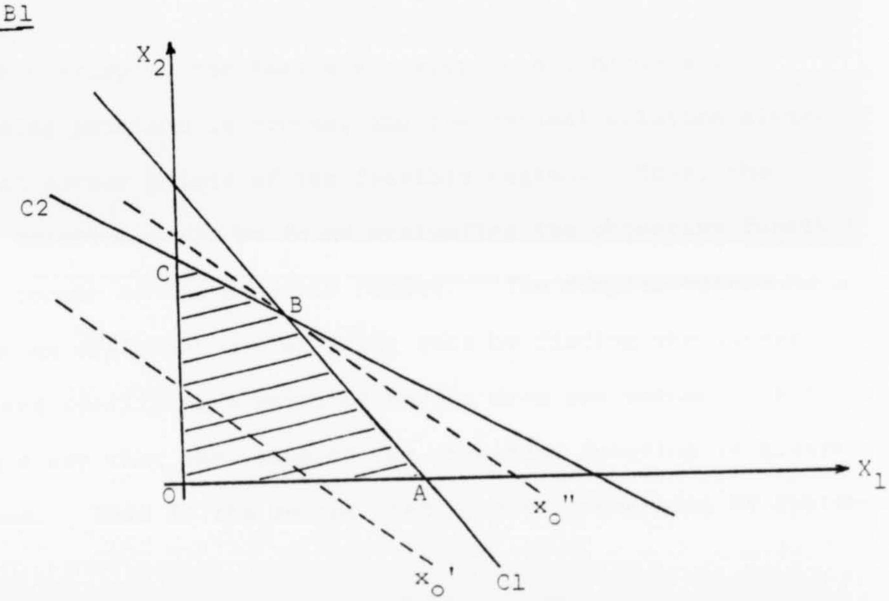
$$(C1) \quad 4x_1 + 3x_2 \leqslant 12$$

$$(C2) \quad 3x_1 + 5x_2 \leqslant 15$$

where x_1 and $x_2 \geqslant 0$. (1)

The graphic representation of this system is shown in Figure B1, in which C1 represents the first constraint, and C2 represents the second constraint. The shaded area OABC is called the feasible region or solution space, and is determined by the two ($\leqslant$) constraints. Any point in the feasible region is called a feasible point and satisfies all the constraints.

Figure B1



There is an infinite number of feasible points. In this problem, we are interested in finding the feasible point which yields the maximum value of x_o. It can be seen on Figure B1 (dotted line), that by increasing the value of x_o, the straight line $x_o = 4x_1 + 5x_2$ moves parallel to itself and away from the origin. In order to satisfy all the constraints, the objective function must have at least one point in the feasible region. Therefore, the maximum value x_o can attain without violating any of the constraints occurs when $x_o = 4x_1 + 5x_2$ touches the feasible point B. The coordinates at this point give the ***optimal solution:*** $x_1 = 1.36$ and $x_2 = 2.18$. The optimal value of Z is obtained by direct substitution in the objective function: $x_o = 16.34$.

Notice that at point B, both constraints are satisfied as strict equalities (they are *binding*). This need not always be the case. For an OF with a different slope (ie if the relative net benefit obtained from x_1 and x_2 changed), the optimum could be located at either A or C, where only one of the constraints is binding and the other one has a positive ***slack*** (the amount of that resource that is left unused).

As in this example, the feasible region in all mathematical programming problems is convex, and the optimal solution always occurs at corner points of the feasible region. Thus, the optimal solution could be found evaluating the objective function at each corner of the feasible region. The ***Simplex method*** provides an efficient way of doing this by finding the corner points and specifying a means of moving from one corner to another in such a way that the value of the objective function is always increased. This is the method used by most commercial MP systems.

B.1.1 The Dual Problem

The general LP problem can be stated as (Taha, 1971):

$$\text{Maximise} \quad x_o = \sum_{j=1}^{n} c_j x_j$$

$$\text{Subject to} \quad \sum_{j=1}^{n} a_{ij} x_j \leqslant b_i, \quad i=1, 2, \ldots, m$$

$$x_j \geqslant o, \quad j=1, 2, \ldots, n$$

This is referred to as the Primal Problem. Every LP problem has a twin problem associated with it, called the Dual Problem. The dual of the problem above is given by:

$$\text{Minimise} \quad y_o = \sum_{i=1}^{m} b_i y_i,$$

$$\text{Subject to} \quad \sum_{i=1}^{n} a_{ij} y_i \geqslant C_j, \quad j=1, 2, \ldots, n$$

$$y_1 \geqslant o, \quad i=1, 2, \ldots, m$$

where $y_1, y_2, \ldots, y_m$ are the dual variables, known too as *shadow prices*.

The dual problem is obtained from the primal (and vice versa) as follows:

1. Each constraint in one problem corresponds to a variable in the other problem.

2. The elements of the right-hand side of the constraints in one problem are equal to the respective coefficients of the objective function in the other problem.

3. One problem seeks maximisation and the other seeks minimisation.

4. The maximisation problem has ($\leq$) constraints, and the minimisation has ($\geq$) constraints.

5. The variables in both problems are non-negative.

Notice that the primal above has only ($\leq$) constraints. If the problem we are seeking to solve has equality constraints (=), the same procedure is followed, with the difference that the dual variable corresponding to the equality constraint must be unrestricted in sign. In the case of ($\geq$) constraints, they can be converted into ($\leq$) constraints multiplying the inequality by - 1.

This procedure can be better understood with a numerical example:

Primal Problem:

Maximise $x_o = 3x_1 + 9x_2 + x_3$

Subject to
$$-x_1 + 5x_2 + 2x_3 \leq 10$$
$$2x_1 + 4x_2 + 7x_3 \geq 5$$
$$x_1 - x_2 + 4x_3 = 8$$
$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$$

It is convenient to break the inequalities using *slack variables*. In this way all the constraints are of the equality type (this is called the standard form of the problem).

Maximise $x_o = 3x_1 + 9x_2 + x_3 + 0\,S_1 + 0\,S_2$

Subject to
$$-x_1 + 5x_2 + 2x_3 + S_1 = 10$$
$$-2x_1 - 4x_2 - 7x_3 + S_2 = -5$$
$$x_1 - x_2 + 4x_3 = 8$$

$$x_1 \geqslant 0,\ x_2 \geqslant 0,\ x_3 \geqslant 0,\ S_1 \geqslant 0,\ S_2 \geqslant 0 \qquad (2)$$

The dual is given by:

Minimise: $y_o = 10y_1 - 5y_2 + 8y_3$

Subject to
$$-y_1 - 2y_2 + y_3 \geqslant 3$$
$$5y_1 - 4y_2 - y_3 \geqslant 9$$
$$2y_1 - 7y_2 + 4y_3 \geqslant 1$$
$$y_1 \geqslant 0,\ y_2 \geqslant 0$$

y_3 unrestricted in sign.

The constraints $y_1 \geqslant 0$ and $y_2 \geqslant 0$ correspond to the slack variables S_1 and S_2 respectively. Notice that only the primal constraint that was initially in equation form has its associated dual variable unrestricted in sign.

The primal and the dual have some important properties at the optimum:

(1) If the primal constraint is not binding (the slack variable is positive), the corresponding dual variable is zero.

(2) If the primal variable has a positive value, the corresponding dual constraint is binding.

(3) The optimal solutions of the primal and dual occur at the same point and have identical values.

Properties (1) and (2) are known as *complementary slackness conditions.*

B.1.2 Integer Programming

This case will be illustrated making use of the same numerical example as above, with the only difference that x_1 and x_2 can only take integer values.

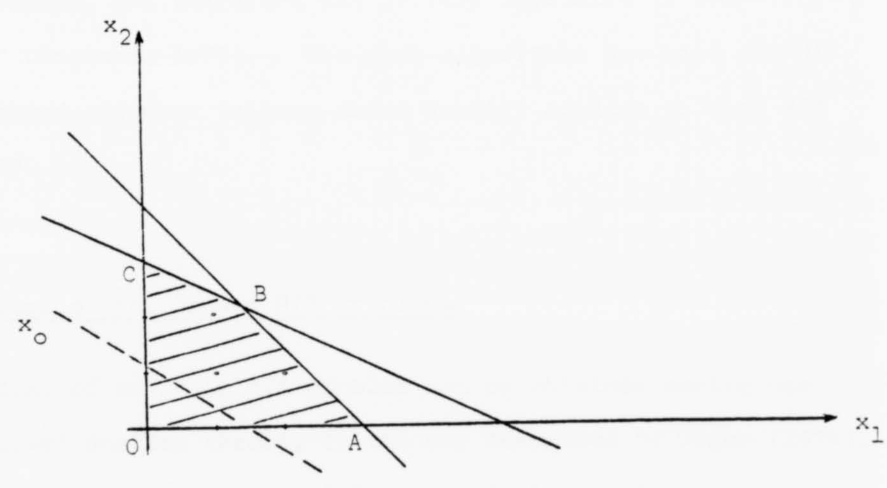
Maximise $x_o = 4x_1 + 5x_2$

Subject to $4x_1 + 3x_2 \leq 12$

$3x_1 + 5x_2 \leq 15$

where $x_1 > 0$, $x_2 > 0$ and integers.

Figure B2



It was shown that, for the continuous case, the maximum lay at point B, where $x_1 = 1.36$ and $x_2 = 2.18$. At this point none of the variables has an integer value. Thus, we are looking for an alternative solution which comes as close as possible to the maximum at point B, and where x_1 and x_2 are integer valued.

Several feasible points satisfy the integrality condition. By observation it can be deduced that the maximum value x_o can attain satisfying the integrality condition lies at point C, where $x_1 = 0$ and $x_2 = 3$. The value of x_o at this point is 15, which is lower than the value obtained for the continuous case (16.34). The maximum value of the objective function for the integer case is always lower than that one for the continuous case, except when both solutions occur at the same point, in which case their values are identical.

Two of the most common computing methods to obtain an integer optimal solution are the Branch and Bound Algorithm suggested by Land and Doig (1960) and the Cutting Plane technique of Gomory (1958). "All integer programming procedures require the repeated solution of LP problems larger than the original LP problem, and therefore can be very expensive in computing time" (England, 1977). The same algorithms are used for MIP. The above and what follows about duality applies to both MIP and IP.

B.1.2.1 The Dual of the IP and MIP Problems

The dual of an IP or MIP problem can be obtained making use of Balas' duality theory, in the way described by Unger (1974). This will be illustrated with a numerical example:

Let us redefine problem (2) above for the particular case where x_1 and x_2 are 0, 1 integer variables:

$$\text{Maximise} \quad x_o = 3x_1 + 9x_2 + x_3 + 0\,S_1 + 0\,S_2$$

$$\text{Subject to} \quad -x_1 + 5x_2 + 2x_3 + S_1 = 10$$

$$-2x_1 - 4x_2 - 7x_3 + S_2 = -5$$

$$x_1 - x_2 + 4x_3 = 8$$

$$x_1,\ x_2 = 0 \text{ or } 1,\ x_3 \geqslant 0,\ S_1 \geqslant 0,\ S_2 \geqslant 0 \qquad (3)$$

Applying Balas' duality concept, the dual to this problem can be written as:

$$\underset{x}{\text{Max}} \quad \underset{y}{\text{Min}} \quad y_o = 10y_1 - 5y_2 + 8y_3 - \mu_1\, x_1 - \mu_2\, x_2$$

$$\text{Subject to} \quad -y_1 - 2y_2 + y_3 - \mu_1 \geqslant 3$$

$$5y_1 - 4y_2 - y_3 - \mu_2 \geqslant 9$$

$$2y_1 - 7y_2 + 4y_3 \geqslant 1$$

$$y_1,\ y_2 \geqslant 0$$

y_3, μ_1, μ_2 unrestricted in sign.

$$x_1,\ x_2 = 0 \text{ or } 1.$$

Except for the unrestricted dual surplus variables, μ_1 and μ_2, this dual is exactly the dual obtained for the continuous case above. μ_1 and μ_2 are subtracted from those dual constraints derived from the coefficients of the primal variables that are constrained to be integer. In the objective function, the negative of these dual surplus variables appears multiplied by their complementary primal variables, x_1 and x_2.

Balas' duality theory for integer programming problems has the following properties:

(1) The dual of the dual is the primal;

(2) Complementary slackness conditions hold;

(3) If an optimal solution exists to the primal, then the dual has an optimal solution and the optimal values of the primal and dual problems are equal.

These properties are identical to those described for the continuous linear programming case.

APPENDIX C

It can be shown that $\overline{\lambda}_t \leqslant 0$, $t = 0, \ldots, T$ for any set of values of $\overline{A}_t$ and $\overline{S}_t$, and that $\overline{\lambda}_t < 0$, $t = 0, \ldots, T$ provided $S_T > 0$.

Dual constraint (3.2.9) holds as an equality if $\overline{S}_t > 0$.

$$\overline{\lambda}_t = - \frac{h}{1 + 1'_t} \qquad \text{(C-1)}$$

If $\overline{S}_t > 0$, $t = 1, \ldots, T - 1$, we get from (3.3.7)

$$\overline{\lambda}_t = - h\overline{\rho}_{t+1} - rh\overline{\beta}_{t+1} \qquad t = 1, \ldots, T - 1 \qquad \text{(C-2)}$$

From the dual problem (3.2) we know that ρ_t, $\beta_t \geqslant 0$ for all t, and from Property 1, $\rho_t > 0$, $t = 1, \ldots, T$. Applying this to (C-2) and using (C-1) we obtain:

$$\overline{\lambda}_t < 0 \text{ for } S_t > 0, \ t = 1, \ldots, T \qquad \text{(C-3)}$$

From (3.8.9) we know that

$$\overline{\lambda}_t = \overline{\lambda}_{t+1} \text{ for } \overline{A}_t > 0, \ t = 1, \ldots, T - 1 \qquad \text{(C-4)}$$

From (3.3.9) $\lambda_T = 0$ if $A_T > 0$. By successive substitution in (C-4) we obtain

$$\overline{\lambda}_t = 0, \ t = 0, \ldots, T$$

if $\overline{A}_t > 0$ for every value of t.

Combining (C-3) and (C-4) we deduce that, given one year k such that $\overline{A}_k > 0$ and $\overline{S}_{k+1} > 0$, then $\overline{\lambda}_t < 0$ for all $t \leqslant k+1$. If we make $k = T - 1$, then we have that $\overline{\lambda}_t < 0$ for all $t \leqslant T$.

Hence, $\overline{\lambda}_t \leqslant 0$, $t = 0, \ldots, T$ for any set of values of $\overline{A}_t$ and $\overline{S}_t$, and $\lambda_t < 0$, $t = 0, \ldots, T$ provided $\overline{S}_t > 0$, Q.E.D.

APPENDIX D

Program of the simulation model

This program, written in Fortran, makes use of three subroutines, two library routines and one sub-program.

Subroutines

- CUM calculates the shift (ie correlations between consecutive values of the inflation rate).

- SMC calls the library routine that generates the random numbers. This subroutine also defines the random estimate of the inflation rate corresponding to each random number.

- SIGNO assignes the appropriate sign to the coefficients and RHS's of the input matrix.

Library Routines

- MPS solves the mixed integer program associated with each input matrix generated by the simulation model.

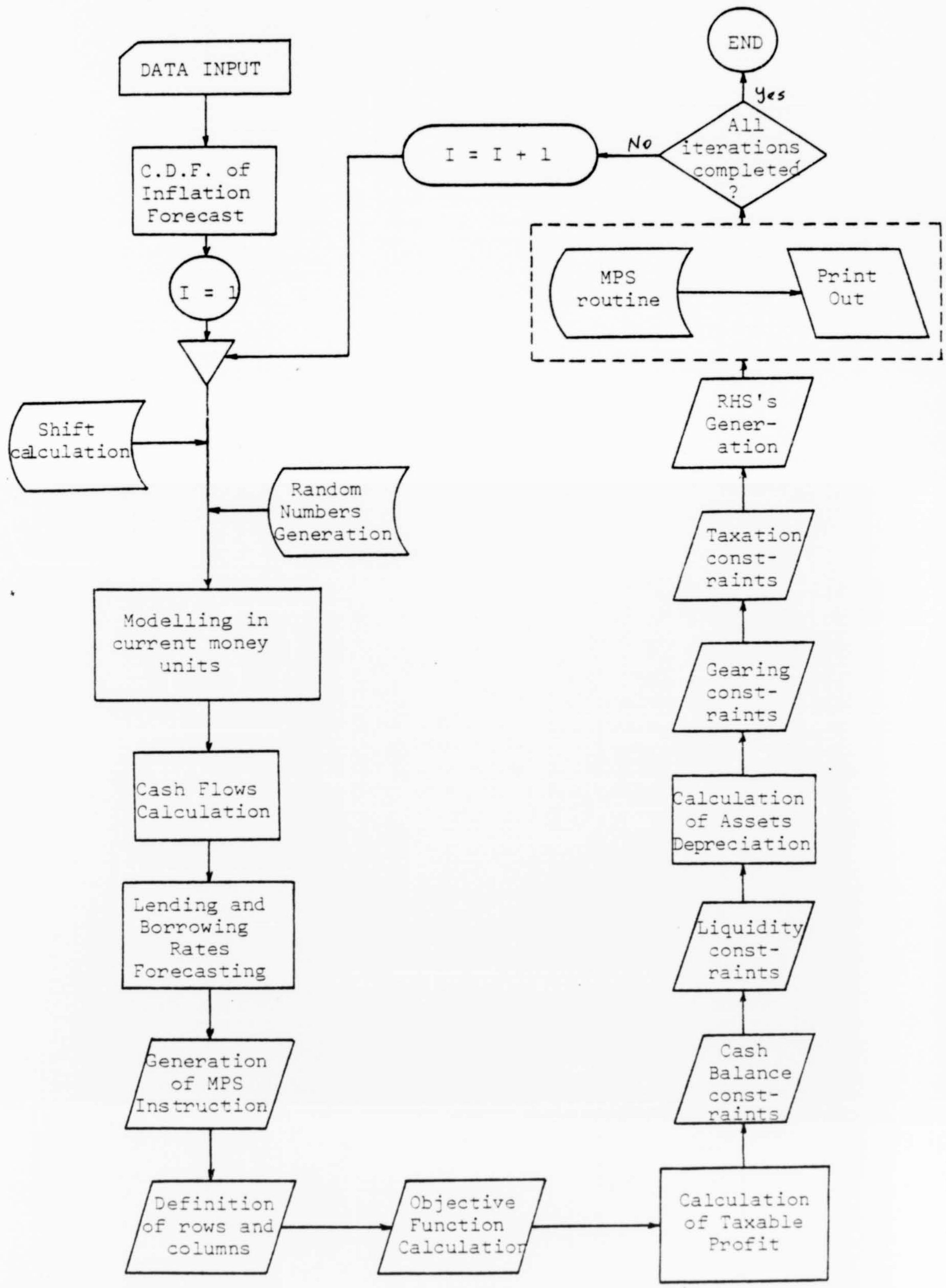
- GD5CAF generates one random number every time it is called from subroutine SMC.

Sub-program

This is a "report generator" used to reduce the simulation output to a manageable size. For each iteration, the MPS routine produces information concerning the control instructions used, characteristics of the input matrix, details about the iterations needed for the solution of the linear and mixed integer programs, and a table of results, with the optimal values of every variable used (see Appendix G). Most of this information is unnecessary for the identification of the most frequently selected portfolios, or for the production of histograms of sensitivity to inflation. Furthermore, as the number of iterations increases, the amount of output produced becomes unmanageable, and a report generator to extract the relevant information is necessary. The report generator is shown in Table D1 at the end of this Appendix.

The flow diagram of the simulation program is shown in Figure D1.

Figure D1 Flow diagram of the simulation program



```
EEA,J ;,9,17,25,33,41,49,57,65,73
$       IDENT   SG#24
```

Listing of the Simulation Program

```
$        OPTION   FORTRAN
$        USE      .GTLIT
$        FORTRAN ASCII
       DOUBLE PRECISION GOSCAF
       DIMENSION C(25,25),F(25,25),H(25,25),FF(25),CC(25),CX(25),CIV(25)
       DIMENSION CF(25,25),XX(25),CT(25),DIR(25),IT(25),ID(25)
       DIMENSION TM(25,25),GC(25,25),GDEP(25,25),GG(25,25),GGG(25)
        DIMENSION HH(25,25),COS(25),DEP(25,25),F1(25,25),C1(25,25)
        DIMENSION R11(25),R22(25),STIN(25),STIP(25),TN(25),CNPV(25)
        DIMENSION FIF(25),CIF(25),CTIF(25),HF(25,25),HC(25,25),CNPV1(25)
        DIMENSION TCF(25,25),CC(25),CCF(25),STN(25),CT1(25),CCT(25,25)
        DIMENSION PRO(25,25),DI(25,25),CM(25,25),DIS(25,25)
        DIMENSION LP(25),CFB(25,25),TP(25,25),CFA(25,25),PV(25)
        DIMENSION C2(25,25),F2(25,25),V1(25,25),V(25,25),F3(25,25)
        INTEGER EINS(25),ZWEI(25),DREI(25),VIER(25),FUNF(7),DIRO(25)
        INTEGER SIGN(25),RT(25),ALL(25),IMP(25),REP(25),PAY(25)
        INTEGER CABAL(25),LIG(25),GEAR(25),DUL(25),WIZ(25),WIR(25),ZIP(25)
        INTEGER RIP(25),TYPE(5),PEN(3)
        DATA EINS/'W1','W2','W3','W4','W5','W6','W7','W8','W9','W10',
       1'Z10',14*' '/
        DATA ZWEI/'X1+','X2+','X3+','X4+','X5 .LE.',20*' '/
        DATA DREI/'Y0','Y1','Y2','Y3','Y4','Y5','Y6','Y7','Y8','Y9','Y10',
       114*' '/
        DATA VIER/'Z0','Z1','Z2','Z3','Z4','Z5','Z6','Z7','Z8','Z9','Z10',
       114*' '/
        DATA FUNF/'=','.LE.','.GE.',' ','0','1','10'/
        DATA DIRO/'X1','X2','X3','X4','X5','X6','X7','X8','X9','X10',
       1'X11','X12','X13','X14','X15','X16','X17','X18','X19','X20',
       2'X21','X22','X23','X24','X25'/
        DATA IPLUS/'+'/,MINUS/'-'/
        DATA ALL/'A0','A1','A2','A3','A4','A5','A6','A7','A8','A9','A10',1
       14*' '/
        DATA IMP/'T0','T1','T2','T3','T4','T5','T6','T7','T8','T9','T10',1
       14*' '/
        DATA REP/'R1','R2','R3','R4','R5','R6','R7','R8','R9','R10',15*' '
       1/
        DATA PAY/'P1','P2','P3','P4','P5','P6','P7','P8','P9','P10',15*' '
       1/
        DATA CABAL/'CB1','CB2','CB3','CB4','CB5','CB6','CB7','CB8','CB9','
       1CB10','CB11',14*' '/
        DATA LIG/'LG1','LG2','LG3','LG4','LG5','LG6','LG7','LG8','LG9','LG
       110','LG11',14*' '/
        DATA GEAR/'GR1','GR2','GR3','GR4','GR5','GR6','GR7','GR8','GR9','G
       1R10','GR11',14*' '/
        DATA DUL/'TX1','TX2','TX3','TX4','TX5','TX6','TX7','TX8','TX9','TX
       110','TX11',14*' '/
        DATA WIZ/'WZ1','WZ2','WZ3','WZ4','WZ5','WZ6','WZ7','WZ8','WZ9','WZ
       110','WZ11',14*' '/
        DATA WIR/'WR1','WR2','WR3','WR4','WR5','WR6','WR7','WR8','WR9','WR
       110','WR11',14*' '/
        DATA ZIP/'ZP1','ZP2','ZP3','ZP4','ZP5','ZP6','ZP7','ZP8','ZP9','ZP
       110','ZP11',14*' '/
        DATA RIP/'RP1'.'RP2','RP3','RP4','RP5','RP6','RP7','RP8','RP9','RP
       110','RP11',14*' '/
        DATA TYPE/'(F)','(F)','( )','(Z)','(I)'/
        DATA PEN/'LGL','STR',','/
        CALL FREDIA(34,2)
        CALL FREDIA(03,2)
        CALL FREDIA(44,2)
C  ***DATA INPUT***
        READ(05,10) [illegible], 1,[illegible],1,[illegible]1,R2,[illegible],[illegible]IV(1),T[illegible],[illegible],[illegible],SR
```

```
 10   FORMAT(4I2,2F4.0,3F10.0,3F5.0,F10.0)
      P=P/SF
      DIV(1)=DIV(1)/SF
      READ(05,6)AV,AE,AC
  6   FORMAT(3F6.3)
      READ(05,14)(OCF(L),L=1,N)
 14   FORMAT(6F8.0)
      DO 450 L=1,N
450   OCF(L)=OCF(L)/SF
      MAN=M+1
      MPN=M+N
      MP1=MAN+1
      READ(05,12)(IT(L),L=1,MAN),(ID(L),L=1,MAN)
      DO 955 L=MP1,MPN
      IT(L)=IT(L-1)+1
955   ID(L)=ID(MAN)
 12   FORMAT(11I3)
      READ(05,17)(CT1(L),L=1,MAN)
 17   FORMAT(5F10.0)
      DO 455 L=1,MAN
455   CT1(L)=CT1(L)/SF
      DO 5 K=1,N1
      READ(05,20)(C1(K,L),L=1,M)
  5   CONTINUE
      DO 15 K=1,N1
      READ(05,20)(F1(K,L),L=1,M)
 15   CONTINUE
      DO 530 K=1,N1
      READ(05,20)(V1(K,L),L=1,M)
530   CONTINUE
 20   FORMAT(10F8.0)
      READ(05,20)(C1(K,MAN),K=1,N1)
      READ(05,20)(F1(K,MAN),K=1,N1)
      READ(05,20)(V1(K,MAN),K=1,N1)
      DO 460 K=1,N1
      DO 460 L=1,MAN
       C1(K,L)=C1(K,L)/SF
       V1(K,L)=V1(K,L)/SF
460    F1(K,L)=F1(K,L)/SF
       READ(05,25)(F1F(L),L=1,MAN)
       READ(05,25)(C1F(L),L=1,MAN)
       READ(05,25)(CT1F(L),L=1,MAN)
 25    FORMAT(11F4.2)
       READ(05,2)N
  2    FORMAT(I6)
       DO 1 I=2,N
       READ(05,23)(PRO(I,K),K=1,13)
  1    READ(05,23)(CI(I,K),K=1,13)
 23    FORMAT(13F5.0)
C  ***CUMULATIVE DENSITY FUNCTION OF THE INFLATION FORECAST***
       CI(1,7) = .15
       CIR(1) = CI(1,7)
       DO 620 I=2,N
       CR(I,1) = PRO(I,1)
       DO 620 K=2,13
620    CR(I,K) = CR(I,K-1) + PRO(I,K)
C  ***ITERATIONS BEGIN***
       ITER = 0
       CALL GUSUCF
500    CONTINUE
       ITER = ITER + 1
```

```
      DO 3 I=2,N
C  ***SUBROUTINES FOR SHIFT CALCULATION AND
C     RANDOM NUMBERS GENERATION***
      CALL CUM(CI,OIR,UIS,I)
      CALL SMC(CM,CIS,OIR,I)
    3 CONTINUE
      WRITE(06,625)ITER,(OIR(L),L=2,N)
  625 FORMAT(1H ,I3,3X,10F8.3)
      LIT = N + 1
      DO 4 K=LIT,N1
    4 OIR(K) = OIR(N)
C  ***MODELLING IN CURRENT MONEY UNITS***
      DO 280 K=1,MAN
      HC(1,K) = 1.0
      HH(1,K) = 1.0
      HF(1,K) = 1.0
      DO 280 J=2,N1
      HC(J,K) = HC(J-1,K)*(1.0+CTIF(K)*OIR(J))
      HF(J,K) = HF(J-1,K)*(1.0+FIF(K)*OIR(J))
  280 HH(J,K) = HH(J-1,K)*(1.0+CIF(K)*OIR(J))
      DO 335 L=1,MAN
      DO 335 K=1,N1
      F2(K,L) = F1(K,L)*HF(K,L)
      V(K,L) = V1(K,L)*HC(K,L)
      F3(K,L) = V(K,L) - F2(K,L)
  335 C2(K,L) = C1(K,L)*HH(K,L)
      DO 950 L=MP1,MPN
      DO 950 K=2,N1
      F2(K,L)=F2(K-1,L-1)*HF(K,MAN)/HF(K-1,MAN)
      V(K,L) = V(K-1,L-1)*HC(K,MAN)/HC(K-1,MAN)
      F3(K,L) = V(K,L) - F2(K,L)
  950 C2(K,L)=C2(K-1,L-1)*HH(K,MAN)/HH(K-1,MAN)
      MM = M - M1 + 1
      IF(MM.GT.M)GO TO 970
      DO 975 L=MM,M
      NIT=N1-IT(L)
      DO 975 K=1,NIT
  975 F(K+IT(L),L)=F1(K,L)*HF(K+IT(L),L)
  970 CONTINUE
C  ***CASH FLOWS CALCULATION***
      DO 99 L=1,MPN
      DO 99 K=1,N1
      F(K,L)=AV*V(K-1,L)+(1-AV)*V(K,L) - AF*F2(K-1,L)-(1-AF)*F2(K,L
      IF(K.EQ.1)F(K,L)=0.0
   99 C(K,L)=AC*C2(K-1,L)+(1-AC)*C2(K,L)
C  ***FORECASTING OF LENDING AND BORROWING RATES***
      R11(1) = R1
      R22(1) = R2
      DO 340 K=2,N1
      IF(OIR(K-1).LT..01)OIR(K-1)=.01
      R11(K) = R11(K-1)*(1+.6*(OIR(K)-OIR(K-1))/OIR(K-1))
      IF(R11(K).LT..06)R11(K)=.06
      R22(K) = R22(K-1)*(1+.6*(OIR(K)-OIR(K-1))/OIR(K-1))
      IF(R22(K).LT..08)R22(K)=.08
  340 CONTINUE
      WRITE(06,444)(R11(K),K=2,N),(R22(K),K=2,N)
  444 FORMAT(1H ,2X,'R11',10F8.3/2X,'R22',10F8.3)
      DO 345 K=1,N1
      ST (K) = R11(K)
      ST (K) = (1.0 + R11(K))
  345 ST1R(K) = R22(K) + 1
```

```
      STP = (1.0+R22(N)*(1-TAX))/(1.0+R11(N)*(1-TAX))
      SNV = (1-TAX)/(1.0+R11(N)*(1-TAX))
      TN(1) = 1.0
      DO 350 K=2,N1
  350 TN(K) = TN(K-1)*(1.0 + R11(K))
      NO = N1 + 1
C  ***CHANGES IN WORKING CAPITAL***
      DO 32 K=1,MAN
      JOT=IT(K)-ID(K)
   32 CT(K)=CT1(K)*HF(JOT,K)
      DO 960 K=MP1,MPN
      JOT=IT(K)-ID(K)
  960 CT(K)=CT1(MAN)*HF(JOT,MAN)
      DO 980 K=1,MAN
      JIT=IT(K)-ID(K)+1
      CCT(JIT,K)=CT1(K)*HF(JIT,K)
      DO 980 L=2,N1
      IF(L.GT.JIT.AND.L.LE.IT(K))GO TO 982
      CCT(L,K)=0.0
      GO TO 980
  982 CCT(L,K)=CCT(L-1,K)*V(L,K)/V(L-1,K)
  980 CONTINUE
      DO 985 K=MP1,MPN
      JIT=IT(K)-ID(K)+1
      CCT(JIT,K)=CT1(MAN)*HF(JIT,MAN)
      DO 985 L=2,N1
      IF(L.GT.JIT.AND.L.LE.IT(K))GO TO 988
      CCT(L,K)=0.0
      GO TO 985
  988 CCT(L,K)=CCT(L-1,K)*V(L,K)/V(L-1,K)
  985 CONTINUE
C  ***CASH FLOWS AFTER THE HORIZON***
      DO 35 K=1,MPN
      FF(K) = 0.0
      IF(IT(K).GE.N)FF(K)=CCT(IT(K),K)*TN(N)/TN(IT(K))
      MR = N + 1
      DO 35 J=MR,NO
      IF(J.EQ.NO) GO TO 22
      IF(J.GT.MR) GO TO 27
      FF(K) = FF(K) + F(J,K)*TN(N)/TN(J)
      GO TO 35
   27 FF(K) = FF(K)+(F(J,K)-TAX*F(J-1,K))*TN(N)/TN(J)
      GO TO 35
   22 FF(K) = FF(K)-TAX*F(J-1,K)*TN(N)/TN(J-1)
   35 CONTINUE
      DO 45 K=MM,M
      RT(K)=IT(K)+ID(K)
      CC(K) = 0.0
      IF(MM.GT.N)GO TO 111
      IF(RT(K).GE.N.AND.RT(K).LE.N1)CC(K)=-CCT(IT(K),K)*TN(N)/TN(RT(K
  111 CONTINUE
      DO 45 J=MR,N1
      IF(J.GT.MR) GO TO 47
      CC(K) = CC(K) + C(J,K)*TN(N)/TN(J)
      GO TO 45
   47 CC(K) = CC(K)+(C(J,K)-TAX*C(J-1,K))*TN(N)/TN(J)
   45 CONTINUE
      NN = N - 1
      IN = NN - 1
C  ***...S CONTROL I STRUCTI...S***
      WRITE(34,405)
```

```
      STP = (1.0+R22(N)*(1-TAX))/(1.0+R11(N)*(1-TAX))
      SNV = (1-TAX)/(1.0+R11(N)*(1-TAX))
      TN(1) = 1.0
      DO 350 K=2,N1
  350 TN(K) = TN(K-1)*(1.0 + R11(K))
      NO = N1 + 1
C  ***CHANGES IN WORKING CAPITAL***
      DO 32 K=1,MAN
      JOT=IT(K)-ID(K)
   32 CT(K)=CT1(K)*HF(JOT,K)
      DO 960 K=MP1,MPN
      JOT=IT(K)-ID(K)
  960 CT(K)=CT1(MAN)*HF(JOT,MAN)
      DO 980 K=1,MAN
      JIT=IT(K)-ID(K)+1
      CCT(JIT,K)=CT1(K)*HF(JIT,K)
      DO 980 L=2,N1
      IF(L.GT.JIT.AND.L.LE.IT(K))GO TO 982
      CCT(L,K)=0.0
      GO TO 980
  982 CCT(L,K)=CCT(L-1,K)*V(L,K)/V(L-1,K)
  980 CONTINUE
      DO 985 K=MP1,MPN
      JIT=IT(K)-ID(K)+1
      CCT(JIT,K)=CT1(MAN)*HF(JIT,MAN)
      DO 985 L=2,N1
      IF(L.GT.JIT.AND.L.LE.IT(K))GO TO 988
      CCT(L,K)=0.0
      GO TO 985
  988 CCT(L,K)=CCT(L-1,K)*V(L,K)/V(L-1,K)
  985 CONTINUE
C  ***CASH FLOWS AFTER THE HORIZON***
      DO 35 K=1,MPN
      FF(K) = 0.0
      IF(IT(K).GE.N)FF(K)=CCT(IT(K),K)*TN(N)/TN(IT(K))
      MR = N + 1
      DO 35 J=MR,NO
      IF(J.EQ.NO) GO TO 22
      IF(J.GT.MR) GO TO 27
      FF(K) = FF(K) + F(J,K)*TN(N)/TN(J)
      GO TO 35
   27 FF(K) = FF(K)+(F(J,K)-TAX*F(J-1,K))*TN(N)/TN(J)
      GO TO 35
   22 FF(K) = FF(K)-TAX*F(J-1,K)*TN(N)/TN(J-1)
   35 CONTINUE
      DO 45 K=MM,M
      RT(K)=IT(K)+ID(K)
      CC(K) = 0.0
      IF(MM.GT.N)GO TO 111
      IF(RT(K).GE.N.AND.RT(K).LE.N1)CC(K)=-CCT(IT(K),K)*TN(N)/TN(RT(K
  111 CONTINUE
      DO 45 J=MR,N1
      IF(J.GT.MR) GO TO 47
      CC(K) = CC(K) + C(J,K)*TN(N)/TN(J)
      GO TO 45
   47 CC(K) = CC(K)+(C(J,K)-TAX*C(J-1,K))*TN(N)/TN(J)
   45 CONTINUE
      NN = N - 1
      IN = NN - 1
C  ***MPS CONTROL INSTRUCTIONS***
      WRITE(34,403)
```

```
903   FORMAT(7X,'TITLE    INVESTMENT')
      WRITE(34,440)ITER
440   FORMAT(7X,'GETGEN   IDENT=',I3,'/ZH')
445   WRITE(34,907)ITER
907   FORMAT(7X,'INPUT    SOURCE=',I3,'/03')
      WRITE(34,913)ITER
913   FORMAT(7X,'SETUP    SOURCE=',I3)
      NTER=ITER+100
      WRITE(34,917)NTER
917   FORMAT(7X,'MAX'/7X,'SET      NFS=L',I3)
      WRITE(34,938)
938   FORMAT(7X,'SET      SMETH=7,IMETH=8,TINT2=0.1,IREOPT=2')
      WRITE(34,948)
948   FORMAT(7X,'SET      IPOIR=0,TINT=0.001,INFR2=3')
      WRITE(34,925)
925   FORMAT(7X,'INTEGER')
      WRITE(34,923)NTER
923   FORMAT(' L',I3,'  OUTPUT')
      IF(ITER.LT.MN1)GO TO 427
      WRITE(34,927)
927   FORMAT(7X,'ENDLP')
427   CONTINUE
C   ***MATRIX GENERATION BEGINS***
      WRITE(03,60)ITER
 60   FORMAT('NAME     ',I3)
      WRITE(03,333)ITER
333   FORMAT('* MATRIZ NUMERO',I3)
C   ***DEFINITION OF ROWS AND COLUMNS***
      WRITE(03,50)TYPE(1)
 50   FORMAT('LGL     TV',A3)
      DO 505 J=1,N
505   WRITE(03,100)PEN(1),CANAL(J),TYPE(2)
      DO 510 I=1,NN
510   WRITE(03,100)PEN(1),LIQ(I),TYPE(3)
      DO 515 L=1,N
515   WRITE(03,100)PEN(1),GEAR(L),TYPE(2)
      DO 520 L=1,N
520   WRITE(03,100)PEN(1),DOL(L),TYPE(4)
      WRITE(03,90)TYPE(2)
 90   FORMAT('LGL     PPJ',A3)
100   FORMAT(A3,4X,A4,A3)
103   FORMAT(A3,4X,2A3)
      DO 545 I=1,MPN
545   WRITE(03,103)PEN(2),OTRO(I),TYPE(5)
      DO 560 I=1,N
560   WRITE(03,103)PEN(2),DREI(I),TYPE(2)
      DO 570 I=1,N
570   WRITE(03,103)PEN(2),VIER(I),TYPE(2)
      DO 575 I=1,N
575   WRITE(03,103)PEN(2),ALL(I),TYPE(2)
      DO 580 I=1,N
580   WRITE(03,103)PEN(2),INF(I),TYPE(2)
C   ***OBJECTIVE FUNCTION CALCULATION***
      DO 55 K=1,MPN
      IF(K.GE.MP) GO TO 65
      CX(K) = FF(K)
      GO TO 55
 65   CX(K) = FF(K)-CC(K)
 55   CONTINUE
      CALL SIGNO(MPN,SIGN,IPLUS,CX,MINUS)
      WRITE(03,110)OTRO(1),SIGN(1),CX(1)
```

```
 110  FORMAT('MATRIX IV,',A2,'=',A1,F15.5)
      DO 585 K=2,MPN
 585  WRITE(03,123)PEN(3),OTRO(K),SIGN(K),CX(K)
 123  FORMAT(7X,A1,A3,'=',A1,F15.5)
      WRITE(03,120)DREI(N)
 120  FORMAT('          ,',A3,'=1')
      WRITE(03,130)VIER(N),MINUS,STP
 130  FORMAT('          ,',A3,'=',A1,F15.5)
      WRITE(03,130)IMP(N),MINUS,SNV
C  ***CALCULATION OF ANNUAL DIVIDENDS***
      DO 210 L=2,N
      DIV(L) = DIV(L-1)*(1.0 + DIR(L))
 210  CC(L) = OCF(L) - DIV(L)
      WRITE(06,1000)(DIV(L),L=2,N)
 1000 FORMAT(1H,2X,'DIV',10F8.3)
      CC(1) = OCF(1) - DIV(1) + P
C  ***CASH FLOWS AND TAXABLE PROFIT***
      DO 75 K=1,MPN
      CCC = CCT(IT(K),K) - CT(K)
      DO 75 J=2,N1
      CF(J,K) = C(J,K) - F(J,K) + CCT(J,K) - CCT(J-1,K)
      IF(IT(K).EQ.J)CF(J,K)=CF(J,K)-CCT(J,K)
      IF(F3(J,K).LT.0.0)F3(J,K)=0.0
      TCF(J,K) = C2(J,K) - F3(J,K)
      IF(IT(K).EQ.J.AND.CCC.GT.0.0)TCF(J,K)=TCF(J,K)-CCC
 75   CONTINUE
      STAX = TAX
C  ***CASH BALANCE CONSTRAINTS***
      WRITE(03,133)CABAL(1),OTRO(1),IPLUS,C(1,1)
 133  FORMAT('MATRIX ',A4,',',A3,'=',A1,F15.5)
      DO 590 K=2,MPN
 590  WRITE(03,123)PEN(3),OTRO(K),IPLUS,C(1,K)
      WRITE(03,142)DREI(1),IPLUS,FUNF(6)
      WRITE(03,142)VIER(1),MINUS,FUNF(6)
      DO 105 I=1,MPN
 105  XX(I) = CF(2,I)
      CALL SIGNO(MPN,SIGN,IPLUS,XX,MINUS)
      WRITE(03,140)CABAL(2),OTRO(1),SIGN(1),XX(1)
 140  FORMAT(7X,A4,',',A3,'=',A1,F15.5)
      DO 502 K=2,MPN
 502  WRITE(03,123)PEN(3),OTRO(K),SIGN(K),XX(K)
      WRITE(03,142)DREI(2),IPLUS,FUNF(6)
      WRITE(03,130)DREI(1),MINUS,STIN(1)
 142  FORMAT('          ,',A3,'=',A1,A4)
      WRITE(03,130)VIER(1),IPLUS,STIP(1)
      WRITE(03,142)VIER(2),MINUS,FUNF(6)
      WRITE(03,130)IMP(1),IPLUS,STAX
      DO 85 J=3,N
      JJ = J - 2
      JV = J - 1
      DO 115 I=1,MPN
 115  XX(I) = CF(J,I)
      CALL SIGNO(MPN,SIGN,IPLUS,XX,MINUS)
      WRITE(03,140)CABAL(J),OTRO(1),SIGN(1),XX(1)
      DO 508 K=2,MPN
 508  WRITE(03,123)PEN(3),OTRO(K),SIGN(K),XX(K)
      WRITE(03,123)PEN(3),VIER(JV),IPLUS,STIP(JV)
      WRITE(03,142)DREI(J),IPLUS,FUNF(6)
      WRITE(03,130)DREI(J-1),MINUS,STIN(J-1)
      WRITE(03,142)VIER(J),MINUS,FUNF(6)
      WRITE(03,130)IMP(J-1),IPLUS,STAX
```

```
 85   CONTINUE
C   ***LIQUIDITY CONSTRAINTS***
      SEL = EL
      SRH = SEL*TAX
      WRITE(03,133)LIQ(1),VIER(1),MINUS,SEL
      WRITE(03,142)DREI(1),IPLUS,FUNF(6)
      DO 215 I=2,NN
      L = I - 1
      WRITE(03,133)LIQ(I),VIER(I),MINUS,SEL
      WRITE(03,130)IMP(I),MINUS,SRH
      WRITE(03,142)DREI(I),IPLUS,FUNF(6)
 215  CONTINUE
C   ***ASSETS DEPRECIATION***
      DO 260 J=1,MPN
      KK = II(J) - ID(J)
      COS(J) = C2(1,J)
      DEP(1,J) = 0.0
      DO 260 I=2,N
      IF(I.GT.KK) GO TO 270
      COS(J) = COS(J)*(1.0+CIF(J)*DIR(I))+C2(I,J)
      DEP(I,J) = 0.0
      GO TO 260
 270  CONTINUE
      IF(J.LE.MAN)GO TO 972
      DEP(I,J) = (COS(J)/ID(J))*HH(I,MAN)/HH(KK,MAN)
      GO TO 260
 972  DEP(I,J) = (COS(J)/ID(J))*HH(I,J)/HH(KK,J)
 260  CONTINUE
      DO 300 J=1,MPN
      GC(1,J) = G*C2(1,J)
      GDEP(1,J) = G*DEP(1,J)
      GG(1,J) = GDEP(1,J) - GC(1,J)
      DO 300 I=2,N
      GC(I,J) = GC(I-1,J) + C2(I,J)*G
      GDEP(I,J) = GDEP(I-1,J) + G*DEP(I,J)
      GG(I,J) = GDEP(I,J) - GC(I,J)
      IF(GG(I,J).GT.0.0)GG(I,J)=0.0
 300  CONTINUE
      SG = G
C   ***GEARING CONSTRAINTS***
      DO 150 I=1,N
      L = I - 1
      DO 305 J=1,MPN
 305  GGG(J) = GG(I,J)
      CALL SIGNO(MPN,SIGN,IPLUS,GGG,MINUS)
      WRITE(03,133)GEAR(I),OTRO(1),SIGN(1),GGG(1)
      DO 532 J=2,MPN
 532  WRITE(03,123)PEN(3),OTRO(J),SIGN(J),GGG(J)
  80  FORMAT(7X,A1,A3,'=',2A1)
      WRITE(03,80)PEN(3),VIER(I),IPLUS,FUNF(6)
 150  WRITE(03,130)DREI(I),MINUS,SG
C   ***TAXATION CONSTRAINTS***
      WRITE(03,133)COL(1),OTRO(1),IPLUS,C2(1,1)
      DO 548 K=2,MPN
 548  WRITE(03,123)PEN(3),OTRO(K),IPLUS,C2(1,K)
      WRITE(03,142)IMP(1),IPLUS,FUNF(6)
      WRITE(03,142)COL(1),MINUS,FUNF(6)
      DO 405 I=2,N
      L = I - 1
      LIP = I-2
      DO 425 J=1,MPN
```

```
425   XX(J) = TCF(I,J)
      CALL SIGNO(MPN,SIGN,IPLUS,XX,MINUS)
      WRITE(03,133)COL(I),OTRO(1),SIGN(1),XX(1)
      DO 552 K=2,MPN
552   WRITE(03,123)PEN(3),OTRO(K),SIGN(K),XX(K)
      WRITE(03,123)PEN(3),VIER(L),IPLUS,R22(I-1)
      WRITE(03,142)IMF(I),IPLUS,FUNF(6)
      WRITE(03,142)ALL(I),MINUS,FUNF(6)
      WRITE(03,142)ALL(L),IPLUS,FUNF(6)
405   WRITE(03,130)DREI(L),MINUS,STN(L)
      WRITE(03,146)OTRO(MAN),FUNF(6)
146   FORMAT('MATRIX PRJ,',A3,'=',A1)
      DO 578 L=MP1,MPN
578   WRITE(03,80)PEN(3),OTRO(L),IPLUS,FUNF(6)
C  ***RHS'S CALCULATION***
144   FORMAT('MATRIX ',A4,',',A3,'=',2A1)
      WRITE(03,70)CABAL(1),DD(1)
 70   FORMAT('RHS     ',A4,',RHS=',F15.5)
      DO 805 J=2,M
805   WRITE(03,245)CABAL(J),DD(J)
245   FORMAT(7X,A4,'=',F15.5)
      DO 810 J=1,MM
810   WRITE(03,155)LIG(J)
155   FORMAT(7X,A4,'=0')
      DO 815 J=1,M
815   WRITE(03,155)GEAR(J)
      DO 820 J=1,M
820   WRITE(03,245)COL(J),OCF(J)
      WRITE(03,175)
175   FORMAT('         PRJ=1')
      WRITE(03,330)
330   FORMAT('END***')
C  ***END OF ITERATION. IF MORE ITERATIONS GO TO 500***
      IF(ITER.LT.MNI)GO TO 500
      STOP
      END
C  ***THIS SUBROUTINE CALCULATES THE "SHIFT"***
      SUBROUTINE COM(OI,OIR,OIS,I)
      DIMENSION OI(25,25),OIR(25),OIS(25,25)
      OIS(1,7) = OIR(1)
      DO 5 L=1,13
      IF(I.EQ.2.OR.OI(I,L).EQ.0)GO TO 10
      SHIFT = .7*(OIR(I-1)-OIS(I-1,7))+.3*(OIR(I-2)-OIS(I-2,7))
      GO TO 5
 10   SHIFT = 0.0
  5   OIS(I,L) = OI(I,L) + SHIFT
      RETURN
      END
C  ***THIS SUBROUTINE GENERATES RANDOM NUMBERS***
      SUBROUTINE SMC(CM,OIS,OIR,I)
      DOUBLE PRECISION G05CAF
      DIMENSION CM(25,25),OIS(25,25),OIR(25)
      RR = G05CAF(Z)
      IF(RR.LT.CM(I,1))OIR(I)=OIS(I,1)
      DO 5 K=2,13
  5   IF(RR.GE.CM(I,K-1).AND.RR.LT.CM(I,K)) OIR(I) = OIS(I,K)
      IF(OIR(I).LT..0)OIR(I) = OIR(I-1)
      RETURN
      END
C  ***THIS SUBR. CHECKS SIGNS(IE. "+" OR "-") FOR MATRIX GENERATIO
      SUBROUTINE SIGNO(MPN,SIGN,IPLUS,XX,MINUS)
```

```
      DIMENSION XX(25)
      INTEGER SIGN(25)
      DO 95 I=1,MPN
      SIGN(I) = IPLUS
      IF(XX(I).GE.0) GO TO 95
      XX(I) = ABS(XX(I))
      SIGN(I) = MINUS
   95 CONTINUE
      RETURN
      END
$      LIBRARY L1
$      EXECUTE
$      PRMFL   L1,R,R,NAG/LIB
$      LIMITS  .08,55K,,30K
$      FILE    03,03S,100L
$      FILE    34,01S,80L
$      FILE    45,09S,80L
$      DATA    05
&111810    .13 .15 500000.                    50000.   .5   .75  .4   100000
 0.25;0.17;0.5
 .0      100000. 110000. 120000. 130000. 140000.
&150000. 160000. 170000. 180000. 190000.
 10 12 13 18 16 13 12 11 16 16  7
  7 11 10  8  6 11  9  9  9 17  6
     60000.  30000.     25000.     70000.      20000.
     75000. 100000.     80000.     25000.      60000.
  40000.
&140000.;130000.;;;;;;;;300000.
&120000.;;;;;200000.;280000.;330000.
 ;;140000.;;;;60000.
 ;;;;;;;;110000.
 ;;;160000.;40000.
 ;30000.;;;;;;;;60000.
 ;30000.;;;;68000.;;78000.;;68000.
&43000.;30000.;26000.;;;45000.;14900.;90000.;;68000.
&96000.;29000.;28000.;;;34000.;130000.;103000.;;60000.
 65000.;29000.;34000.;;;17000.;118000.;90000.;;52000.
 72000.;27000.;40000.;;;14000.;90000.;71000.;;45000.
 51000.;25000.;34000.;;;14000.;60000.;51000.;26000.;38000.
 39000.;21000.;28000.;;;57000.;40000.;32000.;26000.;30000.
 16000.;15000.;22000.;;;45000.;28000.;19000.;26000.;22000.
 ;9000.;17000.;64000.;19000.;34000.;17000.;6500.;25000.;7000.
 ;3000.;12000.;56000.;17000.;23000.;12000.;;22000.;52000.
 ;;5500.;45000.;12000.;12000.;;;16000.;45000.
 ;;;32000.;9000.;;;;13000.;38000.
 ;;;13000.;6500.;;;;6000.;30000.
 ;;;15000.;2700.;;;;2700.;22000.
```

```
5520 ;;;58000.;;;;;;15000.
5530 ;;;64000.;;;;;;7000.
5540
5550 ;53000.;;;;;;;;107000.
5560 ;53000.;;;;135000.;;146000.;;120000.
5570$194000.;53000.;56000.;;;90000.;281000.;170000.;;120000.
5580$180000.;51000.;56000.;;;68000.;259000.;194000.;;107000.
5590$170000.;51000.;68000.;;;34000.;236000.;170000.;;93000.
5600$136000.;48000.;79000.;;;20000.;160000.;134000.;;80000.
5610$107000.;43000.;68000.;;;20000.;135000.;97000.;49000.;67000.
5620 73000.;37000.;56000.;;;113000.;79000.;61000.;49000.;53000.
5630 34000.;29000.;45000.;;;90000.;56000.;36000.;49000.;40000.
5640 ;16000.;34000.;121000.;36000.;68000.;34000.;12000.;47000.;10000.
5650 ;3000.;23000.;101000.;32000.;45000.;23000.;;43000.;93000.
5660 ;;11000.;85000.;22000.;23000.;;;37000.;60000.
5670 ;;;61000.;17000.;;;;28000.;67000.
5680 ;;;24000.;12000.;;;;19000.;53000.
5690 ;;;14000.;5000.;;;;9000.;40000.
5700 ;;;111000.;;;;;;27000.
5710 ;;;121000.;;;;;;13000.
5720 120000.
5730 0.0
5740 ;54000.;60000.;54000.;58000.;67000.;63000.
5750 0.0
5760 ;80000.;97000.;110000.;100000.;90000.;80000.
5770 0.0
5780$1.051.0 1.051.1 1.1 1.0 1.101.2 1.0 1.051.1
5790$1.1 1.1 1.1 1.1 1.1 1.1 1.1 1.1 1.1 1.1 1.2
5800$0.95 .9 0.85 .951.0  .9  .851.0 0.8 0.90 .85
5810       100
5820                          .15  .26  .30  .22  .05
5830                         .13  .14  .15  .16  .17
5840                     .07  .12  .25  .27  .20  .06  .03
5850                    .11  .12  .13  .14  .15  .16  .17
5860                .04  .06  .10  .24  .26  .20  .06  .02  .02
5870               .09  .10  .11  .12  .13  .14  .15  .16  .17
5880           .025 .025 .035 .075 .22  .24  .22  .075 .035 .025 .025
5890          .07  .08  .09  .10  .11  .12  .13  .14  .15  .16  .17
5900 .02  .02  .03  .04  .08  .20  .22  .20  .08  .04  .03  .02  .02
5910 .05  .06  .07  .08  .09  .10  .11  .12  .13  .14  .15  .16  .17
5920 .03  .03  .04  .05  .10  .16  .18  .16  .10  .05  .04  .03  .03
5930 .03  .04  .05  .06  .07  .08  .09  .10  .11  .12  .13  .14  .15
5940 .03  .03  .05  .06  .11  .14  .16  .14  .11  .06  .05  .03  .03
5950 .03  .04  .05  .06  .07  .08  .09  .10  .11  .12  .13  .14  .15
5960 .03  .04  .06  .08  .10  .12  .14  .12  .10  .08  .06  .04  .03
5970 .03  .04  .05  .06  .07  .08  .09  .10  .11  .12  .13  .14  .15
5980 .05  .05  .07  .08  .09  .10  .12  .10  .09  .08  .07  .05  .05
5990 .03  .04  .05  .06  .07  .08  .09  .10  .11  .12  .13  .14  .15
6000 .06  .06  .07  .08  .09  .09  .10  .09  .09  .08  .07  .06  .06
6010 .01  .02  .03  .04  .05  .06  .07  .08  .09  .10  .11  .12  .13
6020$      ENDCOPY
6030$      SELECT   PUBLIB/MPS/JCL
6040$      LIMITS   2.2,60K,,50K
6050$      FILE     ZS,,50R
6060$      FILE     ZH,21S,50R
6070$      FILE     M1,,15R
6080$      FILE     M2,,15R
6090$      FILE     M3,,15R
6100$      FILE     I*,01S
6110$      FILE     03,03S
6120$      FILE     C1,04S
```

```
6130$        PRMFL    P*,R,L,TEMP/SG824/HUNEY
6140$        OPTION   FORTRAN
6150$        SELECT   SG824/MPSSUMRD
6160$        EXECUTE
6170$        LIMITS   .15,40K
6180$        PRMFL    01,R,S,TEMP/SG824/HUNEY
6190$        ENDJOB

*OFF
**COST:  $    1.08 TO DATE:  $ 9825.28= 10%
**ON AT 12.509 - OFF AT 12.574 ON  04/15/81
```

```
LINE TERMINATED - CP
```

Table D1 Report Generator

```
*LIST MPSSUMRY

C       MPS OUTPUT EXTRACTION PROGRAM
C
      PARAMETER MO=01
C
      CHARACTER LINE*132,INTGRX1*10,X10*3
C
      CALL FPARAM(1,132)
C
    1 READ(MO,100,END=199) LINE
  100 FORMAT(A132)
      DECODE(LINE,8000) INTGRX1
 8000 FORMAT(T8,A10)
      IF(INTGRX1.NE.'INTGR   X1') GOTO 1
C
      BACKSPACE MO
      BACKSPACE MO
      BACKSPACE MO
      BACKSPACE MO
      BACKSPACE MO
C
      PRINT 601
  601 FORMAT(/////,1X,131('-'))
    2 READ(MO,100,END=199) LINE
      DECODE(LINE,8001) X10
 8001 FORMAT(T16,A3)
C
      PRINT 600,LINE
  600 FORMAT(1X,A132)
      IF(X10.EQ.'X21') GOTO 1
      GOTO 2
C
  199 STOP
      END
```

APPENDIX E

Samples of the simulation and sensitivity analysis outputs

The simulation program produces two types of output. The first one, illustrated in Table E1, contains four rows of information for each iteration: the first row details the iteration number followed by the inflation rate forecast for each year, the second and third rows exhibit the lending (R11) and borrowing (R22) rates respectively; the fourth row shows the dividends to be paid in every year of the planning period. The other type of output, illustrated in Table E2, details the status of the project selection variables, ie whether they are accepted or rejected (value of 1 or 0 under the X-VALUE column); the value of their dual variables (DJ column); their coefficient in the objective function (COST*SCALE column); the ranges of possible values set by the user or by the Branch and Bound algorithm; and the value of the objective function (top of the Table). Notice that the objective function and all the values in columns DJ and COST*SCALE are expressed in millions of pounds; this is due to the fact that all coefficients and RHS's in the input matrix have been scaled down to minimise computer load.

The sensitivity analysis run produces the same two types of output described above. However, because the portfolio of projects remains unchanged during the run, full details about the projects are unnecessary, and that part of the output is reduced, as shown in Table E3 at the end of this Appendix.

Table E1

7810t 04/16/81 at 15.03.13 by sgb24
*JOUT 7810T
function?ACTI 2
function?EPRI 06

1	0.150	0.130	0.113	0.100	0.100	0.087	0.090	0.110	0.104	0.016
R11	0.130	0.120	0.110	0.103	0.103	0.095	0.097	0.109	0.106	0.060
R22	0.150	0.138	0.127	0.118	0.118	0.109	0.111	0.126	0.122	0.080
, DIV	0.058	0.065	0.072	0.080	0.088	0.095	0.104	0.115	0.127	0.129
2	0.150	0.130	0.123	0.097	0.086	0.097	0.101	0.096	0.090	0.090
R11	0.130	0.120	0.116	0.101	0.094	0.101	0.104	0.101	0.097	0.097
R22	0.150	0.138	0.134	0.117	0.109	0.117	0.120	0.116	0.112	0.112
, DIV	0.058	0.065	0.073	0.080	0.087	0.095	0.105	0.115	0.125	0.137
3	0.150	0.150	0.127	0.136	0.101	0.092	0.091	0.083	0.083	0.107
R11	0.130	0.130	0.118	0.123	0.104	0.098	0.098	0.093	0.093	0.109
R22	0.150	0.150	0.136	0.142	0.120	0.114	0.113	0.107	0.107	0.126
, DIV	0.058	0.066	0.075	0.085	0.093	0.102	0.111	0.120	0.130	0.144
4	0.130	0.116	0.137	0.131	0.076	0.052	0.121	0.072	0.040	0.040
R11	0.120	0.112	0.124	0.121	0.090	0.073	0.132	0.100	0.073	0.073
R22	0.138	0.129	0.143	0.139	0.104	0.084	0.152	0.115	0.084	0.084
, DIV	0.057	0.063	0.072	0.081	0.087	0.092	0.103	0.110	0.115	0.119
5	0.150	0.130	0.123	0.107	0.103	0.067	0.066	0.037	0.029	0.007
R11	0.130	0.120	0.116	0.107	0.104	0.082	0.082	0.060	0.060	0.060
R22	0.150	0.138	0.134	0.123	0.120	0.095	0.094	0.080	0.080	0.080
, DIV	0.058	0.065	0.073	0.081	0.089	0.095	0.101	0.105	0.108	0.109
6	0.160	0.117	0.112	0.071	0.102	0.112	0.100	0.079	0.100	0.091
R11	0.135	0.113	0.110	0.086	0.109	0.115	0.108	0.094	0.109	0.103
R22	0.156	0.131	0.127	0.099	0.126	0.133	0.124	0.109	0.126	0.119
, DIV	0.058	0.065	0.072	0.077	0.085	0.095	0.104	0.112	0.123	0.135
7	0.130	0.126	0.134	0.117	0.086	0.133	0.146	0.112	0.029	0.005
R11	0.120	0.117	0.122	0.113	0.095	0.126	0.133	0.115	0.064	0.060
R22	0.138	0.135	0.141	0.130	0.109	0.145	0.154	0.132	0.080	0.080
. DIV	0.057	0.064	0.072	0.081	0.088	0.099	0.114	0.126	0.130	0.131
8	0.150	0.130	0.103	0.093	0.087	0.100	0.101	0.066	0.089	0.045
R11	0.130	0.120	0.105	0.099	0.095	0.103	0.104	0.082	0.100	0.070
R22	0.150	0.138	0.121	0.114	0.109	0.119	0.120	0.095	0.115	0.081
, DIV	0.058	0.065	0.072	0.078	0.085	0.094	0.103	0.110	0.120	0.125
9	0.140	0.123	0.120	0.117	0.160	0.135	0.142	0.064	0.054	0.008
R11	0.125	0.116	0.114	0.112	0.137	0.124	0.128	0.086	0.078	0.060
R22	0.144	0.134	0.132	0.130	0.158	0.143	0.148	0.099	0.090	0.080
, DIV	0.057	0.064	0.072	0.080	0.093	0.105	0.120	0.128	0.135	0.136
10	0.150	0.150	0.147	0.130	0.073	0.092	0.089	0.092	0.037	0.037
R11	0.130	0.130	0.128	0.120	0.088	0.102	0.100	0.102	0.065	0.065
R22	0.150	0.150	0.148	0.138	0.102	0.118	0.115	0.118	0.080	0.080
, DIV	0.058	0.066	0.076	0.086	0.092	0.100	0.109	0.119	0.124	0.128
11	0.130	0.126	0.124	0.110	0.093	0.090	0.114	0.097	0.139	0.042
R11	0.120	0.117	0.116	0.108	0.098	0.096	0.112	0.102	0.128	0.075
R22	0.138	0.135	0.134	0.125	0.113	0.111	0.129	0.118	0.148	0.086
, DIV	0.057	0.064	0.072	0.079	0.087	0.095	0.105	0.116	0.132	0.137
12	0.130	0.136	0.131	0.123	0.100	0.083	0.137	0.185	0.167	0.072
R11	0.120	0.123	0.120	0.116	0.103	0.092	0.128	0.155	0.146	0.096
R22	0.138	0.142	0.139	0.134	0.119	0.107	0.148	0.179	0.169	0.111
. DIV	0.057	0.064	0.073	0.082	0.090	0.097	0.110	0.131	0.153	0.161
13	0.130	0.126	0.124	0.070	0.075	0.025	0.015	0.087	0.156	0.117
R11	0.120	0.117	0.116	0.086	0.090	0.060	0.060	0.233	0.344	0.292
R22	0.138	0.135	0.134	0.099	0.103	0.080	0.080	0.310	0.458	0.389
. DIV	0.057	0.064	0.072	0.077	0.082	0.084	0.086	0.093	0.108	0.120
14	0.130	0.126	0.084	0.102	0.135	0.074	0.061	0.081	0.044	0.058
R11	0.120	0.117	0.091	0.106	0.127	0.092	0.083	0.099	0.072	0.085
R22	0.138	0.135	0.108	0.122	0.146	0.106	0.095	0.114	0.083	0.099
, DIV	0.057	0.064	0.069	0.076	0.086	0.093	0.098	0.106	0.111	0.117

Table E2

PRNAM=21 : : INFCT= 1.2826963+ OBJ=TV : : RHS=RHS : :

COLUMNS

COL	KJ	TYPE	COLUMN NAME	INDIC.	X-VALUE	DJ	COST*SCALE	RANGES	
	46	INTGR	X1	*ATBND	.	.17311265-	.	.0	.0
	47	INTGR	X2	*ATBND	1.00000000+	.04741659-	.00221000-	.0	+1.0000
	48	INTGR	X3	*ATBND	1.00000000+	.06512189-	.01868000-	.0	+1.0000
	49	INTGR	X4		.	.00087757+	.19084000-	.0	+1.0000
	50	INTGR	X5	*ATBND	1.00000000+	.00055679-	.04681000-	.0	+1.0000
	51	INTGR	X6	*ATBND	1.00000000+	.11140088-	.04303000-	.0	+1.0000
	52	INTGR	X7	*ATBND	1.00000000+	.32028917-	.01353000-	.0	+1.0000
	53	INTGR	X8	*ATBND	.	.10672319-	.00198000-	.0	.0
	54	INTGR	X9		.	.01444219+	.06222000-	.0	+1.0000
	55	INTGR	X10	*ATBND	.	.14620635-	.13200000-	.0	.0
	56	INTGR	X11	*BASIS	.	.	.	.0	+1.0000
	57	INTGR	X12		.	.01631250+	.	.0	+1.0000
	58	INTGR	X13		.	.02245467+	.	.0	+1.0000
	59	INTGR	X14		.	.02429652+	.	.0	+1.0000
	60	INTGR	X15		.	.04902106+	.00709000-	.0	+1.0000
	61	INTGR	X16		.	.06664671+	.01108000-	.0	+1.0000
	62	INTGR	X17		.	.08017523+	.02042000-	.0	+1.0000
	63	INTGR	X18		.	.08831639+	.04129000-	.0	+1.0000
	64	INTGR	X19		.	.09020148+	.07904000-	.0	+1.0000
	65	INTGR	X20		.	.10125418+	.09522000-	.0	+1.0000
	66	INTGR	X21	*ATBND	1.00000000+	.06419744-	.09415000-	.0	+1.0000

Table E3

```
COLUMNS                                                                         DJ                        COST*SCALE          RANGES
COL KJ TYPE         COLUMN NAME       INDIC.        X-VALUE                     .18738683-)               .                 .0        .0
    45 INTGR   X1                                   1.00000000+         (       .04980715-)               .00208000-        +1.00000  +1.00000
    46 INTGR   X2
                                                    1.00000000+         (       .06634184-)               .01722000-        +1.00000  +1.00000
    47 INTGR   X3
                                                    .                           .04171314+                .14366000-        .0        .0
    48 INTGR   X4

--------------------------------------------------------------------------------------------------------------------------------------------

PRNAM=48   :   :         INFCT=  1.3031581+  OBJ=TV   :   :          RHS=RHS   :   :

COLUMNS                                                                         DJ                        COST*SCALE          RANGES
COL KJ TYPE         COLUMN NAME       INDIC.        X-VALUE                     .19353742-)               .                 .0        .0
    45 INTGR   X1                                   .                   (       .05370209-)               .00247000-        +1.00000  +1.00000
    46 INTGR   X2                                   1.00000000+         (
                                                    1.00000000+         (       .07734074-)               .02099000-        +1.00000  +1.00000
    47 INTGR   X3
                                                    .                           .00574524+                .22260000-        .0        .0
    48 INTGR   X4

--------------------------------------------------------------------------------------------------------------------------------------------

PRNAM=49   :   :         INFCT=  1.2931809+  OBJ=TV   :   :          RHS=RHS   :   :

COLUMNS                                                                         DJ                        COST*SCALE          RANGES
COL KJ TYPE         COLUMN NAME       INDIC.        X-VALUE                     .18407832-)               .                 .0        .0
    45 INTGR   X1                                   .                   (       .05186102-)               .00245000-        +1.00000  +1.00000
    46 INTGR   X2                                   1.00000000+         (
                                                    1.00000000+         (       .07079280-)               .02074000-        +1.00000  +1.00000
    47 INTGR   X3
                                                    .                           .00243053+                .22092000-        .0        .0
    48 INTGR   X4

--------------------------------------------------------------------------------------------------------------------------------------------

PRNAM=50   :   :         INFCT=  1.2969762+  OBJ=TV   :   :          RHS=RHS   :   :

COLUMNS                                                                         DJ                        COST*SCALE          RANGES
COL KJ TYPE         COLUMN NAME       INDIC.        X-VALUE                     .17343523-)               .                 .0        .0
    45 INTGR   X1                                   .                   (       .04736399-)               .00226000-        +1.00000  +1.00000
    46 INTGR   X2                                   1.00000000+         (
                                                    1.00000000+         (       .06456058-)               .01914000-        +1.00000  +1.00000
    47 INTGR   X3
                                                    .                   (       .00211142-)               .19832000-        .0        .0
    48 INTGR   X4
```

APPENDIX F

PROJECT SELECTION USING SHORT- AND MEDIUM-TERM DEBT

A variant of the example in Chapter IV is presented in this Appendix to illustrate the use of short- and medium-term debt to finance investment. The use of debt for that example, was deemed not viable in most years because of high interest rates and relatively low project profitability. The low profitability was mostly due to the big differences between the escalation rates of costs and prices (see Table 4.4). The inflation sensitivity factors assumed for this example are shown in Table F.1.

Table F.1

Project	1	2	3	4	5	6	7	8	9	10	11
Costs	0.9	1.0	1.0	1.0	1.0	1.0	1.25	1.1	1.0	1.0	1.05
Prices	1.05	.95	.95	1.0	1.0	.95	.75	1.0	.95	1.0	.95
Fixed Investment	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.2

For project 1, prices escalate faster than costs, whereas for project 7, the difference between the escalation rate of costs and prices is more severe than it was in the example of Chapter IV. This, obviously reduces the profitability of project 7. All other parameters are as defined in Chapter IV. The inflation scenario and the corresponding rates of interest are shown in Table F.2 below.

Table F.2

Year	1	2	3	4	5	6	7	8	9	10
Inflation Rate	.15	.13	.123	.107	.113	.104	.09	.086	.067	.076
Lending Rate	.13	.12	.116	.107	.110	.105	.097	.094	.082	.088
Borrowing Rate	.15	.138	.134	.123	.127	.121	.111	.108	.094	.102

Under this inflation scenario and the other circumstances outlined in Chapter IV, the model selected portfolio 2 (see Table 4.7), ie projects 2, 3, 4, 5, 6, 7, 21, with a Terminal Value of 1,285,000 and borrowing in year 2.

For the sensitivity factors shown in Table F.1, the optimal solution obtained in summarised in Table F.3.

Table F.3

Terminal Value = 1.735

Projects Selected: 1, 2, 3, 4, 5, 9, 10 and 21

Year	0	1	2	3	4	5	6	7	8	9	10
Liquidity	.165	.0	.0	.051	.281	.465	.617	.713	.885	.950	
Gearing	.294	.062	.0	.301	.389	.429	.484	.511	.530	.655	.745
Lending	.165	.223	.353	.051	.408	.661	.736	.934	1.092	.950	.901
Borrowing		.297	.470								
Tax Payable	-.570	-.509	-.504	-.102	.336	.522	.316	.588	.553	-.037	.160

The increased project profitability, reflected in the higher Terminal Value, makes borrowing a viable source of finance. Notice that, because debt is obtained by means of renewable one-year contracts, the amount borrowed in year 2 (£470,000) represents the sum of the amount borrowed in year 1 (£297,000) plus the extra debt actually raised in year 2 (£470,000 - £297,000 = £173,000). The debt contracted in year 1 is not repaid in year 2; instead, it is renewed and increased by £173,000, and the total £470,000 are repaid in year 3. Interest on debt is paid annually. The money borrowed makes liquidity constraints in years 1 and 2, and the gearing constraint in year 2 binding. While, theoretically this is the optimal situation, binding liquidity and gearing constraints are indicative of the fact that the company has stretched its financial resources to the limit. Sensitivity analysis of the optimum is necessary to ensure that fluctuations in the data would not cause the company cash difficulties.

The situation portrayed in this example is not representative of the general case. Firms do not always rely entirely on short-term debt as the only source of external finance. Even though this type of debt is generally less expensive and more flexible than medium- or long-term debt, it subjects the firm to greater risks than other types of debt. One of the reasons for this, as stated by Weston et al (1979), is that, if the firm borrows heavily on a short-term basis, it may find that it has difficulty in repaying the loan and thus develops liquidity problems, or it may find that the lender is unwilling to extend the loan.

In an attempt to bring more realism into the analysis, the model of Chapter III has been improved to include one form of medium-term debt. It is assumed that short-term debt cannot be obtained in two years in succession. However a slightly more expensive form of debt can be obtained for five years, at a fixed interest rate and repayment in five equal instalments. The annuities are calculated using equation F-1 below (Weston et al, 1979):

$$a = \frac{P_t}{\frac{1}{(1 + r)} + \frac{1}{(1 + r)^2} + \ldots + \frac{1}{(1 + r)^5}} \qquad \text{(F-1)}$$

where a is the annual payment

P_t is the amount borrowed in year t

r is the fixed interest rate on the loan

r is derived from the short-term borrowing interest rate, r_t, in the year the loan P_t is taken.

$$r = r_t + .01 \qquad \text{(F-2)}$$

As in the case of short-term debt, interest payments are tax-deductable. In all other respects, the model is as described in Chapter III.

This improved model was used to analyse the example discussed above. The optimal solution is shown in Table F.4.

Table F.4 (£000s)

Terminal Value = 1.821

Projects Selected: 1, 3, 4, 5, 6, 9, 10, 21

Year	0	1	2	3	4	5	6	7	8	9	10
Liquidity	.230	.0	.0	.102	.322	.470	.583	.660	.849	.967	
Gearing	.268	.236	.0	.182	.312	.391	.470	.499	.528	.667	.779
Lending	.230	.020	.239	.144	.478	.701	.710	.869	1.087	.981	.985
Short-term debt			.263								
Medium-term debt		.089	.095								
Tax Payable	-.440	-.629	-.564	-.143	.303	.504	.281	.557	.633	.036	.266

Notice that project 2 has now been replaced by project 6. This project requires a high initial investment in years 1 and 2 (see

Table F.5), but produces a better return than project 2. If all the debt is taken on a short-term basis, full repayment will be required in year 3, making investment in project 3 impossible. The combined use of short- and medium-term debt, however, does not require full repayment in year 3. This has a less damaging effect on liquidity and the joint investment in projects 3 and 6 becomes possible.

Table F.5 Fixed Cash Outlays (£000s)

Project	1	2	3	4	5	6	7	8	9	10	21
Year:											
0	70	65								150	
1	140	48				117	163	192		115	
2	70		93			60	203	136		63	
3			67								
4											
5											
6									116		
7									86		
8											
9				219	55						
10				117	24						195

Projects 1, 2, 3, 6, 7, 8 and 10 compete for the limited funds during the first four years of the planning period. Projects 1 and 10 have priority over all the others, because of their higher profitabilities. The projects with lowest contribution to Terminal Value, in descending order, are 7, 6, 3, 8 and 2. Projects 7 and 8 are rejected because they require very high initial investments, which would absorb capital that could be invested in a set of projects (eg 6 and 2, or 6 and 3) with higher joint profitability. Thus, projects 6, 3 and 2 compete for the funds left after investment in 1 and 10. In the case portrayed in Table F.3, project 2 is preferred to project 6 due to financial constraints. The new form of financing allows, then, for a more profitable investment strategy. As shown in Table F.2, inflation and interest rates are falling during the planning period. Under these circumstances, debt contracted at a fixed interest rate for a period of five years, would not seem to be an attractive choice, and capital in year 3 would be borrowed on a short-term basis only. However, the more favourable timing of cash flows it produces, offsets the relatively higher cost, and a mixture of medium- and short-term debt is the optimal choice.

Table F.6

Project	Horizon Values (£000s)	
	(Short-Term Debt)	(Short and Medium Term Debt)
1	149	245
2	10	45
3	91	111
4	101	110
5	13	15
6	75	129
7	36	148
8		105
9	50	49
10	171	252
21	124	125

Table F.6 shows the horizon value of each project under the two different financing strategies (ie the negative of the dual values for the project selection variables). Observe that those projects competing for funds in years 1 and 2, when the debt is raised (ie projects 1, 2, 3, 6, 7, 8 and 10), have the largest differences in horizon values. Projects 4, 5, 9 and 21, on the other hand, have very similar horizon values for the two financing strategies. This indicates that a project's contribution to total Terminal Value depends on the form of finance used.

APPENDIX G

MIP output and input matrix

(1) MIP output

As mentioned in Appendix D, the MIP output contains details about different aspects of the solution process that are not of direct interest to the user. The relevant part of this output is shown in Tables G1 (a) and (b). The top of Table G1 (a) is similar to the first part of the simulation output described in Appendix E. This part of the output is not actually produced by the MPS package, but by the Matrix Generator. The centre of Table G1 (a) contains details about the rows or logical variables (ie the objective function and the constraints): the L-VALUE column shows the slack value at the optimum; the PI column exhibits the value of their dual variables; and the RHS column contains the right hand side values of the constraints. The bottom of Table G1 (a) and Table G1 (b) contain the optimal values of the columns or structural variables (X-VALUE column); the value of their dual variables (DJ column); and the range of possible values for the integer variables.

Index of Variables

Rows		Columns	
TV	Objective Function	X1-X21	Projects
CB1-CB11	Cash Balance	Y0-Y10	Lending
LQ1-LQ10	Liquidity	Z0-Z10	Borrowing
GR1-GR11	Gearing	A0-A10	Allowances
TX1-TX11	Taxation	T0-T10	Taxable income
PRJ	Constraint on projects X11-X21		

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FUNCTION?EPRI 06

1	0.140	0.133	0.117	0.123	0.134	0.057	0.121	0.087	0.047	0.033
R11	0.125	0.121	0.112	0.116	0.126	0.080	0.134	0.111	0.081	0.066
R22	0.144	0.140	0.130	0.134	0.141	0.042	0.154	0.128	0.043	0.060
DIV	0.057	0.065	0.072	0.081	0.092	0.097	0.104	0.116	0.124	0.126

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INFCT= 1.32422068+ OBJ=IV : : RHS=RHS : :

PRNAM=1 : :

ROWS

ROW KJ	TYPE	ROW NAME	INDIC.	L-VALUE	PI	RHS	RANGES
1	FREE	IV	*BASIS	1.32422080-	1.00000000-	.	
2	PLUS	CH1		.	1.67344620-	.45000000+	
3	PLUS	CH2		.	1.73820339+	.04300000+	
4	PLUS	CB3		.	1.62234693+	.04542000+	
5	PLUS	CB4		.	1.47051474+	.04706000+	
6	PLUS	CB5		.	1.36805682+	.04849000+	
7	PLUS	CH6		.	1.30763297+	.04813000+	
8	PLUS	CH7		.	1.22975408+	.05240000+	
9	PLUS	CH8		.	1.17951300+	.05115000+	
10	PLUS	CH9		.	1.10179168+	.05180000+	
11	PLUS	CH10		.	1.04155444+	.05612000+	
12	PLUS	CH11		.	1.00000000+	.06203000+	
13	MINUS	LW1	*BASIS	.38500000-	.	.	
14	MINUS	LW2	*BASIS	.15240001-	.05783264-	.	
15	MINUS	LQ3		.	.	.	
16	MINUS	LQ4	*BASIS	.15406957-	.	.	
17	MINUS	LQ5	*BASIS	.37572300-	.	.	
18	MINUS	LW6	*BASIS	.53302813-	.	.	
19	MINUS	LW7	*BASIS	.66847720-	.	.	
20	MINUS	LW8	*BASIS	.79672419-	.	.	
21	MINUS	LW9	*BASIS	.95354784-	.	.	
22	MINUS	LW10	*BASIS	1.00649385-	.	.	
23	PLUS	GW1	*BASIS	.20600000+	.	.	
24	PLUS	GW2	*BASIS	.32909400+	.	.	
25	PLUS	GW3	*BASIS	.04863365+	.	.	
26	PLUS	GW4	*BASIS	.37896043+	.	.	
27	PLUS	GW5	*BASIS	.47028544+	.	.	
28	PLUS	GW6	*BASIS	.47548013+	.	.	
29	PLUS	GW7	*BASIS	.46747203+	.	.	
30	PLUS	GW8	*BASIS	.43406074+	.	.	
31	PLUS	GW9	*BASIS	.47040020+	.	.	
32	PLUS	GW10	*BASIS	.53913150+	.	.	
33	PLUS	GW11	*BASIS	.04349474+	.84402812-	.	
34	ZERO	IX1		.	.84402812-	.10000000+	
35	ZERO	IX2		.	.84402812-	.11000000+	
36	ZERO	IX3		.	.84402812-	.12000000+	
37	ZERO	IX4		.	.85341644-	.13000000+	
38	ZERO	IX5		.	.81467455-	.14000000+	
39	ZERO	IX6		.	.58975684-	.15000000+	
40	ZERO	IX7		.	.55089585-	.16000000+	
41	ZERO	IX8		.	.52077473-	.17000000+	
42	ZERO	IX9		.	.50000000-	.18000000+	
43	ZERO	IX10		.	.46399400-	.19000000+	
44	ZERO	IX11		.	.02665141+	1.00000000+	
45	PLUS	PRJ		.			

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INFCT= 1.32422066+ OBJ=IV : : RHS=RHS : :

PRNAM=1 : :

COLUMNS

COL KJ	TYPE	COLUMN NAME	INDIC.	X-VALUE	DJ	COST*SCALE	RANGES	
46	INTGR	X1	*ATBND	1.00000000+	.18805350-	.00235000-	.0	+1.00000
47	INTGR	X2	*ATBND	1.00000000+	.05092933-	.01968000-	.0	+1.00000
48	INTGR	X3	*ATBND	1.00000000+	.07252912-	.20727000-	.0	+1.00000
49	INTGR	X4		.	.00075195+	.05159000-	.0	+1.00000
50	INTGR	X5	*ATBND	1.00000000+	.00188601-	.08674000-	.0	+1.00000
51	INTGR	X6	*ATBND	1.00000000+	.12545564-	.01424000-	.0	+1.00000
52	INTGR	X7	*ATBND	1.00000000+	.34185134-	.00218000-	.0	+1.00000
53	INTGR	X8	*ATBND	.	.12000682-	.00628000-	.0	+1.00000
54	INTGR	X9		.	.00743692+	.14212000-	.0	+1.00000
55	INTGR	X10	*ATBND	.	.15860380-	.	.0	
56	INTGR	X11	*BASIS	.	.			

57	INTGR	X12		.	.01580465+	.	.0	+1.00000
58	INTGR	X13		.	.02035677+	.	.0	+1.00000
59	INTGR	X14		.	.02982627+	.	.0	+1.00000
60	INTGR	X15		.	.05299030+	.00738000-	.0	+1.00000
61	INTGR	X16		.	.07471533+	.00942000-	.0	+1.00000
62	INTGR	X17		.	.08848177+	.01705000-	.0	+1.00000
63	INTGR	X18		.	.11340072+	.03717000-	.0	+1.00000
64	INTGR	X19		.	.12021189+	.07622000-	.0	+1.00000
65	INTGR	X20		.	.12341803+	.04160000-	.0	+1.00000
66	INTGR	X21	*ATBND	1.00000000+	.05600221-	.00872000-	.0	+1.00000

67	PLUS	Y0	*BASIS	.38500000+	.	.
68	PLUS	Y1	*BASIS	.15246001+	.	.
69	PLUS	Y2	*BASIS	.33153895+	.	.
70	PLUS	Y3	*BASIS	.17436731+	.	.
71	PLUS	Y4	*BASIS	.54123840+	.	.
72	PLUS	Y5	*BASIS	.71452034+	.	.
73	PLUS	Y6	*BASIS	.86230507+	.	.
74	PLUS	Y7	*BASIS	.95829196+	.	.
75	PLUS	Y8	*BASIS	1.16034500+	.	.
76	PLUS	Y9	*BASIS	1.23987895+	.	.
77	PLUS	Y10	*BASIS	1.15301196+	.	1.00000000-
78	PLUS	Z0		.	.02088347+	.
79	PLUS	Z1		.	.01782375+	.
80	PLUS	Z2	*BASIS	.44205193+	.	.
81	PLUS	Z3		.	.01268591+	.
82	PLUS	Z4		.	.01734681+	.
83	PLUS	Z5		.	.01209842+	.
84	PLUS	Z6		.	.00773198+	.
85	PLUS	Z7		.	.01196303+	.
86	PLUS	Z8		.	.00926607+	.
87	PLUS	Z9		.	.00639335+	.
88	PLUS	Z10		.	.00871001+	1.00871001+
89	PLUS	A0	*BASIS	.13000000+	.	.
90	PLUS	A1	*BASIS	.50838999+	.	.
91	PLUS	A2	*BASIS	.53348298+	.	.
92	PLUS	A3		.	.04011182+	.
93	PLUS	A4		.	.03903694+	.
94	PLUS	A5		.	.02512272+	.

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PRNAME1 : : INFCT= 1.3242266+ OBJ=TV : : RHS=RHS : :

COLUMNS

97	PLUS	A8		.	.02077472+	.
98	PLUS	A9		.	.01601000+	.
99	PLUS	A10		.	.08399000+	.
100	PLUS	T0		.	.17507354+	.
101	PLUS	T1		.	.11714636+	.
102	PLUS	T2		.	.06241653+	.
103	PLUS	T3	*BASIS	.05412731+	.	.
104	PLUS	T4	*BASIS	.44137494+	.	.
105	PLUS	T5	*BASIS	.48397920+	.	.
106	PLUS	T6	*BASIS	.47287434+	.	.
107	PLUS	T7	*BASIS	.43083404+	.	.
108	PLUS	T8	*BASIS	.55146071+	.	.
109	PLUS	T9	*BASIS	.40769365+	.	.
110	PLUS	T10	*BASIS	.10802985+	.	.08399000+
111	RHS	RHS				

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PRNAME1 : : INFCT= 1.3242266+ OBJ=TV : : RHS=RHS : :

57 ENDLP . 15:37 21

RESTINT POSITION REACHED, USED 4 LLINKS ON 3V FILE 6 POSITIONS IN SVTAB.

END OF 52 .
FUNCTION?HOLD

(2) Input Matrix

The input matrix is shown here in the format required by the MPS routine. The variables names are as previously defined. Before actually stating the objective function and the constraints, the variables are defined using the terminology described below:

LGL	Row or Logical variable (includes the objective function)
STR	Structural variable
(F)	Objective function
(P)	$\leq$ constraint or positive continuous variable
(M)	$\geq$ constraint
(Z)	equality constraint
(I)	0, 1 integer variable.

*CONVERT DAT

Listing of the Input Matrix

```
NAME      1
* MATRIZ NUMERO   1
LGL     TV(F)
LGL     CR1 (P)
LGL     CR2 (P)
LGL     CR3 (P)
LGL     CR4 (P)
LGL     CR5 (P)
LGL     CR6 (P)
LGL     CR7 (P)
LGL     CR8 (P)
LGL     CR9 (P)
LGL     CR10(P)
LGL     CR11(P)
LGL     LG1 (M)
LGL     LG2 (M)
LGL     LG3 (M)
LGL     LG4 (M)
LGL     LG5 (M)
LGL     LG6 (M)
LGL     LG7 (M)
LGL     LG8 (M)
LGL     LG9 (M)
LGL     LG10(M)
LGL     GR1 (P)
LGL     GR2 (P)
LGL     GR3 (P)
LGL     GR4 (P)
LGL     GR5 (P)
LGL     GR6 (P)
LGL     GR7 (P)
LGL     GR8 (P)
LGL     GR9 (P)
LGL     GR10(P)
LGL     GR11(P)
LGL     TX1 (Z)
LGL     TX2 (Z)
LGL     TX3 (Z)
LGL     TX4 (Z)
LGL     TX5 (Z)
LGL     TX6 (Z)
LGL     TX7 (Z)
LGL     TX8 (Z)
LGL     TX9 (Z)
LGL     TX10(Z)
LGL     TX11(Z)
LGL     PRJ(P)
STR     X1 (I)
STR     X2 (I)
STR     X3 (I)
STR     X4 (I)
STR     X5 (I)
STR     X6 (I)
STR     X7 (I)
STR     X8 (I)
STR     X9 (I)
STR     X10(I)
STR     X11(I)
STR     X12(I)
STR     X13(I)
STR     X14(I)
STR     X15(I)
STR     X16(I)
STR     X17(I)
STR     X18(I)
STR     X19(I)
STR     X20(I)
STR     X21(I)
STR     Y0 (P)
STR     Y1 (P)
STR     Y2 (P)
STR     Y3 (P)
STR     Y4 (P)
STR     Y5 (P)
STR     Y6 (P)
STR     Y7 (P)
STR     Y8 (P)
STR     Y9 (P)
STR     Y10(P)
STR     Z0 (P)
STR     Z1 (P)
STR     Z2 (P)
STR     Z3 (P)
STR     Z4 (P)
STR     Z5 (P)
STR     Z6 (P)
STR     Z7 (P)
STR     Z8 (P)
STR     Z9 (P)
STR     Z10(P)
STR     A0 (P)
STR     A1 (P)
STR     A2 (P)
STR     A3 (P)
STR     A4 (P)
STR     A5 (P)
STR     A6 (P)
STR     A7 (P)
STR     A8 (P)
STR     A9 (P)
STR     A10(P)
STR     T0 (P)
STR     T1 (P)
STR     T2 (P)
STR     T3 (P)
STR     T4 (P)
STR     T5 (P)
STR     T6 (P)
STR     T7 (P)
STR     T8 (P)
STR     T9 (P)
STR     T10(P)
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          ,X2 =+             0.00235
          ,X3 =+             0.01986
          ,X4 =+             0.20727
          ,X5 =+             0.05159
          ,X6 =+             0.04674
          ,X7 =+             0.01424
          ,X8 =+             0.00214
          ,X9 =+             0.00626
          ,X10=+             0.14212
          ,X11=+             0.
          ,X12=+             0.
          ,X13=+             0.
          ,X14=+             0.
          ,X15=+             0.00736
          ,X16=+             0.00942
          ,X17=+             0.01705
          ,X18=+             0.03717
          ,X19=+             0.07622
          ,X20=+             0.09154
          ,X21=+             0.08672
          ,Y10=1
          ,Z10=-             1.00671
          ,T10=-             0.46394
MATRIX  CR1 ,X1 =+             0.0700
          ,X2 =+             0.06500
          ,X3 =+             0.
          ,X4 =+             0.
          ,X5 =+             0.
          ,X6 =+             0.
          ,X7 =+             0.
          ,X8 =+             0.
          ,X9 =+             0.
          ,X10=+             0.15000
          ,X11=+             0.05000
          ,X12=+             0.
          ,X13=+             0.
          ,X14=+             0.
          ,X15=+             0.
          ,X16=+             0.
```

```
,X17=+          0.
,X18=+          0.
,X19=+          0.
,X20=+          0.
,X21=+          0.
,Y0 =+1
,Z0 =-1
CB2 ,X1 =+          0.13924
,X2 =+          0.04863
,X3 =+          0.
,X4 =+          0.
,X5 =+          0.
,X6 =+          0.11540
,X7 =+          0.16156
,X8 =+          0.19041
,X9 =+          0.
,X10=+          0.11676
,X11=+          0.04456
,X12=+          0.07008
,X13=+          0.
,X14=+          0.
,X15=+          0.
,X16=+          0.
,X17=+          0.
,X18=+          0.
,X19=+          0.
,X20=+          0.
,X21=+          0.
,Y1 =+1
,Y0 =-          1.13000
,Z0 =+          1.15000
,Z1 =-1
,T0 =+          0.50000
CB3 ,X1 =+          0.06924
,X2 =-          0.02706
,X3 =+          0.09260
,X4 =+          0.
,X5 =+          0.
,X6 =+          0.06064
,X7 =+          0.20124
,X8 =+          0.13660
,X9 =+          0.
,X10=-          0.05611
,X11=-          0.03652
,X12=+          0.05464
,X13=+          0.08126
,X14=+          0.
,X15=+          0.
,X16=+          0.
,X17=+          0.
,X18=+          0.
,X19=+          0.
,X20=+          0.
,X21=+          0.
,Z1 =+          1.14400
,Y2 =+1
,Y1 =-          1.12480
,Z2 =-1
,T1 =+          0.50000
CB4 ,X1 =-          0.09302
,X2 =-          0.02959
,X3 =+          0.06920
,X4 =+          0.
,X5 =+          0.
,X6 =-          0.06780
,X7 =-          0.23044
,X8 =-          0.09775
,X9 =+          0.
,X10=-          0.06529
,X11=-          0.06276
,X12=-          0.03803
,X13=+          0.06602
,X14=+          0.09267
,X15=+          0.
,X16=+          0.
,X17=+          0.
,X18=+          0.
,X19=+          0.
,X20=+          0.
,X21=+          0.
,Z2 =+          1.13968
,Y3 =+1
,Y2 =-          1.12106
,Z3 =-1
,T2 =+          0.50000
CB5 ,X1 =-          0.12731
,X2 =-          0.03131
,X3 =-          0.03719
,X4 =+          0.
,X5 =+          0.
,X6 =-          0.05353
,X7 =-          0.20333
,X8 =-          0.12189
,X9 =+          0.
,X10=-          0.06652
,X11=-          0.04172
,X12=-          0.00679
,X13=-          0.03930
,X14=+          0.07790
,X15=+          0.10635
,X16=+          0.
,X17=+          0.
,X18=+          0.
,X19=+          0.
,X20=+          0.
,X21=+          0.
,Z3 =+          1.12960
,Y4 =+1
,Y3 =-          1.11232
,Z4 =-1
,T3 =+          0.50000
CB6 ,X1 =-          0.13647
,X2 =-          0.03387
,X3 =-          0.04586
,X4 =+          0.
,X5 =+          0.
,X6 =-          0.03525
,X7 =-          0.16820
,X8 =-          0.12610
,X9 =+          0.
,X10=-          0.06368
,X11=-          0.02378
```

```
,X12=-         0.04262
,X13=-         0.07121
,X14=-         0.04040
,X15=+         0.09244
,X16=+         0.12345
,X17=+         0.
,X18=+         0.
,X19=+         0.
,X20=+         0.
,X21=+         0.
,Z4  =+        1.13359
,Y5  =+1
,Y4  =-        1.11577
,Z5  =-1
,T4  =+        0.50000
CB7 ,X1  =-        0.12078
,X2  =-        0.03518
,X3  =-        0.05585
,X4  =+        0.
,X5  =+        0.
,X6  =-        0.01422
,X7  =-        0.14446
,X8  =-        0.11223
,X9  =+        0.11366
,X10=-         0.05818
,X11=-         0.01366
,X12=-         0.02393
,X13=-         0.04431
,X14=-         0.07411
,X15=-         0.04142
,X16=+         0.11011
,X17=+         0.13190
,X18=+         0.
,X19=+         0.
,X20=+         0.
,X21=+         0.
,Z5  =+        1.14075
,Y6  =+1
,Y5  =-        1.12199
,Z6  =-1
,T5  =+        0.50000
CB8 ,X1  =-        0.11226
,X2  =-        0.03300
,X3  =-        0.05689
,X4  =+        0.
,X5  =+        0.
,X6  =-        0.00953
,X7  =-        0.11293
,X8  =-        0.09034
,X9  =+        0.09170
,X10=-         0.05152
,X11=-         0.01319
,X12=-         0.01065
,X13=-         0.02193
,X14=-         0.04353
,X15=-         0.07751
,X16=-         0.04145
,X17=+         0.11996
,X18=+         0.15105
,X19=+         0.
,X20=+         0.
,X21=+         0.
,Z6  =+        1.09223
,Y7  =+1
,Y6  =-        1.07993
,Z7  =-1
,T6  =+        0.50000
CB9 ,X1  =-        0.08115
,X2  =-        0.03174
,X3  =-        0.05066
,X4  =+        0.
,X5  =+        0.
,X6  =-        0.07776
,X7  =-        0.07744
,X8  =-        0.06538
,X9  =-        0.03606
,X10=-         0.04456
,X11=+         0.
,X12=-         0.01388
,X13=-         0.00851
,X14=-         0.02015
,X15=-         0.04416
,X16=-         0.08094
,X17=-         0.04186
,X18=+         0.14053
,X19=+         0.16682
,X20=+         0.
,X21=+         0.
,Z7  =+        1.15436
,Y8  =+1
,Y7  =-        1.13377
,Z8  =-1
,T7  =+        0.50000
CB10,X1  =-        0.04700
,X2  =-        0.02625
,X3  =-        0.04377
,X4  =+        0.21590
,X5  =+        0.05396
,X6  =-        0.09452
,X7  =-        0.05253
,X8  =-        0.04154
,X9  =-        0.03720
,X10=-         0.03726
,X11=+         0.
,X12=+         0.
,X13=-         0.01436
,X14=-         0.00727
,X15=-         0.01942
,X16=-         0.04484
,X17=-         0.08310
,X18=-         0.04217
,X19=+         0.15723
,X20=+         0.17623
,X21=+         0.
,Z8  =+        1.12635
,Y9  =+1
,Y8  =-        1.11122
,Z9  =-1
,T8  =+        0.50000
CB11,X1  =-        0.01226
,X2  =-        0.01670
,X3  =-        0.03409
```

```
              ,X4  =+          0.14235
              ,X5  =+          0.02898
              ,X6  =-          0.07930
              ,X7  =-          0.03519
              ,X8  =-          0.01937
              ,X9  =-          0.03680
              ,X10=-           0.01506
              ,X11=+           0.
              ,X12=+           0.
              ,X13=+           0.
              ,X14=-           0.01461
              ,X15=-           0.00618
              ,X16=-           0.01865
              ,X17=-           0.04497
              ,X18=-           0.08442
              ,X19=-           0.04219
              ,X20=+           0.16737
              ,X21=+           0.18321
              ,Z9  =+          1.09293
              ,Y10=+1
              ,Y9  =-          1.08054
              ,Z10=-1
              ,T9  =+          0.50000
MATRIX LG1  ,Z0  =-                0.75000
              ,Y0  =+1
MATRIX LG2  ,Z1  =-                0.75000
              ,T1  =-          0.37500
              ,Y1  =+1
MATRIX LG3  ,Z2  =-                0.75000
              ,T2  =-          0.37500
              ,Y2  =+1
MATRIX LG4  ,Z3  =-                0.75000
              ,T3  =-          0.37500
              ,Y3  =+1
MATRIX LG5  ,Z4  =-                0.75000
              ,T4  =-          0.37500
              ,Y4  =+1
MATRIX LG6  ,Z5  =-                0.75000
              ,T5  =-          0.37500
              ,Y5  =+1
MATRIX LG7  ,Z6  =-                0.75000
              ,T6  =-          0.37500
              ,Y6  =+1
MATRIX LG8  ,Z7  =-                0.75000
              ,T7  =-          0.37500
              ,Y7  =+1
MATRIX LG9  ,Z8  =-                0.75000
              ,T8  =-          0.37500
              ,Y8  =+1
MATRIX LG10,Z9  =-                 0.75000
              ,T9  =-          0.37500
              ,Y9  =+1
MATRIX GR1  ,X1  =-                0.05600
              ,X2  =-          0.05200
              ,X3  =+          0.
              ,X4  =+          0.
              ,X5  =+          0.
              ,X6  =+          0.
              ,X7  =+          0.
              ,X8  =+          0.
              ,X9  =+          0.
```

```
              ,X10=-           0.12000
              ,X11=-           0.04600
              ,X12=+           0.
              ,X13=+           0.
              ,X14=+           0.
              ,X15=+           0.
              ,X16=+           0.
              ,X17=+           0.
              ,X18=+           0.
              ,X19=+           0.
              ,X20=+           0.
              ,X21=+           0.
              ,Z0  =+1
              ,Y0  =-          0.40000
MATRIX GR2  ,X1  =-                0.11139
              ,X2  =-          0.04654
              ,X3  =+          0.
              ,X4  =+          0.
              ,X5  =+          0.
              ,X6  =-          0.09232
              ,X7  =-          0.12925
              ,X8  =-          0.15233
              ,X9  =+          0.
              ,X10=-           0.11185
              ,X11=-           0.03866
              ,X12=-           0.05606
              ,X13=+           0.
              ,X14=+           0.
              ,X15=+           0.
              ,X16=+           0.
              ,X17=+           0.
              ,X18=+           0.
              ,X19=+           0.
              ,X20=+           0.
              ,X21=+           0.
              ,Z1  =+1
              ,Y1  =-          0.40000
MATRIX GR3  ,X1  =-                0.11139
              ,X2  =-          0.04029
              ,X3  =-          0.07406
              ,X4  =+          0.
              ,X5  =+          0.
              ,X6  =-          0.08270
              ,X7  =-          0.16100
              ,X8  =-          0.13293
              ,X9  =+          0.
              ,X10=-           0.10252
              ,X11=-           0.02782
              ,X12=-           0.04523
              ,X13=-           0.06501
              ,X14=+           0.
              ,X15=+           0.
              ,X16=+           0.
              ,X17=+           0.
              ,X18=+           0.
              ,X19=+           0.
              ,X20=+           0.
              ,X21=+           0.
              ,Z2  =+1
              ,Y2  =-          0.40000
MATRIX GR4  ,X1  =-                0.08921
```

```
           ,X2  =-        0.03323
           ,X3  =-        0.06572
           ,X4  =+        0.
           ,X5  =+        0.
           ,X6  =-        0.07184
           ,X7  =-        0.13843
           ,X8  =-        0.11103
           ,X9  =+        0.
           ,X10=-         0.09198
           ,X11=-         0.01546
           ,X12=-         0.03287
           ,X13=-         0.05266
           ,X14=-         0.07414
           ,X15=+         0.
           ,X16=+         0.
           ,X17=+         0.
           ,X18=+         0.
           ,X19=+         0.
           ,X20=+         0.
           ,X21=+         0.
           ,Z3  =+1
           ,Y3  =-        0.40000
MATRIX GR5 ,X1  =-            0.06402
           ,X2  =-        0.02522
           ,X3  =-        0.05622
           ,X4  =+        0.
           ,X5  =+        0.
           ,X6  =-        0.05951
           ,X7  =-        0.11282
           ,X8  =-        0.08617
           ,X9  =+        0.
           ,X10=-         0.08001
           ,X11=-         0.00128
           ,X12=-         0.01669
           ,X13=-         0.03647
           ,X14=-         0.05996
           ,X15=-         0.08508
           ,X16=+         0.
           ,X17=+         0.
           ,X18=+         0.
           ,X19=+         0.
           ,X20=+         0.
           ,X21=+         0.
           ,Z4  =+1
           ,Y4  =-        0.40000
MATRIX GR6 ,X1  =-            0.03513
           ,X2  =-        0.01603
           ,X3  =-        0.04533
           ,X4  =+        0.
           ,X5  =+        0.
           ,X6  =-        0.04537
           ,X7  =-        0.08343
           ,X8  =-        0.05764
           ,X9  =+        0.
           ,X10=-         0.06826
           ,X11=+         0.
           ,X12=-         0.00623
           ,X13=-         0.02201
           ,X14=-         0.04350
           ,X15=-         0.06662
           ,X16=-         0.09676
           ,X17=+         0.
           ,X18=+         0.
           ,X19=+         0.
           ,X20=+         0.
           ,X21=+         0.
           ,Z5  =+1
           ,Y5  =-        0.40000
MATRIX GR7 ,X1  =-            0.00442
           ,X2  =-        0.00625
           ,X3  =-        0.03376
           ,X4  =+        0.
           ,X5  =+        0.
           ,X6  =-        0.03034
           ,X7  =-        0.05220
           ,X8  =-        0.02733
           ,X9  =-        0.09094
           ,X10=-         0.05169
           ,X11=+         0.
           ,X12=+         0.
           ,X13=-         0.00443
           ,X14=-         0.02591
           ,X15=-         0.05104
           ,X16=-         0.08116
           ,X17=-         0.10552
           ,X18=+         0.
           ,X19=+         0.
           ,X20=+         0.
           ,X21=+         0.
           ,Z6  =+1
           ,Y6  =-        0.40000
MATRIX GR8 ,X1  =+            0.
           ,X2  =+        0.
           ,X3  =-        0.02064
           ,X4  =+        0.
           ,X5  =+        0.
           ,X6  =-        0.01330
           ,X7  =-        0.01681
           ,X8  =+        0.
           ,X9  =-        0.07949
           ,X10=-         0.03516
           ,X11=+         0.
           ,X12=+         0.
           ,X13=+         0.
           ,X14=-         0.00577
           ,X15=-         0.03090
           ,X16=-         0.06104
           ,X17=-         0.08536
           ,X18=-         0.12084
           ,X19=+         0.
           ,X20=+         0.
           ,X21=+         0.
           ,Z7  =+1
           ,Y7  =-        0.40000
MATRIX GR9 ,X1  =+            0.
           ,X2  =+        0.
           ,X3  =-        0.00627
           ,X4  =+        0.
           ,X5  =+        0.
           ,X6  =+        0.
           ,X7  =+        0.
           ,X8  =+        0.
```

```
         ,X9 =-          0.06695
         ,X10=-          0.01705
         ,X11=+          0.
         ,X12=+          0.
         ,X13=+          0.
         ,X14=+          0.
         ,X15=-          0.00865
         ,X16=-          0.03679
         ,X17=-          0.06314
         ,X18=-          0.09860
         ,X19=-          0.13346
         ,X20=+          0.
         ,X21=+          0.
         ,Z8 =+1
         ,Y8 =-          0.40000
MATRIX GR10,X1 =+           0.
         ,X2 =+          0.
         ,X3 =+          0.
         ,X4 =-          0.17272
         ,X5 =-          0.04316
         ,X6 =+          0.
         ,X7 =+          0.
         ,X8 =+          0.
         ,X9 =-          0.05375
         ,X10=+          0.
         ,X11=+          0.
         ,X12=+          0.
         ,X13=+          0.
         ,X14=+          0.
         ,X15=+          0.
         ,X16=-          0.01530
         ,X17=-          0.03964
         ,X18=-          0.07510
         ,X19=-          0.10996
         ,X20=-          0.14098
         ,X21=+          0.
         ,Z9 =+1
         ,Y9 =-          0.40000
MATRIX GR11,X1 =+           0.
         ,X2 =+          0.
         ,X3 =+          0.
         ,X4 =-          0.15035
         ,X5 =-          0.03572
         ,X6 =+          0.
         ,X7 =+          0.
         ,X8 =+          0.
         ,X9 =-          0.04008
         ,X10=+          0.
         ,X11=+          0.
         ,X12=+          0.
         ,X13=+          0.
         ,X14=+          0.
         ,X15=+          0.
         ,X16=+          0.
         ,X17=-          0.01521
         ,X18=-          0.05067
         ,X19=-          0.08553
         ,X20=-          0.11656
         ,X21=-          0.14657
         ,Z10=+1
         ,Y10=-          0.40000
MATRIX TX1 ,X1 =+           0.14000
         ,X2 =+          0.13000
         ,X3 =+          0.
         ,X4 =+          0.
         ,X5 =+          0.
         ,X6 =+          0.
         ,X7 =+          0.
         ,X8 =+          0.
         ,X9 =+          0.
         ,X10=+          0.30000
         ,X11=+          0.12000
         ,X12=+          0.
         ,X13=+          0.
         ,X14=+          0.
         ,X15=+          0.
         ,X16=+          0.
         ,X17=+          0.
         ,X18=+          0.
         ,X19=+          0.
         ,X20=+          0.
         ,X21=+          0.
         ,T0 =+1
         ,A0 =-1
MATRIX TX2 ,X1 =+           0.13848
         ,X2 =-          0.02548
         ,X3 =+          0.
         ,X4 =+          0.
         ,X5 =+          0.
         ,X6 =+          0.23080
         ,X7 =+          0.32312
         ,X8 =+          0.38082
         ,X9 =+          0.
         ,X10=-          0.05166
         ,X11=-          0.02720
         ,X12=+          0.14016
         ,X13=+          0.
         ,X14=+          0.
         ,X15=+          0.
         ,X16=+          0.
         ,X17=+          0.
         ,X18=+          0.
         ,X19=+          0.
         ,X20=+          0.
         ,X21=+          0.
         ,Z0 =+          0.15000
         ,T1 =+1
         ,A1 =-1
         ,A0 =+1
         ,Y0 =-          0.13000
MATRIX TX3 ,X1 =+           0.
         ,X2 =-          0.02607
         ,X3 =+          0.18520
         ,X4 =+          0.
         ,X5 =+          0.
         ,X6 =-          0.08236
         ,X7 =+          0.07937
         ,X8 =-          0.08293
         ,X9 =+          0.
         ,X10=-          0.06241
         ,X11=-          0.04144
         ,X12=-          0.02621
```

```
          ,X13=+          0.16253
          ,X14=+          0.
          ,X15=+          0.
          ,X16=+          0.
          ,X17=+          0.
          ,X18=+          0.
          ,X19=+          0.
          ,X20=+          0.
          ,X21=+          0.
          ,Z1  =+         0.14400
          ,T2  =+1
          ,A2  =-1
          ,A1  =+1
          ,Y1  =-         0.12480
MATRIX TX4  ,X1  =-            0.13859
          ,X2  =-         0.03056
          ,X3  =-         0.03559
          ,X4  =+         0.
          ,X5  =+         0.
          ,X6  =-         0.06050
          ,X7  =-         0.36255
          ,X8  =-         0.10625
          ,X9  =+         0.
          ,X10=-          0.06742
          ,X11=-          0.07000
          ,X12=-          0.04324
          ,X13=-          0.02892
          ,X14=+          0.18535
          ,X15=+          0.
          ,X16=+          0.
          ,X17=+          0.
          ,X18=+          0.
          ,X19=+          0.
          ,X20=+          0.
          ,X21=+          0.
          ,Z2  =+         0.13968
          ,T3  =+1
          ,A3  =-1
          ,A2  =+1
          ,Y2  =-         0.12106
MATRIX TX5  ,X1  =-            0.12596
          ,X2  =-         0.03195
          ,X3  =-         0.03630
          ,X4  =+         0.
          ,X5  =+         0.
          ,X6  =-         0.05016
          ,X7  =-         0.17136
          ,X8  =-         0.13175
          ,X9  =+         0.
          ,X10=-          0.06616
          ,X11=-          0.03599
          ,X12=-          0.07484
          ,X13=-          0.04501
          ,X14=-          0.02947
          ,X15=+          0.21271
          ,X16=+          0.
          ,X17=+          0.
          ,X18=+          0.
          ,X19=+          0.
          ,X20=+          0.
          ,X21=+          0.
```

```
          ,Z3  =+         0.12960
          ,T4  =+1
          ,A4  =-1
          ,A3  =+1
          ,Y3  =-         0.11232
MATRIX TX6  ,X1  =-            0.14281
          ,X2  =-         0.03516
          ,X3  =-         0.05029
          ,X4  =+         0.
          ,X5  =+         0.
          ,X6  =-         0.02774
          ,X7  =-         0.16811
          ,X8  =-         0.12716
          ,X9  =+         0.
          ,X10=-          0.06300
          ,X11=-          0.02132
          ,X12=-          0.03622
          ,X13=-          0.08030
          ,X14=-          0.04673
          ,X15=-          0.02976
          ,X16=+          0.24691
          ,X17=+          0.
          ,X18=+          0.
          ,X19=+          0.
          ,X20=+          0.
          ,X21=+          0.
          ,Z4  =+         0.13359
          ,T5  =+1
          ,A5  =-1
          ,A4  =+1
          ,Y4  =-         0.11577
MATRIX TX7  ,X1  =-            0.11166
          ,X2  =-         0.03509
          ,X3  =-         0.05940
          ,X4  =+         0.
          ,X5  =+         0.
          ,X6  =-         0.00928
          ,X7  =-         0.13194
          ,X8  =-         0.10415
          ,X9  =+         0.22736
          ,X10=-          0.05571
          ,X11=-          0.01110
          ,X12=-          0.02050
          ,X13=-          0.03610
          ,X14=-          0.08270
          ,X15=-          0.04733
          ,X16=-          0.02970
          ,X17=+          0.26380
          ,X18=+          0.
          ,X19=+          0.
          ,X20=+          0.
          ,X21=+          0.
          ,Z5  =+         0.14075
          ,T6  =+1
          ,A6  =-1
          ,A5  =+1
          ,Y5  =-         0.12199
MATRIX TX8  ,X1  =-            0.10935
          ,X2  =-         0.03252
          ,X3  =-         0.05569
          ,X4  =+         0.
```

```
         ,X5  =+        0.
         ,X6  =-        0.00996
         ,X7  =-        0.10374
         ,X8  =-        0.08278
         ,X9  =-        0.03535
         ,X10=-         0.04966
         ,X11=+         0.
         ,X12=-         0.00630
         ,X13=-         0.01842
         ,X14=-         0.03556
         ,X15=-         0.08783
         ,X16=-         0.04845
         ,X17=-         0.02938
         ,X18=+         0.30210
         ,X19=+         0.
         ,X20=+         0.
         ,X21=+         0.
         ,Z6  =+        0.09223
         ,T7  =+1
         ,A7  =-1
         ,A6  =+1
         ,Y6  =-         0.07993
MATRIX   TX9 ,X1  =-          0.06972
         ,X2  =-         0.03096
         ,X3  =-         0.04616
         ,X4  =+         0.
         ,X5  =+         0.
         ,X6  =-         0.11149
         ,X7  =-         0.06264
         ,X8  =-         0.05536
         ,X9  =-         0.03682
         ,X10=-          0.04162
         ,X11=+          0.
         ,X12=+          0.
         ,X13=-          0.00571
         ,X14=-          0.01637
         ,X15=-          0.03472
         ,X16=-          0.09157
         ,X17=-          0.04697
         ,X18=-          0.02880
         ,X19=+          0.33364
         ,X20=+          0.
         ,X21=+          0.
         ,Z7  =+         0.15436
         ,T6  =+1
         ,A6  =-1
         ,A7  =+1
         ,Y7  =-          0.13377
MATRIX   TX10,X1  =-           0.03414
         ,X2  =-          0.02626
         ,X3  =-          0.04101
         ,X4  =+          0.43181
         ,X5  =+          0.10795
         ,X6  =-          0.09304
         ,X7  =-          0.04626
         ,X8  =-          0.03339
         ,X9  =-          0.03763
         ,X10=-           0.03401
         ,X11=+           0.
         ,X12=+           0.
         ,X13=+           0.
         ,X14=-           0.00404
         ,X15=-           0.01500
         ,X16=-           0.03406
         ,X17=-           0.09360
         ,X18=-           0.04912
         ,X19=-           0.02833
         ,X20=+           0.35246
         ,X21=+           0.
         ,Z8  =+          0.12633
         ,T9  =+1
         ,A9  =-1
         ,A8  =+1
         ,Y8  =-          0.11122
MATRIX   TX11,X1  =+           0.
         ,X2  =-          0.01441
         ,X3  =-          0.03056
         ,X4  =-          0.11716
         ,X5  =-          0.03900
         ,X6  =-          0.07211
         ,X7  =-          0.02651
         ,X8  =-          0.01087
         ,X9  =-          0.03646
         ,X10=-           0.00467
         ,X11=+           0.
         ,X12=+           0.
         ,X13=+           0.
         ,X14=+           0.
         ,X15=-           0.00275
         ,X16=-           0.01393
         ,X17=-           0.03350
         ,X18=-           0.09502
         ,X19=-           0.04916
         ,X20=-           0.02792
         ,X21=+           0.36642
         ,Z9  =+          0.09293
         ,T10=+1
         ,A10=-1
         ,A9  =+1
         ,Y9  =-          0.08054
MATRIX   P90,X11=1
         ,X12=+1
         ,X13=+1
         ,X14=+1
         ,X15=+1
         ,X16=+1
         ,X17=+1
         ,X18=+1
         ,X19=+1
         ,X20=+1
         ,X21=+1
RHS      CR1 ,RHS=           0.45000
         CR2 =            0.04300
         CR3 =            0.04542
         CR4 =            0.04766
         CR5 =            0.04899
         CR6 =            0.04813
         CR7 =            0.05290
         CR8 =            0.05115
         CR9 =            0.05168
         CR10=            0.05612
         CR11=            0.06203
```

```
        LG1 =0
        LG2 =0
        LG3 =0
        LG4 =0
        LG5 =0
        LG6 =0
        LG7 =0
        LG8 =0
        LG9 =0
        LG10=0
        GR1 =0
        GR2 =0
        GR3 =0
        GR4 =0
        GR5 =0
        GR6 =0
        GR7 =0
        GR8 =0
        GR9 =0
        GR10=0
        GR11=0
         TX1 =          0.
         TX2 =          0.10000
         TX3 =          0.11000
         TX4 =          0.12000
         TX5 =          0.13000
         TX6 =          0.14000
         TX7 =          0.15000
         TX8 =          0.16000
         TX9 =          0.17000
         TX10=          0.18000
         TX11=          0.19000
         PRJ=1
END***
```

GLOSSARY OF VARIABLES

A_j	Value of the fixed investment on project j in the first year of operation.
A_t	Total unrelieved balance of capital allowances up to and including year t.
a_j	Inflation sensitivity factor for the fixed assets of project j.
$\hat{a}_j$	Value of post-horizon flows from project j discounted back to the horizon at the after tax lending rate of interest.
$a_{t,j}$	Flow in year t from project j.
B_t	Amount borrowed in year t.
b_t	Before tax borrowing interest rate.
b'_t	After tax borrowing interest rate.
C_j	After tax cash flow in year j, in terms of costs and prices prevailing in year zero.
C'_j	After tax cash flow in year j, in terms of costs and prices expected to prevail in that year.
C_{jBT}	Before tax cash flow in year j in terms of existing prices.
$C_j{}'_{BT}$	Before tax cash flow in year j in current money units.
$C_{k,j}$	Fixed investment on project j made in year k.
$CCT_{i,j}$	Working capital required for project j in year i.
$CF_{i,j}$	Net annual cash flow from project j in year i.
C	Delay in the payment of capital investment.
$D_{k,j}$	Depreciation on plant and equipment due to project j in year k.
d_j	Duration of project j (in years).
$E_{j,m}$	Total expenditure in current money units for project m in year j.
e	Delay in the payment of total expenses (in years); in Chapter II only, e is the market lending rate of interest after tax.
F_t	Slack variables associated with cash balance constraints.

$F_{i,j}$	Operating cash flow from project j in year i.
f_m	Inflation sensitivity factor for the total expenses on project m.
G_t	Slack variable associated with the gearing constraints.
g	Gearing ratio. Proportion of debt used to finance a project.
h	Corporate tax rate (fractional).
$I_{i,j}$	Capital investment cash flow from project j in year i.
i	Annual inflation rate.
i'	Particular inflation rate affecting the initial investment of a particular project.
i_k	Inflation rate in year k.
L_t	Amount lent in year t.
l_t	Before tax lending interest rate.
l'_t	After tax lending interest rate.
m	Market or nominal rate of interest.
NPV	Net Present Value.
n	Duration of the project (in years).
P	Fixed investment.
$P_{j,m}$	Total expenditure in terms of present time costs and prices, for project m in year j.
P_t	Cash expected to be available in year t from old projects, net of dividends and debt commitments existing at the outset.
P_t'	Cash expected to be available in year t from old projects.
Q_t	Slack variable associated with liquidity constraints.
R	Slack variable associated with the constraint on number of project versions that can be accepted.
r	Liquidity ratio. In Chapter II only, r is the annual interest rate on loan, before tax.
S_t	Taxable income in year t.
S_j	Start-up year for project j.

T_j	Taxable profit from operations in year j.
TV	Terminal Value.
TCF_i	Total taxable profit minus allowances in year i.
t	Fractional tax rate.
$V_{i,j}$	Value of sales due to project j in year i, expressed in current money units.
v	Delay in the payment of sales (in years).
x_j	0, 1 integer variable denoting rejection or acceptance of project j.
α	Annual after tax return required on invested capital (in real terms).
β_t	Dual variable associated with the liquidity constraint in year t.
γ_t	Dual variable associated with the gearing constraint in year t.
δ	Dual variable associated with the constraint on project versions $x_{11} - x_{21}$.
λ_t	Dual variable associated with the taxation constraint in year t.
μ_j	Dual variable associated with the x_j integer variables.
ρ_t	Dual variable of cash balance constraint in year t.
Φ_j	Forecast of the inflation rate in year j.
Ψ_j	Expected value of the inflation rate in year j.

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