

Thesis  
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The Efficiency of the London Traded Options  
Market: the Implications of Volatility, Volume,  
and Bid-Ask Spreads

by

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Thesis submitted for the degree of Ph.D.  
in Finance of the University of Stirling,  
April 1993.

## Abstract

This study is a test of the efficiency of the London Traded Options Market. Because it uses the Black-Scholes Option Pricing Model, it is also a test of option pricing. In the process of examining call option price behaviour it investigates the effects of three empirical factors.

First, it investigates the effect of a non-constant share price volatility. Hitherto, there has been no agreed procedure on modelling or forecasting the future share price volatility. This study shows that the GARCH process has the best forecasting accuracy. The ex ante GARCH volatility estimate is then incorporated in the Black-Scholes model. Because the volatility is assumed constant in the Black-Scholes model, the consideration of adapting the GARCH volatility into the model sheds insight on bridging empirical results and theoretical requirements.

Second, because the London Traded Options Market is thinly traded the quoted prices may not reflect prices at which trade did or could take place. However, information on call option trading volume may not be available. This study develops and implements an analytical criterion to select the most actively traded call options. The call options selected by this criterion bear the basic characteristics of those frequently traded call options where trading volume is available.

Third, this study uses the bid and ask quotations for shares and call options to test the efficiency of the London Traded Options Market. By incorporating the bid-ask spread directly in the establishment of arbitrage portfolios, an accurate assessment of transactions data can be made.

The results of incorporating these factors in the test for market efficiency reveal that, despite the identification of mispriced call options, it would not have been possible to exploit the mispricing by setting up arbitrage portfolios. It must therefore be concluded that the London Traded Options Market was trading efficiently over the period of this study.

## Acknowledgements

I am indebted to my supervisor, Professor Charles Ward. I thank him for his stimulating advices and encouragement. Professor Robin Limmack also was very helpful to me and my wife and created a very supportive environment for us in the University.

The full support and understanding of my wife Josephine has sustained me in times of weariness and imparted a peaceful mind in the writing up of the thesis.

Finally, this thesis could not have been finished without the study leave granted by the Hong Kong Baptist College. I would like to thank my students KK Pang, CH Ma, YT Fok, SB Chau and YL Ho who assisted in the collection of data.

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# Chapter 1

## Introduction

### 1.1 Introduction

One of the most important concepts of finance theory is the analysis of market prices of traded assets and securities. Researchers and investors have developed a large number of models which are used to derive values for securities but many of these models are normative in that they set out to reflect in a consistent way the beliefs and expectations of the individual investor. A new class of models in finance has been developed over the last thirty years based on less subjective expectations. These models, which may have some linkage with the normative models derived earlier, seek to answer the question of what would be the observable results in a market dominated by rational and efficient participants. These 'positive' models have been derived on the basis of strong assumptions on market efficiency but have proved powerful tools in illuminating the ways in which markets operate and securities are priced.

One important assumption in this approach has been the Principle or Law of One Price. Modigliani and Miller (1958) pointed out that, in equilibrium, portfolios of financial claims which are in essence equivalent, must earn the same return. The same principle applies equally to derivative securities such as call options. A call option is a contract giving its owner the right, but not the obligation, to buy a

fixed number of shares of a specified common stock at a fixed price at any time on or before a given date. As the exercise price is fixed, the call option is more valuable the higher the share price. Black and Scholes (1973) used the Law of One Price to derive a closed form valuation model for call options. Central in their model is an equilibrium relationship between call option prices and the prices of the underlying shares. The call option price is perfectly correlated with its underlying share price so that a riskfree hedge portfolio can be created. If the portfolio is rebalanced continuously so as to remain riskfree, it must by the Law of One Price earn the riskfree interest rate.

As noted by Merton (1973), because the dependent variables of the Black-Scholes model are directly observable (except the share price volatility) the model can be tested empirically. Unfortunately, if the tests reveal that market prices are not explained by the model, two inferences may follow. Firstly, the model may simply be incorrect or that the market does not operate in a way which could or should be described as efficient. In terms of the Black and Scholes model tests, if the excess return of the hedge portfolio exceeds the riskfree interest rate, the market is usually inferred to be inefficient. However, it is also argued that the model is incorrect because the model values often differ from the actual prices in a systematic way. Researchers (e.g., Cox and Rubinstein 1985, Bhattacharya 1980) have identified that apparent mispricing of a call option might not have correctly indicated the possibility of obtaining abnormal profits for

four reasons:

1. The prices published for the call option and share markets were not applicable simultaneously.
2. The bid-ask spread was not taken into account.
3. The market lacked sufficient depth to ensure trading opportunities.
4. The share price volatility was non-stationary and misestimated by previous researchers/investors.

In looking at these reasons, it should be recognised that researchers assume the Law of One Price will operate simultaneously in the market for options and the market for shares. The implication of the first reason for test failure is clearly a failure of the simultaneity assumption. The second and third reasons are also statements about the efficiency of the information process. If researchers assume that published prices correctly indicate the prices at which investors can buy or sell, they should take care to ensure that actions could indeed follow the identification of breakdowns in the Law of One Price. Institutional factors and deficiencies could well reconcile apparent differences between theoretical and actual prices. The fourth reason concerns the most difficult empirical component of the tests of the Black and Scholes model. The component is the variability of the underlying share price. Since the component is theoretically an expected variable, unknown at the time at which the options and shares are traded, strong assumptions have to be made about how investors can have

common expectations regarding the variable. For various reasons, the strongest assumptions result in simple but inefficient estimates of the actual share price variability. Thus researchers have sought to extend the analysis of option prices by more sophisticated estimates of the ex ante variability. The results of these problems will be discussed in later chapters.

## 1.2 The aims of this study

In facing these contemporary empirical problems, this study examines three issues. First, this study examines the implication of the bid-ask quotes on market efficiency. The call option and share bid-ask quotes are used to identify mispriced call options and further used to test the efficiency of the London Traded Options Market. The results are important and relevant to investors who must incur the bid-ask spread. More importantly, it is pointed out that the sum of call option and share percentage spreads differs across companies. This implies that the procedure adopted by Phillips and Smith's (1980) using an overall average spread as a measure of trading cost in testing market efficiency is very crude. Future researchers, even using synchronous data, may have to use call and share bid-ask quotes directly.

Second, for call options lacking market depth, this study develops an analytical criterion for selecting the most actively traded call options. The criterion is particularly useful to researchers to whom call option trading volume is not available or too costly to obtain. Furthermore, this study

examines the implications of thin trading in terms of large spreads on the persistence of market efficiency.

The third empirical issue examined in this study is the share price volatility. This study recognises that the volatility is persistent over time and models the volatility by the GARCH process. The GARCH estimate is *ex ante* and is therefore relevant to testing market efficiency. Because the volatility is assumed constant in the Black-Scholes model, the consideration of adapting the GARCH volatility into the model sheds insight on bridging empirical results and theoretical requirements.

### 1.3 Organisation of this study

The rest of this study is organised as follows: Chapter 2 is a review of past theoretical and empirical researches in option pricing. Chapter 3 and Chapter 4 discuss two distinct aspects of the GARCH process and are therefore split into two individual chapters. Chapter 3 compares the forecasting accuracy on share price volatilities by the GARCH, ARCH, EWMA and the naive models. The GARCH model is found to have superior forecasting accuracy among the four models. Chapter 4 discusses how the GARCH process which allows a changing conditional volatility might be adapted to the Black-Scholes formula which only accommodates a constant volatility.

Chapter 5 examines the trading activity of a call option in two major steps. First, a proxy is defined for near-the-moneyness within which a call option is expected to be actively traded. Second, an analytical model is derived for identifying

actively traded call option series. Chapter 6 is a critique of the issue on ex-dividend share price decline. Chapter 7 introduces the London Traded Options Market (LTOM), presents the data source and the period of study, and discusses the rational boundary conditions. Chapter 8 tests the efficiency of the LTOM, contrasting between the results of using bid-ask quotes and mid prices. Chapter 9 examines three major areas of contemporary finance issues: the implication of large bid-ask spreads on the thinly traded LTOM, the empirical issues on using the Black-Scholes model, and the special attributes of mispriced call options. Finally, chapter 10 concludes the thesis, outlines the limitations, and points out the possible extensions and implications of this study for future research.

## Chapter 2

# An Overview of Option Pricing Models and Empirical Studies

### 2.1 Introduction

Black and Scholes (1973) derived a model for pricing call options which has been widely used by investors and researchers. However, as noted by the previous chapter, the model generates model values which are systematically different from the actual prices. For example, the model tends to overprice out-of-the-money near-maturity call relative to at-the-money middle-maturity calls on the same underlying share (Rubinstein 1985). These results have stimulated interest in pursuing alternative models. Special emphases have focused on relaxing the assumptions that (1) the share price volatility is constant, (2) the share price follows a continuous path through time, and (3) the share pays no dividends, and finally (4) the call option cannot be exercised early.

The aim of this chapter is to give an overview of past development of option models and to outline major areas of empirical studies in option pricing.

### 2.2 A review of option pricing models

This section reviews option pricing models which relax the assumptions of constant volatility, share price continuity, no dividend payments and no early exercise. In particular, the GARCH option pricing model recently proposed by Duan (1991) is discussed in section 2.2.2.

### 2.2.1 Changing share price volatility

The original Black-Scholes model assumes that the share price volatility is constant. However, Black (1975) points out that the share price volatility tends to increase as the share price decreases. Cox (1975) incorporates this concept in his constant elasticity of variance model where the share price volatility is given by:

$$\sigma(S_t, t) = \sigma S_t^{\psi-1} \text{ where } 0 \leq \psi < 1$$

and is therefore inversely related to the share price.

The only major empirical study of this model is by MacBeth and Merville (1980). They conclude that the Cox model values are closer to market prices than the Black-Scholes prices. The additional difficulty of applying this model is that both  $\sigma$  and  $\psi$  have to be estimated and the parameter  $\psi$  is found to vary over time (MacBeth 1981).

Geske's (1979) compound option model takes a similar approach where the partial derivative of the instantaneous share price volatility with respect to the share price is

$$\frac{\partial \sigma_s}{\partial S} = -\frac{V}{S^2} N_1(k + \sigma_v \sqrt{T_2}) \sigma_v < 0$$

where  $S$  is the share price,  $V$  is the market value of the firm and  $\sigma_v$  is the volatility of the assets of the firm. In Geske's model, the variance is inversely proportional to the share price. He argues that as the share price falls (rises), the firm's debt-equity ratio rises (falls). The increased (decreased) risk induced is reflected by a rise (drop) in the variance of the share returns. This model has not been empirically tested.

### 2.2.2 The GARCH option pricing model

The time series of share returns have long been recognised as heteroscedastic and leptokurtic. Duan (1991) used the GARCH process to capture these two important return characteristics and developed the GARCH option pricing model based on the Risk-Neutral Valuation Principle. Under the GARCH(p,q) specification, a European call option with underlying share price  $S$  and exercise price  $X$  maturing at time  $T$  has the value at time  $t$  (Note 2.1, p.2.20):

$$C_t^{GH} = \exp[-(T-t)r_c] E^Q[\max(S_T - X, 0) | \mathcal{F}_t].$$

The Black-Scholes option value can be viewed as a GARCH option value with  $S_T$  constructed as a *homoscedastic* lognormal process and its conditional variance equals the *stationary* variance of the GARCH process. Duan found that the GARCH option pricing model prices out-of-the-money options higher than does the Black-Scholes model because the specification of heteroscedasticity and leptokurtosis induce an out-of-the-money option more likely to end up in-the-money. Duan pointed out that the GARCH model could explain some well-documented systematic biases in the Black-Scholes model. Unfortunately, the GARCH model has no analytic solution and must be examined via a control-variate Monte Carlo simulation procedure.

### 2.2.3 Discontinuity of the share price

The original Black-Scholes model excludes large jumps in share prices. Merton's (1976) jump-diffusion formula includes a jump component and therefore recognises that the path of share price can be discontinuous over time. It is out of the

scope of this review to discuss his formula. Merton's (1976) own empirical work on his model shows that the jump component seems to be not very significant for common shares. In other words, the mis-specification error of the underlying share price return on option pricing is quite small if the jump component is ignored.

#### 2.2.4 Dividend correction models

The original Black-Scholes model assumes that no dividend is paid during the life of the option. However, because options span up to nine months, there will usually be at least one dividend during the life of the option. Without correcting for the dividend, the model value will be overstated. There are two competing dividend correction approaches. The first approach assumes that the share pays a finite number of known dividends over the life of the option. The second approach assumes that stochastic dividends are paid continuously over the life of the option.

The first approach was proposed by Black (1975) and confirmed by Jarrow and Rudd (1983). Jarrow and Rudd examine the case where there are two known dividend payments before the option matures, at times  $t_1$  and  $t_2$  where the current time =  $t < t_1 < t_2 < T$  = maturity date. They assume that the market price of the share is the total market price  $S_t$  less the discounted escrowed dividend, i.e.,

$$\hat{S}_t = S_t - \sum_{i=1}^2 D_i e^{-rt_i}$$

where  $\tau_i = t_i - t$  for  $i=1,2$ . Then

$$C_t = \hat{S}_t N(d_1) - XN(d_2) e^{-rT} \quad (2.1)$$

where

$$d_1 = \frac{\ln\left(\frac{\hat{S}_t}{X}\right) + (r+\sigma^2) T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

They argue that this formula is appropriate because if the option is not exercised early, then the option buyer will not receive the dividends, therefore he should subtract the present value of the dividends from the share price. This formula is an exact valuation formula either for a European option or for an American option when early exercise is not optimal.

The alternative approach is proposed by Merton (1973). Merton assumes that dividends are paid continuously such that the dividend yield  $D$  is constant and he adjusts the current share price  $S_t$  as

$$\hat{S}_t = S_t e^{-Dt}$$

which is interpreted as the current market price of the share minus the present value of the stochastic dividends paid over the life of the option. Merton's dividend model is (Jarrow and Rudd 1983, p.132):

$$\begin{aligned} C_t &= S_t e^{-Dt} N(d_1) - XN(d_2) e^{-rT} \\ &= \hat{S}_t N(d_1) - XN(d_2) e^{-rT}. \end{aligned} \quad (2.2)$$

Chiras and Manaster (1978) adopt Merton's model in their empirical research but they point out that the constant dividend yield assumption does not conform to actual firm

dividend policy. Jarrow and Rudd (1983) remark that this solution is not exact but is only an approximation.

#### 2.2.5 Models with the possibility of early exercise

There are two approaches to account for the possibility of early exercise. One is Black's (1975) ad hoc revision of the European call option model. The other is Roll's (1977) American call option pricing model. When there is a possibility of exercising the option before the last ex dividend date, Black (1975) suggests an approach to value the option which is later labelled by Rubinstein and Cox (referred by Jarrow and Rudd 1983, p.127) as the pseudo-American Black-Scholes model. The step is first to calculate an option value according to equation (2.1) and then to calculate an option value by assuming that the option expires just before the last ex-dividend date. The higher of these two option values will be taken as the fair value. (The second calculation subtracts the present values of all dividends except the last and uses a maturity date just before the last ex-dividend date.) The solution of this approach is not exact but it gives a precise lower bound for the value of an American call option where the underlying share pays known dividends.

Roll (1977) derives an exact valuation formula for an American call option with one known dividend which accounts for the possibility of early exercise. Geske (1979) later improves Roll's result to a more compact expression. Finally, Whaley (1981) corrects a minor mistake in both Roll's and Geske's formula. The call option formula of their combined effort is

therefore referred to as the Roll-Geske-Whaley (RGW) formula. The solution to the American call option pricing formula, as given in Whaley (1982), is

$$\begin{aligned}
 C(S, T, X) = & S \left[ N_1(b_1) + N_2(a_1, -b_1; -\sqrt{t/T}) \right] \\
 & - X e^{-rT} \left[ N_1(b_2) e^{r(T-t)} + N_2(a_2, -b_2; -\sqrt{t/T}) \right] \quad (2.3) \\
 & + \alpha D e^{-rt} N_1(b_2),
 \end{aligned}$$

where

$$\begin{aligned}
 a_1 = & [\ln(S/X) + (r+1/2\sigma^2)T] / \sigma\sqrt{T}, \quad a_2 = a_1 - \sigma\sqrt{T}, \\
 b_1 = & [\ln(S/S^*) + (r+1/2\sigma^2)t] / \sigma\sqrt{t}, \quad b_2 = b_1 - \sigma\sqrt{t},
 \end{aligned}$$

and  $N_2(a, b; \rho)$  is the bivariate cumulative normal density function with upper integral limits  $a$  and  $b$ , and correlation coefficient  $\rho$ .  $S^*$  at time  $t$  is the ex-dividend share price determined by

$$c(S_t^*, T-t, X) = S_t^* + \alpha D - X,$$

above which the call option will be exercised just before the ex-dividend instant.  $\alpha D$  is the proportionate ex-dividend share price decline.

Whaley (1982) first tests this model on CBOE call options for the period 17 January 1975 to 3 February 1978. He concludes that the model better describes the observed structure of call option prices than Black's dividend correction model or the pseudo-American call option formula. However, this model tends to underprice options on low volatility shares and overprice options on high volatility shares. Sterk (1983) develops Whaley's (1982) study on CBOE options for the month of October 1979 by noting that the model performs better when the size of the dividend on the underlying share is larger.

## 2.3 A review of previous empirical tests

As noted in the last section, models which specify a different stochastic process for the share price volatility could involve more difficulty for empirical researchers, e.g., Cox's (1975) model. Some models are theoretically appealing but are not ready for empirical study, e.g., Geske's (1979) compound option. Merton's (1976) own empirical work on his model shows that the jump component seems not to be very significant for common shares. The simple Black-Scholes model, in which the variables are directly observable (except the volatility), has attracted most of the empirical studies.

In this section, past empirical studies on the Black-Scholes model are reviewed in seven areas: (1) direct comparison of market prices and model values; (2) hedge returns and market efficiency; (3) rational boundary conditions; (4) simulations; (5) put-call parity; (6) information content of call options; and (7) the call option bid-ask spreads. Finally, some important empirical studies in the UK market are reviewed.

### 2.3.1 Direct comparison of market prices and model values

The aim of comparing the difference between actual and model prices is to examine if the model values are unbiased estimates of actual prices.

MacBeth and Merville (1979) first report such a comparison on the Chicago listed option market for the period 31 December 1975 to 31 December 1976. Using the simple Black-Scholes model they examine the standardised difference between the actual price and model value as a function of the degree of in-the-

moneyness:

$$\frac{C_t^A - C_t^M}{C_t^M} = f \left[ \frac{S_t - Xe^{-rT}}{Xe^{-rT}} \right]$$

They find that the Black-Scholes model underestimates in-the-money options and overestimates out-of-the-money options. Their results contradict those reported by Black (1975). They attribute the conflicting results to the non-stationarity of volatility. Gultekin, Rogalski and Tinic (1982) use the RGW model (equation 2.3) to study CBOE options for the period 1975 to 1976. They find that the model overvalues options written on high volatility shares and undervalues options issued on low volatility shares. The model tends to overestimate in-the-money options and underestimate out-of-the-money options. They also point out that the RGW model will produce better estimates of actual prices as the maturity of the option decreases.

### 2.3.2 Hedge returns and market efficiency

The primary idea of examining a share-option hedge (or option-option hedge) is to see whether riskfree arbitrage profits can be exploited by buying and/or selling mispriced options and hence make inferences about the efficiency of the options market. Galai (1977) first published his results on listed CBOE options market for the period 26 April 1973 to 30 August 1973. He used Black's dividend correction model (equation 2.1) and ex ante tests in testing market efficiency. The returns on the hedged positions for an underpriced and overpriced call option are defined as

$$R_{H,t+2} = (C_{t+2}^A - C_{t+1}^A) - N(d_{1,t}) (S_{t+2} - S_{t+1}), \quad \text{and}$$

$$R_{H,t+2} = N(d_{1,t}) (S_{t+2} - S_{t+1}) - (C_{t+2}^A - C_{t+1}^A)$$

respectively, where  $C^A$  and  $S$  are actual (market) call option and share prices. The excess dollar return for a call option is defined by

$$[\Delta C - N(d_{1,t}) \Delta S] - [C - N(d_{1,t}) S] r \Delta t$$

where  $\Delta C$  is the change in the market price of the call option between trading days;  $\Delta S$  is the change in the share price;  $r$  is the interest rate and  $\Delta t$  is the time interval (1 day). He finds that the ex ante returns show a strong tendency to be positive. However, profit opportunities disappear once transaction costs are included. His study suffers from the drawback that it uses closing prices but on the other hand his insight of using ex ante tests to examine market efficiency has been well accepted by subsequent researchers.

### 2.3.3 Rational boundary conditions

Galai (1978) tests the lower boundary conditions for CBOE traded options. He finds that positive profits can be exploited on the violation of the boundary conditions. However, Phillips and Smith (1980) point out that his reported profits will disappear once the bid-ask spread is accounted for. Halpern and Turnbull (1985) test the boundary conditions for Toronto Stock Exchange options over the period 1978 to 1979. They find that for the sample period, the Toronto market was inefficient even after taking into account the transaction costs (including the bid-ask spread).

#### 2.3.4 Simulations

Simulation tests are used to examine the robustness of a model if the model assumptions do not hold. Figlewski (1989) carries out a very comprehensive study in examining the impact of market imperfection on option arbitrage. He adopts the Black-Scholes model. To control the true price generating process, he uses simulated option and shares price data. He then forms option-share hedges and rebalances them at most once a day. His major observations are that: errors in forecasting volatility cause both option values and hedge ratios to be inaccurate. However, the effect on hedging accuracy is relatively slight. The effect of option contract indivisibility causes the hedge ratio to be inexact and therefore renders the hedge portfolio to be risky. However, the expected hedge return is not strongly affected. Transaction costs incurred in the hedging process are very large. To reduce costs by rebalancing the hedged position less frequently will have the trade-off of a large increase in risk.

#### 2.3.5 Put-call parity

Klemkosky and Resnick (1979) examine CBOE and Philadelphia options for the period July 1977 to June 1978. They conclude that put-call parity with dividend correction holds for the sample period and therefore supports that aspect of efficiency. Zivney (1991) uses the CBOE's S&P 100 index option data for the year 1985 through deviations from the put-call parity relationship to determine the value of early exercise. He finds that the actual value of early exercise is both economically

and statistically significant. In addition, the value of early exercise for put options is greater than for call options.

#### 2.3.6 The information content of call options

Chiras and Manaster (1978) use Merton's dividend correction model (equation 2.2) to compute implied volatility. They find that these volatilities are better forecasts of future share price volatilities than those calculated from historical share price data. A trading strategy using the information content of these implied volatilities yields abnormally high returns. Manaster and Rendleman (1982) use Black's dividend correction model (equation 2.1) to infer the implied share values. A comparison of the implied values with observed share prices show that closing prices of call options contain information about equilibrium share prices that are not fully reflected in the closing prices of its underlying share. However, since Manaster and Rendleman use closing prices they note that the information content could be caused by the non-synchronicity of option and share markets.

Stephan and Whaley (1990) translate intraday CBOE call option price changes into implied share price changes using the RGW call option pricing model (equation 2.3) for the first quarter of 1986. By comparing the intraday share price changes with the actual share price changes using the causality test developed by Sims (1972), their result indicates that price changes in the share market lead the option market by as much as fifteen minutes.

### 2.3.7 Bid-ask spread

The theoretical determination of option spreads will be discussed in Chapter 8. On the empirical side, researchers focus on finding the explanatory variables for option spreads, spread-induced information and spread-induced volatilities.

Explanatory variables. In examining the effects of multiple listing, Neal (1987) develops a model for option spreads. He postulates that: (1) *The trading volume is negatively related to the spread.* (A high trading volume will raise the frequency of transactions and thereby decreases the time a dealer will hold a position. Therefore, the spread will be smaller.) (2) *Competitively traded options usually have high trading volume and thus have narrower spreads.* (3) *The spread is positively related to the option price.* (Spreads can be regarded as one type of transaction costs. As option prices increase, the transaction costs and therefore the spreads increase.) and (4) *The volatility of daily option returns is positively related to the spread.* (For risk averse dealers, the risk of providing a liquidity service is compensated through requiring a wider spread.)

Neal's study covers the period from September 1985 to April 1986 and uses AMEX data. He finds that the call option bid-ask spread is significantly negatively related to volume and measures of competition (whether the option is single listed or multiple listed) and significantly positively related to price. However, the relation between spreads and option return volatility is inconclusive.

Spread-induced information. Bhattacharya (1983) performs lower boundary condition tests on CBOE traded call options based on the bid-ask prices of the calls and the share. He finds that small and infrequent violations of the boundaries were observed but the average positive returns became losses after transaction costs. In another major work, Bhattacharya (1987) uses the option quotations to define mispricing instances of share prices. Given the observed bid-ask prices for a pair of call options identical in all respects except their exercise prices, the pseudo-American Black-Scholes model is used to estimate the implied share quotation. This is compared with the concurrent market share prices. When an implied share quotation interval overlaps with the market share quotation, it indicates that arbitrage is not possible, i.e., the market share prices are correctly priced. When implied values are higher than market values (the share is expected to rise),

$$\text{Implied } \hat{S}_{\text{bid}} > \text{market } S_{\text{ask}}$$

the situation signals a buy strategy. When implied values are lower than market prices,

$$\text{Implied } \hat{S}_{\text{ask}} < \text{market } S_{\text{bid}}$$

the situation suggests a sell strategy. He concludes that while option prices do contain additional information not contained in the contemporary share prices, the information is insufficient to overcome the bid-ask spread, among other costs.

Spread-induced volatility. Choi and Shastri (1989) study the spread-induced volatility. They argue that dealers require the bid-ask spread compensation in executing a transaction.

Consequently, the observed return on a share consists of two components - the 'true' return and the spread-induced return, i.e.,

$$\hat{r}_t = r_t + r_t^s$$

where the first term on the right hand side is the true return and the second term is the spread-induced return. They further prove that the covariance between the true return and the spread-induced return is zero and therefore the volatility of observed returns on a share is the sum of the true volatility and the spread-induced volatility:

$$\hat{\sigma}^2 = \sigma^2 + \sigma_s^2.$$

This implies that the observed volatility overestimates the true volatility because it includes the spread-induced volatility. Their empirical results indicate that the magnitude of the spread-induced volatility increases with the level of the volatility. However, the overestimation is not sufficient to explain the volatility bias exhibited by the Black-Scholes model as options on high volatility shares are still overpriced relative to those on low volatility shares. In other words, the volatility bias exhibited by the Black-Scholes model is composed of yet other unexplained components.

## 2.4 Empirical research in the UK

There are several important empirical studies on different aspects of option pricing in the UK.

### 2.4.1 Put-call parity

With the introduction of put contracts in May 1981 into the London Traded Options Market, Goh and Allen (1984) test put-call parity with dividend correction in the UK market. Their results indicate that the put-call parity holds. The recent study of Nisbet (1992) extends Klemkosky and Resnick's (1979) work to account for transaction costs. She uses UK data for the half year period 27 June to 22 December 1988. She finds that when option spread alone is considered, the put-call parity is frequently violated, but when commission costs on options and shares are considered, none of the deviations could be exploited.

### 2.4.2 Ex-dividend share price decline

Kaplanis (1986) uses option prices to study the ex-dividend share price behaviour in the UK market over 1979 to 1984. She concludes that the average expected share price drop implicit in option prices is around 55% to 60% of the dividend and significantly different from it. The fall-off is inversely proportional to the dividend yield and therefore her result supports the "tax clientele hypothesis". This feature will be further explored in Chapter 6.

### 2.4.3 Implied volatility

Gemmill (1986) compares various implied standard deviations (ISD) derived from the Black-Scholes model with historical estimates as alternative predictors of the volatility on the London Traded Options Market for the period from May 1978 to July 1983. He concludes that in-the-money ISD is the best forecast of subsequent share price volatility. However, he points out that as the LTOM has matured, the improvement in the forecasting performance of the ISDs has been minimal.

### 2.4.4 Market efficiency

Gemmill and Dickens (1986) examine the efficiency of the LTOM using month-end data for the period from April 1978 to July 1983. Their option-option hedges generate persistent and significant excess abnormal returns. However, these riskfree profits do not exceed transaction costs and therefore the LTOM cannot be rejected as efficient. More importantly, they point out that

*"the bid-ask spread in this market (LTOM) is very large and, when it was applied to our trading strategy, it converted a significant profit into a significant loss."*

### 2.4.5 Pricing ability of the RGW model

Ho (1990) first tests the pricing ability of the RGW model (equation 2.3) on the LTOM for the one year period from 1 July 1981 to 30 June 1982. He finds that the model prices actual

prices better than the revised Black-Scholes model which assumes that the call option matures on the ex-dividend date. However, the model tends to undervalue actual call prices, regardless of the volatility estimates used (either historical, actual or EWMA volatilities). In particular, the RGW model tends to underprice options on low volatility shares, out-of-the-money options and short-lived options and, overprices options on high volatility shares, in-the-money options and long-lived options.

## 2.5 Concluding remarks

In this chapter, alternative option pricing models which relax the assumptions of constant volatility, the continuity of share prices, no payment of dividends and no early exercise of the call options have been reviewed. Because the underlying shares of the call options adopted in this study pay up to two dividends during the call options' lives, the Black-Scholes model with known dividend adjustment is adopted in testing the efficiency of the LTOM.

Seven areas of past empirical studies have been reviewed. This study contributes to the empirical research literature by using the bid-ask quotes to study market efficiency and thereby to infer the spread-induced implicit trading cost.

This study adopts the Black-Scholes model with dividend correction (equation 2.1) because: (1) Alternative models to the Black-Scholes model such as Geske's (1979) compound option model may have not been empirically tested. Cox's (1975) model has to estimate two parameters which vary over time. On the

other hand, Galai (1983, p.46) points out that the Black-Scholes model explains the actual behaviour of option prices over time more closely than other alternative models. Figlewski (1989, p.1289) notes that the Black-Scholes model has had the greatest impact on securities trading in the actual markets.

(2) In this study, to focus on examining the different effects of using bid-ask quotes and mid prices on market efficiency, all call option series are required not to be exercised early. The RGW model (2.3) might have stronger pricing ability than equation (2.1). But the RGW model is most useful when a call option is likely to be exercised early. When a call option will not be exercised early, the parsimonious equation (2.1) is theoretically justifiable (Jarrow and Rudd 1983, p.124) and computationally more efficient.

Note 2.1:

Duan assumes that :  $S_t/S_{t-1}|\phi_{t-1}$  is lognormally distributed and

$$E^Q\left(\frac{S_t}{S_{t-1}}|\phi_{t-1}\right) = 1 + r$$

under pricing measure  $Q$  and  $r_c = \ln(1+r)$ . The Risk-Neutral Valuation Relationship defined by Duan implies that, under pricing measure  $Q$ ,

$$\ln\left(\frac{S_t}{S_{t-1}}|\phi_t\right) = r_c - \frac{1}{2}h_t^* + \xi_t,$$

$$\xi_t|\phi_{t-1} \sim N(0, h_t^*),$$

$$h_t^* = \alpha_0 + \sum_{i=1}^q \alpha_i \xi_{t-i}^2 - \sum_{i=1}^p \beta_i h_{t-i}^*, \text{ and}$$

$$S_T = S_t \exp\left[(T-t)r_c - \frac{1}{2} \sum_{s=t+1}^T h_s^* + \sum_{s=t+1}^T \xi_s\right].$$

## Chapter 3

# Share Price Volatility : Comparison of the Forecasting Accuracy of Competing Models

### 3.1 Introduction

The estimation of the share price volatility is of central importance in financial research. Not only is it required in the modelling of asset pricing but more specifically it is an important element in the pricing of derivative securities such as options. This chapter reviews past approaches in tackling a changing volatility. Because the aim of this study is to test market efficiency, the forecasting accuracy of four frequently used models for estimating *ex ante* volatilities are compared. It is found that the GARCH model outperforms the others. In the next chapter, the GARCH volatility estimates will be adapted to be used in the Black-Scholes model.

### 3.2 Previous approaches in handling a changing volatility

Black and Scholes (1973) assume in their seminal study that the distribution of share prices is lognormal and the variance of the rates of return on the share is constant. However, the assumption that the share price volatility is constant is recognised to be heroic. In the past, some option researchers have tried to make adjustments for the simplistic assumption by measuring the historical volatility over a

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Tables and figures which are not put within the text can be found at the back of this chapter.

reasonably short and proximate period. Cox and Rubinstein (1985) suggest that researchers should use at least daily data in forecasting volatility over periods of less than one year. Galai (1977) purges his volatility estimates from weekend effects and holidays effects.

Parkinson (1980), followed by Beckers (1983), demonstrates that the use of extreme values (i.e., the high and low prices, etc.) greatly improves the volatility estimate when compared with the use of closing prices. Unfortunately, the use of extreme values is very vulnerable both to discontinuous trading during the day and to deviations from lognormality (Cox and Rubinstein 1985, p.277). Latané and Rendleman (1976), Chiras and Manaster (1978) and Whaley (1982) among others impute the implied volatility from the option formula with varying degrees of rigour. Schmalensee and Trippi (1978) find only a weak relationship between changes in the average implied standard deviations and the ex post time-series standard deviations. Moreover, Beckers (1981) notes that there is a basic inconsistency in employing the Black-Scholes formula to find presumably nonstationary future volatilities. Gemmill (1986) notes that the forecasting performance of the implied volatility was weak as the London Traded Options Market has matured. Finally, Akgiray (1989) uses the ARCH and the GARCH models to forecast future volatilities for CRSP indices.

It can be seen from the foregoing review that although the problem of a changing or nonstationary volatility is well recognised, there is little agreement on how best to forecast share price volatility in the face of such non-stationarity.

### 3.3 The persistence of volatility

It might appear that it would be possible to find out what causes volatility to change. An extensive study conducted by Schwert (1989) on this issue highlights indirectly that volatility is not closely related to other measures of economic volatility such as, for example, the volatility of inflation. Poterba and Summers (1986) review recent findings and point out that market volatility cannot be explained by fundamental variables such as cash flows. These studies suggest that share price volatility cannot be easily inferred from other economic factors. As a result, it is necessary to investigate the dynamics and movements of the volatilities themselves.

Mandelbrot (1963, p.418) notes seminally that for certain speculative prices:

*"..., large changes tend to be followed by large changes - of either sign - and small changes tend to be followed by small changes,..."*

Black (1976) also finds that changes in volatility tend to be maintained.

The ARCH model introduced by Engle (1982) and subsequently extended by Bollerslev (1986) to the GARCH model recognises explicitly the persistence of volatility in share prices. This chapter therefore examines the case for applying these models to a sample of UK daily share price returns. To evaluate the validity of the application, the fit and the forecasting accuracy of the ARCH and GARCH models are compared with two naive models; one derived by fitting an exponentially weighted moving average to the share returns data, the other by

estimating the historical volatility of the preceding period. Accordingly the bases of the four competing models are described.

### 3.4 Outline of the four competing models

#### 3.4.1 The Naive model

In deriving their option pricing model, Black and Scholes (1973) assume the volatility  $\sigma_t$  is constant. Empirically this implies that

$$\sigma_t = \sigma + \epsilon_t$$

where  $\sigma$  is a constant and  $\epsilon_t$  is a strict white noise. The naive model is equivalent to an ARIMA(0,0,0) model (Makridakis, Wheelwright and McGee 1983, p.358). Forecasts of  $\sigma_t$  can be made by the historical average, i.e.,  $E(\sigma_t) = \sigma$ . In the absence of any identified return generating process, the naive model will of course provide unbiased errors in forecasting future volatility. The task is seen to improve on this forecasting process.

#### 3.4.2 The Exponentially Weighted Moving Average model (EWMA)

The EWMA method is justified as a forecasting procedure if the volatility is generated by a process such as

$$\sigma_t - \sigma_{t-1} = (1 - \theta_1 B) a_t, \text{ where } a_t = \sigma_t - \hat{\sigma}_t$$

where  $\theta_1$  is a constant. This process is not stationary but the series of its first difference behaves like a first order moving average process. In Box-Jenkins terminology,  $\sigma_t$  would be explained by an ARIMA(0,1,1) model. The forecasting model appropriate for such a process is (cf. McKenzie 1984):

$$\begin{aligned}
\sigma_t - \sigma_{t-1} &= (1 - \theta_1 B) a_t \\
&= a_t - \theta_1 a_{t-1} \\
&= (\sigma_t - \hat{\sigma}_t) - \theta_1 (\sigma_{t-1} - \hat{\sigma}_{t-1}) \\
\Rightarrow \hat{\sigma}_t &= (1 - \theta_1) \sigma_{t-1} + \theta_1 \hat{\sigma}_{t-1}
\end{aligned} \tag{3.1}$$

Letting  $\theta = 1 - \theta_1$ , equation (3.1) becomes

$$\hat{\sigma}_t = \theta \sigma_{t-1} + (1 - \theta) \hat{\sigma}_{t-1} \tag{3.2}$$

which is an exponentially weighted moving average process. The forecasting constant  $\theta$  in equation (3.2) is obtained by minimising the mean square error

$$MSE(\theta) = \frac{1}{94} \sum_{t=1}^{94} (\sigma_t - \hat{\sigma}_t)^2$$

where  $\sigma_t$  in this study is the sample standard deviation over every twenty days and  $\hat{\sigma}_t$  is the EWMA forecast standard deviation. The estimation period is from 1 August 1977 to 10 October 1986 and  $\theta$  ranges from 0.025, 0.05, ..., to unity.

### 3.4.3 The ARCH Model

The ARCH( $q$ ) model is given by

$$h_t = \alpha_0 + \sum_{i=1}^q \beta_i \varepsilon_{t-i}^2, \quad \alpha_0 > 0, q > 0, \beta_i \geq 0, \tag{3.3}$$

where

$$\varepsilon_t = y_t - \mu, \text{ and } \varepsilon \sim N(0, h_t).$$

The conditional error distribution of  $\varepsilon_t$  is normal with the conditional variance  $h_t$  a linear function of past squared errors. Since the right hand side of equation (3.3) is positive, there is a tendency for extreme (large or small) values to be followed by other extreme values. Therefore, the ARCH model allows volatility shocks to persist over time.

### 3.4.4 The GARCH Model

The weakness of the ARCH model is that it has no memory for any variance innovations, i.e., past variance innovations are not accounted for in the ARCH model. The more general GARCH(p,q) model can include past conditional variance in the current conditional variance equation.

$$h_t = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j h_{t-j} \quad (3.4)$$

where

$$\varepsilon_t = y_t - \mu, \text{ and } \varepsilon \sim N(0, h_t),$$

and  $q > 0$ ,  $p \geq 0$ ,  $\alpha_0 \geq 0$ ,  $\alpha_i, \beta_j \geq 0$  for all  $i, j$ . In equation (3.4),  $\mu$  is a constant representing the long-run average on the assumption that returns are approximately uncorrelated over time.

The simple GARCH(1,1) model has intuitive appeal and is widely used in empirical studies.

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} \quad (3.5)$$

The parameter  $\alpha_1$  represents the magnitude of innovations in the conditional variance  $\varepsilon_t$  and  $\beta_1$  determines the persistence of such innovations in the following conditional variances.

Rearranging the terms in equation (3.5)

$$\begin{aligned} h_t &= \alpha_0 + \lambda h_{t-1} + (\alpha_1 \varepsilon_{t-1}^2 - \alpha_1 h_{t-1}), \quad \lambda = \alpha_1 + \beta_1 \\ &= \alpha_0 + \lambda h_{t-1} + \alpha_1 (\varepsilon_{t-1}^2 - h_{t-1}) \\ &= \alpha_0 + \lambda h_{t-1} + \alpha_1 v_{t-1} \end{aligned} \quad (3.6)$$

where

$$v_{t-1} \equiv \varepsilon_{t-1}^2 - h_{t-1}$$

and is a serially uncorrelated innovation.  $\lambda = \alpha_1 + \beta_1$  in equation (3.6) thus measures the persistence of variance. As  $\lambda$  approaches unity, the greater is the persistence of shocks in the variance. Bollerslev (1986) defines the GARCH(1,1) model to be wide-sense stationary when  $\alpha_1 + \beta_1$  is less than 1, and the unconditional variance of the residuals  $\epsilon_t$  (or,  $y_t$  return series, as  $\mu$  is a constant) is given by

$$\sigma^2 = E(\epsilon_t^2) = \frac{\alpha_0}{(1-\alpha_1-\beta_1)}$$

A number of empirical studies (French, Schwert and Stambaugh 1987, Bollerslev 1987) finds that  $\alpha_1 + \beta_1$  is close to but slightly less than unity. The Berndt, Hall, Hall and Hausman (1974) algorithm is used in fitting the parameters of both the ARCH and GARCH models. The algorithm maximises recursively the likelihood function

$$f_t(\theta) = -\frac{1}{2} \log_e(h_t) - \frac{1}{2} \frac{\epsilon_t^2}{h_t}$$

where  $\theta$  is the vector of parameters in the mean and conditional variance equations.

### 3.5. Data analysis

#### 3.5.1 Data source

Daily closing share prices of eighteen British companies are collected from Datastream. The names and acronyms of the eighteen companies are given in Chapter 7. The eighteen companies were chosen on the basis that the options of each company were actively traded throughout the sample period. The prices  $S_t$  are converted to continuously compounded rates of

return  $y_t = \log_e(S_t/S_{t-1})$ . The sample period spans from 1 August 1979 through 3 June 1988; 2,328 observations were available. For bank or public holidays, the missing returns are interpolated by an average of the returns five weeks before and five weeks after that date on the same week days (cf. Liu and Hudak (1986)).

### 3.5.2 Summary statistics of the return series

Most of the return series  $y_t$  are uncorrelated over time. The Ljung-Box portmanteau test statistics for up to the tenth order serial correlation are less than the Chi square statistic at 1 percent level of significance for 14 of the 18 return series. On the other hand, most of the squared returns are clearly not uncorrelated over time, as evidenced by the highly significant Ljung-Box test statistics  $Q^2(10)$  (Table 3.1). This implies that the return series lend themselves to modelling by the ARCH model, and in particular, the GARCH model (cf. Bollerslev 1987).

### 3.5.3 The forecasting procedure

Initially, a 7-year period from 1 August 1979 to 24 October 1986 is chosen to estimate the ARCH and GARCH parameters. These parameters are then used to forecast the actual volatility over the next twenty days. The starting and ending dates are then moved forward twenty days by dropping the oldest twenty observations and adding in twenty new observations. The model parameters are re-estimated and used to forecast the volatility over the next twenty days. The

process is repeated and rolled forward until 3 June 1988 so that there are 22 forecast volatilities. This out-of-the-sample one-period ahead forecasting process allows the model parameters to change in successive periods.

The conditional volatility of the return series is first fitted by the most parsimonious ARCH(1) and GARCH(1,1) models. The basic requirement of the validity of the model is that the estimated parameters should be positive and statistically significant. The same order of the ARCH and GARCH model is kept unchanged in each of the twenty-two forecasting periods.

Most of the return series are found to be well fitted by the simple ARCH(1) and GARCH(1,1) models. The exceptions are the return series of LSMR for the ARCH model, and BCHM, GEC, GKN and PO for the GARCH model. By a sequential search process, the higher order ARCH and GARCH models which well fit these series are found to be

$$h_t = \alpha_0 + \alpha_2 \varepsilon_{t-2}^2$$

$$h_t = \alpha_0 + \alpha_3 \varepsilon_{t-3}^2 + \beta_1 h_{t-1}$$

$$h_t = \alpha_0 + \alpha_2 \varepsilon_{t-2}^2 + \beta_3 h_{t-3}$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-2}$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_3 h_{t-3}$$

respectively. The sample sizes of the return series of BCHM and GKN have to be increased for two more years for their GARCH models to get rid of any negative betas for all twenty-two forecasting periods. This leaves only in the fourteenth period one value of  $\beta_1=1.41$  for BCHM. The estimate however is not significantly different from zero. The returns series of GMET

is the one which presents most difficulty to modelling. The Lagrangian multiplier test for its ARCH(1) model is not significant for most of the twenty-two periods. For the GARCH(1,1) model there are negative betas at the seventh and thirteenth forecasting periods. Despite trying different variates of the GARCH process, no satisfactory model was found which appeared to fit the process over the whole twenty-two periods. Accordingly the second best solution of using GARCH(1,1) was adopted. For forecasting purposes, the parameter estimates for the two 'rogue' periods are replaced by the estimates for their immediate preceding periods.

### 3.6 Results of parameter estimates

The results of the ARCH and GARCH model parameter estimates are reported in Table 3.2 and Table 3.3 respectively. The numbers are the averages of the alphas, betas, t-ratios, and loglikelihood function values across the twenty-two forecasting periods. For the ARCH model, the t-statistics of  $\alpha_0$  and  $\alpha_1$  ( $\alpha_2$  for LSMR) are highly significant. Engle (1982) points out that the existence of an ARCH effect can be measured by the Lagrangian multiplier test statistic

$$\chi_1^2 = n \cdot R^2$$

where  $n$  is the sample size and  $R^2$  is the multiple regression  $r$ -squared for the squared returns on one lag of itself. The test statistic will be asymptotically distributed as Chi square with one degree of freedom when the null hypothesis  $\alpha_1 = 0$  is true. All return series possess a large Lagrangian multiplier test statistic (except GMET), indicating that they have a strong

ARCH effect. The loglikelihood function values of the ARCH models are consistently less than that of the GARCH model for the same return series (except BCHM, where they are very close). This suggests that the GARCH model represents the volatility process better than the ARCH model does.

The EWMA forecasting constants (Table 3.4) are estimated in the first forecasting period, i.e., from 1 August 1979 to 24 October 1986. (Those of BCHM and GKN start from 24 October 1977). The constants are all less than 0.5 and have an average of 0.215.

### 3.7. Diagnostic tests

The appropriateness of the ARCH and GARCH models in modelling the volatility process can be checked by a number of diagnostic tests. Akgiray (1989) among others examines the standardised residuals

$$\frac{\varepsilon_t}{\sqrt{\hat{h}_t}} \quad (3.7)$$

for unit normality. The means of the ARCH or GARCH standardised residuals are found all close to zero and not significantly different from zero. The standard deviations of both the ARCH and GARCH standardised residuals are very close to one. The exceptions are those of BCHM and GKN. However, these are not surprising as these two series require higher GARCH orders and extra sample data to estimate their model parameters just to meet the basic requirements.

Normality is measured by the correlation between the standardised residuals and their own normal scores. Most of the

standardised residuals have a correlation coefficient of larger than 0.985 and therefore the hypothesis that they are normal cannot be rejected (Table 3.5). It is interesting to observe that the series whose ARCH standardised residuals which are rejected as normal (e.g., GKN) also have their GARCH standardised residuals rejected as normal. Such residuals also have very large kurtoses (Table 3.6).

The Ljung-Box portmanteau test statistics for up to the tenth order,  $Q(10)$ , for the ARCH and GARCH standardised residuals are not significant for most series except BARC, LRHO, and SHEL (Tables 3.6 and 3.7). However, when compared with the Ljung-Box test statistics  $Q(10)$  of their original return series  $y_t$  (or  $\hat{\epsilon}_t$ ), the Ljung-Box test statistics  $Q(10)$  of BARC and LRHO have been reduced and that of SHEL has been maintained at approximately the same level. Therefore, it can be concluded that there is little evidence of serial dependence in the first order standardised residuals. It is important to note that the Ljung-Box test statistics  $Q^2(10)$  for all the GARCH squared standardised residuals

$$\epsilon^2 / \hat{h}_t$$

are not significant. On the other hand, more than half of the ARCH squared standardised residuals have very large Ljung-Box test statistics. This implies that some time varying second order effects still persist in most of the ARCH squared standardised residuals. The GARCH model, which is dominated by the parameter  $\beta$  (the measure of persistence), has captured all present and in particular, past volatility persistence (cf. McCurdy and Morgan 1988). Finally, all kurtoses of both the

ARCH and GARCH standardised residuals (Equation 3.7) are larger than 3, and some are even substantially larger than 10. This suggests that the assumption of normality for the residuals  $\epsilon_t$  may not be appropriate. A conditional t-distribution for the residuals  $\epsilon_t$  may be a better specification for the return series (cf. Bollerslev 1987).

To conclude, the standardised residuals of both models are unit normal in general. They possess no further first order serial dependence. The GARCH squared standardised residuals have absorbed all second order serial dependence whilst most of the ARCH squared standardised residuals still have significant time varying second order effects.

### 3.8 Comparison between actual and forecasted volatilities

The actual and the forecasted volatilities by the four methods are shown in Figures 3.1. There are 21 forecasting periods, 2 to 22. The level of the volatilities lies between 0 and 0.1.

The actual volatilities are exceptionally high around the 1987 October crash. These are represented by the two spikes at periods 13 and 14, specifically from 28 September to 20 November 1987.

The GARCH and ARCH models depict quite accurately the pattern of the actual volatilities. The GARCH model is superior to the ARCH model in that the GARCH volatilities rise and fall within a range closely approximating the actual ones, whilst the ARCH volatilities move up and down only moderately.

The EWMA volatilities only capture the overall trend of

the volatilities movement. They move upward gently across forecasting periods, bounce up steeply around the 1987 crash and move downward gradually. They fail to capture the short-term transitory movement of the volatilities. The naive model though reflects the volatility of the immediate past fails to capture any persistence of volatilities in earlier periods.

These findings reveal that the conditional heteroscedasticity models realistically depict the intertemporal and transitory changes of the actual volatilities.

### 3.9 Comparison of forecasting accuracy

The forecasted volatilities of the four models are evaluated and compared through three error functions: root mean square error (RMSE), mean absolute error (MAE), and mean absolute percent error (MAPE). They are defined as follows:

$$RMSE = \sqrt{\frac{\sum_{t=1}^{21} (\hat{\sigma}_t - F_t)^2}{21}}$$

$$MAE = \frac{\sum_{t=1}^{21} |\hat{\sigma}_t - F_t|}{21}$$

$$MAPE = \frac{\sum_{t=1}^{21} \frac{|\hat{\sigma}_t - F_t|}{21\hat{\sigma}_t}}$$

where  $\hat{\sigma}$  is the historical volatility (standard deviation) and  $F_t$  denotes the forecasted value. Of these three statistics, the RMSE tends to penalise outliers while the MAE provides a linear measure of the errors. The purpose of using three statistics is to examine the sensitivity of the ranking of forecasting methods to the choice of error functions. The detailed results

are reported in Table 3.8. The table below is a summary of the relative forecasting accuracy among the four methods.

Table 3.8.1 Number of dominant rankings

Method	RMSE	MAE	MAPE
GARCH	11	10	8
ARCH	3	2	2
EWMA	3	4	5
Naive	1	2	3

where each entry contains the number of times a least error measurement is observed when the row forecasting method is measured by the column statistic.

Based on the relative values of these statistics, the accuracy of the GARCH forecasts dominates the other three. The naive method is the worst. If the GARCH, the ARCH, and the EWMA models are compared pairwise, the analogous reduced tables are as follows:

Table 3.8.2 Number of dominant rankings

Method	RMSE	MAE	MAPE
GARCH	13	13	13
ARCH	5	5	5

Table 3.8.3 Number of dominant rankings

Method	RMSE	MAE	MAPE
GARCH	11	13	11
EWMA	7	5	7

Table 3.8.4 Number of dominant rankings

Method	RMSE	MAE	MAPE
ARCH	12	9	4
EWMA	6	9	14

The GARCH method is superior to both the ARCH and the EWMA methods. The EWMA method is slightly better than the ARCH method.

### 3.10 Concluding remarks

In this chapter, we have reviewed previous approaches in estimating the share price volatility and their weaknesses. It is found that UK share price volatilities can be modelled by the conditional variance models. The out-of-the-sample forecasting procedures allow the model parameters to be re-estimated over successive forecasting periods. The parameter estimates of both the ARCH and the GARCH models are shown to be very significant. Diagnostic tests reveal that the GARCH squared standardised residuals have absorbed all second order serial dependence, whilst most of the ARCH squared standardised residuals still carry significant ARCH effects.

A graphical comparison of the actual and forecasted volatilities reveals that the GARCH specification is superior to the other three models in depicting the pattern of actual volatilities (Figures 3.1). Furthermore, a comparison of the forecasting accuracy of the four methods also singled out the GARCH forecasts as far better than the other three. As a result, the claim of Dimson and Marsh (1990) that:

*"...a simple model such as the moving average might outperform the more sophisticated ones on a out-of-the-sample basis"*

need not generally be true.

The implication of our results to option pricing is that instead of refining any historical volatility measures, the GARCH model can be used to generate ex ante share price volatilities (cf. Akgiray 1989). However, the GARCH volatility estimates cannot be applied directly in the Black-Scholes model as volatility is assumed to be constant in that model. The next chapter shows how to adapt the GARCH volatility estimate into the Black-Scholes model.

Table 3.1 - Summary Statistics

Return  $y_t$ , or  $\epsilon_t$  in  $y_t = \mu + \epsilon_t$

Series	Q(10)	Q <sup>2</sup> (10)	$\kappa$
BARC	31.66	58.29	4.36
BCHM	10.16	22.14	6.84
BP	15.95	80.50	4.49
CGLD	14.71	88.71	4.99
CTLD	8.48	14.59	12.41
CUAC	15.90	43.33	5.50
GEC	11.65	19.97	5.99
GKN	18.17	32.69	7.84
GMET	20.08	16.22	4.21
ICI	5.19	89.52	5.82
LAND	9.63	230.96	4.48
LRHO	38.35	145.87	5.44
LSMR	12.20	81.14	4.71
MKS	10.80	71.18	4.37
P.O.	15.21	76.99	18.45
RCAL	25.80	34.56	13.25
RTZ	19.33	118.45	4.89
SHEL	29.63	79.64	5.22

Note: In Tables 3.1, 3.6 and 3.7, Q(10) and Q<sup>2</sup>(10) denote the Ljung-Box portmanteau test statistics of the first and second orders respectively.  $\kappa$  is the kurtosis test centred on 3.

Table 3.2 - Maximum Likelihood Estimations and Tests (ARCH)

	$\mu$	$\alpha_0$	$\alpha_1$	$nR^2$	LLFV
BARC	$4.73 \times 10^{-4}$ (1.30)	$2.26 \times 10^{-4}$ (36.13)	0.109 (5.43)	135.23 0.0000*	6886
BCHM	$4.74 \times 10^{-4}$ (1.32)	$2.81 \times 10^{-4}$ (56.36)	0.111 (7.53)	99.51 0.0254	8314
BP	$4.16 \times 10^{-4}$ (1.09)	$2.59 \times 10^{-4}$ (33.71)	0.072 (3.99)	33.94 0.0300	6786
CGLD	$5.48 \times 10^{-4}$ (1.33)	$2.69 \times 10^{-4}$ (31.32)	0.170 (11.30)	24.24 0.0003	6666
CTLD	$10.34 \times 10^{-4}$ (2.34)	$3.28 \times 10^{-4}$ (42.49)	0.127 (5.67)	19.05 0.0003	6522
CUAC	$1.74 \times 10^{-4}$ (0.45)	$2.59 \times 10^{-4}$ (37.16)	0.159 (7.25)	42.16 0.0000	6716
GEC	$3.94 \times 10^{-4}$ (0.98)	$2.72 \times 10^{-4}$ (39.48)	0.110 (4.87)	52.50 0.0521	6709
GKN	$-0.56 \times 10^{-4}$ (-0.14)	$3.53 \times 10^{-4}$ (53.36)	0.172 (9.75)	17.24 0.0001	7989
GMET	$8.69 \times 10^{-4}$ (2.36)	$2.38 \times 10^{-4}$ (38.28)	0.074 (4.47)	56.81 0.1239	6865
ICI	$7.29 \times 10^{-4}$ (2.24)	$1.97 \times 10^{-4}$ (42.34)	0.110 (6.05)	53.94 0.0001	7015
LAND	$4.50 \times 10^{-4}$ (1.55)	$1.39 \times 10^{-4}$ (34.09)	0.202 (7.05)	155.45 0.0000	7264
LRHO	$5.60 \times 10^{-4}$ (1.55)	$2.29 \times 10^{-4}$ (36.78)	0.236 (9.41)	117.62 0.0000	6776
LSMR	$-4.25 \times 10^{-4}$ (-0.73)	$5.94 \times 10^{-4}$ (37.16)	0.076 (4.86)	11.26 0.0342	5995
MKS	$9.12 \times 10^{-4}$ (2.49)	$2.30 \times 10^{-4}$ (30.58)	0.143 (6.79)	72.91 0.0000	6838
P.O.	$6.29 \times 10^{-4}$ (1.69)	$2.29 \times 10^{-4}$ (70.18)	0.290 (16.93)	58.62 0.0000	6777
RCAL	$2.97 \times 10^{-4}$ (0.61)	$3.93 \times 10^{-4}$ (59.70)	0.149 (7.46)	33.62 0.0000	6347
RTZ	$5.69 \times 10^{-4}$ (1.46)	$2.36 \times 10^{-4}$ (40.74)	0.173 (8.55)	66.34 0.0000	6795
SHEL	$4.90 \times 10^{-4}$ (1.47)	$1.92 \times 10^{-4}$ (45.17)	0.115 (5.02)	50.40 0.0020	7036

Note: Numbers in parentheses are t-ratios. \* = p-value.  
LLFV = loglikelihood function value.

Table 3.3 - Maximum Likelihood Estimations (GARCH)

Company	$\mu$	$\alpha_0$	$\alpha_1$	$\beta_1$	$\alpha_1 + \beta_1$	LLFV
BARC	$6.45 \times 10^{-4}$ (1.82)	$0.31 \times 10^{-4}$ (5.82)	0.081 (6.60)	0.799 (29.74)	0.880	6905
BCHM	$5.81 \times 10^{-4}$ (1.63)	$0.77 \times 10^{-4}$ (8.09)	0.068 (7.23)	0.717 (41.62)	0.785	8294
BP	$3.40 \times 10^{-4}$ (0.93)	$0.40 \times 10^{-4}$ (7.91)	0.079 (6.29)	0.778 (39.45)	0.857	6809
CGLD	$5.06 \times 10^{-4}$ (1.22)	$0.77 \times 10^{-4}$ (8.07)	0.150 (11.86)	0.625 (19.49)	0.775	6683
CTLD	$11.72 \times 10^{-4}$ (2.83)	$0.49 \times 10^{-4}$ (9.30)	0.094 (8.85)	0.781 (51.81)	0.875	6541
CUAC	$4.02 \times 10^{-4}$ (1.04)	$0.51 \times 10^{-4}$ (5.57)	0.079 (6.81)	0.753 (25.56)	0.832	6729
GEC	$6.19 \times 10^{-4}$ (1.66)	$0.31 \times 10^{-4}$ (7.64)	0.069 (6.21)	0.834 (49.99)	0.902	6713
GKN	$1.01 \times 10^{-4}$ (0.27)	$0.35 \times 10^{-4}$ (9.22)	0.110 (10.58)	0.811 (51.37)	0.921	8043
GMET	$9.66 \times 10^{-4}$ (2.79)	$0.35 \times 10^{-4}$ (6.96)	0.075 (6.59)	0.788 (33.28)	0.862	6895
ICI	$7.23 \times 10^{-4}$ (2.22)	$0.19 \times 10^{-4}$ (9.07)	0.081 (9.04)	0.836 (57.45)	0.917	7061
LAND	$4.91 \times 10^{-4}$ (1.71)	$0.28 \times 10^{-4}$ (6.89)	0.107 (6.88)	0.726 (25.47)	0.833	7300
LRHO	$6.60 \times 10^{-4}$ (1.81)	$0.48 \times 10^{-4}$ (8.32)	0.160 (9.21)	0.682 (24.26)	0.841	6819
LSMR	$-4.85 \times 10^{-4}$ (-0.89)	$0.51 \times 10^{-4}$ (8.35)	0.061 (6.43)	0.862 (66.53)	0.923	6029
MKS	$7.83 \times 10^{-4}$ (2.15)	$0.43 \times 10^{-4}$ (5.74)	0.101 (7.15)	0.740 (23.69)	0.841	6855
P.O.	$5.56 \times 10^{-4}$ (1.64)	$1.16 \times 10^{-4}$ (13.35)	0.322 (16.28)	0.349 (11.31)	0.671	6786
RCAL	$3.70 \times 10^{-4}$ (0.83)	$0.46 \times 10^{-4}$ (10.16)	0.132 (9.98)	0.784 (47.53)	0.915	6391
RTZ	$4.63 \times 10^{-4}$ (1.24)	$0.33 \times 10^{-4}$ (9.53)	0.105 (10.63)	0.778 (50.19)	0.883	6846
SHEL	$5.64 \times 10^{-4}$ (1.77)	$0.20 \times 10^{-4}$ (9.47)	0.083 (7.22)	0.828 (57.88)	0.911	7075

Note: Number in parentheses are t-ratios.  
LLFV = Loglikelihood function value.

Table 3.4 - EWMA Forecasting Constants

BARC	0.100	ICI	0.200
BCHM	0.250	LAND	0.425
BP	0.450	LRHO	0.125
CGLD	0.050	LSMR	0.300
CTLD	0.100	MKS	0.175
CUAC	0.150	P.O.	0.100
GEC	0.275	RCAL	0.275
GKN	0.275	RTZ	0.250
GMET	0.050	SHEL	0.325

Table 3.5 - Diagnostic Tests for Unit Normality

Series	ARCH			GARCH		
	Mean <sup>1</sup>	Stdev	Normality <sup>2</sup>	Mean	Stdev	Normality
BARC	0	1.0003	0.993	0	1.0000	0.994
BCHM *	0	1.0621	0.981	0	1.0527	0.980
BP	0	1.0003	0.992	0	0.9976	0.993
CGLD	0	1.0003	0.992	0	0.9999	0.993
CTLD *	0	0.9996	0.965	0	0.9973	0.970
CUAC	0	1.0002	0.987	0	0.9998	0.986
GEC *	0	1.0003	0.984	0	0.9964	0.986
GKN *	0	1.0516	0.981	0	1.0314	0.980
GMET	0	1.0003	0.994	0	0.9989	0.994
ICI	0	1.0003	0.983	0	0.9999	0.986
LAND	0	1.0002	0.992	0	1.0000	0.993
LRHO	0	1.0002	0.979	0	1.0001	0.981
LSMR	0	1.0001	0.991	0	0.9982	0.993
MKS	0	1.0002	0.994	0	1.0001	0.994
P.O. *	0	0.9939	0.952	0	0.9965	0.956
RCAL *	0	0.9978	0.958	0	0.9970	0.966
RTZ	0	1.0003	0.987	0	0.9993	0.986
SHEL	0	1.0003	0.987	0	0.9994	0.990

Note:

1. The t-statistics for the means are very close to zero.
2. This value is the correlation between the standardised residuals and its normal score. We reject the hypothesis of normality if this value falls below 0.985 (marked with \*).

Table 3.6 - Diagnostic Test  
ARCH Squared Standardised Residuals

Series	Q(10)	Q <sup>2</sup> (10)	$\kappa$
BARC	28.50	24.63	4.22
BCHM	9.86	14.97	7.39
BP	16.40	58.57	4.42
CGLD	17.52	28.34	4.36
CTLD	7.42	1.95	12.82
CUAC	14.65	18.95	5.22
GEC	11.77	14.73	6.17
GKN	15.18	12.23	8.38
GMET	20.14	12.61	4.20
ICI	3.48	56.10	5.92
LAND	10.65	52.02	4.26
LRHO	29.64	62.02	5.26
LSMR	12.44	56.90	4.53
MKS	11.98	37.87	3.99
P.O.	12.67	9.51	14.89
RCAL	16.61	11.30	14.43
RTZ	19.75	61.23	5.07
SHEL	29.91	46.71	5.43

Table 3.7 - Diagnostic Test  
 GARCH Squared Standardised Residuals

Series	Q(10)	Q <sup>2</sup> (10)	$\kappa$
BARC	29.64	3.92	4.21
BCHM	7.52	5.92	8.00
BP	18.43	8.02	4.42
CGLD	21.49	5.91	4.37
CTLD	7.52	2.27	9.72
CUAC	13.69	8.62	5.26
GEC	12.73	6.14	5.69
GKN	11.17	3.29	9.68
GMET	23.11	2.16	4.23
ICI	1.77	7.10	5.30
LAND	12.45	4.91	4.10
LRHO	35.38	5.39	4.98
LSMR	14.79	8.20	4.14
MKS	13.67	7.18	3.95
P.O.	14.33	6.71	12.68
RCAL	20.01	4.83	11.03
RTZ	22.17	16.12	5.24
SHEL	30.34	6.51	5.05

Table 3.8 - Comparison of Forecasting Accuracy

RMSE	MAE	MAPE	RMSE	MAE	MAPE
BARC			ICI		
0.00995 <sup>1</sup>	0.00656	0.36900	0.00744	0.00427	0.23983
0.00941 <sup>2</sup>	0.00674	0.34753	0.00861	0.00581	0.34280
0.00955 <sup>3</sup>	0.00635	0.31615	0.00910	0.00572	0.31039
0.01096 <sup>4</sup>	0.00754	0.40727	0.00907	0.00481	0.24480
BCHM			LAND		
0.01145	0.00731	0.43727	0.00681	0.00446	0.22706
0.01194	0.00802	0.48470	0.00715	0.00476	0.24994
0.01211	0.00737	0.39956	0.00679	0.00501	0.28935
0.01263	0.00731	0.36241	0.00714	0.00535	0.30809
BP			LRHO		
0.00613	0.00469	0.26599	0.01023	0.00691	0.39920
0.00625	0.00473	0.26998	0.01140	0.00832	0.49486
0.00633	0.00502	0.30240	0.01176	0.00791	0.37007
0.00786	0.00597	0.34214	0.01228	0.00755	0.41256
CGLD			LSMR		
0.00902	0.00590	0.25029	0.00951	0.00814	0.35178
0.00911	0.00600	0.26027	0.00973	0.00759	0.31984
0.01010	0.00615	0.24458	0.01106	0.00920	0.39530
0.00930	0.00564	0.23330	0.01141	0.00940	0.39585
CTLD			MKS		
0.00717	0.00508	0.29834	0.00704	0.00485	0.21745
0.00841	0.00628	0.35838	0.00775	0.00507	0.22157
0.00911	0.00591	0.28523	0.00709	0.00517	0.26113
0.00896	0.00630	0.31030	0.00798	0.00558	0.27512
CUAC			P.O.		
0.00651	0.00449	0.24153	0.01593	0.00918	0.58574
0.00826	0.00576	0.29990	0.01189	0.00804	0.55989
0.00904	0.00596	0.29760	0.00937	0.00475	0.25896
0.00856	0.00552	0.28394	0.00991	0.00535	0.28656

Table 3.8 - Comparison of Forecasting Accuracy (Continuation)

RMSE	MAE	MAPE	RMSE	MAE	MAPE
GEC			RCAL		
0.00824	0.00486	0.20326	0.01686	0.01156	0.43113
0.00870	0.00553	0.22898	0.01342	0.00903	0.30861
0.00793	0.00474	0.21825	0.01452	0.01103	0.45329
0.00935	0.00597	0.25506	0.01878	0.01293	0.51896
GKN			RTZ		
0.01310	0.00867	0.34906	0.01122	0.00819	0.36633
0.01211	0.00874	0.39949	0.01558	0.01064	0.44175
0.01207	0.00896	0.38636	0.01474	0.00914	0.35908
0.01119	0.00827	0.36722	0.01429	0.01024	0.41671
GMET			SHEL		
0.00784	0.00443	0.26816	0.00922	0.00663	0.43752
0.01001	0.00621	0.34895	0.00831	0.00622	0.42853
0.01071	0.00635	0.34658	0.00854	0.00609	0.44277
0.01020	0.00596	0.36118	0.00988	0.00685	0.42410

Note: Forecasting accuracy, using 1. GARCH, 2. ARCH, 3. EWMA, 4. Naive models.

Figure 3.1

Comparison between Actual and Forecasted Volatilities

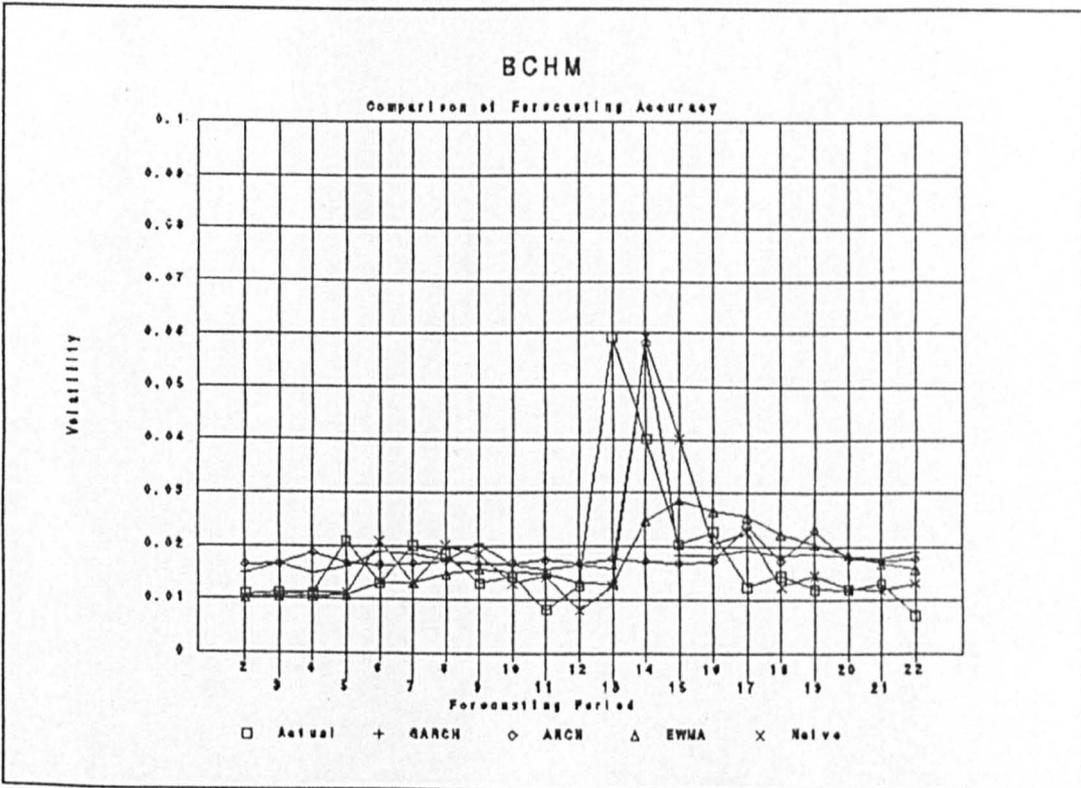
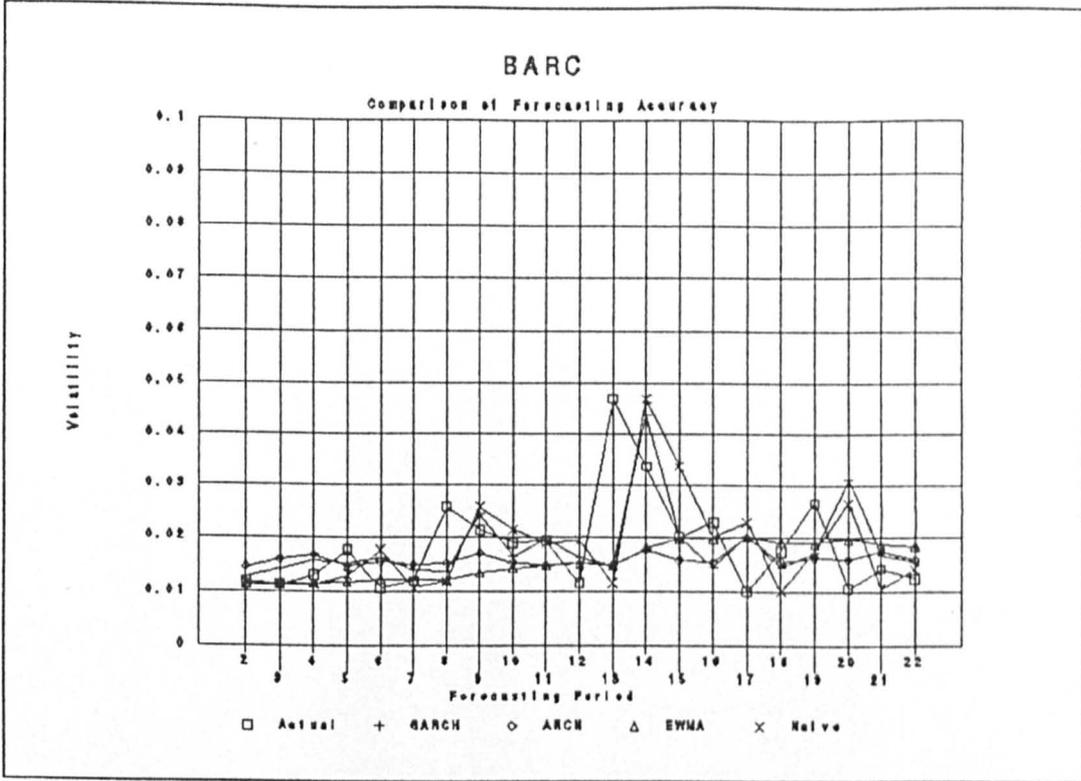


Figure 3.1 (continued)

Comparison between Actual and Forecasted Volatilities

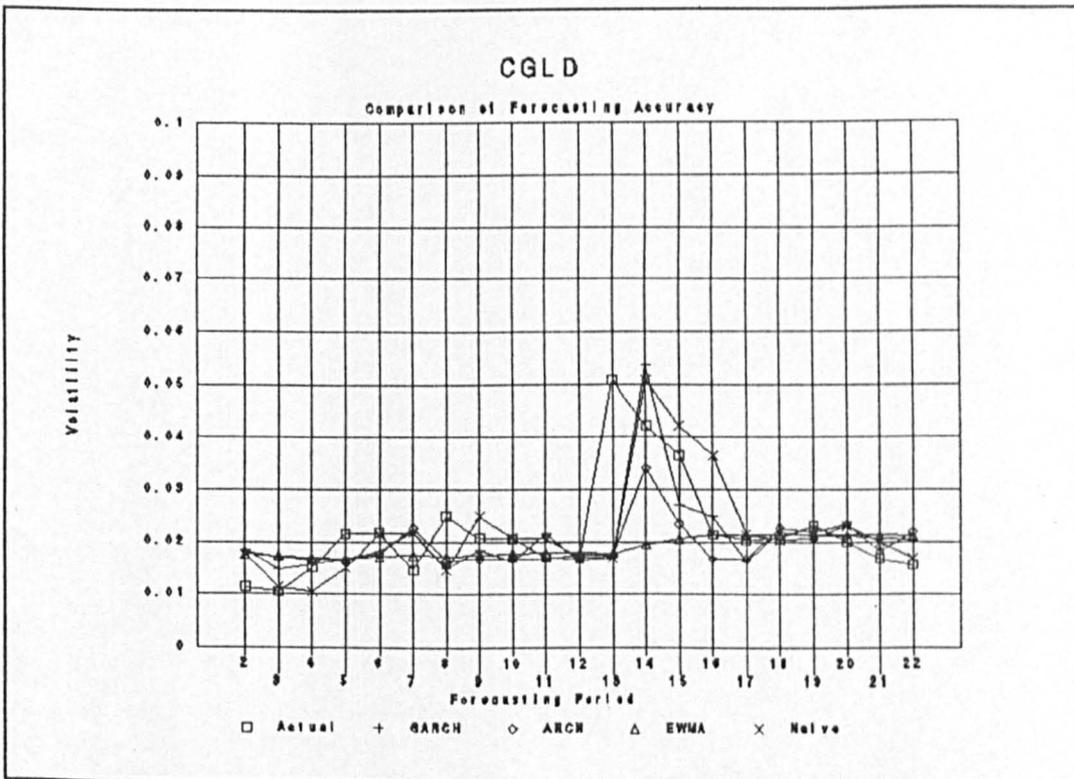
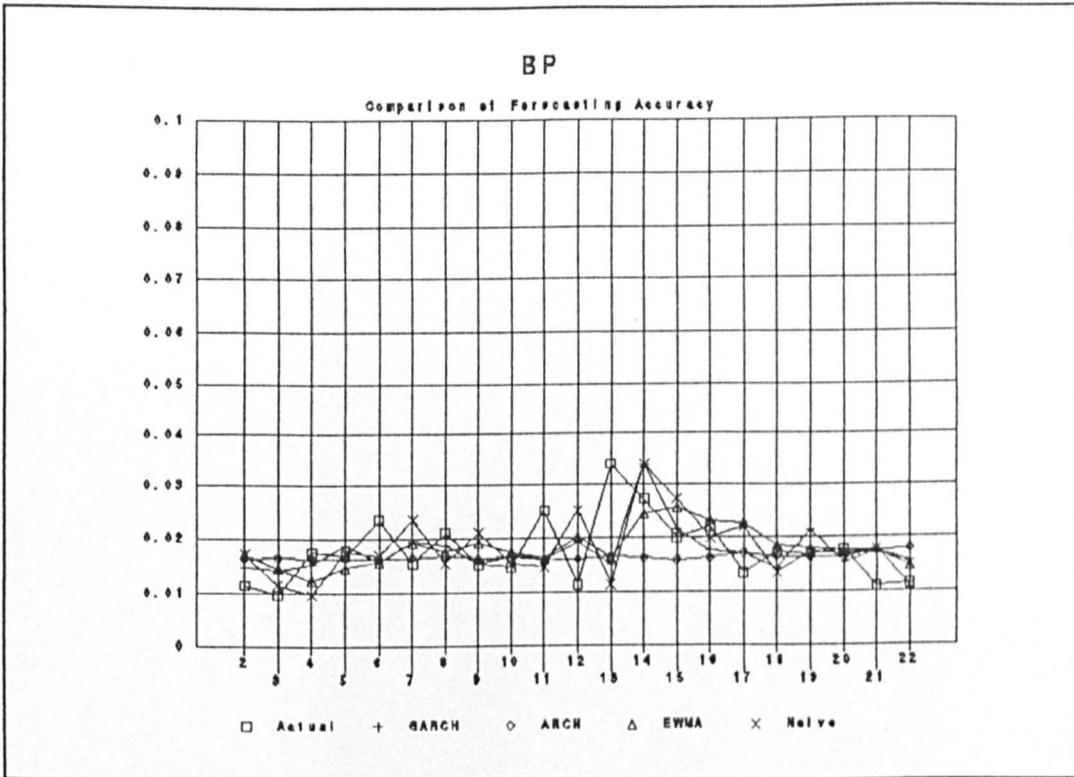


Figure 3.1 (continued)

Comparison between Actual and Forecasted Volatilities

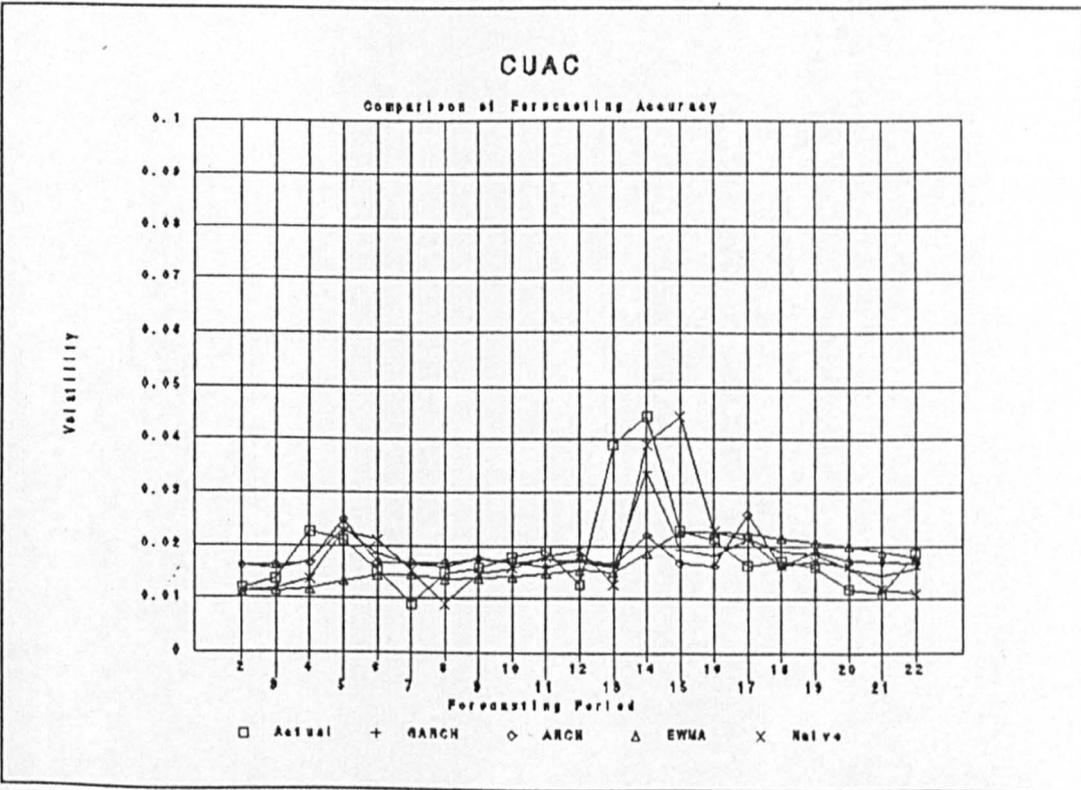
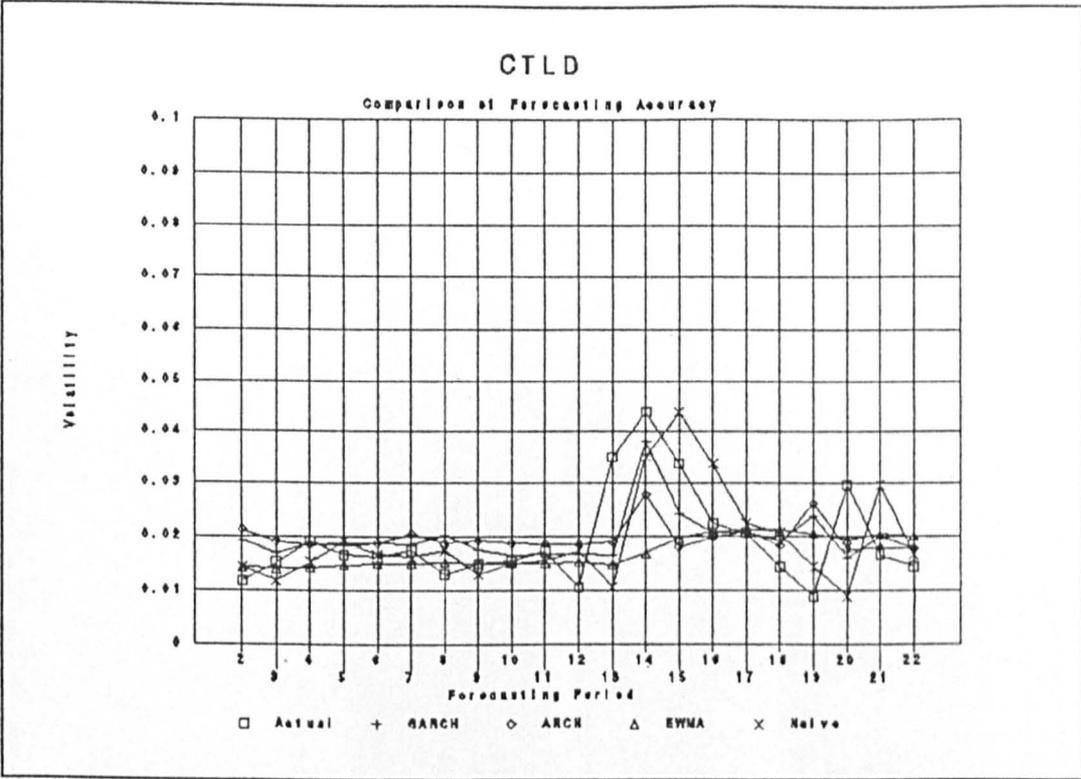


Figure 3.1 (continued)

Comparison between Actual and Forecasted Volatilities

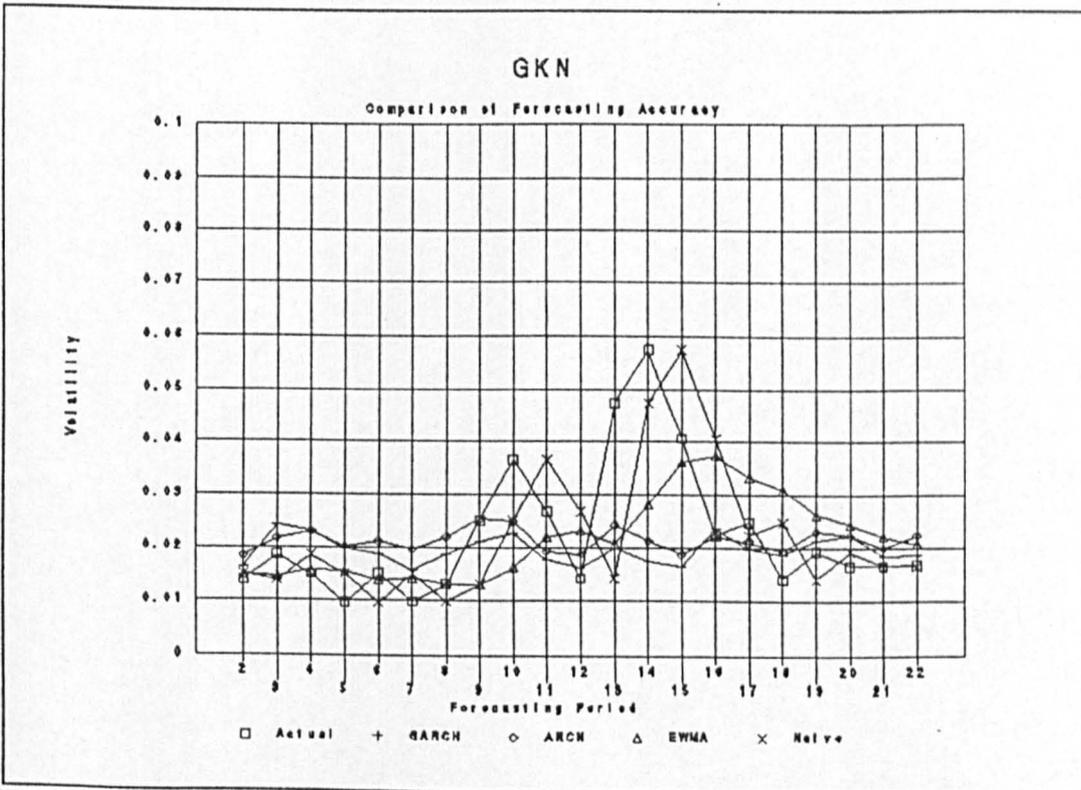
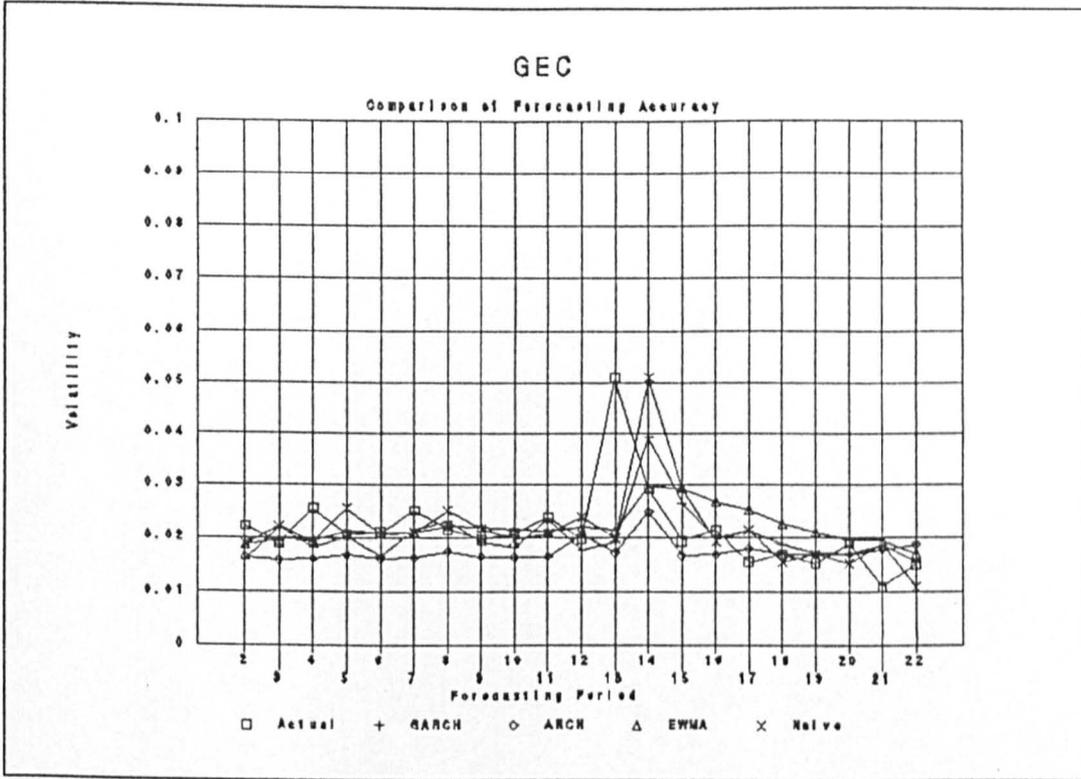


Figure 3.1 (continued)

Comparison between Actual and Forecasted Volatilities

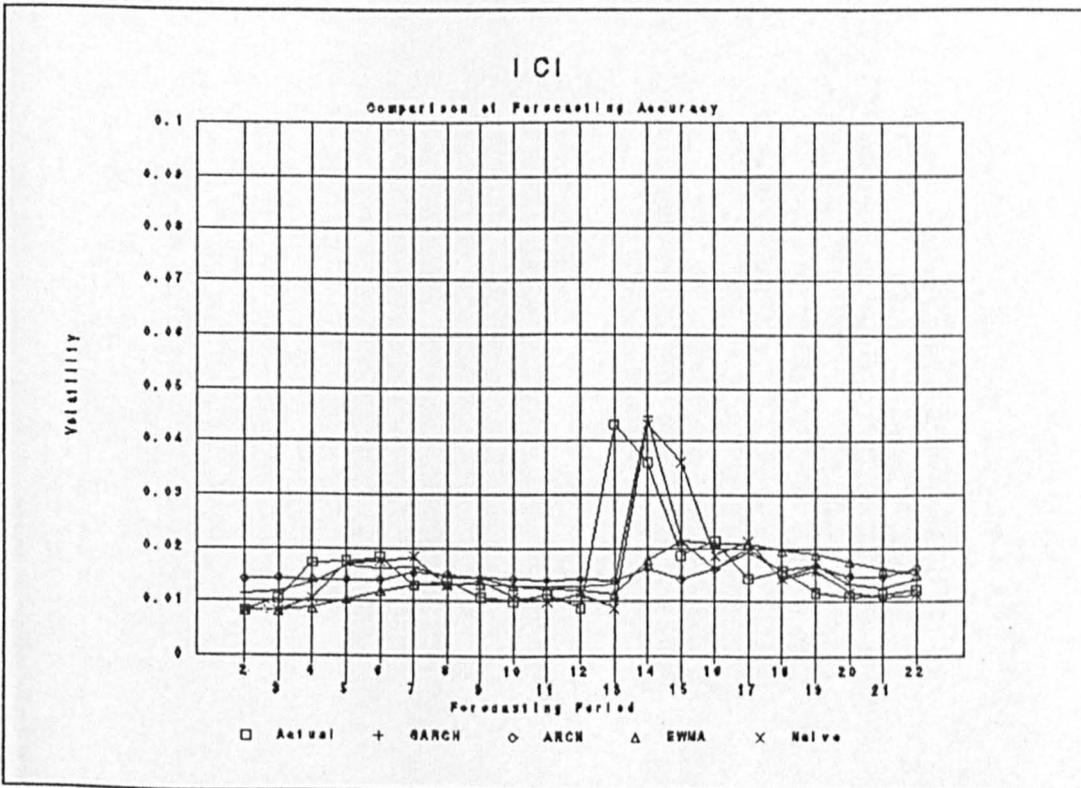
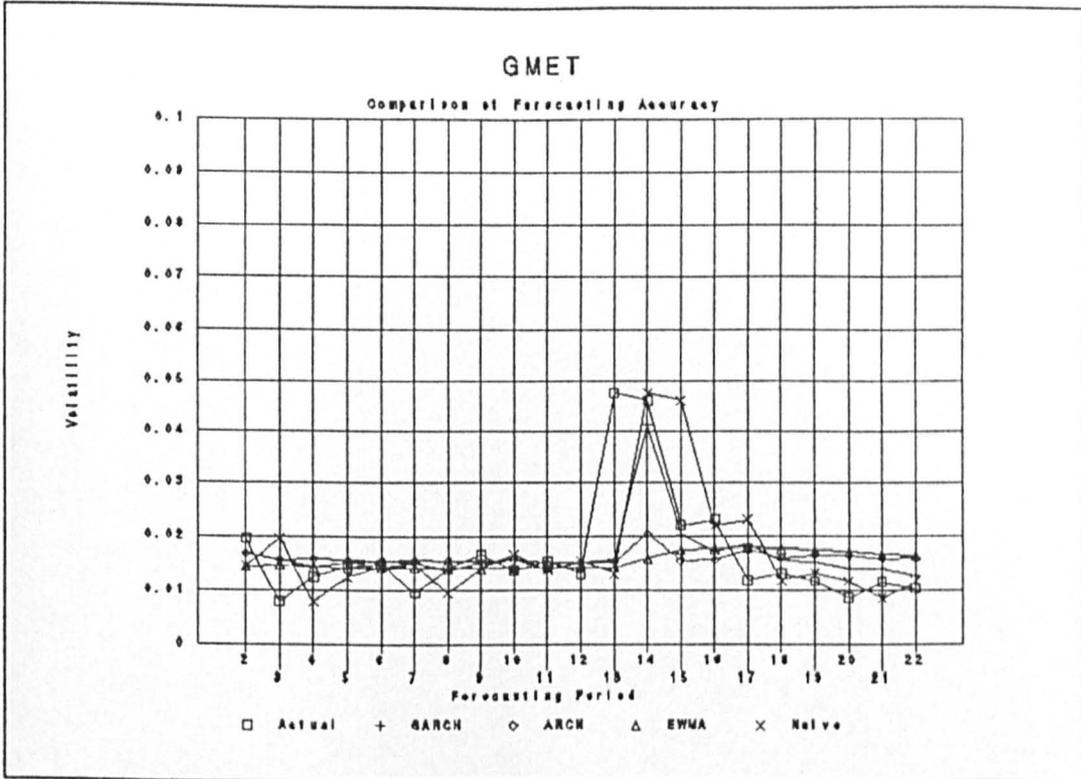


Figure 3.1 (continued)

Comparison between Actual and Forecasted Volatilities

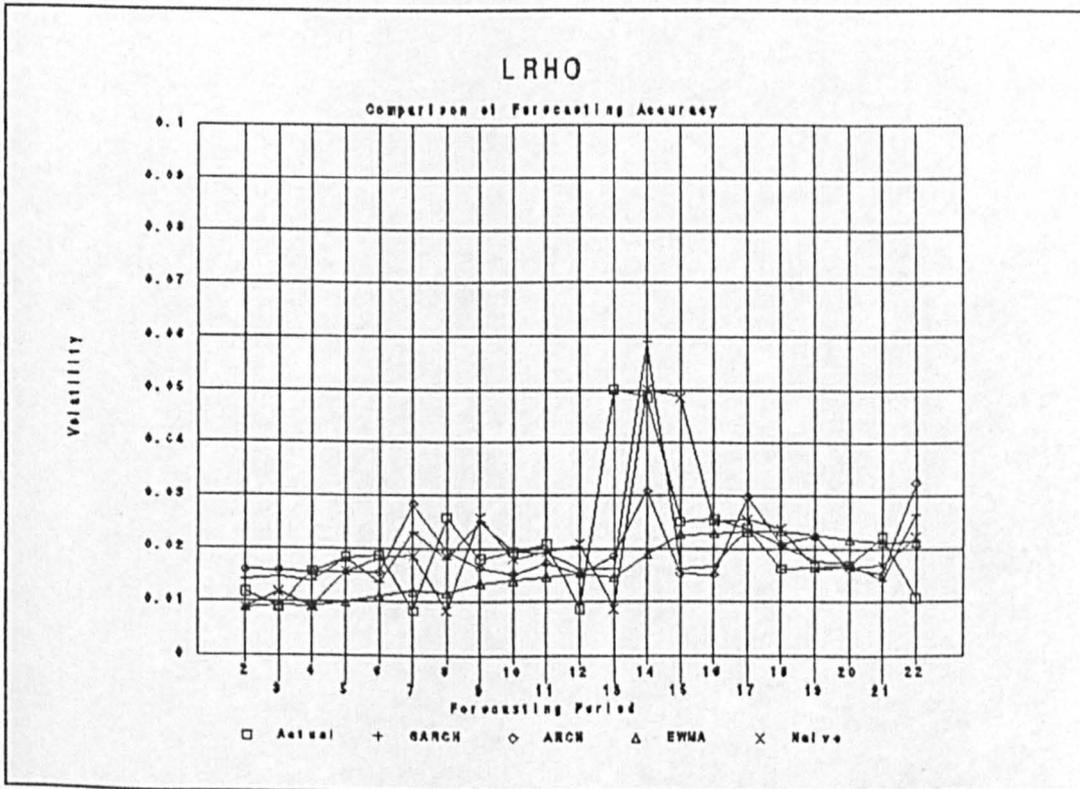
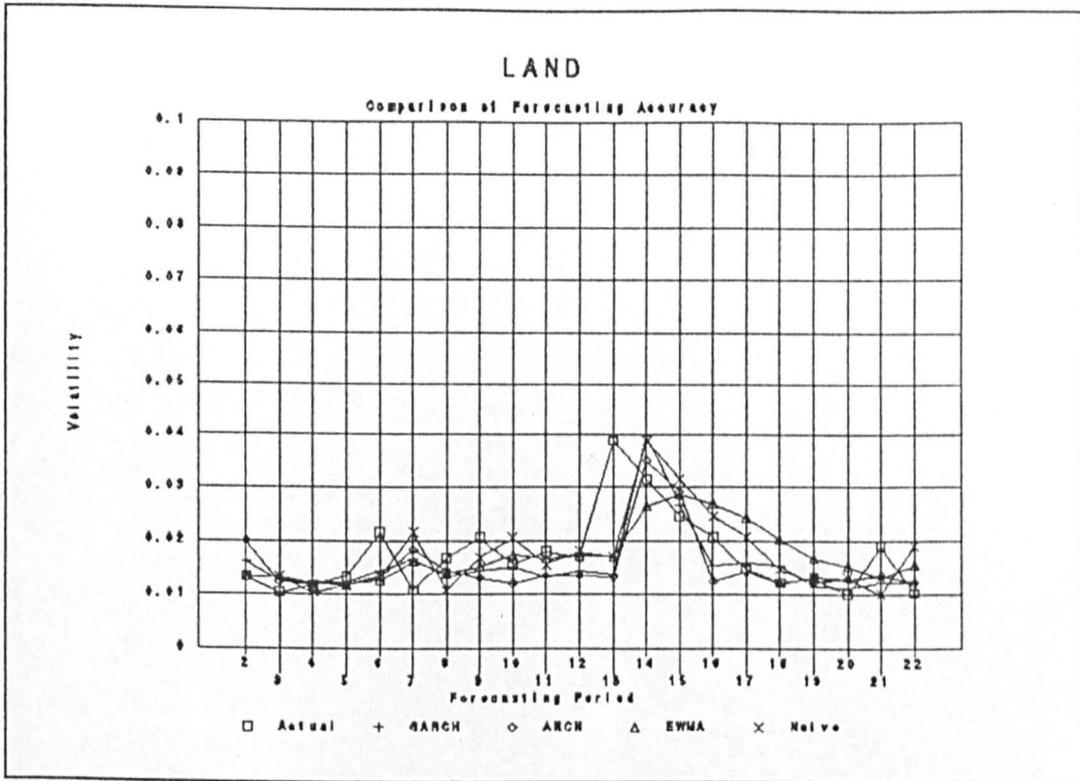


Figure 3.1 (continued)

Comparison between Actual and Forecasted Volatilities

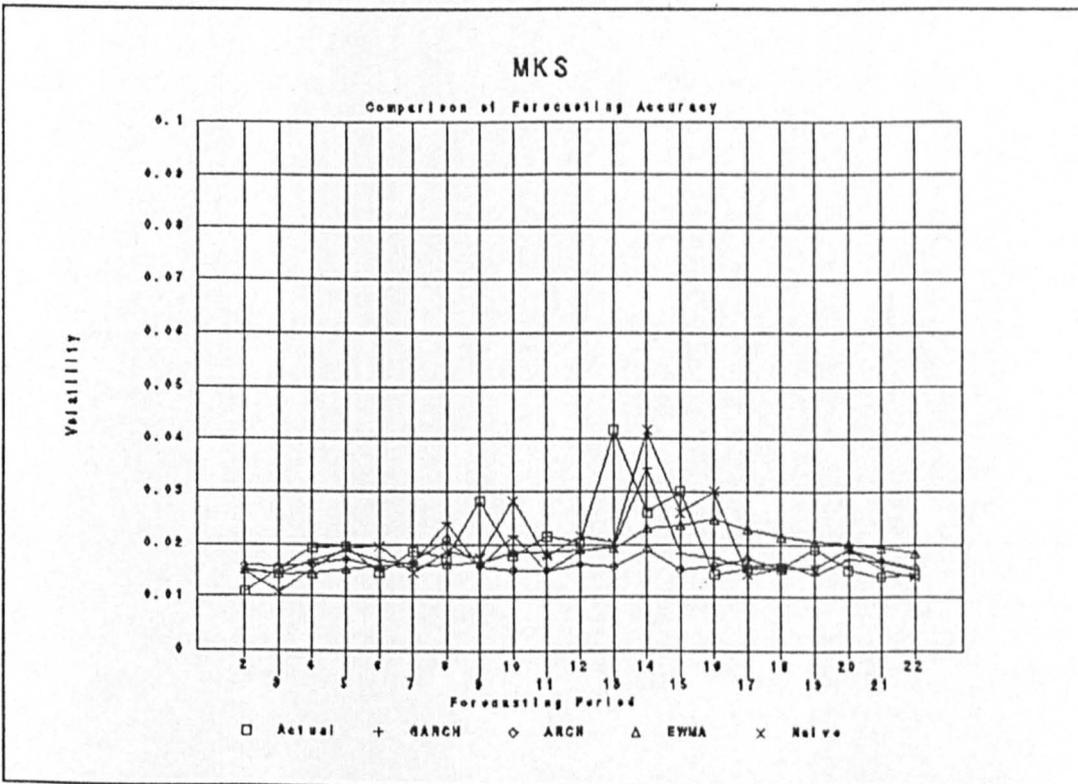
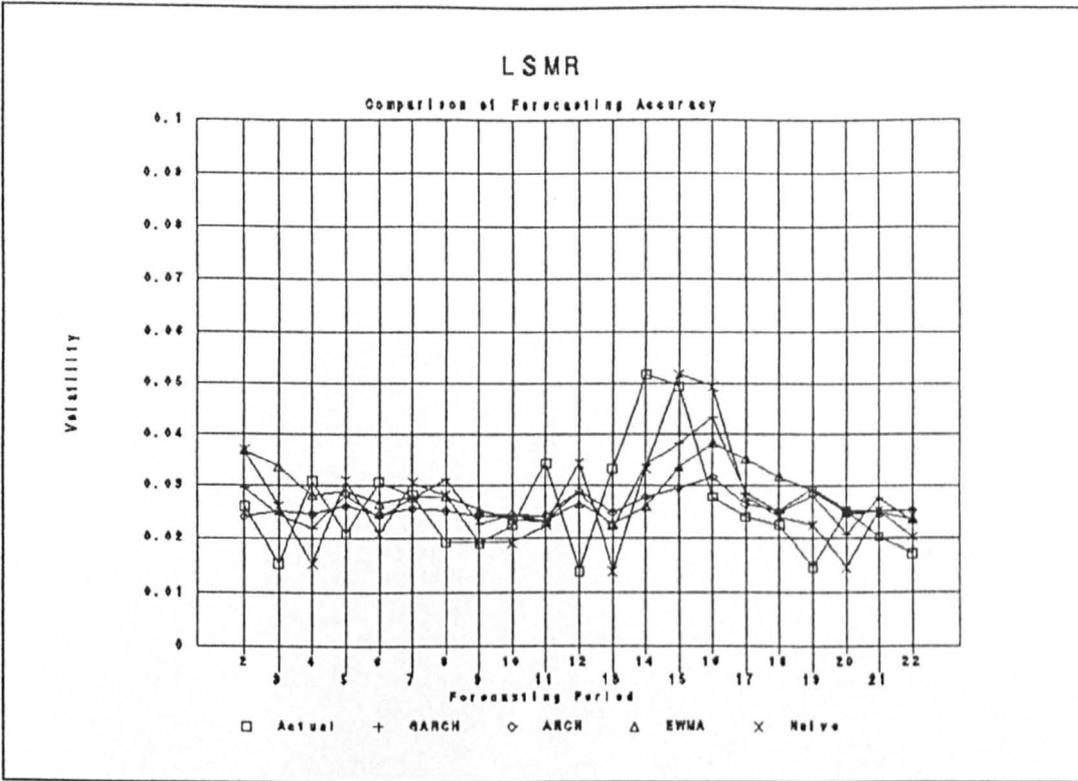


Figure 3.1 (continued)

Comparison between Actual and Forecasted Volatilities

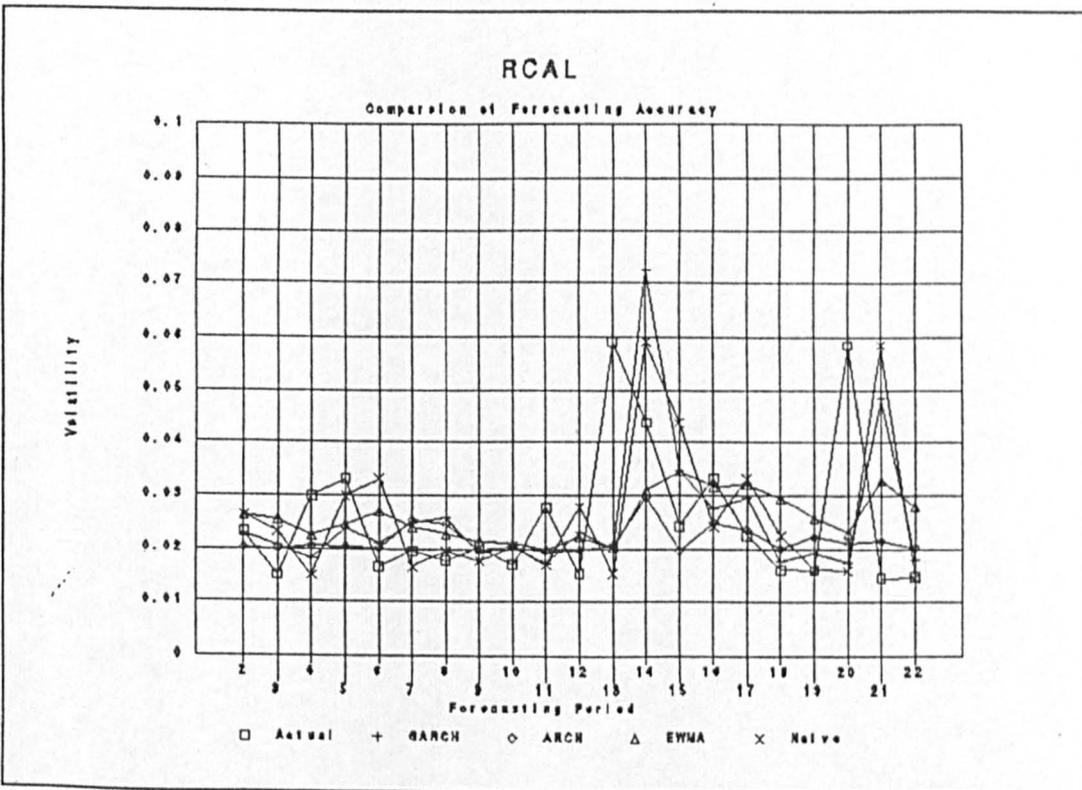
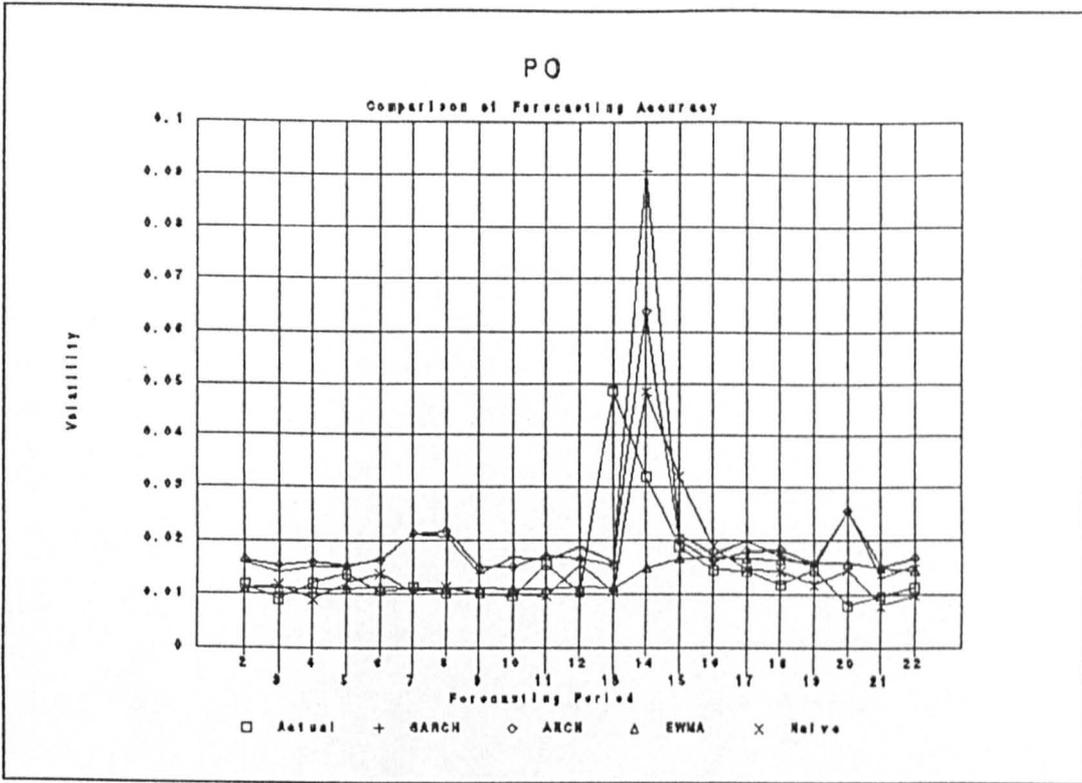
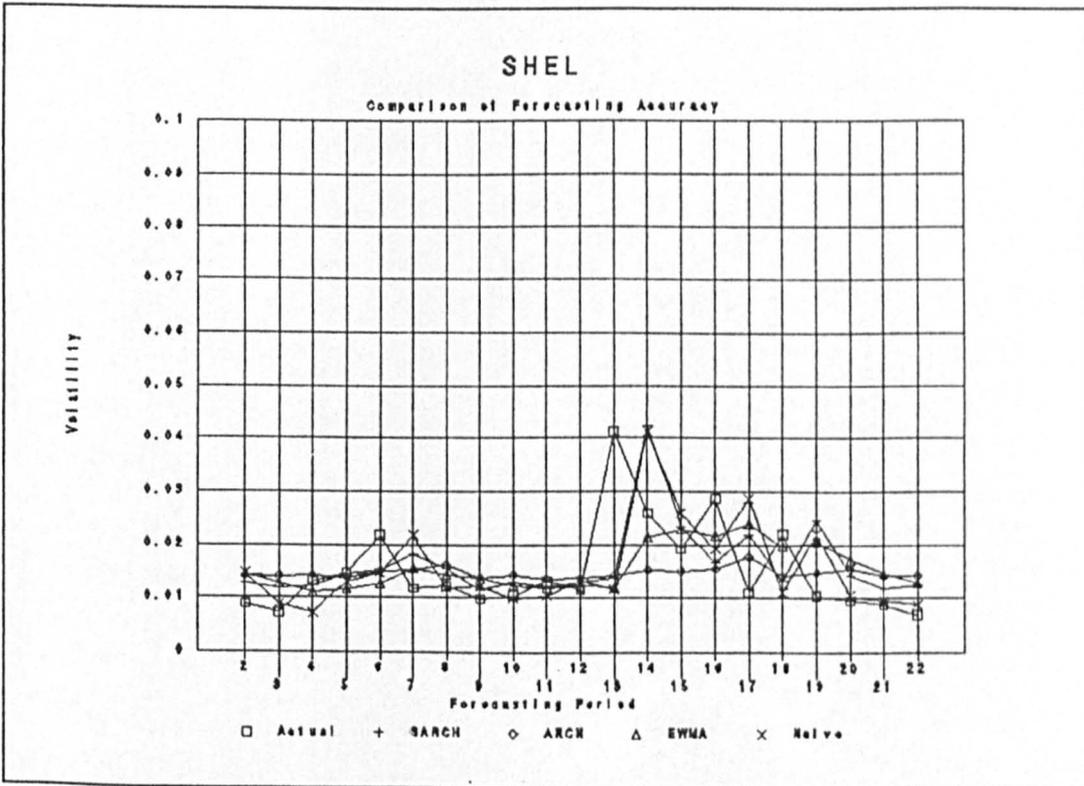
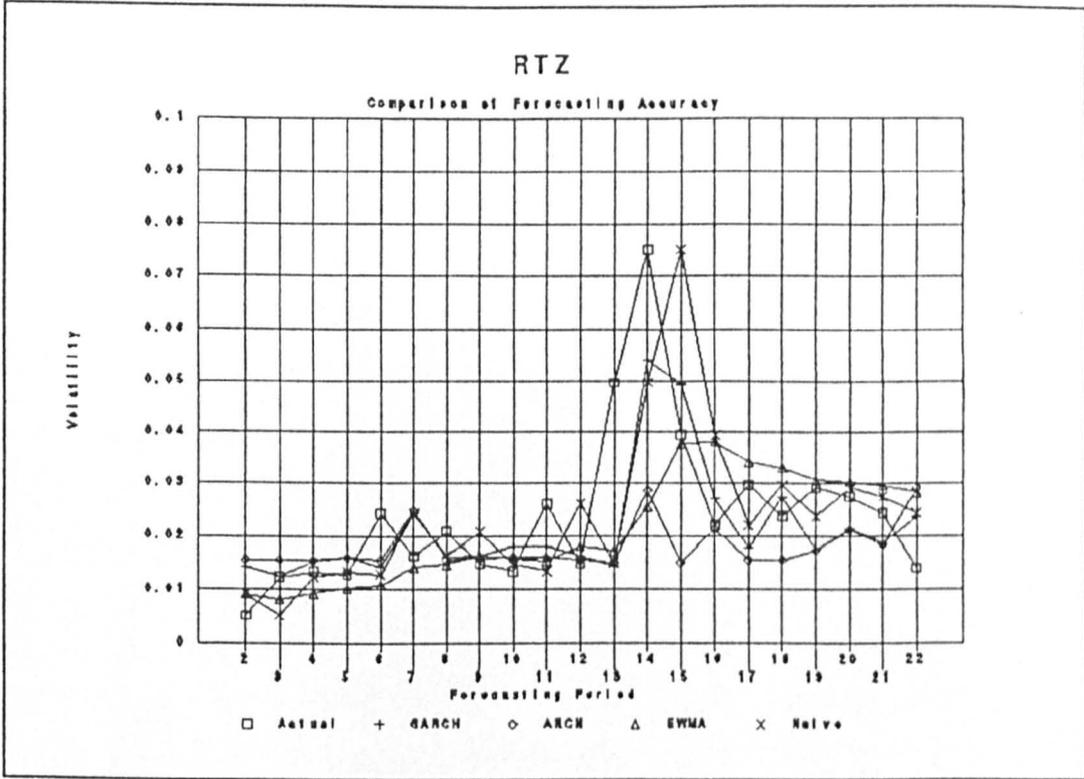


Figure 3.1 (continued)

Comparison between Actual and Forecasted Volatilities



## Chapter 4

# The Use of the GARCH Volatility in the Black-Scholes Model

### 4.1 Introduction

In the last chapter, the GARCH model is shown to be superior to the ARCH model, the EWMA model and the naive model in forecasting share price volatilities. This chapter discusses how the GARCH volatility estimate is adapted in the Black-Scholes model. It is further pointed out that the structure of the GARCH process agrees with Black's (1975) comment on the movement of expected volatilities. Finally, statistical confidence intervals are constructed for the GARCH volatility estimates and it is found that more than 75% of them lie within the 95% confidence interval of the "true" variance.

### 4.2 Application of the GARCH process in the Black-Scholes model

The GARCH process

$$h_t = \alpha_0 + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1}, \quad \text{where } \varepsilon_t = y_t - \mu \quad (4.1)$$

generates ex-ante forecasts of daily conditional variance. However, the Black-Scholes model assumes that the variance is constant over the life of the call option. This implies that the correct GARCH volatility has to be estimated before the call option's life and held constant during the call option's life. The problem arises as values of the residuals

$$\varepsilon_{t-1}^2$$

for  $1 \leq t \leq T$  are not yet known. Taking expected values on both sides of equation (4.1),

$$h_t = \alpha_0 + \alpha E(\varepsilon_{t-1}^2) + \beta h_{t-1}. \quad (4.2)$$

There are two approaches to estimate

$$E(\varepsilon_{t-1}^2). \quad (4.3)$$

One way is to compute the average residual squares for the period which the GARCH parameters, i.e.,  $\alpha_0$ ,  $\alpha$ , and  $\beta$  are estimated. The alternative way is to use Bollerslev's (1986) definition for the unconditional variance

$$E(\varepsilon_{t-1}^2) = \sigma^2 = \frac{\alpha_0}{1 - \alpha - \beta}.$$

The unconditional variance  $\sigma^2$  can therefore be substituted for expression (4.3) where the forecasting period for  $\alpha_0$ ,  $\alpha$ , and  $\beta$  is the one which is closest to the beginning of the call option's life. The GARCH process thus becomes a recursive equation only in the conditional variance

$$\begin{aligned} h_t &= \alpha_0 + \alpha \sigma^2 + \beta h_{t-1} \\ &= (\alpha_0 + \alpha \sigma^2) + \beta h_{t-1} \\ &= \text{constant} + \beta h_{t-1} \quad \text{where } t=1, 2, 3, \dots, T. \end{aligned} \quad (4.4)$$

The GARCH standard deviation can then be defined as

$$\sigma_G = \sqrt{\frac{\sum_{i=1}^T h_i}{T}} \quad (4.5)$$

to be used as the volatility in the Black-Scholes model. Since  $\sigma_G$  is a function of the maturity date  $T$ , there is a different volatility estimate for each maturity date of a call option series (cf. Black 1975, pp.41).

### 4.3 Initial and limiting values of the GARCH process

In this study, the GARCH(p,q) models particularly take the forms

$$h_t = \alpha_0 + \alpha \varepsilon_{t-q}^2 + \beta h_{t-p}, \quad \text{where } 1 \leq p, q \leq 3.$$

Thus, the GARCH models used in this study can be expressed as

$$h_t = \alpha_0 + \alpha \sigma^2 + \beta h_{t-p}, \quad \text{where } 1 \leq p \leq 3. \quad (4.6)$$

It is most critical to set the initial value  $h_0$  for equation (4.6). Black(1975) comments that if a present volatility is high (low) it will maintain to be high (low) for some time and then converge gradually to the general equilibrium level. Because the GARCH model preserves the persistence of shocks to variance, a present high or low variance would induce subsequent volatilities to remain high or low for some time. It is therefore argued that the initial value for  $h_0$  is the GARCH volatility estimate immediately before the call option's life. It will take only a very short time for the GARCH process to converge to the long-run historical (unconditional) volatility. In the following, it is proved that the conditional variance  $h_t$  will converge to  $\sigma^2$  independent of  $p$ , as  $t$  becomes large:

$$\begin{aligned} h_t &= \alpha_0 + \alpha \sigma^2 + \beta h_{t-p} \\ &= (\alpha_0 + \alpha \sigma^2) + \beta (\alpha_0 + \alpha \sigma^2 + \beta h_{t-2p}) \\ &= (\alpha_0 + \alpha \sigma^2) (1 + \beta) + \beta^2 h_{t-2p} \\ &= (\alpha_0 + \alpha \sigma^2) (1 + \beta + \beta^2 + \dots + \beta^m) + \beta^{m+1} h_{t-(m+1)p} \\ &= (\alpha_0 + \alpha \cdot \frac{\alpha_0}{1 - \alpha - \beta}) \cdot \frac{1}{1 - \beta} + 0, \quad \text{for som large } m \\ &= \frac{\alpha_0}{1 - \alpha - \beta} = \sigma^2. \end{aligned}$$

Empirically,  $m$  is found to be around 22 for the last term to approach zero. For a call option with longer life,  $h_t$  converges to  $\sigma^2$  comparatively more quickly.

#### 4.4 Confidence intervals for the "true" volatility

Having derived the GARCH volatility estimate, it is important to see how close it is to the "true" volatility. Ncube and Satchell (1991) have provided a  $(1-\theta)100\%$  confidence interval for the true variance. Let

$$\hat{\sigma}_T^2 = \sum_{i=1}^T (y_i - \mu)^2 / (T-1) \quad (4.7)$$

be the actual sample variance of share returns over  $T$  days. They show that this statistic is unconditionally distributed as

$$\frac{\sigma^2 \chi_{T-1}^2}{T-1}$$

and is an unbiased estimator of the instantaneous (true) variance  $\sigma^2$  specified in the Black-Scholes model. This implies that, given a  $(1-\theta)100\%$  confidence interval of

$$\chi_{T-1}^2,$$

there exist two constants  $\gamma$  and  $\beta$  such that

$$\text{Prob} \left[ \gamma \leq \frac{(T-1) \hat{\sigma}_T^2}{\sigma^2} \leq \beta \right] = 1 - \theta. \quad (4.8)$$

The constants  $\gamma$  and  $\beta$  in equation (4.8) are determined by  $\theta$  and the degrees of freedom  $T-1$ . For example, let  $\theta$  be 5 percent and  $T-1 = 90$ . Then  $\gamma$  and  $\beta$  have the values

$$\chi_{90,0.975}^2 = 65.647 \quad \text{and} \quad \chi_{90,0.025}^2 = 118.136$$

respectively. Now let equation (4.7) be the actual sample variance of the underlying share of a call option over the call option's life. Then the  $(1-\theta)100\%$  confidence interval of the "true" variance  $\sigma^2$  is

$$\text{Prob} \left[ \frac{(T-1) \hat{\sigma}_T^2}{\beta} \leq \sigma^2 \leq \frac{(T-1) \hat{\sigma}_T^2}{\gamma} \right] = 1 - \theta.$$

#### 4.5 Empirical results and concluding remarks

In this study, it is found that 75% of the GARCH estimates are within 95% confidence intervals of the "true" variance (Table 4.1). The majority of the GARCH estimates which are outside the 95% confidence interval are characterised by an over-estimation of the true variances. As Figlewski (1989) notes that the impact of errors in forecasting volatility is only slight on hedging accuracy and as the GARCH estimate developed in this chapter is sufficiently close to the "true" volatility, the GARCH volatility estimate is suitable for testing market efficiency. The implications of using the GARCH volatility and the actual volatility on market efficiency tests will be discussed in section 9.3.2.

Table 4.1

## Confidence Intervals for the GARCH Volatility Estimates

Legend: lb and ub denote the lower bound and upper bound of a 95% confidence interval for the "true" volatility of a call option series. 0 denotes that either a GARCH variance (annualised) is larger than ub or less than lb respectively. 1 denotes that a GARCH variance is within the 95% confidence interval of the "true" variance.

	Series	Nobs	Actual Variance	lb	GARCH Variance	ub	Signal
<u>BARC</u>	BA2750	78	0.03813	0.02852	0.05869	0.05362	0
	BA68395	30	0.02250	0.01437	0.05923	0.04020	0
	BA68429	30	0.02250	0.01437	0.05923	0.04020	0
	BA2755	78	0.03813	0.02852	0.05869	0.05362	0
	BA5760	69	0.05700	0.04191	0.05980	0.08206	1
	BA8765	27	0.08278	0.05175	0.06196	0.15338	1
	BA3846	101	0.12064	0.09323	0.09406	0.16227	1
	BA6836	30	0.02250	0.01437	0.06850	0.04020	0
	BA12742	35	0.14729	0.09689	0.09257	0.25062	0
	BA5750	144	0.04635	0.03727	0.05913	0.05922	1
<u>BCHM</u>	B03750	43	0.07799	0.05325	0.07455	0.12517	1
	B03755	23	0.09615	0.03511	0.07698	0.08254	1
	B12642	36	0.03254	0.02152	0.07765	0.05488	0
	B03746	98	0.05141	0.03958	0.07776	0.06949	0
	B06755	89	0.08629	0.06565	0.07884	0.11852	1
	B06760	64	0.08431	0.06133	0.07905	0.12322	1
	B12739	39	0.23527	0.15789	0.09332	0.38781	1
	B12742	40	0.25186	0.16977	0.09329	0.41232	1
	B03846	102	0.14027	0.10853	0.09451	0.18837	1
	B12646	36	0.03254	0.02152	0.07765	0.05488	0
	B06750	109	0.08086	0.06307	0.07888	0.10745	1
	B12633	36	0.03254	0.02152	0.07765	0.05488	0
	B12636	36	0.03254	0.02152	0.07765	0.05488	0
	B12639	36	0.03254	0.02152	0.07765	0.05488	0
	B03742	98	0.05141	0.03958	0.07776	0.06949	0
	B09755	153	0.07112	0.05754	0.07854	0.09018	1
	B12736	33	0.19747	0.12850	0.09398	0.34193	1
B03842	102	0.14027	0.10853	0.09451	0.18837	1	
B06846	163	0.10227	0.08328	0.09462	0.12861	1	
<u>BP</u>	BP1770	58	0.04895	0.03508	0.06829	0.07308	1
	BP1824	55	0.11254	0.07999	0.09076	0.17006	1
	BP7736	50	0.07580	0.05307	0.06854	0.11713	1
	BP1826	57	0.11267	0.08053	0.09594	0.16886	1
	BP4775	77	0.07879	0.05882	0.07317	0.11105	1
	BP4780	67	0.08446	0.06184	0.07328	0.12229	1
	BP4826	120	0.08771	0.06915	0.08141	0.11494	1
	BP77217	55	0.07656	0.05441	0.06857	0.11568	1
	BP77233	55	0.07656	0.05441	0.06857	0.11568	1
	BP7725	55	0.07656	0.05441	0.06857	0.11568	1
	BP77267	55	0.07656	0.05441	0.06857	0.11568	1
	BP77283	55	0.07656	0.05441	0.06857	0.11568	1
	BP7730	55	0.07656	0.05441	0.06857	0.11568	1
	BP77317	55	0.07656	0.05441	0.06857	0.11568	1
	BP7733	54	0.07757	0.05498	0.06833	0.11769	1
	BP4770	121	0.06803	0.05369	0.07186	0.08904	1
	BP1765	58	0.04895	0.03508	0.06829	0.07308	1
	BP1822	55	0.11254	0.07999	0.08810	0.17006	1

Table 4.1 (continued)

<u>CGLD</u>	CG1765	58	0.05251	0.03763	0.07764	0.07839	1	
	CG1770	58	0.05251	0.03763	0.07764	0.07839	1	
	CG77100	67	0.11769	0.08618	0.07766	0.17041	1	
	CG77105	65	0.11964	0.08722	0.07686	0.17428	1	
	CG77110	43	0.13881	0.09477	0.07567	0.22277	1	
	CG77115	42	0.13718	0.09328	0.07567	0.22155	1	
	CG1875	55	0.22063	0.15682	0.10603	0.33339	1	
	CG1880	56	0.22502	0.16040	0.11183	0.33861	1	
	CG1885	57	0.22610	0.16160	0.11490	0.33887	1	
	CG1890	58	0.25595	0.18343	0.11467	0.38213	1	
	CG4775	67	0.09749	0.07138	0.07652	0.14115	1	
	CG4780	61	0.10605	0.07658	0.07650	0.15660	1	
	CG4785	31	0.10876	0.06991	0.07650	0.19214	1	
	CG4770	121	0.07895	0.06230	0.07709	0.10334	1	
	CG4885	120	0.15254	0.12026	0.10924	0.19991	1	
	CG4890	121	0.16751	0.13219	0.11229	0.21924	1	
	CG7795	79	0.11655	0.08730	0.07570	0.16351	1	
	CG1870	39	0.19942	0.13383	0.10259	0.32871	1	
	CG7780	124	0.11125	0.08804	0.07569	0.14508	1	
	CG7785	94	0.11381	0.08718	0.07567	0.15487	1	
	CG4880	119	0.15148	0.11931	0.10783	0.19874	1	
	<u>CTLD</u>	CT4736	67	0.07733	0.05662	0.10082	0.11197	1
		CT4739	56	0.07567	0.05394	0.10028	0.11387	1
CT4742		53	0.06891	0.04870	0.10026	0.10501	1	
CT7750		34	0.05423	0.03549	0.10655	0.09306	0	
CT1733		58	0.04760	0.03411	0.09847	0.07106	0	
CT4733		114	0.06313	0.04949	0.10010	0.08334	0	
CT7742		116	0.06350	0.04988	0.12293	0.08361	0	
CT7746		79	0.06222	0.04660	0.12549	0.08728	0	
CT1833		55	0.26501	0.18836	0.12087	0.40044	0	
CT1836		57	0.27767	0.19847	0.12510	0.41616	0	
CT4833		118	0.15789	0.12425	0.09605	0.20739	0	
CT1830		50	0.23973	0.16783	0.10562	0.37045	0	
CT1730		58	0.04760	0.03411	0.09847	0.07106	0	
<u>CUAC</u>		CU1730	58	0.04041	0.02896	0.07416	0.06034	0
		CU1733	58	0.04041	0.02896	0.07416	0.06034	0
	CU1736	58	0.04041	0.02896	0.07416	0.06034	0	
	CU1726	58	0.04041	0.02896	0.07416	0.06034	0	
	CU1728	58	0.04041	0.02896	0.07416	0.06034	0	
	CU4730	121	0.06828	0.05388	0.07531	0.08937	1	
	CU4736	44	0.06795	0.04657	0.07539	0.10840	1	
	CU7733	113	0.07129	0.05583	0.07456	0.09422	1	
	CU7736	107	0.06424	0.04999	0.07453	0.08560	1	
	CU1724	30	0.03863	0.02467	0.07444	0.06902	0	
	CU1828	50	0.18901	0.13232	0.08938	0.29207	0	
	CU4728	121	0.06828	0.05388	0.07531	0.08937	1	
	CU4733	121	0.06828	0.05388	0.07531	0.08937	1	
<u>GEC</u>	G1718	58	0.09016	0.06462	0.07259	0.13461	1	
	G1720	58	0.09016	0.06462	0.07259	0.13461	1	
	G4722	57	0.09480	0.06776	0.07727	0.14208	1	
	G7722	120	0.09799	0.07725	0.07868	0.12842	1	
	G7724	98	0.09567	0.07366	0.07844	0.12931	1	
	G1814	28	0.09736	0.06131	0.10440	0.17808	1	
	G1816	55	0.11561	0.08217	0.12090	0.17469	1	
	G4720	121	0.10363	0.08178	0.07703	0.13564	1	
	G4816	118	0.08120	0.06390	0.10173	0.10666	1	
	G1722	58	0.09016	0.06462	0.07259	0.13461	1	
	G4724	35	0.08017	0.05274	0.07650	0.13641	1	
	G4718	121	0.10363	0.08178	0.07703	0.13564	1	
	G4818	124	0.09874	0.07814	0.10159	0.12876	1	

Table 4.1 (continued)

<u>GKN</u>	GK12630	36	0.04216	0.02789	0.11021	0.07112	0
	GK12626	36	0.04216	0.02789	0.11021	0.07112	0
	GK12628	36	0.04216	0.02789	0.11021	0.07112	0
	GK3728	98	0.07227	0.05565	0.09666	0.09768	1
	GK3730	98	0.07227	0.05565	0.09666	0.09768	1
	GK3733	98	0.07227	0.05565	0.09666	0.09768	1
	GK1826	57	0.21691	0.15504	0.11780	0.32510	1
	GK1828	57	0.21691	0.15504	0.11780	0.32510	1
	GK6733	105	0.08465	0.06573	0.10609	0.11314	1
	GK4828	120	0.15437	0.12170	0.12107	0.20231	1
	GK3726	98	0.07227	0.05565	0.09666	0.09768	1
	GK12624	36	0.04216	0.02789	0.11021	0.07112	0
	GK3736	98	0.07227	0.05565	0.09666	0.09768	1
	GK3739	98	0.07227	0.05565	0.09666	0.09768	1
	<u>GMET</u>	GM7760	33	0.06022	0.03919	0.06103	0.10428
GM1746		58	0.04367	0.03129	0.06321	0.06519	1
GM1839		57	0.21729	0.15531	0.09458	0.32566	1
GM4746		121	0.04628	0.03652	0.06199	0.06058	0
GM4750		84	0.03960	0.02991	0.06305	0.05494	0
GM4755		40	0.05042	0.03399	0.06305	0.08255	1
GM7755		103	0.05464	0.04233	0.06118	0.07326	1
GM4846		124	0.13382	0.10590	0.07437	0.17450	1
GM1836		55	0.17444	0.12399	0.07083	0.26360	1
GM4842		124	0.13382	0.10590	0.07437	0.17450	1
GM7750		147	0.04679	0.03770	0.06115	0.05963	0
<u>ICI</u>		I17110	58	0.02833	0.02031	0.05096	0.04230
	I17115	58	0.02833	0.02031	0.05096	0.04230	0
	I18100	55	0.16184	0.11503	0.08625	0.24454	0
	I18105	57	0.17287	0.12356	0.08990	0.25909	0
	I18110	57	0.17287	0.12356	0.08990	0.25909	0
	I18115	59	0.18515	0.13303	0.09369	0.27540	0
	I47110	121	0.04922	0.03884	0.05366	0.06442	1
	I47115	121	0.04922	0.03884	0.05366	0.06442	1
	I47130	53	0.06852	0.04843	0.05550	0.10442	1
	I47135	51	0.05572	0.03913	0.05550	0.08568	1
	I47140	51	0.05572	0.03913	0.05550	0.08568	1
	I47145	36	0.05787	0.03827	0.05550	0.09761	1
	I47150	36	0.05787	0.03827	0.05550	0.09761	1
	I77135	114	0.04712	0.03694	0.05448	0.06219	1
	I77140	114	0.04712	0.03694	0.05448	0.06219	1
	I77145	99	0.04731	0.03647	0.05442	0.06384	1
	I77150	99	0.04731	0.03647	0.05442	0.06384	1
	I48105	120	0.10767	0.08488	0.07448	0.14110	1
	I17105	58	0.02833	0.02031	0.05096	0.04230	0
	I1895	54	0.13425	0.09515	0.07359	0.20369	0
	I47120	67	0.06545	0.04792	0.05539	0.09476	1
	I47125	57	0.07251	0.05183	0.05547	0.10868	1
	I48100	118	0.10192	0.08020	0.07045	0.13388	1
	I48110	120	0.10767	0.08488	0.07448	0.14110	1
<u>LAND</u>	LA4739	40	0.07929	0.05344	0.04026	0.12980	1
	LA7755	35	0.08690	0.05717	0.04234	0.14788	1
	LA1733	58	0.02834	0.02031	0.04068	0.04230	1
	LA1736	58	0.02834	0.02031	0.04068	0.04230	1
	LA7742	80	0.07551	0.05665	0.04102	0.10570	1
	LA7746	57	0.07964	0.05692	0.04104	0.11935	1
	LA7750	45	0.08475	0.05831	0.04104	0.13445	1
	LA4736	121	0.04431	0.03497	0.04023	0.05800	1
	LA7739	103	0.07749	0.06003	0.04103	0.10389	1
	LA4846	124	0.09601	0.07598	0.06648	0.12520	1

Table 4.1 (continued)

<u>LRHO</u>	LR67273	39	0.08834	0.05929	0.07290	0.14562	1
	LR12720	35	0.18570	0.12216	0.13861	0.31598	1
	LR12722	35	0.18570	0.12216	0.13861	0.31598	1
	LR68223	35	0.21298	0.14011	0.07417	0.36240	1
	LR68257	35	0.21298	0.14011	0.07417	0.36240	1
	LR3724	98	0.03620	0.02787	0.07744	0.04892	0
	LR3726	98	0.03620	0.02787	0.07744	0.04892	0
	LR9733	34	0.04765	0.03118	0.07593	0.08177	1
	LR3822	97	0.13168	0.10126	0.11087	0.17827	1
	LR3728	23	0.04761	0.00868	0.07819	0.01528	0
	LR3824	99	0.19513	0.15043	0.10326	0.26331	0
	LR67218	39	0.08834	0.05929	0.07290	0.14562	1
	LR67236	39	0.08834	0.05929	0.07290	0.14562	1
	LR67255	39	0.08834	0.05929	0.07290	0.14562	1
	LR68171	35	0.21298	0.14011	0.07361	0.36240	1
	LR68189	35	0.21298	0.14011	0.07361	0.36240	1
	LR68206	35	0.21298	0.14011	0.07361	0.36240	1
	LR97273	103	0.08148	0.06312	0.07274	0.10924	1
	LR6824	160	0.17969	0.14605	0.08383	0.22650	1
<u>LSMR</u>	LS2718	27	0.16838	0.10525	0.15935	0.31197	1
	LS8736	27	0.18033	0.11272	0.16383	0.33411	1
	LS2716	67	0.45226	0.33114	0.16554	0.65481	1
	LS2822	71	0.36796	0.27162	0.18595	0.52675	1
	LS5718	93	0.17384	0.13300	0.16238	0.23699	1
	LS5720	76	0.17837	0.13292	0.16243	0.25199	1
	LS5724	45	0.20651	0.14207	0.16244	0.32760	1
	LS5726	40	0.20339	0.13710	0.16244	0.33298	1
	LS8726	100	0.16767	0.12941	0.15915	0.22590	1
	LS8728	70	0.14297	0.10532	0.15900	0.20525	1
	LS8730	69	0.14268	0.10490	0.15900	0.20541	1
	LS8733	35	0.18720	0.12315	0.15900	0.31853	1
	LS2713	78	0.46193	0.34542	0.16394	0.64950	1
	LS2714	78	0.46193	0.34542	0.16394	0.64950	1
	LS5722	50	0.19622	0.13737	0.16244	0.30321	1
	LS5828	139	0.26639	0.21344	0.18401	0.34192	1
<u>MKS</u>	M1816	32	0.11288	0.07302	0.08144	0.19736	1
	M4724	28	0.05328	0.03355	0.06598	0.09745	1
	M7728	26	0.13072	0.08107	0.07115	0.24550	1
	M1718	58	0.04839	0.03468	0.06683	0.07225	1
	M1720	58	0.04839	0.03468	0.06683	0.07225	1
	M4720	121	0.05962	0.04705	0.06574	0.07803	1
	M7724	91	0.10540	0.08041	0.06605	0.14422	1
	M7726	52	0.12564	0.08852	0.06607	0.19231	1
	M1818	50	0.15378	0.10766	0.07605	0.23763	1
	M4818	113	0.10265	0.08040	0.07525	0.13568	1
	M1722	58	0.04839	0.03468	0.06683	0.07225	1
	M4722	121	0.05962	0.04705	0.06574	0.07803	1
	M4820	123	0.10566	0.08354	0.08902	0.13795	1
<u>P. &amp; O.</u>	P87688	46	0.03281	0.02266	0.08502	0.05175	0
	P8775	45	0.03347	0.02302	0.08525	0.05309	0
	P2850	81	0.15817	0.11887	0.11263	0.22088	0
	P2855	81	0.15817	0.11887	0.11263	0.22088	0
	P2750	78	0.02833	0.02118	0.08636	0.03983	0
	P2755	78	0.02833	0.02118	0.08636	0.03983	0
	P5760	80	0.03819	0.02865	0.08197	0.05346	0
	P5765	67	0.03952	0.02894	0.08193	0.05722	0
	P5855	143	0.10877	0.08740	0.10461	0.13910	1
	P87448	46	0.03281	0.02266	0.08502	0.05175	0
	P87488	46	0.03281	0.02266	0.08502	0.05175	0
	P87538	46	0.03281	0.02266	0.08502	0.05175	0
	P87588	46	0.03281	0.02266	0.08502	0.05175	0
	P5755	144	0.03373	0.02712	0.08171	0.04309	0

Table 4.1 (continued)

<u>RCAL</u>	RC05724	71	0.12494	0.09223	0.11822	0.17886	1
	RC05726	59	0.07871	0.05655	0.11768	0.11708	0
	RC02718	78	0.11801	0.08825	0.13282	0.16593	1
	RC02720	78	0.11801	0.08825	0.13282	0.16593	1
	RC02722	78	0.11801	0.08825	0.13282	0.16593	1
	RC05722	80	0.12105	0.09082	0.11781	0.16944	1
	RC08726	119	0.10027	0.07898	0.12083	0.13156	1
	RC08728	43	0.13475	0.09200	0.12095	0.21626	1
	RC02822	78	0.22500	0.16825	0.19044	0.31636	1
	RC08724	131	0.12298	0.09792	0.12084	0.15911	1
	RC05720	144	0.12480	0.10035	0.11807	0.15946	1
<u>RTZ</u>	Z2770	78	0.03488	0.02608	0.06921	0.04904	0
	Z2775	25	0.03969	0.02441	0.06817	0.07563	1
	Z87110	65	0.11022	0.08035	0.06535	0.16056	1
	Z87115	29	0.15054	0.09548	0.06418	0.27206	1
	Z87120	27	0.15189	0.09495	0.06418	0.28142	1
	Z87125	23	0.16663	0.08873	0.06418	0.26299	1
	Z5775	91	0.08621	0.06577	0.06657	0.11796	1
	Z5780	85	0.08893	0.06726	0.06654	0.12311	1
	Z5785	36	0.12567	0.08312	0.06653	0.21199	1
	Z5790	27	0.11346	0.07092	0.06653	0.21021	1
	Z5795	26	0.11613	0.07202	0.06653	0.21810	1
	Z2846	78	0.31862	0.23826	0.11535	0.44799	1
	Z2848	78	0.31862	0.23826	0.11535	0.44799	1
	Z2850	78	0.31862	0.23826	0.11535	0.44799	1
	Z2852	78	0.31862	0.23826	0.11535	0.44799	1
	Z2854	78	0.31862	0.23826	0.11535	0.44799	1
	Z2856	74	0.31309	0.23247	0.09661	0.44456	1
	Z87100	78	0.10561	0.07897	0.06429	0.14849	1
	Z87105	70	0.10942	0.08061	0.06420	0.15709	1
	Z2833	77	0.32207	0.24043	0.10926	0.45392	1
	Z2836	78	0.31862	0.23826	0.11535	0.44799	1
	Z2838	78	0.31862	0.23826	0.11535	0.44799	1
	Z2840	78	0.31862	0.23826	0.11535	0.44799	1
	Z2842	78	0.31862	0.23826	0.11535	0.44799	1
	Z2844	78	0.31862	0.23826	0.11535	0.44799	1
	Z2828	71	0.25779	0.19029	0.08947	0.36903	1
	Z2830	77	0.32207	0.24043	0.10926	0.45392	1
	Z5770	144	0.06633	0.05333	0.06684	0.08475	1
	Z8780	145	0.09279	0.07466	0.06424	0.11846	1
<u>SHEL</u>	S1795	58	0.02648	0.01898	0.05264	0.03953	0
	S17100	42	0.01728	0.01175	0.05504	0.02791	0
	S77130	80	0.03756	0.02818	0.05339	0.05258	0
	S77135	45	0.02679	0.01843	0.05319	0.04250	0
	S77140	45	0.02679	0.01843	0.05319	0.04250	0
	S1895	57	0.13040	0.09320	0.10085	0.19544	1
	S18100	59	0.13967	0.10035	0.10561	0.20775	1
	S18105	59	0.13967	0.10035	0.10561	0.20775	1
	S47120	26	0.06043	0.03747	0.05701	0.11348	1
	S47100	105	0.03887	0.03018	0.05681	0.05195	0
	S47110	61	0.05484	0.03960	0.05700	0.08097	1
	S47115	50	0.05861	0.04104	0.05700	0.09057	1
	S77125	84	0.04205	0.03176	0.05340	0.05834	1
	S48105	122	0.08837	0.06980	0.07542	0.11552	1
	S1790	58	0.02648	0.01898	0.05264	0.03953	0
	S1775	57	0.02476	0.01770	0.05298	0.03711	0
	S47105	67	0.05031	0.03683	0.05689	0.07284	1
	S77115	113	0.04439	0.03477	0.05335	0.05868	1
	S77120	89	0.04131	0.03143	0.05319	0.05674	1
	S48100	122	0.08837	0.06980	0.07542	0.11552	1

## Chapter 5

### Call Option Trading Activity

#### 5.1 Introduction

To study the pricing of call options, it is obviously important to ensure as far as possible that prices quoted represent prices at which trading is feasible. Researchers therefore select only those options which have a positive trading volume in their studies (Galai 1974, Bhattacharya 1983, Kalay and Subrahmanyam 1984, and Rubinstein 1985). French (1984), Gemmill and Dickens (1986) studied call options with a period corresponding to a high trading volume.

The significance of a high call option trading volume is that it enhances the reliability of intra-day holding period returns and hypotheses testing (Bhattacharya 1987). On the other hand, infrequent trading raises serious problems for option pricing. Cox and Rubinstein (1985) point out that low volumes, especially for the case of extreme in-the-money and out-of-the-money options, have greater non-synchronicity between the call option and share price quotations and have larger pricing errors (cf. Ritchken 1987, p.227). This implies that an apparently mispriced call option identified by the Black-Scholes formula might have been a false signal in that it did not represent an opportunity in which an investor could exploit the apparent inefficiency.

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Tables and figures which are not put within the text can be found at the back of this chapter.

However, call option trading volume may not readily be available. In the UK, the number of call option contracts traded is neither reported along with call option quotations in the Daily Official List nor is available in Datastream on a daily basis. When faced with a huge call option database, it is both costly and time-consuming to collect the accompanying call option trading volume. Moreover, the London Traded Options Market is thinly traded. A call option is characterised by frequent zero trading volumes over its life. It is thus necessary to model call option trading activity before empirical tests are carried out.

## 5.2 Methodology

The research in call option trading activity is still meagre. Karpoff's (1988) methodology in studying share trading volume is first followed in trying to explain call option trading volume by share and call returns and many other variables defined in section 5.4. Unfortunately, it is found that none of the variables is significantly correlated with the number of call option contracts traded. It is further found that the previous trading history of a call option cannot explain the movement of its trading volume today. Moreover, call option trading volume data cannot be well-fitted by four probability distributions.

This study therefore proposes to examine call option trading activity in terms of the likelihood of an instance (occurrence) of a call transaction. Based on this concept, the call option trading activity is examined in two major steps.

The first step sets out to search for a condition where a call option price will be most likely traded (sections 5.4 to 5.6). The second step further models the trading activity of a call option series over its life analytically (section 5.7).

### 5.3 Data

Daily closing prices and the numbers of contracts traded of call and put options and the closing prices of their underlying shares are collected from Datastream. The data of three British companies, namely, the Beecham Group, General Electric Company and London & Scottish Marine Oil are chosen because their options have differing life cycles so that the behaviour of call option trading activity can be examined over a wide spectrum. The data cover the period from October 1986 to June 1988 and consist of 140 option series of differing maturity dates and exercise prices (Note 5.1).

At the first step of searching for a condition for actively traded call options, it is noted that some of the call option series contain so few data points (e.g., very short maturity) that could render statistical tests biased. As a result, call option series of each company having differing exercise prices but the same maturity date are merged into eighteen grouped data files (Table 5.1). The data collected on non-trading days (e.g., a bank holiday, etc.) are excluded lest instances of no trades are overstated. The same database will be modified slightly at the second step of modelling the trading activity analytically for a call option series.

A preliminary examination of the data reveals that the majority of call option series are thinly traded. If the trading frequency of a call option is defined as the percentage of days having a positive transaction over the life of the option, then the trading frequency of more than half of the 140 option series is below 25 percent (Table 5.2), i.e., call options are thinly traded.

Table 5.2

Thin Trading in the London Traded Options Market

Trading Frequency	Number of Series	Percentage
0% - 25%	80	57%
25% - 50%	33	24%
50% - 75%	20	14%
75% - 100%	7	5%
Total	140	100%

#### 5.4 The search of a condition for actively traded call options

This section finds that call option trading activity cannot be inferred from call option trading volume data in terms of the number of contracts traded but has to be examined through the likelihood of an instance of a transaction. A call option will be actively traded if it is near-the-money.

##### 5.4.1 Karpoff's model

Past researchers usually focused on share trading volume (Jain 1988, Karpoff 1986, 1987, 1988) or futures trading volume (Grammatikos and Saunders 1986, Martell and Wolf 1987). Anthony

(1988) examines the *interrelationship* of share and option trading volumes, but not the option trading volume per se.

Karpoff(1988) summarises previous empirical findings on share trading volume in two typical results:

- (1) The correlation between volume and the absolute value of the price change is positive in both equity and futures markets;
- (2) The correlation between volume and the price change per se is positive in equity markets.

However, while the above results may hold for share trading volume, they do not necessarily hold for call option trading volume. To verify this hypothesis, the relationship between call option trading volume and share as well as call returns is studied through the following regressions:

$$CNCT_t = \alpha_i + \beta_i X_{it} + \varepsilon_{it}, \quad i = 1, 2, 3, 4$$

where on day  $t$ ,  $CNCT_t$  is number of call option contracts traded,  $X_{1t}$  is share return ( $SR_t$ ),  $X_{2t}$  is call return ( $CR_t$ ),  $X_{3t}$ , and  $X_{4t}$  are absolute values of  $X_{1t}$  ( $ASR_t$ ) and  $X_{2t}$  ( $ACR_t$ ) respectively.

There is a total of 96 regression results (Table 5.3a). It is found that most of the R-squareds are less than one percent with a mean of only 0.33 percent. Only two out of the 96  $\beta$  coefficients are significant at the 5% level.

Similarly, the R-squareds are very small in the regressions

$$CNCT_t = \alpha + \beta Y_t + \varepsilon_t$$

where  $Y_t$  represents in turn the share price  $S_t$ , the intrinsic value  $(S-X)_t$ , the call price  $C_t$ , the put price  $P_t$  and the number of put option contracts traded  $PNCT_t$ , although some of the  $\beta$  coefficients are significant (Table 5.3b)

The empirical evidence therefore imply that the number of call option contracts traded is neither linearly correlated with share or call returns, nor with any of the variables defined above. The scatter plot of call option trading volume against share price, taking the BCHM 500 March 1987 series for instance, reveals that while the share price is continuous, the number of call option contracts traded has frequent zeros and unsystematic jumps over the life of the call option (Figure 5.1). As a result, the relationship between the share price and the number of call option contracts traded is not linear.

#### 5.4.2 Fitting probability distributions to the number of call option contracts traded data

If the distribution of the number of call option contracts traded can be specified by a standard probability distribution, then the likelihood that a certain number of call option contracts has been traded can be estimated. The data of the number of call option contracts traded is thus fitted to four probability distribution functions: the exponential and Poisson distributions which are characterised by the arrival of information, the student's t distribution which captures fat

tails and the Chi-squared distribution.

Graphically, the exponential curve appears to be the best fit as the number of call option contracts traded is characterised by an overwhelmingly high frequency of zero volumes and an exponentially sharp decline to positive but very low volumes. However, the distribution of the number of call option contracts traded is shown to be significantly different from all four probability distributions (Table 5.4). Thus, the number of call option contracts traded is not characterised by the four probability distributions.

#### 5.4.3 Instances of a call transaction

Although the magnitude of call option contracts traded can neither be inferred from regression analyses nor probability distribution fittings, a high likelihood that a call option has been transacted is a good proxy for an actual transaction. The number of call option contracts traded is thus transformed into either a positive trade or no trade, i.e., a dummy series  $d_t$  of 0 and 1, where 0 denotes no trade and 1 denotes an instance of trade. Then the relationship between  $d_t$  and the effect of the previous trading history of a call option and other variables are examined.

Previous trading history. The previous trading history of a call option is defined as the series of the cumulative percentage of the previous instances of trade. For example, suppose the historical series of the instances of trade is

$$d_t = 0, 1, 1, 0, 1, 0, \dots$$

then the trading history is

$$th_t = 0, 1/2, 2/3, 2/4, 3/5, \dots$$

If an instance of a call transaction today is affected by an earlier transaction, then the coefficient  $b$  in the regression

$$d_t = a + b(th_t) + e_t$$

is expected to be significantly different from zero. The regression results show that more than 85% of the  $b$  coefficients are not significant while only 17% of the R-squareds are larger than 10%. Therefore the instance of a call option transaction cannot be accepted to be correlated with the past trading history of the call option (Table 5.5).

Relationship with other variables. The regressions between the instances of a call option transaction and the share return, the call option return, the share price, the absolute share return, the absolute call option return, the intrinsic value, the call option price, the put option price, and the put option trading volume are also characterised by very small R-squareds and insignificant  $\beta$  coefficients (Table 5.6). It is therefore concluded that the instance of a call option transaction cannot be explained by these variables or, that the linear regression model is not a relevant model for examining the behaviour of trading instances.

## 5.5 The logit model

A logit model is useful for estimating the parameters  $\alpha$  and  $\beta$  for a dummy dependent variable  $Y_t$  and any real independent variable  $W_t$ . In the context of this study,  $Y_t$  is the series of instances of trade  $d_t$  and

$$d_t = \alpha + \beta W_t + \varepsilon_t$$

where  $W_t$  will be specified later in the chapter. Since  $d_t$  can be either 0 or 1, the expected value of  $d_t$ , given  $W_t$ , is the probability  $p_t$  of  $d_t$  being 1, i.e., the probability of a positive value:

$$p_t = E(d_t = 1 | W_t) \quad (5.1)$$

Note that the aim here is not at predicting how  $w_t$  will bring about  $d_t=1$  with perfect accuracy but at capturing the likelihood of a positive trade, given  $W_t$ .

Gujarati (1988) points out that the relationship (5.1) can best be described by the cumulative logistic distribution

$$p_t = \frac{1}{1 + e^{-z_t}} \quad (5.2)$$

or the cumulative probit distribution

$$p_t = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z_t} e^{-s^2/2} ds \quad (5.3)$$

where  $z_t = \alpha + \beta W_t$ .

### 5.5.1 The strength of the logit model

Both the logit model and the probit model have non-linear, S-shaped curves and map the entire real line (all possible values of  $W_t$ ) to the unit interval [0,1]. In particular, the logit model has got many strong and desirable statistical properties. Pindyck and Rubinfeld (1981) point out that the strength of the logit model as follows:

- (1) the parameters  $\alpha$  and  $\beta$  can be estimated by a maximum likelihood estimation and it can be proved that a unique maximum always exists for the logit model;
- (2) all parameter estimates are consistent and efficient for large samples; and
- (3) all parameter estimates are known to be normal so that the analogy of the regression t-test can be applied.

In addition, the logit model is computationally more efficient while the probit model involves non-linear estimations. The logit model is therefore adopted in this study instead of the probit model.

#### 5.5.2 The interpretation of the logit model

If  $p_t$ , the probability of having a trade, is given by equation (5.2), then the probability of not having a trade, is:

$$1 - p_t = \frac{1}{1 + e^{z_t}}$$

This implies that the odds ratio in favour of a trade is

$$\frac{p_t}{1 - p_t} = \frac{1 + e^{z_t}}{1 + e^{-z_t}} = e^{z_t}$$

It follows that the log of the odds ratio, i.e., the logit

$$L_t = \ln \left[ \frac{p_t}{1 - p_t} \right] = z_t = \alpha + \beta W_t$$

is not only linear in  $W_t$ , but also linear in the parameters. If  $\beta$  is significantly positive, then larger values of  $W_t$  will correspond to a higher log-odds in favour of having a trade.

The significance of the entire logit model is indicated by the likelihood ratio test:

$$\chi^2 = -2 \cdot \log(\lambda) = -2 \cdot [\log(L_0) - \log(L_{\text{Max}})]$$

which follows a Chi-square distribution with degree of freedom  $n$ , where  $\log(L_0)$  is the value of the likelihood function when all parameters except the constant are set equal to zero (constrained),  $\log(L_{\text{Max}})$  is the maximum value of the likelihood function (unconstrained), and  $n$  is the number of parameters except the constant.

If the likelihood ratio test is significant, then the logit model implies that the likelihood of a positive trade is significantly explained by the independent variable  $W_t$ .

### 5.5.3 The application of the logit model to trading activity

Hypothesis. In applying the logit model, it is postulated that the likelihood of a positive call option transaction will be explained by the call option price being near-the-money. In-the-money options have positive intrinsic values while out-of-the-money options can be used to attain a high gearing. However, when the underlying share price is fluctuating, new option series will be created to be at- or near-the-money. The market is presumably structured to introduce options which are more likely to be traded - there would be little point in introducing options which are not traded. Correspondingly there

is a presumption that if the share price has changed radically, options which have become far-removed from at-the-money will cease to be frequently traded. Deep in-the-money call options have very little gearing benefit therefore option holders will likely close their positions or sell the options. Therefore, it is hypothesised that deep in- and deep out-of-the-money options will be infrequently traded and only at- or near-the-money options will be actively traded.

The independent variable. The logit model

$$p(d_t=1) = \frac{1}{1 + e^{-Z_t}} \quad (5.4)$$

is adopted to test the above hypothesis where

$$Z_t = \alpha + \beta W_t = \alpha + \beta \left| \frac{S_t - X}{S_t} \right| \quad (5.5)$$

and  $W_t$  is the absolute value of the intrinsic value normalised by the share price and is taken as a measure for near-the-moneyness. By taking the absolute value of the normalised intrinsic values, it is no longer distinguishable between whether a call option is in-the-money or out-of-the-money but how far a call option is from at-the-money. A smaller  $W_t$  indicates that the call option is near-the-money. A larger  $W_t$  indicates that either the option is (deep) in-the-money or (deep) out-of-the-money. The resultant logistic curve becomes a restricted portion of the original curve (Figure 5.2).

Results. The proposed hypothesis holds if the following conditions are satisfied:

- (C1) the entire logit model is significant;
- (C2) the t-ratio for  $\beta$  is significant; and
- (C3) the sign of  $\beta$  is negative.

A negative  $\beta$  implies that when the intrinsic value is smaller (near-the-money), there is a higher likelihood that the call option will be traded.

The logit results for the eighteen grouped call option series satisfy all conditions (C1-C3), i.e., the logit model are all significant at the 0.001 level and all  $\beta$  coefficients are significantly negative at the 0.001 level. This implies that the likelihood of an instance of a call option transaction is significantly explained by a smaller normalised intrinsic value or, a near-the-money call option. Thus, it is concluded that a call option will be most actively traded when it is near-the-money (Table 5.7).

Table 5.7

## Logit Results for Grouped Call Option Series

Grouped Series	Sample Size	$\alpha$	$t(\alpha)$	$\beta$	$t(\beta)$	Ratio Test
B37	758	-0.1164	-0.79	-11.83	-7.57	77.06
B97	1001	0.2912	2.01	-16.67	-11.23	201.40
B127	826	-0.8583	-5.97	-5.24	-4.45	24.56
B38	939	-0.9211	-5.71	-10.02	-6.55	60.70
B68	1161	-0.9679	-5.98	-11.03	-7.64	79.19
G17	625	0.3238	2.30	-6.20	-5.63	35.67
G47	604	0.8185	5.51	-5.55	-4.79	23.89
G77	768	1.2344	8.79	-10.12	-10.32	131.66
G18	878	0.7366	5.65	-9.67	-10.69	202.27
G48	1050	-0.3620	-2.79	-7.09	-8.98	145.98
G78	909	-0.5504	-3.58	-8.60	-7.83	94.89
L27	1008	0.2593	2.25	-4.74	-8.50	82.12
L57	851	0.9192	6.35	-8.32	-12.63	236.41
L87	1177	0.3934	3.53	-6.81	-12.29	210.82
L117	1375	0.8929	7.78	-10.16	-15.36	380.82
L28	1280	0.0671	0.55	-9.44	-11.44	199.18
L58	1380	-0.3875	-3.11	-8.49	-10.16	157.72
L88	1404	-0.8532	-5.60	-10.69	-9.00	143.50

Note:  $t(\alpha)$  and  $t(\beta)$  are t-ratios for the parameters  $\alpha$  and  $\beta$  respectively.

5.5.4 The uniqueness of the normalised intrinsic value in explaining call option trading activity

In the last section, the logit model is applied to the 18 grouped call option series. The logit model is further used to estimate the parameters  $\alpha$  and  $\beta$  for all 140 individual call option series, each having distinct maturity date and exercise price. The logit results (Table 5.8) show that the signs of the  $\beta$  coefficients are negative for 80% of the 140 series (i.e., 112 series). Of these 112 series, 75 series have a significant  $\beta$  coefficient with the logit model per se being simultaneously significant (Table 5.9). Thus, more than half (75) of the 140 series show that near-the-money call options will be most likely traded.

Table 5.9

Uniqueness of the Absolute Normalised Intrinsic Value as Indicated by  $\beta$

	$\beta$			Total
	-	+	*	
Signs	112	26	2	140
Model & $\beta$ significant	75	4	-	79

Note: \* denotes that the logit model fails to run for two series (L5730 and L5842).

However, the logit results for the variables: the share return, the call option return, the call option price, the put option price and the put option trading volume show that most of the likelihood ratio tests are not significant, i.e., the logit model is not significant. In cases for which the

likelihood ratio tests are significant the signs of the corresponding  $\beta$  coefficients are however quite heterogeneous (Table 5.10). Taking the put option price for instance, 13 out of the 24 call option series have a significant likelihood ratio test but seven  $\beta$  coefficients carry positive signs whilst six  $\beta$  coefficients carry negative signs. Therefore, none of these variables is able to explain the likelihood of a call transaction in a consistent manner. In summary, it is found that the absolute normalized intrinsic value of a call option uniquely explains the trading activity of a call option.

#### 5.5.5 The implication of the Black-Scholes hedge ratio on call option trading activity

Using the Black-Scholes model, a riskfree hedge can be formed by writing  $1/N(d_1)$  call options for each share bought:

$$H = S - \frac{1}{N(d_1)} C$$

$$\text{where } N(d_1) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{d_1} e^{-s^2/2} ds.$$

It is interesting to note that the hedge ratio actually follows a probit model (equation (5.3), Figure 5.3).

Cox and Rubinstein (1985) point out that the slope of the  $N(d_1)$  curve is greatest at the exercise price of the call option and the curve has flat tails. The hedge ratio will be near to zero for deep out-of-the-money call options and near unity for deep in-the-money call options. Therefore, it is only when the share price is near to the exercise price that a change in the share price will be associated with a substantial

change in the hedge ratio. In other words, the hedge ratio will be adjusted more frequently only for near-the-money call options. This implies that a near-the-money call option will be more frequently traded. This analysis agrees with actual trading activities observed in the LTOM that

*"Not all series will be "active" at any one time; option series which are far in-the-money or far out-of-the-money are much less likely to be traded than series which are near to or at-the-money." (Quality of Markets Quarterly, Summer 1987, p.27).*

Although the logit model and the probit model have different mathematical expressions, they are similar in shapes and in their properties. From this similarity it is inferred that the logit model has a direct relevance in modelling call option trading activity.

#### 5.6 A measure for near-the-moneyness

The logit model only shows conceptually that a call option will be most likely traded when the call option is near-the-money. Practically it is important to derive an explicit measure for near-the-moneyness.

It would be convenient to classify the normalised intrinsic values into a number of classes (Note 5.2), and then define an interval which will serve as an appropriate measure for near-the-moneyness:

$$\left| \frac{S - X}{S} \right| \leq \delta\% \quad (5.6)$$

### 5.6.1 Classification of the trading frequency and the average trading volume

The trading volume (in terms of number of call option contracts traded) and the corresponding converted instance of a transaction are classified into N specified classes of the normalised intrinsic values. The observations are required to fall into each class equally likely, i.e., with probability  $1/N$ . These N classes can be interpreted as degrees of at-the-moneyness. It is convenient to define N as an odd integer. N has been set equal to 3, 5, 7 and 9 but the following discussion only limits to  $N = 7$ . Since there are at least six hundred observations in each of the 18 grouped call option series containing the variables S, S-X, CNCT, and d, it can be assumed that the ex ante normalised intrinsic value is normally distributed. The boundary values which divide the standard normal distribution into seven equal areas of probability =  $1/7$  are given in Table 5.11.

Table 5.11  
Definition of Classes

z-score range	Prob.	Class*	In-the-moneyness
$z < -1.068$	1/7	-3	Out-of-the-money
$-1.068 < z < -0.566$	1/7	-2	
$-0.566 < z < -0.18$	1/7	-1	
$-0.18 < z < 0.18$	1/7	0	Near-the-money
$0.18 < z < 0.566$	1/7	+1	In-the-money
$0.566 < z < 1.068$	1/7	+2	
$z > 1.068$	1/7	+3	

-1 denotes an out-of-the-money class, -2 denotes a deeper out-of-the-money class, -3 denotes a very deep out-of-the-money class. The same analogy applies to the in-the-money classes.

For each class it is defined that

$$\text{Trading frequency (TF)} = \frac{\Sigma d_t}{\Sigma TD_t} \times 100\%$$

$$\text{Average trading volume (ATV)} = \frac{\Sigma CNCT_t}{\Sigma TD_t} \times 100\%.$$

where  $\Sigma TD$  is the total number of trading days,  $\Sigma d_t$  is the total number of trading instances and  $\Sigma CNCT$  is the total trading volume in a class. The class which contains the highest trading frequency and the highest average trading volume is defined as the one where the call option is most actively traded.

#### 5.6.2 The consistency of both the highest trading frequency and average trading volume in depicting trading activity

It is found that the distributions of the trading frequency and the average trading volume over the seven classes are both triangle-shaped with one or two modes (Table 5.12, Figures 5.4). For the highest trading frequency of the eighteen grouped series, ten fall into the classes which contains a zero intrinsic value (i.e., near-the-money), seven fall into the immediate adjacent classes and only one falls into the second adjacent class.

Moreover, for most of the eighteen grouped series, the class which has the highest trading frequency also has the highest average trading volume. Thus, call option trading activity is consistently depicted by the highest trading frequency and the highest average trading volume.

### 5.6.3 Defining a proxy for near-the-moneyness

As the highest trading frequency and the highest average trading volume of most grouped series fall into classes containing the zero intrinsic value, the interval:

$$\left| \frac{S - X}{S} \right| \leq \delta\%$$

where  $\delta > 0$  can be defined as a proxy for near-the-moneyness. However, the value of  $\delta$  cannot be analytically determined. This section aims at suggesting a value for  $\delta$  through examining historical call option data.

First, it is noted that the trading intensity (TF) of a call option is an increasing function of  $\delta$ . This is empirically verified to be true both for the 18 grouped call option series and the separate 140 call option series. For the eighteen grouped series:

$\delta$	5%	10	15	20	25	30
TF( $\geq$ )	35%	60	75	85	90	95

and for the 140 separate call option series:

$\delta$	5%	10	15	20	25	30
TF( $\geq$ )	29%	52	66	77	85	90

The figures (Figures 5.5) illustrate how rapidly the trading frequency of a call option converges to 100% over the call option's life as  $\delta$  increases from 5, 10 to around 50. Each curve in a figure denotes the trading frequency of a call option series with differing maturity and exercise price over increasing values of  $\delta$  for the shares BCHM, GEC and LSMR.

Second, for the data used in this study, it is found that both the highest trading frequency and the highest average trading volume are captured by the interval:

$$\left| \frac{S - X}{S} \right| \leq 15\%.$$

Moreover, for the eighteen grouped call option series more than 75% of the trading frequency over the life of a call option occurred in this interval. A larger value of  $\delta$  will include more trading instances but will however embrace more non-trading instances. It is therefore suggested to adopt the above interval as a proxy for near-the-moneyness.

#### 5.6.4. A critique of different definitions for near-the-moneyness

At present there is no agreed criterion for the near-the-moneyness of call options. Bhattacharya (1980, p.1089) defined a ratio of  $X/S = 1.25$  for deep out-of-the-money call options and a ratio of  $X/S = 0.875$  for deep in-the-money call options. These are very close to the definition established earlier in this chapter. In another study, Bhattacharya (1983, p.170) however defined the moneyness of a call option in a different form: a call option is deep in-the-money if  $S_{bid}/X > 1.30$  and deep out-of-the-money if  $S_{bid}/X < 0.75$ . Choi and Shastri (1989) defined an around-the-money call option as one which satisfies the ratio  $0.9 < S/X < 1.1$  which is equivalent to

$$\left| \frac{S - X}{X} \right| < 10\%.$$

Stephan and Whaley (1990) chose call options whose moneyness falls inside the range:

$$e^{rT-0.7\sqrt{T}} \leq \frac{S}{X} \leq e^{rT+0.7\sqrt{T}}.$$

They argued that if the current share price equals the exercise price, then there is a 95% chance that the moneyness of the call option will lie in this range at the maturity date. The value "0.7" is twice the standard deviation of return of a typical stock on the NYSE. However, this interval is very conservative, i.e., too large. For example, a value of  $T = 25$ , and an interest rate of 10% would imply

$$-0.16 \leq \frac{S - X}{X} \leq 0.21.$$

For larger values of  $T$ , this range will include almost all call option prices.

Barone-Adesi and Whaley (1986) filtered call options data by requiring that exactly one dividend is paid on the underlying share during the life of the call option and that price quotations be more than 1/8th. The selected database was summarised by the degree of in-the-moneyness and by the call option's time to maturity. It was found that most of the transactions are approximately at-the-money and have a short time to maturity. Specifically, 84% of the call option transactions falls in the interval:

$$0.85 < \frac{S}{X} < 1.15 \Leftrightarrow \left| \frac{S - X}{X} \right| < 15\%.$$

It can be seen that researchers use either the share price or the exercise price to normalise the intrinsic value. However, the difference of using either is minimal, for

$$\frac{S - X}{S} - \frac{S - X}{X} = (S - X) \left( \frac{X - S}{SX} \right) = - \frac{(S - X)^2}{SX}$$

would be very small for near-the-money call options.

### 5.7 An analytical model for identifying actively traded call option series

It can be seen from the last section that the definitions adopted for near-the-moneyness are not unique. The width ( $\delta$ ) of the near-the-moneyness interval adopted by different researchers varies and lacks theoretical justification. More importantly, while the interval captures a call option price which is near-the-money, it does not distinguish whether a call option series was actively traded over its life. Faced with the enormous number of call option series, an analytical form for call option trading activity is needed as a priori criterion to identify a call option series which have been actively traded.

It is postulated that call option trading frequency is a function of the number of trading days over the call option's life ( $\tau$ ) and the percentage of in-the-moneyness instances over these  $\tau$  days ( $P$ ), i.e.,

$$TF = f(\tau, p).$$

When  $P$  is either very large or very small, it is inferred that the option is deep-in- or deep-out-of-the-money and  $TF$  will be low.

#### 5.7.1 Data

The database is the same as that used in the first step with some minor modification. Originally, there are 140 series with 33, 34 and 73 series in the three classes BCHM, GEC and LSMR respectively. As LSMR has 73 series which account for more than half of the total number of series, to avoid the possible dominating effect of LSMR, five series are selected at random from each of the seven maturity days (strata) of LSMR so that the number of series in each class is approximately the same. The total number of call option series becomes 101 (Table 5.13).

#### 5.7.2 Methodology

Stepwise regressions are used to model  $TF$  in terms of the moments of  $\tau$  and  $P$  and the interactions between them. It is found that for the data of each of the three sample companies and for the pooled data of the three companies,  $TF$  is consistently explained by a function linear in  $\tau$  and quadratic in  $P$ , i.e.,

$$TF = b_0 + b_1\tau + b_2P + b_3P^2$$

There are several strong and coherent properties among the four stepwise regression results: firstly, the signs of the constant term and the variables  $\tau$ ,  $P$ , and  $P^2$  are consistently positive, negative, positive and negative respectively; secondly, the

t-ratios for all variables are statistically significant; and thirdly, the explanatory power of the regression model is at least 49% which shows that the model chosen is plausible (Table 5.14).

Table 5.14  
Stepwise Regression Results for  
 $TF = b_0 + b_1\tau + b_2P + b_3P^2$

Company	$b_0$	$b_1$	$b_2$	$b_3$	F	R <sup>2</sup>
BCHM	0.5989	-0.0026	0.0082	-0.000104	23.2	70.6%
	(6.47)	(-5.70)	(3.17)	(-4.38)		
GEC	0.5523	-0.0022	0.0135	-0.000139	9.29	49.0%
	(4.71)	(-3.34)	(4.14)	(-4.26)		
LSMR	0.6240	-0.0026	0.0101	-0.000122	31.1	75.0%
	(5.31)	(-5.73)	(2.83)	(-4.27)		
ALL.	0.5927	-0.0025	0.0108	-0.000126	51.4	61.4%
	(9.74)	(-7.95)	(6.34)	(-7.92)		

### 5.7.3 Testing for model assumptions

For the regression model of the three companies pooled together

$$TF = 0.5927 - 0.00248\tau + 0.0108P - 0.000126P^2 + \epsilon$$

to be valid, the residual  $\epsilon$  must be normally distributed with zero mean and constant variance. The following statistical tests show that neither of these assumptions can be rejected.

Normality. When the residual  $\epsilon$  is fitted to a normal curve, it is found that the Chi-square statistic is  $X^2 = 9.79$

with degrees of freedom = 9 and an observed prob-value = 0.368. Therefore, the residuals  $\epsilon$  cannot be rejected as normally distributed. Furthermore, the residual  $\epsilon$  has a t-ratio of zero and hence the hypothesis that the population mean of  $\epsilon$  is not significantly different from zero also cannot be rejected.

Homoscedasticity. The data TF,  $\tau$  and P are pooled from three companies. It is important to check whether the variance of the residuals  $\epsilon$  is homoscedastic. Homoscedasticity might imply that the regression model was not company-specific. The equation

$$TF = f(\tau, P)$$

can therefore be used as a filter to select actively traded options data.

The Goldfeld-Quandt test is used to check for homoscedasticity. It is postulated that the residual variance is linearly related to  $\tau$  (or P). The procedures of the test is summarised below:

Step 1. Rank the 101 observations according to values of  $\tau$  (or P), beginning with the lowest  $\tau$  (or P) value.

Step 2. Omit  $C = 21$  central observations (Note 5.3) and divide the remaining 80 observations into two groups each of 40 observations.

Step 3. Fit separate OLS regression to the first 40 observations and the last 40 observations, and obtain the separate residual sum of squares  $RSS_1$  and  $RSS_2$ . These RSSs each have degrees of freedom =  $(101-21)/2 - 4 = 36$ .

Step 4. The test statistic

$$\lambda = \frac{RSS_2/36}{RSS_1/36} = \frac{RSS_2}{RSS_1} \sim F_{0.05; 36, 36}$$

will, on the assumption of homoscedasticity, follow the F distribution with (36,36) degrees of freedom. If  $\lambda < 1$ , then the null hypothesis cannot be rejected.

The empirical results show that the value of  $\lambda$  for both  $\tau$  and P are less than 1 (Table 5.15).

Table 5.15

Goldfeld-Quandt Test Results for  $\tau$  and P

Explanatory variable	$\lambda$
$\tau$	0.5005 < 1
P	0.7489 < 1

Therefore, it cannot be rejected that the residual variance is homoscedastic. To conclude, the regression equation satisfies the basic assumptions that the residuals are normally distributed and have a homoscedastic variance.

#### 5.7.4 The properties of the analytical model

The analytical model for call option trading activity

$$TF = 0.5927 - 0.00248\tau + 0.0108P - 0.000126P^2$$

is linear in  $\tau$  and quadratic in P. For the sample data, the variables  $\tau$  and P lie within the intervals

$$30 < \tau < 190 \quad \text{and} \quad 0 < P < 100$$

respectively. As TF is linear in and inversely related with  $\tau$ , a call option with a longer time to maturity will imply a lower trading frequency. If  $\tau$  is constant, then the global maximum of the curve TF occurs at:

$$\frac{\partial TF}{\partial P} = 0.0108 - 0.000252P = 0 \Rightarrow P = 42.86$$

Therefore, considering P alone, TF will be large when slightly less than half of the intrinsic values is positive (Figure 5.6). This is equivalent to stating that 57.14% of the intrinsic values are negative, i.e., the call option is slightly out-of-the-money.

This theoretical property agrees with Stephan and Whaley's (1990) empirical observations. They collect call option transaction data from the CBOE for the period 2 January 1986 through 31 March 1986. Their data include the time, the price, and the number of contracts traded. After screening the data for rational boundary conditions and eliminating the deep in-the-money and deep out-of-the-money call options, they found that short-term call options are the most actively traded. Moreover, slightly out-of-the-money call options are traded more frequently than the other moneyness categories. In particular, they point out that out-of-the-money call options tend to be more active than in-the-money call options, with the proportions of trading activity being close to 57 and 43 respectively. It is surprising to note that these proportions for all call option transactions agree with the theoretical proportions (57.14% and 42.86%) derived earlier for the out-of-the-money and in-the-money instances of a call option which is

most actively traded.

Finucane (1991) studies the put-call parity relationship for S & P 100 Index option transaction prices over the period 2 December 1985 to 30 November 1988. He also notes that (p.448, note 7):

*"the short time to maturity, at-the-money calls and puts tend to have the greatest combined volume and the most frequently recorded quotes".*

### 5.8 Concluding remarks

Many researchers in call options require that the option volume must be positive. Zero volume will invalidate some of the major Black-Scholes model assumptions, misvalue the model value, as well as decrease the reliability of the holding returns. Traded call options in the LTOM are thinly traded. In carrying out empirical researches, it is important to ensure that the call option series chosen will be actively traded. Cox and Rubinstein (1985, p.286) note that the call option trading volume is concentrated in near-the-money, short-term options. Barone-Adesi and Whaley (1986, p.96) note that extreme in-the-money and out-of-the-money call options are infrequently traded, although they did not suggest a priori criterion by which to select option series.

This chapter points out that call option trading activity cannot be inferred from the number of call option contracts traded but has to be examined through the likelihood of an instance of a call transaction. It is found that a call option will be most likely traded when it is near-the-money. An

analytical criterion is further developed for selecting actively traded call options over their lives. The model implies that long-lived options will be less actively traded and that trading is more active for slightly out-of-the-money call option series. The properties of the model is consistent with many empirical evidences in identifying actively traded call options with call option trading volume available (cf. Stephan and Whaley 1990).

## Notes

### Note 5.1:

The database used in the empirical tests in sections 5.4.1, 5.4.2, 5.4.3, and 5.6.3 are a subset of the 140 series. They are the individual series of B68, G77, and L88 having differing maturities and exercise prices and each consisting of six variables: the share price, the intrinsic value, the call option trading volume (number of contracts traded), the call option price, the put option price and the put option trading volume.

### Note 5.2:

The eighteen grouped series have been classified into three, five, seven and nine classes of the intrinsic values but the results reported refer to a 7-class classification. Two examples are illustrated to show the appropriateness of the 7-class case. If the series B97 (Table 5.16) is classified into three classes, then the highest trading frequency falls in the class with an intrinsic value of  $-\infty < S-X < 24.70$ . Thus there is no lower bound for the class to contain the highest trading frequency of 42.78% and the highest average trading volume of 35.40 contracts. If the series B97 is classified into nine classes, then the highest trading frequency of 52.78% and the higher average number of trading volume of 33.83 contracts in the 7-class case will be split and contained into two classes ( $-31.83 < S-X < 0.79$  and  $0.79 < S-X < 24.70$ ) in the 9-class case. If the series G48 (Table 5.17) is classified into five classes, then because more than eighty percent of the intrinsic

values are negative, all class limits are negative. It is impossible to find a positive upper bound for the class unless the series is classified into seven or more classes.

Note 5.3 The choice of C in Goldfeld-Quandt test. Johnston (1984) notes that the power of the test will depend, among other things, on the number of central observations excluded. The power will be small if the omitted observations are large (so that the  $RSS_1$  and  $RSS_2$  have very few degrees of freedom) or too small (so that the difference between the residual sums of squares is reduced). Harvey and Phillips (1974, p.312) suggested C equals one third of the sample size while Kmenta (1990) suggested one-sixth of the sample size. The choice here of  $C=21$ , or 21% of the sample size closely agrees with Pindyck and Rubinfeld's (1991) recommendation of one-fifth of the total sample size.

Table 5.1

## Database

Company	Maturity Month	Year	Code
Beecham Group	March	1987	B37*
	September	1987	B97
	December	1987	B127
	March	1988	B38
	June	1988	B68
General Electric Company	January	1987	G17
	April	1987	G47
	July	1987	G77
	January	1988	G18
	April	1988	G48
	July	1988	G78
London & Scottish Marine Oil	February	1987	L27
	May	1987	L57
	August	1987	L87
	November	1987	L117
	February	1988	L28
	May	1988	L58
	August	1988	L88

## Note

B37 denotes all Beecham Group call option series the maturing in March 1987.

For the test results in the following tables, we use the highlighted numbers 1, 2, 3 and 4 to label that  $\beta$  or the likelihood ratio test is significant at the 0.05, 0.01, 0.005 and 0.001 levels respectively. We also use \* to denote the instances that a R-squared is greater than 8%.

Table 5.3a Regression Results between:  
Call Option Trading Volume and Share Return

Series	Nobs.	CNCT = $\alpha + \beta(SR)$				
		$\alpha$	t-Ratio	$\beta$	t-Ratio	R-Squared
B6836	154	1.7706	2.7108	-0.2716	-0.2025	0.0003
B6839	160	0.4157	1.8310	-0.0832	-0.1740	0.0002
B6842	161	0.3520	1.1231	-0.1089	-0.1646	0.0002
B6846	161	36.2020	3.7426	-5.7777	-0.2829	0.0005
B6850	161	39.6100	3.3209	-6.8596	-0.2723	0.0005
B6855	179	2.5215	2.1993	-0.3816	-0.1587	0.0001
B6860	178	0.5934	1.0005	-0.1010	-0.0815	0.0000
G7716	182	16.4360	3.0468	-1.5097	-0.1055	0.0001
G7718	182	23.2520	3.3191	-1.2662	-0.0681	0.0000
G7720	164	80.0410	5.9261	-7.6054	-0.2275	0.0003
G7722	117	214.4200	6.6353	-18.8500	-0.2868	0.0007
G7724	96	336.8000	5.4726	-51.6180	-0.4659	0.0023
G7726	21	14.8070	1.6824	-2.7303	-0.3702	0.0072
L8820	171	3.1937	1.1783	0.6336	-0.0968	0.0001
L8822	173	0.0058	0.9904	0.0005	0.0342	0.0000
L8824	173	0.0351	1.1841	-0.0116	-0.1600	0.0001
L8826	173	3.6982	1.4997	0.3597	0.0598	0.0000
L8828	156	0.5021	1.3936	-0.0545	-0.0678	0.0000
L8830	156	1.2777	1.1041	-0.3791	-0.1468	0.0001
L8833	134	3.8446	1.9266	-0.6815	-0.1700	0.0002
L8836	86	5.7279	2.4570	-0.7495	-0.2017	0.0005
L8839	66	19.1200	2.4754	-3.2462	-0.3132	0.0015
L8842	66	40.9510	3.4495	-6.8083	-0.4273	0.0028
L8846	39	241.6900	3.7508	-37.6920	-0.5758	0.0089

Call Option Trading Volume and Absolute Share Return

CNCT = $\alpha + \beta(ASR)$						
B6836	154	1.7757	2.7124	-0.3080	-0.2291	0.0003
B6839	160	0.4166	1.8302	-0.0797	-0.1661	0.0002
B6842	161	0.3494	1.1118	-0.0301	-0.0454	0.0000
B6846	161	36.3000	3.7435	-6.2769	-0.3065	0.0006
B6850	161	39.7280	3.3226	-7.4898	-0.2966	0.0006
B6855	179	2.5352	2.2043	-0.5228	-0.2168	0.0003
B6860	178	0.5962	1.0020	-0.1228	-0.0988	0.0001
G7716	182	16.4480	3.0376	-1.3088	-0.0911	0.0000
G7718	182	23.2800	3.3106	-1.4899	-0.0798	0.0000
G7720	164	80.2380	5.9191	-9.3770	-0.2794	0.0005
G7722	117	214.8500	6.6255	-21.0150	-0.3187	0.0009
G7724	96	337.7400	5.4683	-52.6400	-0.4734	0.0024
G7726	21	14.8080	1.6765	-2.5764	-0.3481	0.0063
L8820	171	3.1923	1.1734	0.4762	0.0725	0.0000
L8822	173	0.0058	0.9898	-0.0000	-0.0017	0.0000
L8824	173	0.0348	1.1705	-0.0028	-0.0389	0.0000
L8826	173	3.7092	1.4984	0.0363	0.0060	0.0000
L8828	156	0.5045	1.3959	0.0856	-0.1062	0.0001
L8830	156	1.2721	1.0958	-0.1772	-0.0684	0.0000
L8833	134	3.8345	1.9162	-0.3603	-0.0896	0.0001
L8836	86	5.7319	2.4533	-0.6993	-0.1877	0.0004
L8839	66	19.1530	2.4742	-3.1891	-0.3070	0.0015
L8842	66	41.0210	3.4476	-6.6913	-0.4191	0.0027
L8846	39	242.1700	3.7516	-38.1270	-0.5814	0.0046

Table 5.3a (continued)  
Regression Results between:  
Call Option Trading Volume and Call Return

$$\text{CNCT} = \alpha + \beta(\text{CR})$$

B6836	154	1.7676	2.7062	-0.2558	-0.1480	0.0001
B6839	160	0.4167	1.8366	-0.1534	-0.2492	0.0004
B6842	161	0.3630	1.1602	-0.6148	-0.6575	0.0027
B6846	161	35.7360	3.7047	15.6410	0.5336	0.0018
B6850	161	39.3480	3.3128	-20.6390	-0.6138	0.0024
B6855	179	2.5160	2.2028	1.9695	0.6768	0.0026
B6860	178	0.5773	0.9787	-1.6148	-1.0436	0.0062
G7716	182	16.3730	3.0310	0.7184	0.0337	0.0000
G7718	182	22.8620	3.2608	15.6630	0.5191	0.0015
G7720	164	77.1570	5.7835	117.7200	2.1035	1 0.0266
G7722	117	209.1400	6.5271	173.1400	1.4352	0.0176
G7724	96	333.8200	5.4462	-4.8672	-0.0411	0.0000
G7726	21	12.6790	1.4779	-14.0230	-0.9104	0.0418
L8820	171	3.0735	1.1332	4.4053	0.5208	0.0016
L8822	173	0.0056	0.9579	0.0062	0.3194	0.0006
L8824	173	0.0357	1.2006	0.0312	-0.3002	0.0005
L8826	173	3.4701	1.4071	7.8916	0.8877	0.0046
L8828	156	0.4970	1.3773	0.0915	0.0821	0.0000
L8830	156	1.3128	1.1333	-1.5436	-0.4181	0.0011
L8833	134	3.8195	1.9121	-0.1686	-0.0601	0.0000
L8836	86	5.5289	2.3610	2.7393	0.4081	0.0020
L8839	66	19.3830	2.5128	-8.8778	-0.5716	0.0051
L8842	66	41.2560	3.4863	-17.2110	-0.6808	0.0072
L8846	39	237.9000	3.7209	-44.4370	-0.4249	0.0049

Call Option Trading Volume and Absolute Call Return

$$\text{CNCT} = \alpha + \beta(\text{ACR})$$

B6836	154	1.8099	2.7172	-0.6233	-0.3535	0.0008
B6839	160	0.4277	1.8219	-0.1548	-0.2431	0.0004
B6842	161	0.3235	0.9832	0.2307	0.2346	0.0003
B6846	161	31.0800	2.9213	34.6970	1.0732	0.0072
B6850	161	41.9280	3.0955	-15.2310	-0.3972	0.0010
B6855	179	2.4214	1.9493	0.5672	0.1792	0.0002
B6860	178	0.4148	0.6526	1.2440	0.7462	0.0032
G7716	182	16.1970	2.8971	2.7771	0.1259	0.0001
G7718	182	21.2510	2.8316	22.8570	0.7077	0.0028
G7720	164	71.9950	4.6973	67.5990	1.0514	0.0068
G7722	117	189.3600	4.9562	166.5300	1.1577	0.0115
G7724	96	240.7500	3.5760	369.9500	2.8420	2 0.0791
G7726	21	12.4590	1.1712	4.9939	0.2615	0.0036
L8820	171	2.9566	1.0703	3.7986	0.4409	0.0011
L8822	173	0.0055	0.9229	0.0038	0.1909	0.0002
L8824	173	0.0351	1.1507	-0.0057	-0.0535	0.0000
L8826	173	3.2439	1.2701	6.0667	0.6589	0.0025
L8828	156	0.5065	1.3674	-0.0815	-0.0713	0.0000
L8830	156	1.2472	1.0415	0.1847	0.0484	0.0000
L8833	134	3.5186	1.7008	2.9401	0.5070	0.0019
L8836	86	5.1541	2.1294	4.9869	0.7189	0.0061
L8839	66	19.3000	2.4227	-3.5704	-0.2226	0.0008
L8842	66	41.5270	3.3355	-8.0226	-0.3017	0.0014
L8846	39	245.9800	3.5997	-46.3800	-0.4149	0.0046

Table 5.3b  
Regression Results between:  
Call Option Trading Volume and Share Price

$$\text{CNCT} = \alpha + \beta(S)$$

B6836	154	35.8350	2.1164	-0.0744	-2.0139	1	0.0260
B6839	160	-0.8639	-0.1742	0.0028	0.2577		0.0004
B6842	161	4.7177	0.7008	-0.0096	-0.6499		0.0026
B6846	161	-464.9800	-2.2767	1.1004	2.4555	1	0.0365
B6850	161	-240.3800	-0.9404	0.6144	1.0956		0.0075
B6855	179	3.6844	0.2633	-0.0025	-0.0843		0.0000
B6860	178	2.9210	0.3941	-0.0050	-0.3155		0.0006
G7716	182	-125.2200	-2.7846	0.6846	3.1706	2	0.0529
G7718	182	-1.3574	-0.0226	0.1188	0.4125		0.0009
G7720	164	-135.0600	-1.0920	1.0212	1.7472		0.0185
G7722	117	-867.5300	-1.9258	4.8796	2.4057	1	0.0479
G7724	96	-3511.4000	-4.6495	17.2280	5.1047	2	0.2170*
G7726	21	-242.7800	-0.8581	1.0812	0.9083		0.0416
L8820	171	11.3910	0.8692	-0.0229	-0.6374		0.0024
L8822	173	0.0511	1.8579	-0.0001	-1.6849		0.0163
L8824	173	0.1482	1.0520	-0.0003	-0.8241		0.0040
L8826	173	21.9400	1.8817	-0.0513	-1.5988		0.0147
L8828	156	1.5214	0.7620	-0.0028	-0.5200		0.0018
L8830	156	-2.2182	-0.3459	0.0094	0.5518		0.0020
L8833	134	24.2230	1.9877	-0.0534	-1.6970		0.0214
L8836	86	28.5040	1.2534	-0.0543	-1.0091		0.0120
L8839	66	222.9200	1.6370	-0.4635	-1.5011		0.0340
L8842	66	-535.9600	-2.6731	1.3086	2.8786	2	0.1146*
L8846	39	-12888.0000	-2.9960	28.6530	3.0511	2	0.2010*

Call Option Trading Volume and the Intrinsic Value

$$\text{CNCT} = \alpha + \beta(S-X)$$

B6836	154	9.0425	2.4620	-0.0744	-2.0139	1	0.0260
B6839	160	0.2288	0.3059	0.0028	0.2577		0.0004
B6842	161	0.6863	1.1304	-0.0096	-0.6499		0.0026
B6846	161	41.1980	4.2465	1.1004	2.4555	1	0.0365
B6850	161	66.8380	2.4087	0.6144	1.0956		0.0075
B6855	179	2.2916	0.8147	-0.0025	-0.0843		0.0000
B6860	178	-0.0950	-0.0422	-0.0050	-0.3155		0.0006
G7716	182	15.6790	-1.3768	0.6846	3.7060	2	0.0529
G7718	182	20.0250	1.9226	0.1188	0.4125		0.0009
G7720	164	69.1740	4.7201	1.0212	1.7472		0.0185
G7722	117	205.9800	6.5274	4.8796	2.4057	1	0.0479
G7724	96	623.2700	7.9442	17.2280	5.1047	2	0.2170*
G7726	21	38.3460	1.3694	1.0812	0.9083		0.0416
L8820	171	6.8105	1.0896	-0.0229	-0.6374		0.0024
L8822	173	0.0230	1.9614	-0.0001	-1.6849		0.0163
L8824	173	0.0715	1.3356	-0.0003	-0.8241		0.0040
L8826	173	8.6039	2.1987	-0.0513	-1.5988		0.0147
L8828	156	0.7456	1.2573	-0.0028	-0.5200		0.0018
L8830	156	0.6147	0.3736	0.0094	0.5518		0.0020
L8833	134	6.6054	2.5767	-0.0534	-1.6970		0.0214
L8836	86	8.9694	2.2448	-0.0543	-1.0091		0.0120
L8839	66	42.1710	2.4395	-0.4635	-1.5011		0.0340
L8842	66	13.6390	0.9432	1.3086	2.8786	2	0.1146*
L8846	39	293.0500	4.8745	28.6530	3.0511	2	0.2010*

Table 5.3b (continued)  
 Regression Results between:  
 Call Option Trading Volume and Call Premium

$$\text{CNCT} = \alpha + \beta(C)$$

B6836	154	9.9559	1.5265	-0.0733	-1.2630		0.0104
B6839	160	-3.0328	-1.8272	0.0400	2.0947	1	0.0270
B6842	161	-1.6746	-1.0702	0.0319	1.3187		0.0108
B6846	161	133.8300	5.4172	-2.5441	-4.2629	2	0.1026*
B6850	161	62.4650	3.2995	-0.9063	-1.5616		0.0151
B6855	179	3.1638	2.0474	-0.0356	-0.6298		0.0022
B6860	178	0.9570	1.2484	-0.0334	-0.7512		0.0032
G7716	182	-28.1290	-1.7521	0.7984	2.9346	2	0.0457
G7718	182	29.4550	1.6368	-0.1612	-0.3762		0.0008
G7720	164	64.6160	2.0385	0.5921	0.5285		0.0017
G7722	117	64.8050	0.7783	8.5272	1.9315		0.0314
G7724	96	209.0100	1.8129	15.2620	1.2742		0.0170
G7726	21	1.9512	0.1519	11.2740	1.2377		0.0746
L8820	171	7.1397	1.0034	-0.0238	-0.5958		0.0021
L8822	173	0.0243	1.7340	-0.0001	-1.4494		0.0121
L8824	173	0.0969	1.4444	-0.0005	-1.0320		0.0062
L8826	173	10.5370	1.9992	-0.0604	-1.4613		0.0123
L8828	156	1.0600	1.3331	-0.0053	-0.7888		0.0040
L8830	156	0.4673	0.1961	0.0089	0.3815		0.0009
L8833	134	9.8099	2.4879	-0.0805	-1.7539		0.0228
L8836	86	12.4120	1.9805	-0.0904	-1.1556		0.0156
L8839	66	77.2170	2.4497	-0.9369	-1.9069		0.0538
L8842	66	-55.8710	-1.2948	2.3862	2.3103	1	0.0770
L8846	39	221.1300	1.0670	0.7274	0.0743		0.0001

Call Option Trading Volume and Put Premium

$$\text{CNCT} = \alpha + \beta(P)$$

B6836	154	0.2119	0.2630	0.2508	3.0942	2	0.0593
B6839	160	0.2904	0.9943	0.0102	0.6600		0.0027
B6842	161	0.0377	0.0862	0.0162	1.0093		0.0064
B6846	161	82.4290	5.2077	-1.3922	-3.6202	2	0.0762
B6850	161	81.7700	2.8654	-0.7329	-1.6325		0.0165
B6855	179	3.4802	0.9900	-0.0105	-0.2923		0.0005
B6860	178	0.0590	0.0245	0.0038	0.2277		0.0003
G7716	182	24.2010	3.4186	-3.4651	-1.6789		0.0154
G7718	182	30.7240	3.3220	-1.3036	-1.2317		0.0084
G7720	164	111.3600	5.9698	-3.0091	-2.4009	1	0.0344
G7722	117	299.4900	5.5056	-7.5009	-1.9433	1	0.0318
G7724	96	817.8800	7.8788	-21.4010	-5.4392	2	0.2394*
G7726	21	51.5710	1.5414	-1.6430	-1.1565		0.0658
L8820	171	2.4365	0.7569	0.1414	0.4444		0.0012
L8822	173	-0.0039	-0.5728	0.0011	2.5121	1	0.0356
L8824	173	0.0340	0.9218	0.0001	0.0314		0.0000
L8826	173	0.3752	0.1210	0.2096	1.7385		0.0174
L8828	156	0.6476	1.3842	-0.0098	-0.4913		0.0016
L8830	156	2.1589	1.3863	-0.0441	-0.8535		0.0047
L8833	134	1.6999	0.6300	0.1003	1.1510		0.0099
L8836	86	4.2615	1.4320	0.1258	0.7526		0.0067
L8839	66	7.2588	0.6635	1.2876	1.4593		0.0322
L8842	66	82.3050	4.5574	-2.4197	-2.9426	2	0.1192*
L8846	39	496.3300	2.9971	-12.2650	-1.6948		0.0720

Table 5.3b (continued)  
 Regression Results between:  
 Call Option Trading Volume and Put Option Trading Volume  
 $CNCT = \alpha + \beta(PNCT)$

B6836	154	1.7740	2.7143	-0.0392	-0.2432	0.0004
B6839	160	0.4142	1.8200	-0.0013	-0.0714	0.0000
B6842	161	0.3627	1.1371	-0.0026	-0.2302	0.0003
B6846	161	32.1200	3.2827	0.4602	1.8014	0.0200
B6850	161	38.8100	3.2422	2.1111	0.3794	0.0009
B6855	179	2.5265	2.2015	-0.0114	-0.1865	0.0002
B6860	178	-	-	-	-	-
G7716	182	16.5160	3.0603	-0.7623	-0.2669	0.0004
G7718	182	23.6480	3.3560	-1.9273	-0.4474	0.0011
G7720	164	65.3160	4.6775	0.3901	2.9986	2 0.0526
G7722	117	211.7000	6.3591	0.1040	0.2147	0.0004
G7724	96	225.5300	4.2681	2.0806	6.7350	2 0.3255*
G7726	21	-	-	-	-	-
L8820	171	3.2593	1.1991	-0.5650	-0.1377	0.0001
L8822	173	0.0058	1.0022	-0.0002	-0.1009	0.0001
L8824	173	0.0349	1.1784	-0.0019	-0.0942	0.0001
L8826	173	3.7715	1.5230	-2.0953	-0.1930	0.0002
L8828	156	0.5032	1.3976	-0.1258	-0.1119	0.0001
L8830	156	1.2848	1.1049	-0.0699	-0.1444	0.0001
L8833	134	3.8839	1.9371	-0.1242	-0.2609	0.0005
L8836	86	5.7439	2.4653	-0.0586	-0.2711	0.0009
L8839	66	19.4040	2.4672	-0.2743	-0.3309	0.0017
L8842	66	34.7390	3.0160	1.7533	2.4287	1 0.0844*
L8846	39	186.3300	4.2024	4.7840	6.4934	2 0.5326*

Table 5.4  
 Fitting Call Option Trading Volume to Four Probability Distributions

Series	Poisson		Chi-Square		Exponential		Student's t	
	$\chi^2$	SL DF	$\chi^2$	SL DF	$\chi^2$	SL DF	$\chi^2$	SL DF
B6836	644.12	0.00 1	258.52	0.00 1	295.69	0.00 1	52.62	0.00 1
B6839	67.37	0.00 1	32.66	0.00 2	-	-	280.44	0.00 3
B6842	59.72	0.00 1	29.03	0.00 1	-	-	1.19	0.00 3
B6846	-	-	-	-	691.00	0.00 2	-	-
B6850	-	-	-	-	34.75	0.00 1	-	-
B6855	-	-	-	-	-	-	-	-
B6860	-	-	-	-	-	-	13.77	0.00 1
G7716	-	-	-	-	356.83	0.00 1	-	-
G7718	-	-	-	-	35.98	0.00 1	-	-
G7720	-	-	-	-	109.90	0.00 3	-	-
G7722	-	-	-	-	35.69	0.00 5	-	-
G7724	-	-	-	-	22.28	0.00 3	-	-
G7726	-	-	-	-	-	-	-	-
L8820	-	-	-	-	-	-	-	-
L8822	-	-	-	-	-	-	-	-
L8824	-	-	-	-	-	-	-	-
L8826	-	-	-	-	-	-	-	-
L8828	92.04	0.00 1	47.00	0.00 2	82.55	0.00 1	292.17	0.00 3
L8830	-	-	-	-	-	-	-	-
L8833	-	-	-	-	-	-	-	-
L8836	-	-	-	-	43.06	0.00 1	-	-
L8839	-	-	-	-	93.87	0.00 1	-	-
L8842	-	-	-	-	19.25	0.00 2	-	-
L8846	-	-	-	-	23.75	0.00 2	-	-

NOTE : SL - Significance Level, DF - Degree of Freedom  
 Most of the probability distribution fittings to CNCT fails because of insufficient degrees of freedom.

Table 5.5

Regression Results between: the Instance of  
a Call Transaction and Trading History of a  
Call

$$d = \alpha + \beta(th)$$

Series	Nobs.	$\alpha$	t-Ratio	$\beta$	t-Ratio	R-Squared
B6836	154	0.1033	1.6818	-0.3797	-0.6600	0.0029
B6839	160	0.0221	0.9248	0.1967	0.8283	0.0043
B6842	161	0.0394	2.1026	-1.4399	-1.3470	0.0113
B6846	161	-0.0573	-0.6588	2.3151	4.1217	2 0.0965*
B6850	161	0.1008	2.1193	1.2273	3.6694	2 0.0781
B6855	179	0.0616	2.6928	-0.1422	-0.7085	0.0028
B6860	178	0.0080	1.1934	-1.1903	-0.6538	0.0024
G7716	182	0.1046	1.4720	0.3181	1.1791	0.0077
G7718	182	0.1739	1.4342	0.3439	1.1025	0.0067
G7720	164	0.2117	2.1106	0.9356	4.3825	2 0.1060*
G7722	117	1.0588	5.3904	-0.1319	-0.6077	0.0032
G7724	96	0.9653	2.1990	-0.2752	-0.4452	0.0021
G7726	21	0.1832	1.0298	-0.1169	-0.2542	0.0034
L8820	171	0.0279	2.0162	-0.9591	-1.4558	0.0124
L8822	173	0.0079	1.1652	-0.0086	-0.6012	0.0021
L8824	173	0.0166	1.6261	-0.1719	-0.8214	0.0039
L8826	173	0.0157	0.4296	0.3400	0.9339	0.0051
L8828	156	0.0226	1.3544	-0.2389	-0.2682	0.0005
L8830	156	0.0584	2.0426	-1.4111	-1.2777	0.0105
L8833	134	0.0655	1.0509	0.1448	0.2881	0.0006
L8836	86	0.0997	1.2665	0.1170	0.2347	0.0007
L8839	66	0.0997	0.6028	0.3799	0.8124	0.1020*
L8842	66	0.1262	1.6786	1.7867	4.6783	2 0.2548*
L8846	39	0.3187	2.1547	0.8927	3.1813	2 0.2148*

Table 5.6 Regression Results between  
The Instance of a Call Transaction and Share Return

Series	Nobs.	$d = \alpha + \beta(SR)$				
		$\alpha$	t-Ratio	$\beta$	t-Ratio	R-Squared
B6836	154	0.0654	3.2601	-0.0107	-0.2606	0.0004
B6839	160	0.0378	2.4918	-0.0073	-0.2280	0.0003
B6842	161	0.0188	1.7497	-0.0050	-0.2184	0.0003
B6846	161	0.2751	7.7683	-0.0473	-0.6328	0.0025
B6850	161	0.2314	6.9186	-0.0404	-0.5715	0.0020
B6855	179	0.0566	3.0766	-0.0081	-0.2352	0.0003
B6860	178	0.0057	1.0005	-0.0010	-0.0815	0.0000
G7716	182	0.1762	6.2020	0.1694	2.2466	1 0.0273
G7718	182	0.2977	8.7355	0.1497	1.6554	0.0150
G7720	164	0.6259	16.4500	-0.1171	-1.2431	0.0094
G7722	117	0.9398	42.3070	0.0073	0.1609	0.0002
G7724	96	0.7685	17.6590	0.0412	0.5263	0.0029
G7726	21	0.1503	1.8346	-0.0284	-0.4132	0.0089
L8820	171	0.0117	1.4089	0.0000	0.0002	0.0000
L8822	173	0.0058	0.9904	0.0005	0.0342	0.0000
L8824	173	0.0116	1.4173	-0.0019	0.0962	0.0001
L8826	173	0.0463	2.8692	-0.0003	-0.0065	0.0000
L8828	156	0.0193	1.7415	-0.0029	-0.1187	0.0001
L8830	156	0.0258	23.0207	-0.0049	-0.1727	0.0002
L8833	134	0.0827	3.4472	-0.0131	-0.2712	0.0006
L8836	86	0.1175	3.3391	-0.0169	-0.3016	0.0011
L8839	66	0.2309	4.3847	-0.0394	-0.5575	0.0048
L8842	66	0.3848	6.3280	-0.0649	-0.7955	0.0098
L8846	39	0.7631	10.9160	-0.1238	-1.7435	0.0759

The Instance of a Call Transaction and Absolute  
Share Return

$d = \alpha + \beta(ASR)$						
B6836	154	0.0655	3.2605	-0.0113	-0.2744	0.0005
B6839	160	0.0379	2.4911	-0.0071	-0.2228	0.0003
B6842	161	0.0188	1.7393	-0.0025	-0.1094	0.0001
B6846	161	0.2757	7.7665	-0.0479	-0.6389	0.0026
B6850	161	0.2319	6.9188	-0.0417	-0.5884	0.0022
B6855	179	0.0508	3.0755	-0.1014	-0.2921	0.0005
B6860	178	0.0057	1.0020	-0.0012	-0.0988	0.0001
G7716	182	0.1738	6.0933	0.1709	2.2473	1 0.0273
G7718	182	0.2957	8.6417	0.1458	1.6052	0.0141
G7720	164	0.6275	16.4310	0.0116	-1.2279	0.0092
G7722	117	0.9395	42.1480	0.0102	0.2242	0.0004
G7724	96	0.7675	17.5780	0.0448	0.5704	0.0034
G7726	21	0.1502	1.8265	-0.0264	-0.3832	0.0077
L8820	171	0.0117	1.4028	0.0002	0.7497	0.0000
L8822	173	0.0058	0.9898	-0.0000	-0.0017	0.0000
L8824	173	0.0116	1.4071	-0.0006	-0.2904	0.0000
L8826	173	0.0464	2.8692	-0.0037	-0.0931	0.0001
L8828	156	0.0194	1.7421	-0.0034	-0.1386	0.0001
L8830	156	0.0258	2.0152	-0.0039	-0.1348	0.0001
L8833	134	0.0827	3.4369	-0.0100	-0.2067	0.0003
L8836	86	0.1176	3.3334	-0.0155	-0.2754	0.0009
L8839	66	0.2312	4.3796	-0.0376	-0.5311	0.0044
L8842	66	0.3857	6.3288	-0.0657	-0.8029	0.0100
L8846	39	0.7645	10.9210	-0.1247	-1.7531	0.7670*

Table 5.6 (continued)  
 Regression Results between the Instance of a  
 Call Transaction and Call Return

$$d = \alpha + \beta(CR)$$

B6836	154	0.0653	3.2552	-0.0108	-0.2037	0.0003
B6839	160	0.0379	2.5024	-0.0150	-0.3641	0.0008
B6842	161	0.0193	1.8002	-0.0276	-0.8616	0.0046
B6846	161	0.2752	7.8108	-0.1231	-1.1500	0.0082
B6850	161	0.2298	6.9068	-0.0881	-0.9349	0.0050
B6855	179	0.0563	3.0657	0.0151	0.3601	0.0007
B6860	178	0.0052	0.9787	-0.1538	-1.0436	0.0062
G7716	182	0.1751	6.1588	0.2567	2.2887	1 0.0283
G7718	182	0.2949	8.6923	0.3253	2.2284	1 0.0268
G7720	164	0.6215	16.2430	0.0187	0.1165	0.0001
G7722	117	0.9418	42.4970	-0.0639	-0.7660	0.0051
G7724	96	0.7711	17.8040	0.0363	0.4338	0.0020
G7726	21	0.1301	1.6259	-0.1263	-0.7879	0.0391
L8820	171	0.0115	1.3807	0.0068	0.2626	0.0004
L8822	173	0.0056	0.9579	0.0062	0.3194	0.0006
L8824	173	0.0117	1.4250	-0.0050	-0.1753	0.0002
L8826	173	0.0440	2.7383	0.0719	1.2396	0.0089
L8828	156	0.0193	1.7393	-0.0036	-0.1041	0.0001
L8830	156	0.0260	2.0307	-0.0110	-0.2704	0.0005
L8833	134	0.0827	3.4432	-0.0162	-0.2402	0.0004
L8836	86	0.1156	3.2654	0.0138	0.1362	0.0002
L8839	66	0.2322	4.4143	-0.0774	-0.7304	0.0083
L8842	66	0.3869	6.4018	-0.1483	-1.1484	0.0202
L8846	39	0.7545	11.0470	-0.2270	-2.0316	1 0.1004*

The Instance of a Call Transaction and  
 Absolute Call Return

$$d = \alpha + \beta(ACR)$$

B6836	154	0.0668	3.2681	-0.0233	-0.4311	0.0012
B6839	160	0.0380	2.4249	-0.0053	-0.1254	0.0001
B6842	161	0.0179	1.5889	0.0065	0.1925	0.0002
B6846	161	0.2626	6.7193	0.0757	0.6374	0.0025
B6850	161	0.2424	6.3848	-0.0743	-0.6913	0.0030
B6855	179	0.0506	2.8358	-0.0021	-0.0457	0.0000
B6860	178	0.0040	0.6526	0.0118	0.7462	0.0032
G7716	182	0.1587	5.4319	0.3258	2.8266	1 0.0425
G7718	182	0.2708	7.4630	0.3655	2.3403	1 0.0295
G7720	164	0.6388	14.7070	-0.1463	-0.8029	0.0040
G7722	117	0.9414	35.6450	-0.0084	-0.0848	0.0001
G7724	96	0.7413	15.0590	0.1174	1.2331	0.0159
G7726	21	0.1340	1.3503	0.0272	0.1526	0.0012
L8820	171	0.0112	1.3265	0.0069	0.2599	0.0004
L8822	173	0.0055	0.9229	0.0038	0.1909	0.0002
L8824	173	0.0117	1.3890	-0.0023	-0.0786	0.0000
L8826	173	0.0419	2.5132	0.0513	0.9371	0.0051
L8828	156	0.0198	1.7299	-0.0066	-0.1860	0.0002
L8830	156	0.0258	1.9474	-0.0014	-0.0339	0.0000
L8833	134	0.0776	3.1226	0.0446	0.6395	0.0031
L8836	86	0.1094	2.9916	0.0660	0.6296	0.0047
L8839	66	0.2338	4.3027	-0.0487	-0.4446	0.0031
L8842	66	0.3961	6.2184	-0.1146	-0.8425	0.0110
L8846	39	0.7890	10.6450	-0.2060	-1.6992	0.0724

Table 5.6 (continued)  
 Regression Results between: the Instance of  
 a Call Transaction and Share Price

$$d = \alpha + \beta(S)$$

B6836	154	1.0047	1.9279	-0.0021	-1.8046	0.0210
B6839	160	0.6453	1.9693	-0.0013	-1.8568	0.0214
B6842	161	-0.0398	-0.1721	0.0001	0.2529	0.0004
B6846	161	-1.8061	-2.4260	0.0046	2.7961	2 0.0469
B6850	161	-2.5359	-3.6989	0.0061	4.0385	2 0.0930*
B6855	179	0.1427	0.7106	-0.0002	-0.4618	0.0012
B6860	178	0.0278	0.3941	0.0000	-0.3155	0.0006
G7716	182	0.5937	2.4250	-0.0020	-1.6958	0.0157
G7718	182	0.6846	2.3388	-0.0018	-1.3153	0.0095
G7720	164	-0.9969	-3.0272	0.0077	4.9446	2 0.1311*
G7722	117	0.4357	1.3885	0.0023	1.6117	0.0221
G7724	96	-1.5919	-2.8837	0.0106	4.2911	2 0.1638*
G7726	21	-1.0473	-0.3911	0.0050	0.4446	0.0103
L8820	171	0.0131	0.3269	-0.0000	-0.3662	0.0000
L8822	173	0.0511	1.8579	-0.0001	-1.6849	0.0163
L8824	173	0.0705	1.8166	-0.0002	-1.5531	0.0139
L8826	173	0.2607	3.4790	-0.0006	-2.9266	2 0.0477
L8828	156	0.0291	0.4723	-0.0000	-0.1628	0.0002
L8830	156	0.0442	0.6233	-0.0001	-0.2659	0.0005
L8833	134	0.4337	2.9946	-0.0009	-2.4600	2 0.0438
L8836	86	0.5039	1.4702	-0.0009	-1.1368	0.0152
L8839	66	3.9079	4.7280	-0.0084	-4.4598	2 0.2371*
L8842	66	-4.4125	-4.3918	0.0102	4.8025	2 0.2649*
L8846	39	-2.4342	-0.4523	0.0069	0.5906	0.0093

The Instance of a Call Transaction and the  
 Intrinsic Value

$$d = \alpha + \beta(S-X)$$

B6836	154	0.2658	2.3512	-0.0021	-1.8046	0.0210
B6839	160	0.1250	2.5285	-0.0013	-1.8568	0.0214
B6842	161	0.0141	0.6762	0.0001	0.2529	0.0004
B6846	161	0.2949	8.3406	0.0046	2.7961	2 0.0469
B6850	161	0.5016	6.7398	0.0067	4.0385	2 0.0930*
B6855	179	0.0332	0.8233	-0.0002	-0.4618	0.0012
B6860	178	-0.0009	-0.0422	-0.0000	-0.3155	0.0006
G7716	182	0.2747	4.4307	-0.0020	-1.6958	0.0517
G7718	182	0.3518	6.9214	-0.0018	-1.3153	0.0095
G7720	164	0.5420	13.8890	0.0077	4.9446	2 0.1311*
G7722	117	0.9366	42.6140	0.0023	1.6117	0.0221
G7724	96	0.9487	16.5420	0.0106	4.2911	2 0.1638*
G7726	21	0.2552	0.9630	0.0050	0.4446	0.0103
L8820	171	0.0123	0.6433	-0.0000	0.0366	0.0000
L8822	173	0.0230	1.9614	-0.0001	-1.6849	0.0163
L8824	173	0.0307	2.0804	-0.0002	-1.5531	0.0139
L8826	173	0.1038	4.1276	0.0006	-2.9266	2 0.0477
L8828	156	0.0216	1.1806	-0.0000	-0.1628	0.0002
L8830	156	0.0291	1.5996	-0.0001	-0.2659	0.0005
L8833	134	0.1302	4.2734	-0.0009	-2.4600	1 0.0438
L8836	86	0.1722	2.8597	-0.0009	-1.1368	0.0152
L8839	66	0.6484	6.1795	-0.0084	-4.4598	2 0.2371*
L8842	66	0.1703	2.5415	0.0102	4.8025	2 0.2649*
L8846	39	0.7575	10.0710	0.0069	0.5906	0.0093

Table 5.6 (continued)  
 Regression Results between: the Instance of  
 a Call Transaction and Call Premium

$$d = \alpha + \beta(C)$$

B6836	154	0.1406	0.6989	-0.0007	-0.3780	0.0009
B6839	160	-0.0636	-0.5670	0.0012	0.9098	0.0052
B6842	161	0.0395	0.7311	-0.0003	-0.3937	0.0010
B6846	161	0.6950	7.8509	-0.0110	-5.1274	2 0.1419*
B6850	161	0.3490	6.6931	-0.0047	-2.9230	2 0.0510
B6855	179	0.0473	2.1274	0.0002	0.2018	0.0002
B6860	178	0.0091	1.2484	-0.0003	-0.7512	0.0032
G7716	182	0.3302	3.7960	-0.0027	-1.8110	0.0179
G7718	182	0.4239	4.8339	-0.0031	-1.5055	0.0124
G7720	164	0.3416	3.9523	0.0109	3.5829	2 0.0734
G7722	117	0.8255	14.4860	0.0066	2.1766	1 0.0396
G7724	96	0.5889	7.4330	0.0222	2.7021	2 0.0721
G7726	21	0.0643	0.5267	0.0729	0.8420	0.0360
L8820	171	0.0122	0.5582	-0.0000	-0.0239	0.0000
L8822	173	0.0243	1.7340	-0.0001	-1.4494	0.0121
L8824	173	0.0374	2.0240	-0.0002	-1.5570	0.0140
L8826	173	0.1283	3.7794	-0.0007	-2.7270	2 0.0417
L8828	156	0.0296	1.2046	-0.0001	-0.4718	0.0014
L8830	156	0.0390	1.4834	-0.0002	-0.5812	0.0022
L8833	134	0.1821	3.8837	-0.0013	-2.4602	1 0.0438
L8836	86	0.2409	2.5567	-0.0017	-1.4219	0.0235
L8839	66	1.1059	5.8117	-0.0141	-4.7523	2 0.2608*
L8842	66	-0.2756	-1.2833	0.0162	3.1580	2 0.1348*
L8846	39	1.2951	6.0960	-0.0274	-2.7288	2 0.1675*

The Instance of a Call Transaction and Put  
 Premium

$$d = \alpha + \beta(P)$$

B6836	154	0.0227	0.9103	0.0068	2.7360	2 0.0469
B6839	160	0.0023	0.1232	0.0029	2.9170	2 0.0511
B6842	161	0.0231	1.5362	-0.0002	-0.4252	0.0011
B6846	161	0.4612	8.0242	-0.0056	-4.0337	2 0.0928*
B6850	161	0.5464	7.2007	-0.0055	-4.5817	2 0.1166*
B6855	179	0.0405	0.8015	0.0001	0.2054	0.0002
B6860	178	0.0006	0.0245	0.0000	0.2277	0.0003
G7716	182	0.1051	2.8332	0.0338	3.1242	2 0.0514
G7718	182	0.2385	5.3058	0.1106	2.1501	1 0.0250
G7720	164	0.8288	17.0790	-0.0197	-6.0456	2 0.1841*
G7722	117	0.9853	26.1780	-0.0039	-1.4758	0.1860*
G7724	96	0.7480	16.6790	0.0004	1.6706	0.1398*
G7726	21	0.3775	1.1886	-0.0103	-0.7628	0.0297
L8820	171	0.0117	1.1872	-0.0000	-0.0029	0.0000
L8822	173	-0.0039	-0.5728	0.0011	2.5121	1 0.0356
L8824	173	0.0030	0.2914	0.0007	1.4155	0.0116
L8826	173	0.0092	0.4624	0.0023	3.0025	2 0.0501
L8828	156	0.0269	1.8660	-0.0005	-0.8268	0.0044
L8830	156	0.0277	1.6057	-0.0001	-0.1761	0.0002
L8833	134	0.0319	0.9990	0.0024	2.3056	1 0.0387
L8836	86	0.0939	2.0906	0.0020	0.7902	0.0074
L8839	66	0.0027	0.3946	0.0223	4.0985	2 0.2079*
L8842	66	0.7230	8.7558	-0.0198	-5.2762	2 0.3031*
L8846	39	1.1507	6.4222	-0.0192	-2.4476	1 0.1393*

Table 5.6 (continued)  
 Regression Results between: the Instance of  
 a Call Transaction and Put Option Trading  
 Volume

$$d = \alpha + \beta(\text{PNCT})$$

B6836	154	0.0655	3.2630	-0.0014	-0.2924	0.0006
B6839	160	0.0372	2.4462	0.0002	0.1980	0.0002
B6842	161	0.0194	1.7750	-0.0001	-0.3594	0.0008
B6846	161	0.2478	7.0744	0.0030	3.3252	0.0650
B6850	161	0.2282	6.7957	0.0062	0.3992	0.0010
B6855	179	0.0506	3.0728	-0.0002	-0.2603	0.0004
B6860	178	-	-	-	-	-
G7716	182	0.1827	6.3435	-0.0084	-0.5533	0.0017
G7718	182	0.3043	8.8095	-0.0092	-0.4352	0.0011
G7720	164	0.6019	14.9200	0.0005	1.4372	0.0126
G7722	117	0.9359	40.9900	0.0002	0.7210	0.0045
G7724	96	0.7480	16.6790	0.0004	1.6706	0.0288
G7726	21	-	-	-	-	-
L8820	171	0.0119	1.4237	-0.0021	-0.1634	0.0002
L8822	173	0.0058	1.0022	-0.0002	-0.1009	0.0001
L8824	173	0.0116	1.4188	-0.0006	-0.1134	0.0001
L8826	173	0.0470	2.9039	-0.0261	-0.3680	0.0008
L8828	156	0.0194	1.7434	-0.0048	-0.1396	0.0001
L8830	156	0.0261	2.0310	-0.0014	-0.2655	0.0005
L8833	134	0.0836	3.4703	-0.0027	-0.4674	0.0017
L8836	86	0.1177	3.3475	-0.0012	-0.3681	0.0016
L8839	66	0.2360	4.4075	-0.0041	-0.0724	0.0081
L8842	66	0.3570	5.9153	0.0068	1.8099	0.0487
L8846	39	0.7358	10.1510	0.0008	0.6258	0.0105

Table 5.8

Logit Results between the Instance of a Call Transaction and the Absolute Normalised Intrinsic Value (140 Individual Series)

$$p(d_t = 1) = \frac{1}{1 + e^{-z_t}}$$

$$z_t = \alpha + \beta \left| \frac{S_t - X}{S_t} \right|$$

Series	Nobs.	$\alpha$	t-Ratio	$\beta$	t-Ratio	Likelihood Ratio Test
<b>B37</b>						
360	174	-4.0072	-2.2687	-2.6798	-0.2650	0.0757
390	174	-2.0648	-4.6516	-3.8453	-0.9735	1.0668
420	172	-0.9709	-4.0656	-1.4759	-0.5792	0.3449
460	174	1.2231	3.0594	-17.6690	-4.6368	25.7267 4
500	41	3.8572	2.5354	-31.6950	-1.7374	4.1964 1
550	23	3.7957	2.8134	-48.6170	-2.0363	1 5.3567 1
<b>B97</b>						
390	185	-7.8985	-1.6436	13.8830	0.8284	0.9613
420	185	-5.2876	-2.3085	8.0416	0.7932	0.7768
460	185	-1.4377	-2.6105	-10.0070	-2.1281	1 4.4273 1
500	169	-1.4186	-3.1321	0.5284	0.0969	0.0094
550	151	1.0624	3.9270	-23.2730	-3.6044	2 16.0978 4
600	126	1.8361	3.3242	-21.7230	-4.0751	2 23.0505 4
<b>B127</b>						
360	34	-25.0470	-1.2291	111.1700	1.1330	3.1273
390	41	1.5717	1.7229	-36.6670	-3.0066	2 12.7437 4
420	41	-0.9034	-1.3630	34.5620	2.2242	1 6.1964 2
460	166	0.2413	0.4803	-15.4330	-3.8325	2 15.6872 4
500	174	-3.4128	-6.0864	10.3680	2.8128	2 7.6671 2
550	185	-1.0132	-4.6173	-1.6999	-1.0473	1.1693
600	185	-0.5829	-1.8739	-5.9746	-2.7929	2 11.5718 4
<b>B38</b>						
360	94	-5.5438	-0.8615	4.9094	0.1612	0.0275
390	100	-0.1166	-0.1406	-29.3310	-2.8971	2 10.7342 3
420	101	-2.5636	-3.2443	8.4801	0.9309	0.9033
460	101	0.0075	0.0256	-14.7440	-2.3738	1 7.4866 2
500	181	-0.0849	-0.1656	-13.0460	-2.5398	1 8.3834 3
550	181	-3.0374	-5.7838	2.5037	0.9963	0.9769
600	181	-0.8253	-1.4098	-15.5150	-2.5110	1 16.4236 4
<b>B68</b>						
360	155	0.3279	0.1951	-14.6500	-1.7523	2.7517
390	161	-1.4329	-1.5117	-14.2010	-1.8742	3.0698
420	162	-4.0272	-2.4468	0.6891	0.0371	0.0014
460	162	-0.7868	-3.3640	-6.3666	-1.2135	1.6677
500	162	0.9881	1.7056	-25.7520	-3.6090	2 21.4380 4
550	180	-2.4894	-2.9002	-1.7928	-0.4223	0.1803
600	179	-6.1171	-1.7406	2.9875	0.2902	0.0853

Table 5.8 (continued)  
 Logit Results between the Instance of a Call  
 Transaction and the Absolute Normalised  
 Intrinsic Value (140 Individual Series)

G17							
160	88	0.5364	1.2096	-4.1474	-0.9161	0.8461	
180	180	0.9240	3.0267	-12.4480	-3.0620 2	10.1327 3	
200	180	-1.0433	-3.6816	11.4290	4.5008 2	23.1394 4	
220	177	-0.3169	-0.6577	-9.3357	-3.2376 2	12.1157 4	
G47							
160	149	-0.0073	-0.0224	-2.4629	-1.2729	1.6400	
180	182	-0.1295	-0.4957	1.3323	0.5000	0.2504	
200	182	1.4347	4.4750	-7.4609	-2.9809 2	9.2662 3	
220	56	4.3757	3.7518	-37.9400	-2.9929 2	13.4152 4	
240	35	4.9163	3.0310	-32.2970	-3.0978 2	18.4136 4	
G77							
160	183	-0.7403	-1.7222	-3.8059	-1.9100	3.6648	
180	183	-0.4299	-1.3910	-3.1334	-1.5297	2.3799	
200	165	1.0122	3.1860	-5.0253	-1.8910	3.6503	
220	118	3.2292	3.9274	-9.1703	-0.8886	0.7925	
240	97	2.6195	5.2019	-13.3490	-3.7640 2	16.2149 4	
260	12	-1.2747	-0.9032	-2.5458	-0.1745	0.0301	
G18							
140	28	5.7043	1.5900	-61.1060	-1.9285	5.5165 1	
160	55	1.2515	2.5997	-24.7090	-2.6129 2	9.1653 3	
180	184	1.2216	2.6945	-18.6850	-5.5494 2	42.4809 4	
200	153	-0.0178	-0.0501	-5.2988	-2.2771 1	5.4532 1	
220	143	1.0464	3.7116	-12.5530	-4.5394 2	51.1526 4	
240	171	1.7345	6.1661	-8.7683	-6.7993 2	76.6308 4	
260	144	0.1095	0.2946	-5.5457	-3.7660 2	21.1361 4	
G48							
140	90	-0.6689	-0.9533	-4.8708	-0.6886	0.4750	
160	117	-0.4506	-1.3149	-10.8780	-1.6571	3.0123	
180	122	-0.3703	-0.7583	-6.8306	-1.9322	3.8227	
200	172	-0.6242	-1.5214	-6.2887	-2.8912 2	9.1053 3	
220	183	0.2473	0.8384	-8.1945	-5.5443 2	44.7118 4	
240	183	0.0310	0.0807	-9.2746	-4.2949 2	39.7401 4	
260	183	-0.7754	-1.4003	-5.5098	-3.0761 2	13.6805 4	
G78							
140	157	-1.8013	-2.9021	-0.1461	-0.0224	0.0005	
160	184	-0.2495	-0.9400	-5.7487	-1.2867	1.6909	
180	189	-0.8879	-1.6148	-9.8162	-2.4936 1	6.2642 1	
200	189	-0.2018	-0.2342	-9.7936	-2.7744 2	7.4312 2	
220	190	0.2025	0.1797	-9.0740	-2.7980 2	7.4563 2	
L27							
90	145	-1.9186	-2.5762	-0.5069	-0.2399	0.0571	
100	155	-0.9079	-2.0468	-4.1645	-2.2430 1	5.3731 1	
110	179	-1.3793	-4.0457	-0.8070	-0.5087	0.2614	
120	179	-0.9575	-3.4765	2.2972	1.6831	2.8587	
130	179	0.4167	1.4666	-3.7213	-2.4817 1	6.5585 1	
140	68	0.1600	0.2790	6.6109	1.6167	2.8306	
160	67	3.8256	2.5306	6.0717	0.2849	0.0877	
180	27	2.0522	1.7150	0.5067	0.0264	0.0007	
200	9	0.7555	0.6540	-11.7000	-0.5836	0.3530	

Table 5.8 (continued)

Logit Results between the Instance of a  
Call Transaction and the Absolute  
Normalised Intrinsic Value (140 Individual  
Series)

L57						
110	101	-0.3911	-0.1403	-7.1108	-1.0598	1.2787
120	101	-2.4861	-1.1104	-2.4706	-0.4478	0.2030
130	101	-0.0060	-0.0052	-7.4326	-1.9743	1 4.6872
140	101	0.4781	0.7980	-4.8908	-2.5442	2 6.9512
160	131	1.0669	3.4280	-5.9000	-4.2593	2 20.7282
180	91	2.2001	4.2691	-13.4980	-5.1824	2 39.6184
200	73	1.7876	2.8777	-13.6360	-3.5188	2 16.2661
220	50	0.8460	1.4066	-10.6910	-2.1660	1 5.3506
240	44	0.0055	0.0107	4.1898	0.6509	0.4328
260	39	1.6198	2.3784	-18.0010	-2.2211	1 5.7990
280	10	-2.2365	-1.5173	21.6840	0.7934	0.6641
300	9	-	-	-	-	-
L87						
140	183	-0.8252	-1.9208	-3.8515	-2.8859	2 9.0856
160	183	-0.5593	-1.8011	-3.6229	-3.2548	2 11.6972
180	150	1.2708	3.3261	-8.7286	-5.7810	2 46.8702
200	132	0.6242	1.3777	-9.3386	-4.1611	2 24.2013
220	109	-0.0888	-0.2054	-5.7729	-2.7483	2 8.7183
240	103	0.3415	0.9481	-7.3956	-3.3747	2 14.0612
260	98	1.1424	3.0205	-10.0480	-3.9937	2 20.4301
280	69	1.7436	3.3922	-13.0980	-3.7206	2 17.9866
300	68	1.4164	2.5499	-8.3155	-2.0792	1 4.5812
330	35	-3.3967	-2.1895	7.0299	0.4725	0.2275
360	26	2.0204	2.0967	-23.9840	-1.8405	4.2569
390	21	0.9489	1.3949	-10.2220	-1.6905	3.2713
L117						
180	182	-0.0098	-0.0170	-8.5636	-3.9322	2 18.7409
200	182	0.3904	0.7719	-11.0950	-4.2979	2 29.0374
220	169	-0.3836	-0.9531	-8.0625	-3.7401	2 18.2517
240	163	-0.3553	-1.1178	-5.9028	-3.4895	2 13.8665
260	159	0.1190	0.3773	-6.9639	-3.7866	2 16.1036
280	129	1.1536	2.7502	-12.1050	-4.7267	2 27.4243
300	128	1.3390	3.1105	-8.9147	-3.4084	2 14.2666
330	96	1.7068	4.0150	-11.5590	-3.3582	2 25.5082
360	86	2.3488	5.6537	-9.3212	-4.1498	2 38.2153
390	81	1.3946	3.9332	-6.0512	-3.5025	2 23.3586
L28						
200	68	-2.4530	-2.2456	-0.3281	-0.0810	0.0065
220	70	-2.4154	-2.4913	-4.3096	-0.7747	0.6086
240	180	-0.9406	-1.7024	-10.4780	-2.9675	2 11.0449
260	180	-0.2235	-0.5542	-10.2020	-3.8048	2 17.5854
280	170	-0.1278	-0.3611	-10.6920	-4.0400	2 20.4280
300	166	0.3792	1.2973	-8.2587	-4.0346	2 21.5886
330	157	1.2443	3.0679	-16.6390	-4.2070	2 46.5780
360	147	-0.1326	-0.5098	-4.5998	-3.4237	2 17.5783
390	142	1.0383	2.8641	-10.6690	-4.9528	2 55.5829

Table 5.8 (continued)

Logit Results between the Instance of a Call Transaction and the Absolute Normalised Intrinsic Value (140 Individual Series)

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L58							
200	128	-2.3645	-2.2936	-2.7374	-0.8386		0.6612
220	130	-2.5761	-2.4146	-7.9644	-1.3603		2.0952
240	133	-1.2523	-1.9009	-7.5092	-2.0395	1	4.6484 1
260	135	-4.6950	-3.1709	4.5299	0.7326		0.5265
280	135	-1.0235	-2.1931	-9.1095	-2.4193	1	7.5902 2
300	181	-0.3336	-1.2200	-4.3349	-2.5548	1	7.3967 2
330	180	0.4817	1.6849	-9.2879	-3.9901	2	31.0533 4
360	174	-0.2777	-1.0306	-5.9232	-3.4569	2	20.3954 4
390	171	0.7098	1.7088	-17.0650	-4.3247	2	52.2356 4
420	13	-	-	-	-		-
L88							
200	172	-4.6628	-1.9543	0.5300	0.0977		0.0097
220	174	-1.7169	-1.2714	-24.3370	-1.0432		5.0762 1
240	174	-1.8952	-1.3422	-11.8470	-1.4654		3.1794
260	174	-0.9068	-1.2882	-11.4630	-2.5382	1	10.2384 3
280	157	-3.9894	-3.5981	0.2329	0.0547		0.0030
300	157	-3.4418	-4.2394	-1.2037	-0.3012		0.0922
330	135	-1.7811	-3.5853	-4.7916	-1.4333		2.2204
360	87	-1.0575	-1.6208	-7.3556	-1.5767		2.5361
390	67	0.8351	1.3492	-21.7830	-3.4065	2	13.9323 4
420	67	-4.1344	-3.4609	51.8510	3.3667	2	19.2218 4
460	40	0.8114	1.4156	3.2397	0.0781		0.0061

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Table 5.10  
 Logit Results between the Instance of a  
 Call Transaction ( $d_t$ ) and Share Return (SR)

Series	Nobs.	$\alpha$	t-Ratio	$\beta$	t-Ratio	Likelihood Ratio Test
B6836	154	-2.6599	-8.1304	-0.7057	-0.1750	0.1267
B6839	160	-3.2718	-7.6085	-17.6420	-0.7880	0.7022
B6842	161	-4.5298	-5.4192	-61.2240	-2.0587	1 4.1756 1
B6846	161	-0.9667	-5.4556	-4.9125	-0.5236	0.9178
B6850	161	-1.1992	-6.3870	-6.6502	-0.6644	0.9723
B6855	179	-2.9347	-8.5606	-1.2385	-0.1139	0.1140
B6860	178	-5.1799	-5.1241	-3.4718	-0.1156	0.0229
G7716	182	-1.6062	-7.8260	14.9220	1.7154	6.3903 1
G7718	182	-0.9171	-5.4200	16.5070	2.1938	1 7.3906 2
G7720	164	0.5152	3.1813	-0.8312	-0.6519	1.7319
G7722	117	2.7485	7.0371	0.1892	0.1747	0.0309
G7724	96	1.1997	4.9317	0.6645	0.3506	0.4535
G7726	21	-1.8039	-2.7146	-21.3320	-0.7299	0.8738
L8820	171	-4.4368	-6.2143	0.0003	0.0002	0.0000
L8822	173	-5.1504	-5.1097	0.0706	0.0358	0.0010
L8824	173	-4.4420	-6.2378	0.3041	-0.1125	0.0116
L8826	173	-3.0263	-8.3288	-0.0059	-0.0066	0.0000
L8828	156	-3.9253	-6.7251	-0.2720	-0.1370	0.0175
L8830	156	-3.6181	-7.1324	-5.6497	-0.2369	0.1082
L8833	134	-2.4057	-7.6349	-0.4257	-0.2338	0.1196
L8836	86	-2.0159	-5.9790	-0.3219	-0.2997	0.1344
L8839	66	-1.2024	-4.0571	-14.0790	-0.7944	1.1666
L8842	66	-0.4606	-1.7983	-11.3240	-0.7474	1.5281
L8846	39	1.1694	3.0649	-1.1902	-0.3665	2.7663

$d_t$  and Absolute Share Return ASR)

B6836	154	-2.4836	-5.2194	-14.8730	-0.4773	0.3837
B6839	160	-2.7543	-4.5675	-45.1340	-0.9015	1.1998
B6842	161	-3.9558	-6.7618	-0.2281	-0.1244	0.0147
B6846	161	-0.7944	-3.1533	-13.5260	-0.9352	1.5845
B6850	161	-0.8817	-3.2576	-26.0480	-1.4957	3.1618
B6855	179	-1.9812	-3.7869	-107.1600	-1.7020	5.3583 1
B6860	178	-3.9030	-2.6817	-173.1800	-0.7312	1.0112
G7716	182	-2.0605	-6.6726	28.1790	2.4307	1 9.1081 3
G7718	182	-1.2116	-4.8598	20.5780	1.9723	1 6.2747 1
G7720	164	0.5265	3.2226	-0.7838	-0.7251	1.6729
G7722	117	2.7395	6.9799	0.4046	0.2292	0.0787
G7724	96	0.8768	2.3117	19.7260	1.0402	1.7040
G7726	21	-1.7263	-2.7409	-0.6218	-0.2753	0.2651
L8820	171	-4.4374	-6.1913	0.0126	0.0075	0.0001
L8822	173	-5.1473	-5.0940	-0.0042	-0.0017	0.0000
L8824	173	-4.4458	-6.2067	-0.0592	-0.0296	0.0009
L8826	173	-3.0219	-8.2898	-0.1056	-0.0968	0.0098
L8828	156	-3.9125	-6.5189	-0.7714	-0.0945	0.0368
L8830	156	-3.6277	-7.1249	-0.2654	-0.1547	0.0225
L8833	134	-2.4055	-7.6054	-0.2008	-0.2264	0.0513
L8836	86	-2.0141	-5.9552	-0.2642	-0.2714	0.1068
L8839	66	-1.1944	-4.0189	-0.7316	-0.2789	0.4819
L8842	66	0.1114	0.2674	-44.9820	-1.6984	4.0711 1
L8846	39	1.3869	2.2918	-19.6260	-0.4802	3.0274

Table 5.10 (continued)  
 Logit Results between the Instance of a  
 Call Transaction ( $d_t$ ) and Call Return (CR)

B6836	154	-2.6620	-8.1336	-0.3044	-0.2023	0.0576
B6839	160	-3.2750	-7.6133	-2.8241	-0.6672	0.5155
B6842	161	-4.9865	-4.8420	-14.6100	-2.2579	1 5.8602 1
B6846	161	-0.9891	-5.5107	-1.2976	-1.2360	2.1057
B6850	161	-1.2237	-6.4476	-0.6905	-0.9454	1.0847
B6855	179	-2.9419	-8.5788	0.2223	0.3607	0.1021
B6860	178	-6.1707	-3.7284	-5.2765	-1.5866	2.2483
G7716	182	-1.5593	-7.8932	2.0889	0.9985	4.0512 1
G7718	182	-0.8962	-5.3697	3.1864	1.9286	6.1915 1
G7720	164	0.4961	3.0678	0.0809	0.1171	0.0139
G7722	117	2.7956	6.9754	-0.7620	-0.7455	0.4472
G7724	96	1.2176	4.9966	0.2082	0.4369	0.1932
G7726	21	-1.9908	-2.7713	-0.9745	-0.8872	0.7848
L8820	171	-4.4541	-6.1850	0.3113	0.2559	0.0436
L8822	173	-5.1774	-5.0683	0.4510	0.3008	0.0531
L8824	173	-4.4347	-6.2240	-2.7739	-0.2710	0.1004
L8826	173	-	-	-	-	-
L8828	156	-3.9251	-6.7146	-0.2871	-0.1144	0.0130
L8830	156	-3.6313	-7.1490	-2.1360	-0.4023	0.2073
L8833	134	-2.4065	-7.6323	-0.3481	-0.2393	0.0783
L8836	86	-2.0351	-5.9673	0.1188	0.1396	0.0171
L8839	66	-1.2161	-4.0730	-2.9396	-0.9868	1.5159
L8842	66	-0.4903	-1.8828	-2.9050	-1.3861	3.0276
L8846	39	1.0924	2.8028	-2.9585	-1.1838	4.3119 1

$d_t$  and Absolute Call Return (ACR)

B6836	154	-2.1515	-4.6442	-12.5940	-1.2478	2.2312
B6839	160	-3.2263	-7.3876	-0.2117	-0.1352	0.0186
B6842	161	-3.9935	-6.6052	0.2539	0.2547	0.0155
B6846	161	-1.0265	-5.2766	0.3316	0.6133	0.3683
B6850	161	-1.1148	-4.9048	-0.6107	-0.6823	0.6054
B6855	179	-2.9315	-7.8442	-0.0468	-0.0461	0.0022
B6860	178	-	-	-	-	-
G7716	182	-2.0531	-7.2117	8.5433	2.8290	2 11.4241 4
G7718	182	-1.3063	-5.3194	5.9820	2.5981	2 9.4301 3
G7720	164	0.5696	3.0509	-0.6184	-0.7534	0.6328
G7722	117	2.7751	6.0335	-0.1382	-0.0861	0.0069
G7724	96	0.9084	2.8235	1.5974	1.2074	2.5139
G7726	21	-1.8652	-2.3693	0.2140	0.1602	0.0251
L8820	171	-4.4652	-6.1299	0.3123	0.2532	0.0425
L8822	173	-5.1795	-5.0255	0.3441	0.1886	0.0226
L8824	173	-4.4288	-5.9614	-0.3097	-0.0865	0.0074
L8826	173	-3.8580	-8.2096	0.5637	0.8235	0.4754
L8828	156	-3.7609	-4.6638	-3.2333	-0.2721	0.1199
L8830	156	-3.6324	-6.8844	-0.0632	-0.0343	0.0012
L8833	134	-2.4573	-7.5194	0.3528	0.5996	0.2829
L8836	86	-2.0808	-5.9026	0.4226	0.5972	0.2969
L8839	66	-1.1714	-3.7403	-0.4812	-0.4005	0.2761
L8842	66	-0.3445	-1.0321	-1.3184	-0.5856	1.1056
L8846	39	1.3306	3.0806	-1.2404	-0.9648	2.5353

Table 5.10 (continued)  
 Logit Results between the Instance of a  
 Call Transaction ( $d_t$ ) and Share Price (S)

B6836	154	9.8383	1.3857	-0.0276	-1.7467		2.7885
B6839	160	8.7295	1.3023	-0.0267	-1.7600		2.6923
B6842	161	-7.5281	-0.5346	0.0078	0.2541		0.0694
B6846	161	-14.7280	-2.8106	0.0300	2.6407	2	8.8334 3
B6850	161	-28.8970	-3.7860	0.0601	-3.6584	2	20.7251 4
B6855	179	-0.8103	-0.1774	-0.0046	-0.4646		0.2303
B6860	178	-0.3790	-0.0259	-0.0105	-0.3245		0.1156
G7716	182	1.2589	0.7670	-0.0135	-1.6767		2.8689
G7718	182	0.9734	0.7038	-0.0088	-1.3115		1.7342
G7720	164	-6.9043	-4.1105	0.0355	4.3980	2	22.2871 4
G7722	117	-6.4314	-1.1027	0.0423	1.5484		2.6274
G7724	96	-12.9770	-3.4029	0.0648	3.6671	2	16.1124 4
G7726	21	-11.0230	-0.5492	0.0387	0.4615		0.2080
L8820	171	-4.3125	-1.2540	-0.0003	-0.0368		0.0014
L8822	173	6.2223	0.8833	-0.0413	-1.3610		3.8299
L8824	173	1.2878	0.3558	-0.0185	-1.4293		2.7217
L8826	173	2.4138	1.2659	-0.0172	-2.6228	2	9.2466 3
L8828	156	-3.4077	-1.0577	-0.0014	-0.1640		0.0270
L8830	156	-2.8923	-1.0313	-0.0020	-2.6699	2	0.0723
L8833	134	2.7606	1.2417	-0.0144	-2.2284	1	6.3484 1
L8836	86	1.4395	0.4714	-0.0084	-1.1256		1.2376
L8839	66	20.1080	3.3732	-0.0493	-3.5288	2	15.3945 4
L8842	66	-30.7260	-3.3217	0.0677	3.3144	2	21.4639 4
L8846	39	-15.9370	-0.5634	0.0371	0.6006		0.3686

$d_t$  and Intrinsic Value (S-X)

B6836	154	-0.0815	-0.0562	-0.0276	-1.7467		2.7885
B6839	160	-1.6696	-1.8810	-0.0267	-1.7600		2.6923
B6842	161	-4.2513	-3.2407	0.0078	0.2541		0.0694
B6846	161	-0.9253	-5.0927	0.0300	2.6407	2	8.8334 3
B6850	161	1.1351	1.8529	0.0601	3.6584	2	20.7251 4
B6855	179	-3.3462	-3.4629	-0.0046	-0.4646		0.2303
B6860	178	-6.6697	-1.3696	-0.0105	-0.3252		0.1156
G7716	182	-0.9073	-2.3292	-0.0135	-1.6767		2.8689
G7718	182	-0.6097	-0.6232	-0.0088	-1.3115		1.7342
G7720	164	0.1965	1.0808	0.0355	4.3980	2	22.2871 4
G7722	117	2.8743	6.5595	0.0423	1.5484		2.6274
G7724	96	2.5737	5.0316	0.0648	3.6671	2	16.1124 4
G7726	21	-0.9501	-0.5076	0.0387	0.4615		0.2080
L8820	171	-4.3823	-2.6848	-0.0003	-0.0368		0.0014
L8822	173	-2.8721	-2.6861	-0.0413	-1.3610		3.8299
L8824	173	-3.1549	-3.7455	-0.0185	-1.4293		2.7217
L8826	173	-2.0657	-5.0614	-0.0172	-2.6228	2	9.2466 3
L8828	156	-3.8092	-4.1104	-0.0014	-0.1640		0.0270
L8830	156	-3.5061	-5.1256	-0.0020	-0.2670		0.0723
L8833	134	-1.9772	-5.9831	-0.0144	-2.2284	1	6.3484 1
L8836	86	-1.5710	-3.1769	-0.0084	-1.1256		1.2376
L8839	66	0.8936	1.4682	-0.0493	-3.5288	2	15.3945 4
L8842	66	-2.3038	-3.1426	0.0677	3.3144	2	21.4639 4
L8846	39	1.1514	2.8531	0.0371	0.6006		0.3686

Table 5.10 (continued)  
 Logit Results between the Instance of a  
 Call Transaction ( $d_t$ ) and Call Premium (C)

B6836	154	-1.4310	-0.4401	-0.0111	-0.3800		0.1443
B6839	160	-6.1218	-1.8754	0.0326	0.9090		0.8343
B6842	161	-2.8372	-0.9967	-0.0182	-0.3952		0.1580
B6846	161	1.2384	2.4913	-0.0626	-4.4807	2	24.2453 4
B6850	161	-0.3311	-1.0178	-0.0404	-2.9181	2	10.7945 4
B6855	179	-3.0004	-6.4542	0.0032	0.2036		0.0395
B6860	178	-3.7540	-2.6840	-0.5391	-0.5687		1.7560
G7716	182	-0.5250	-0.9328	-0.0183	-1.7827		3.2866
G7718	182	-0.2626	-0.6420	-0.0151	-1.4974		2.2840
G7720	164	-0.7226	-1.8742	0.0493	3.3871	2	12.4139 4
G7722	117	0.8267	0.9317	0.1318	2.0184	1	4.8991 1
G7724	96	0.1393	0.3107	0.1494	2.5347	1	7.6381 2
G7726	21	-2.4361	-2.2701	0.5207	0.8431		0.6831
L8820	171	-4.3951	-2.3524	-0.0003	-0.0240		0.0006
L8822	173	-0.0359	-0.0137	-0.0634	-1.3551		3.7578
L8824	173	-1.2238	-0.6823	-0.0384	-1.4203		3.6309
L8826	173	-0.5522	-0.6167	-0.0312	-2.3043	1	9.9710 3
L8828	156	-3.3796	-22.7246	-0.0057	-0.4692		0.2363
L8830	156	-3.0982	-3.1065	-0.0066	-0.5769		0.3609
L8833	134	-1.0488	-1.7918	-0.0232	-2.1937	1	6.7334 2
L8836	86	-0.9468	-1.1913	-0.0156	-1.3910		1.9516
L8839	66	3.9455	2.9143	-0.0905	-3.6312	2	17.7834 4
L8842	66	-3.7894	-3.0519	0.0795	2.7907	2	9.5433 3
L8846	39	4.9420	2.6294	-0.1794	-2.2162	1	7.1681 2

$d_t$  and Put Premium (P)

B6836	154	-3.3520	-6.7618	0.0789	2.4726	1	5.5072 1
B6839	160	-4.1727	-6.1916	0.0500	2.5304	1	5.6216 1
B6842	161	-3.7028	-4.6826	-0.0157	-0.4258		0.2080
B6846	161	0.1591	0.4900	-0.0401	-3.6507	2	18.2485 4
B6850	161	1.4890	2.3577	-0.0541	-4.0187	2	25.8604 4
B6855	179	-3.1434	-2.9705	0.0022	0.2068		0.0422
B6860	178	-6.1173	-1.4211	0.0066	0.2311		0.0530
G7716	182	-2.0492	-7.0753	0.2020	2.9503	2	8.5320 3
G7718	182	-1.1444	-5.0972	0.0500	2.1086	1	4.3976 1
G7720	164	1.5162	5.6240	-0.0941	-4.8608	2	31.1670 4
G7722	117	3.6713	4.3919	-0.0678	-1.4313		2.1068
G7724	96	2.9970	4.7055	-0.0691	-3.4204	2	13.5826 4
G7726	21	0.0059	0.0026	-0.0825	-0.7745		0.6123
L8820	171	-4.4354	-5.2297	-0.0002	-0.0029		0.0000
L8822	173	-6.6914	-3.6513	0.0774	1.8652		2.9125
L8824	173	-5.2626	-4.5373	0.0442	1.2891		1.4941
L8826	173	-3.9819	-6.3938	0.0388	2.6602	2	6.7979 2
L8828	156	-3.5256	-5.2227	-0.0399	-0.7782		0.8658
L8830	156	-3.5566	-5.3289	-0.0042	-0.1775		0.0321
L8833	134	-3.2054	-5.7615	0.0286	2.1621	1	4.7739 1
L8836	86	-2.2466	-4.9204	0.0174	0.7892		0.5877
L8839	66	-2.6645	-4.3918	0.1274	3.3528	2	13.2668 4
L8842	66	1.2900	2.6410	-0.1282	-3.6105	2	24.2960 4
L8846	39	3.8444	2.6758	-0.1198	-2.1604	1	5.9336 1

Table 5.10 (continued)  
 Logit Results between the Instance of a  
 Call Transaction ( $d_t$ ) and Put Option  
 Trading Volume (PNCT)

B6836	154	-2.6569	-8.1188	-0.0866	-0.3452	0.1574
B6839	160	-3.2538	-7.7490	0.0050	0.2255	0.0260
B6842	161	-3.8787	-6.6471	-0.2385	-0.3201	0.4890
B6846	161	-1.1194	-5.9629	0.0160	2.4259	1 9.6037 3
B6850	161	-1.2181	-6.4444	0.0303	0.3990	0.1403
B6855	179	-2.9288	-8.5559	-0.0191	-0.3193	0.1285
B6860	178	-	-	-	-	-
G7716	182	-1.4939	-7.7555	-4.7696	-0.0001	0.8051
G7718	182	-0.8263	-5.0742	-0.0540	-0.4450	0.2116
G7720	164	0.4006	2.3206	0.0030	1.3767	2.4236
G7722	117	2.5084	6.3820	16.7770	0.0025	3.3273
G7724	96	0.8857	3.4079	0.0500	1.2191	9.7668 3
G7726	21	-	-	-	-	-
L8820	171	-4.4188	-6.2119	-22.6520	0.0000	0.0712
L8822	173	-5.1328	-5.1118	-0.1452	-0.1337	0.0208
L8824	173	-4.4417	-6.2440	-0.1163	-0.1402	0.0165
L8826	173	-3.0082	-8.3057	-24.1890	0.0000	0.2866
L8828	156	-3.9253	-6.7326	-6.5093	0.0000	0.0390
L8830	156	-3.6178	-7.1395	-0.2635	-0.2568	0.1453
L8833	134	-2.3728	-7.5266	-20.2350	-0.0004	0.8737
L8836	86	-2.0139	-5.9812	-0.0345	-0.3998	0.2318
L8839	66	-1.1662	-3.8982	-0.0455	-0.6307	0.7670
L8842	66	-0.6162	2.3201	0.0624	1.1331	4.0432 1
L8846	39	0.9555	2.5679	6.5772	0.0010	1.8623

$d_t$  and trading history of a call option (th)

B6836	154	-2.0389	-2.1018	-6.4360	-0.6653	0.4490
B6839	160	-3.5531	-5.9316	3.5525	0.8037	0.5094
B6842	161	-2.6381	-3.0704	-131.8900	-1.4456	2.5919
B6846	161	-2.8951	-5.1295	12.5880	3.7969	2 16.1335 4
B6850	161	-2.0802	-5.9475	7.1084	3.4186	2 12.6242 4
B6855	179	2.2008	0.7781	-80.2380	-1.7354	4.9390 1
B6860	178	-4.8122	-4.7927	-4301.3000	-0.0001	0.7255
G7716	182	-1.9700	-4.3613	1.8667	1.1621	1.2645
G7718	182	-1.4266	-2.5179	1.5668	1.0948	1.1849
G7720	164	-1.2923	-2.7129	4.1743	3.9851	2 17.6613 4
G7722	117	12.1980	0.9102	-10.3140	-0.7098	0.9288
G7724	96	2.2875	0.9473	-1.5162	-0.4484	0.1982
G7726	21	-1.4113	-0.9251	-1.1391	-0.2648	0.7968
L8820	171	-2.2125	-2.4149	-238.7500	-1.9174	4.8017 1
L8822	173	0.0000	0.0000	-4161.5000	-0.0007	9.5282 3
L8824	173	0.0787	0.0571	-325.3100	-2.7241	8.4748 3
L8826	173	-3.6500	-4.5420	6.4795	0.9330	0.7787
L8828	156	-3.7672	-4.6270	-12.6150	-0.2689	0.0725
L8830	156	-2.6768	-3.2774	-48.4510	-1.2310	1.4576
L8833	134	-2.6430	-3.0732	1.9649	0.2902	0.0855
L8836	86	-2.1969	-2.7642	1.1740	0.2376	0.0574
L8839	66	-2.0320	-1.9259	2.3521	0.8157	0.7104
L8842	66	-1.7959	-3.8356	8.3735	3.7948	2 17.4688 4
L8846	39	-0.9295	-1.1135	4.6183	2.5821	1 7.9803 3

Table 5.12

## Classification of Trading Frequencies

Columns (3), (4), (5), (6), and (7) represent total trading instances, total trading days, average trading frequency (percentage) in the class, average trading volume (number of contracts) in the class, and class mid-point in terms of  $(S-X)/S$

Series	Class	(3)	(4)	(5)	(6)	(7)
B37	1	23	135	17.04	7.99	-0.1184
	2	39	106	36.79	23.95	-0.0306
	3	43	84	51.19	68.63	0.0158
	4	22	87	25.29	22.83	0.0547
	5	24	102	23.53	35.46	0.0937
	6	18	110	16.36	12.82	0.1400
	7	11	134	8.21	5.16	0.2335
B97	1	59	179	32.96	33.09	-0.0988
	2	71	133	53.38	34.72	-0.0051
	3	31	91	34.07	28.65	0.0496
	4	9	115	7.83	3.01	0.0954
	5	10	111	9.01	4.53	0.1413
	6	6	152	3.95	0.24	0.1959
	7	6	220	2.73	0.72	0.2963
B38	1	7	168	4.17	0.99	-0.2925
	2	7	91	7.69	2.49	-0.1645
	3	23	123	18.70	19.63	-0.0919
	4	28	107	26.17	13.39	-0.0309
	5	26	130	20.00	16.26	0.0302
	6	20	174	11.49	3.66	0.1027
	7	3	146	2.05	0.21	0.2258
B68	1	4	225	1.78	0.95	-0.3086
	2	7	143	4.90	2.44	-0.1727
	3	27	130	20.77	41.95	-0.0951
	4	23	109	21.10	15.14	-0.0298
	5	33	130	25.38	39.02	0.0355
	6	7	216	3.24	0.38	0.1131
	7	9	207	4.35	1.24	0.2454

Table 5.12 (continued)

## Classification of Trading Frequencies

B127	1	15	112	13.39	5.82	-0.2607
	2	16	91	17.58	19.14	-0.1432
	3	40	128	31.25	24.3	-0.0722
	4	35	115	30.43	27.31	-0.0125
	5	38	121	31.40	16.17	0.0472
	6	11	165	6.67	4.56	0.1182
	7	5	94	5.32	0.94	0.2272
G17	1	29	128	22.66	10.66	-0.2480
	2	31	82	37.80	19.68	-0.1492
	3	39	71	54.93	89.35	-0.0974
	4	44	73	60.27	71.66	-0.0539
	5	28	56	50.00	66.73	-0.0104
	6	48	115	41.74	53.63	0.0415
	7	39	100	39.00	39.89	0.1323
G47	1	44	94	46.81	47.46	-0.1769
	2	66	109	60.55	102.21	-0.0831
	3	52	71	73.24	217.77	-0.0263
	4	50	84	59.52	159.08	0.0215
	5	52	89	58.43	90.12	0.0692
	6	32	61	52.46	75.21	0.1260
	7	41	96	42.71	52.46	0.2137
G77	1	63	135	46.67	58.33	-0.1340
	2	84	110	76.36	213.13	-0.0329
	3	77	99	77.78	213.09	0.0239
	4	53	86	61.63	113.63	0.0715
	5	36	85	42.35	91.42	0.1192
	6	39	105	37.14	41.51	0.1760
	7	28	148	18.92	23.06	0.2795
G18	1	9	134	6.72	3.53	-0.4491
	2	14	78	17.95	10.06	-0.2884
	3	37	100	37.00	48.72	-0.1704
	4	88	136	64.71	86.35	-0.0712
	5	114	188	60.64	50.59	0.0281
	6	36	180	20.00	10.88	0.1461
	7	4	62	6.45	2.08	0.2984

Table 5.12 (continued)

## Classification of Trading Frequencies

G48	1	2	183	1.09	0.08	-0.6216
	2	2	123	1.63	1.71	-0.4545
	3	4	87	4.60	7.71	-0.3408
	4	15	120	12.50	15.47	-0.2452
	5	35	153	22.88	18.87	-0.1496
	6	69	208	33.17	32.77	-0.0359
	7	47	176	26.70	30.42	0.1449
G78	1	2	177	1.13	3.81	-0.4455
	2	7	128	5.47	4.09	-0.3108
	3	6	97	6.19	6.35	-0.2302
	4	14	100	14.00	20.46	-0.1624
	5	23	99	23.23	26.42	-0.0947
	6	50	130	38.46	119.79	-0.0141
	7	26	178	14.61	13.84	0.1182
L27	1	67	179	37.43	19.66	-0.1569
	2	81	142	57.04	54.53	-0.0122
	3	49	113	43.36	55.94	0.0607
	4	44	92	47.83	47.91	0.1219
	5	40	112	35.71	24.68	0.1831
	6	36	142	25.35	7.44	0.2560
	7	42	228	18.42	7.83	0.4106
L57	1	79	143	55.24	51.68	-0.0683
	2	68	102	66.67	45.96	0.0624
	3	38	82	46.34	31.10	0.1468
	4	25	92	27.17	6.05	0.2178
	5	23	116	19.83	7.77	0.2888
	6	15	119	12.61	2.81	0.3733
	7	4	197	2.03	0.71	0.5305
L87	1	74	156	47.44	18.31	-0.1013
	2	107	207	51.69	22.25	0.0335
	3	54	165	32.73	7.64	0.1251
	4	51	150	34.00	8.52	0.2021
	5	29	135	21.48	3.53	0.2791
	6	16	154	10.39	1.38	0.3707
	7	7	210	3.33	0.64	0.5127

Table 5.12 (continued)

## Classification of Trading Frequencies

L117	1	41	139	29.50	9.00	-0.2094
	2	182	289	62.98	27.16	-0.0289
	3	106	258	41.09	14.24	0.1077
	4	56	267	20.97	5.16	0.2225
	5	14	243	5.76	0.95	0.3373
	6	4	179	2.23	1.27	0.4738
	7	0	0	0.00	0.00	0.5783
L28	1	19	227	8.37	1.23	-0.3554
	2	27	112	24.11	4.17	-0.1509
	3	84	164	51.22	19.14	-0.0480
	4	69	186	37.10	17.90	0.0386
	5	42	196	21.43	12.90	0.1251
	6	20	251	7.97	1.42	0.2280
	7	4	144	2.78	0.88	0.4045
L58	1	13	291	4.47	0.75	-0.3421
	2	33	137	24.09	12.08	-0.1213
	3	47	145	32.41	19.21	-0.0332
	4	51	159	32.08	22.74	0.0408
	5	36	170	21.18	13.83	0.1148
	6	22	199	11.06	5.12	0.2029
	7	11	279	3.94	1.26	0.4050
L88	1	42	221	19.00	34.15	-0.0608
	2	40	172	23.26	33.83	0.0594
	3	13	174	7.47	6.24	0.1348
	4	9	149	6.04	2.21	0.1982
	5	3	165	1.82	1.44	0.2616
	6	2	197	1.02	2.30	0.3369
	7	1	326	0.31	0.31	0.4840

Table 5.13

## Data Used in the Stepwise Regressions

BCHM			GEC			LSMR		
B37 <sup>1</sup>	6 <sup>2</sup>	758 <sup>3</sup>	G17	4	625	L27	5	597
B97	6	1001	G47	5	604	L57	5	416
B127	7	826	G77	5	756	L87	5	683
B38	7	939	G18	7	878	L117	5	704
B68	7	1161	G48	7	1050	L28	5	726
			G78	5	909	L58	5	800
						L88	5	593
Total	33	4685		33	4822		35	4519

## Note:

1. B37 denotes the call option series of the Beecham Group with a maturity date in March 1987. The other acronyms have the similar meaning.
2. There are six call option series of different exercise prices but have the same maturity in March 1987.
3. This is the total number of prices (or trading volumes) for call options of this maturity date.
4. The total number of call option series is 140.

Table 5.16

## Different Classification of the Intrinsic Values

B97 (Beecham Group - September 1987 series)

Total observations = 1001.

N	1	2	3	4	5
3	151	353	42.78	35.40	24.70
	28	276	10.14	5.08	86.35
	13	372	3.49	0.79	
5	83	233	35.62	33.41	-4.71
	78	169	46.15	31.75	37.39
	14	174	8.05	2.94	73.65
	11	186	5.91	2.01	115.75
	6	239	2.51	0.66	
7	59	174	33.91	34.05	-20.88
	76	144	52.78	33.83	15.02
	26	106	24.53	22.20	42.64
	13	123	10.57	4.03	68.40
	8	122	6.56	3.05	96.02
	5	148	3.38	0.45	131.92
	5	184	2.72	0.59	
9	46	141	32.62	26.40	-31.83
	46	112	41.07	42.89	0.79
	59	100	59.00	39.69	24.70
	11	86	12.79	8.17	45.52
	8	92	8.70	3.22	65.52
	9	98	9.18	4.11	86.35
	5	107	4.67	1.14	110.25
	4	137	2.92	0.50	142.88
	4	128	3.13	0.80	

Columns N=number of classes, column 1=total trading instances, column 2=total trading days, column 3=trading frequency within the class, column 4=average trading volumes in terms of contracts, and column 5=the class limits along the intrinsic values respectively.

Table 5.17  
Different Classification of the Intrinsic Values

G48 (GEC April 1988 series)  
Total observations =1051.

N	1	2	3	4	5
3	8	355	2.25	2.52	-53.65
	52	301	17.28	15.86	-20.31
	115	395	29.11	30.75	
5	4	250	1.60	0.90	-69.54
	6	166	3.61	4.41	-46.78
	23	156	14.74	20.50	-27.18
	64	221	28.96	31.91	-4.41
	78	258	30.23	25.61	
7	1	200	0.50	0.02	-78.29
	4	124	3.23	7.02	-58.88
	6	121	4.96	0.77	-43.94
	16	98	16.33	28.60	-30.01
	39	146	26.71	16.29	-15.08
	60	182	32.97	34.57	4.33
	49	180	27.22	29.86	
9	1	159	0.63	0.03	-84.21
	3	109	2.75	2.02	-66.57
	4	87	4.60	7.71	-53.65
	6	103	5.83	4.30	-42.38
	12	77	15.58	31.36	-31.57
	34	121	28.10	15.83	-20.31
	22	99	22.22	47.74	-7.39
	58	162	35.80	21.24	10.25
	35	134	26.12	29.69	

Columns N=number of classes, column 1=total trading instances, column 2=total trading days, column 3=trading frequency within the class, column 4=average trading volumes in terms of contracts, and column 5=the class limits along the intrinsic values respectively.

Figure 5.1

Scatter Plot of Call Option Trading Volume against Share Price

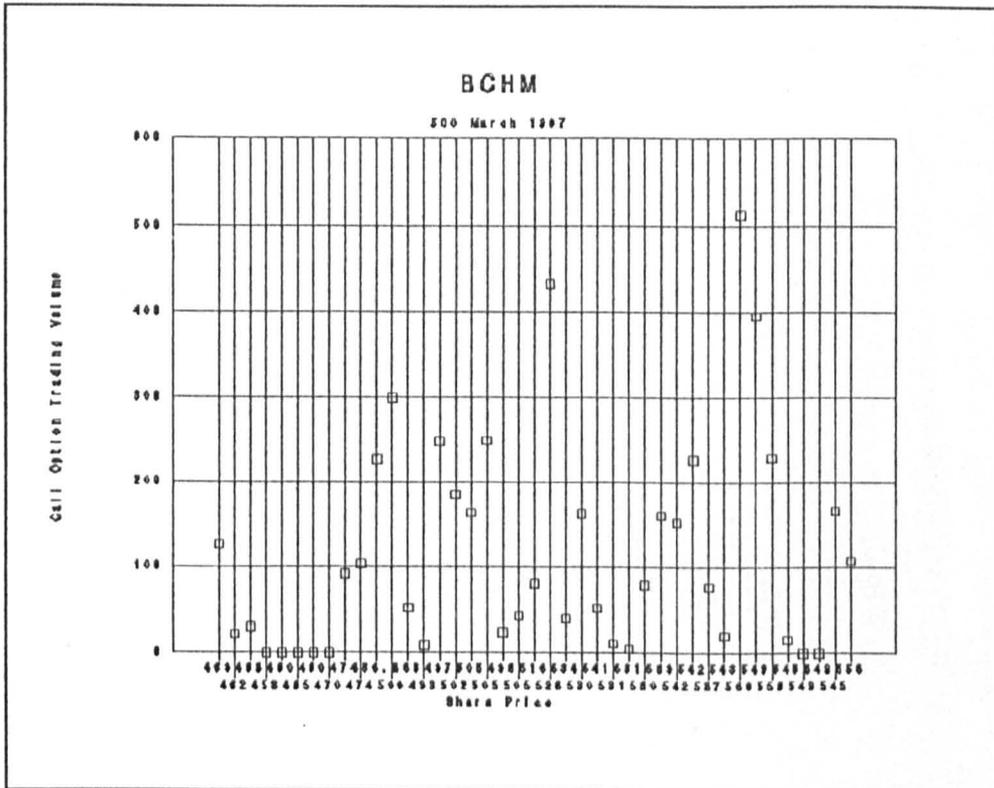


Figure 5.2

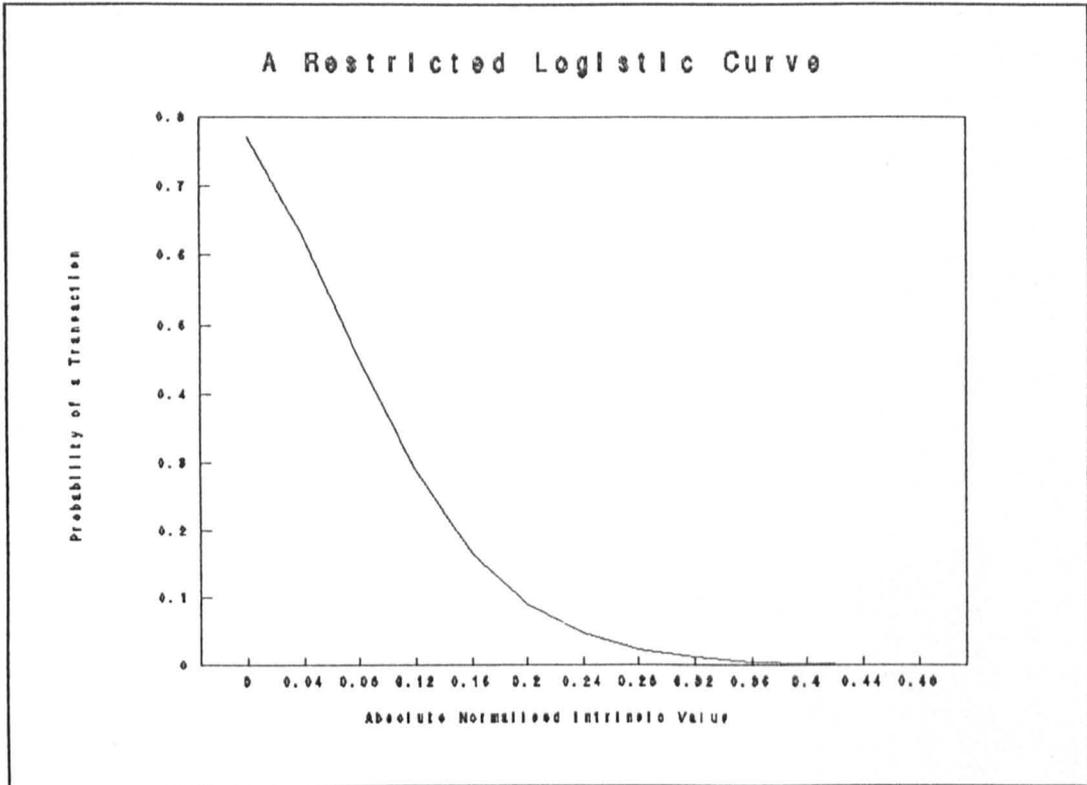


Figure 5.3

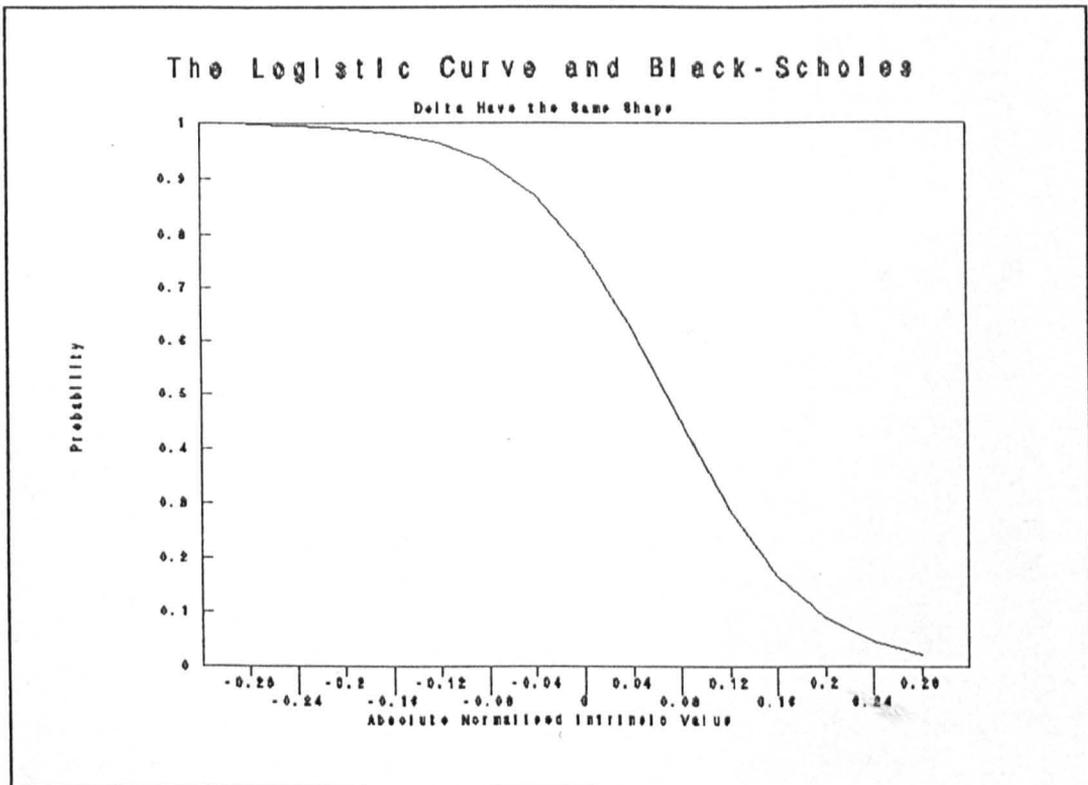


Figure 5.4

The highest average trading frequency (ATF) and the highest average trading volume (ATV) are contained in a 15% interval of the absolute normalised intrinsic values.

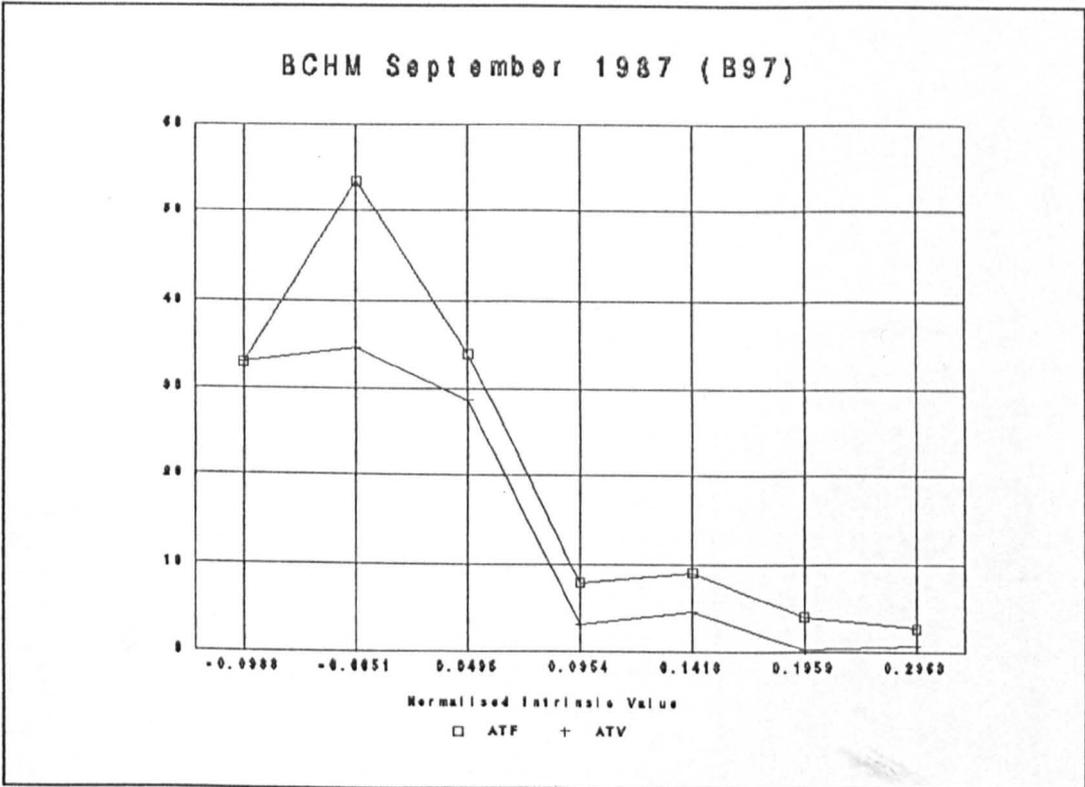
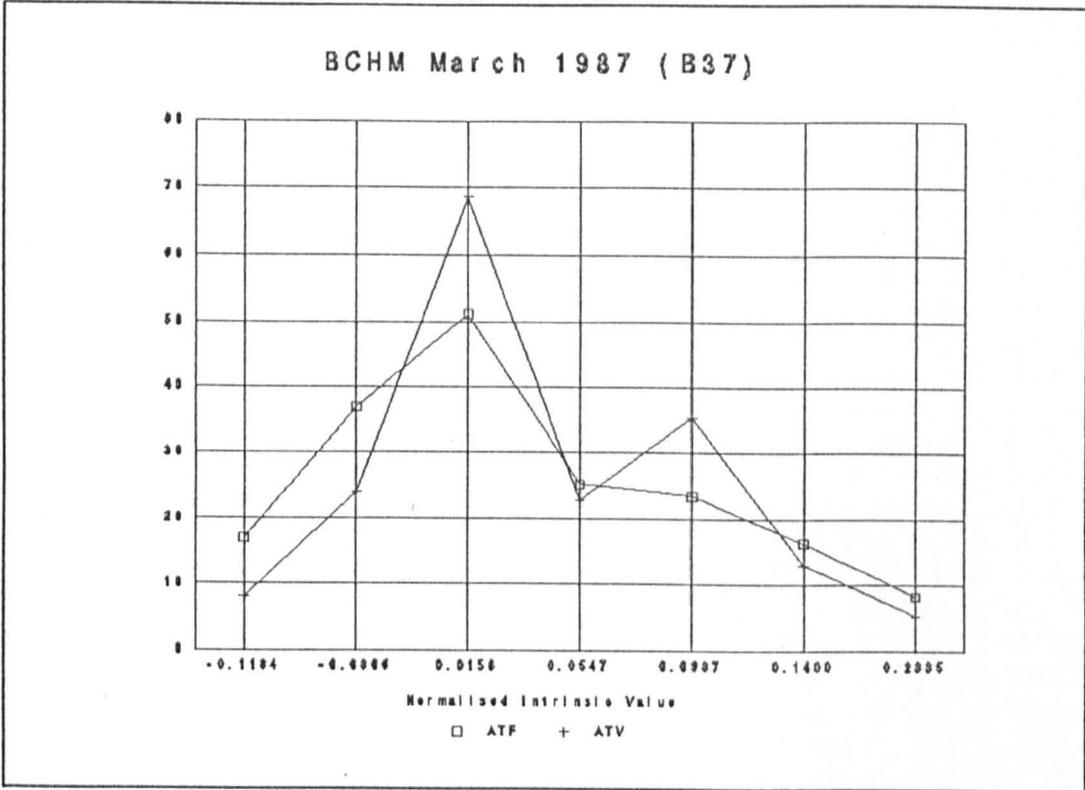


Figure 5.4 (continued)

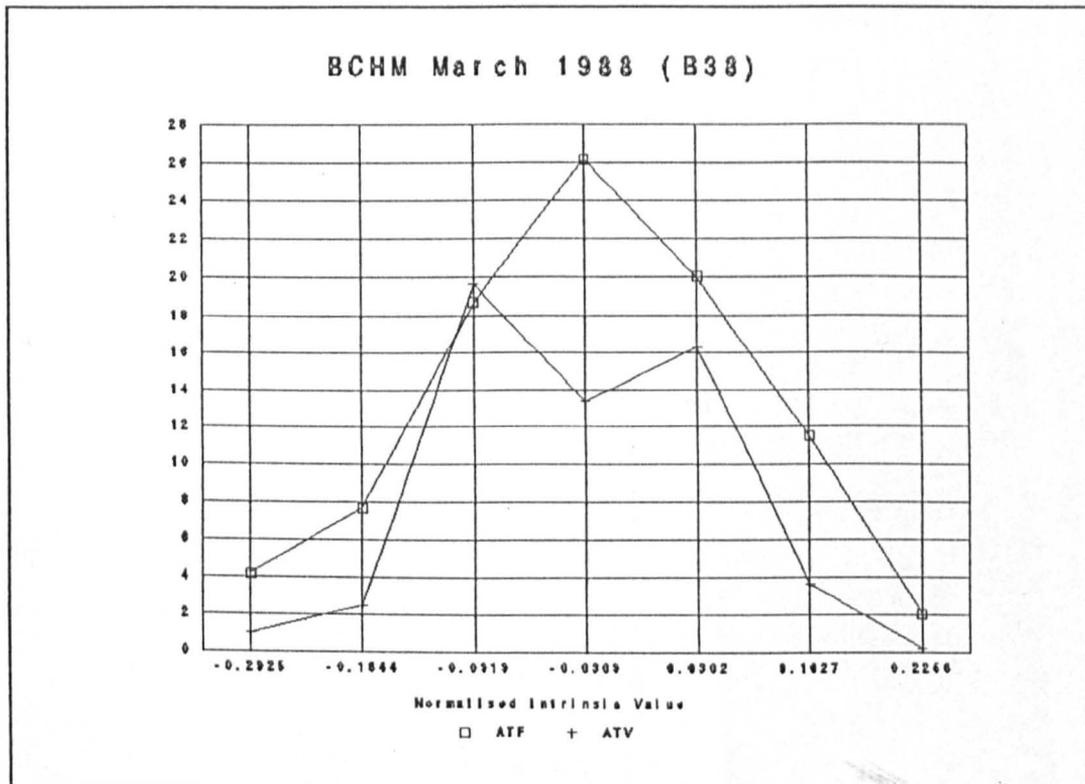
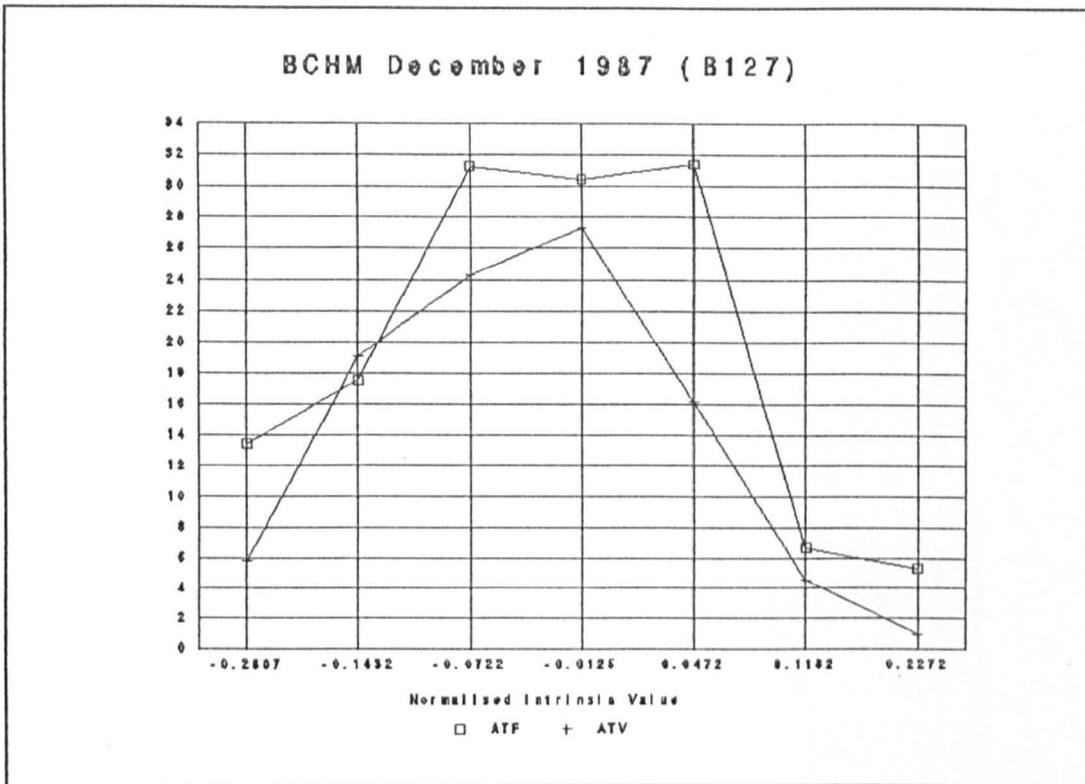


Figure 5.4 (continued)

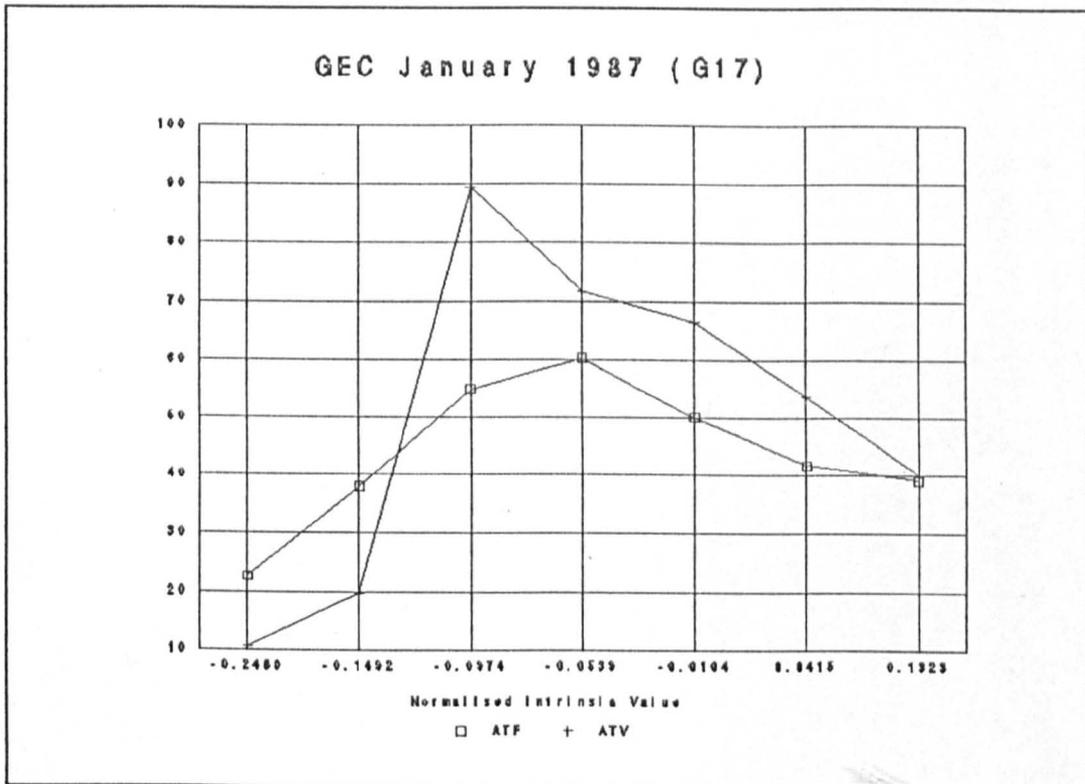
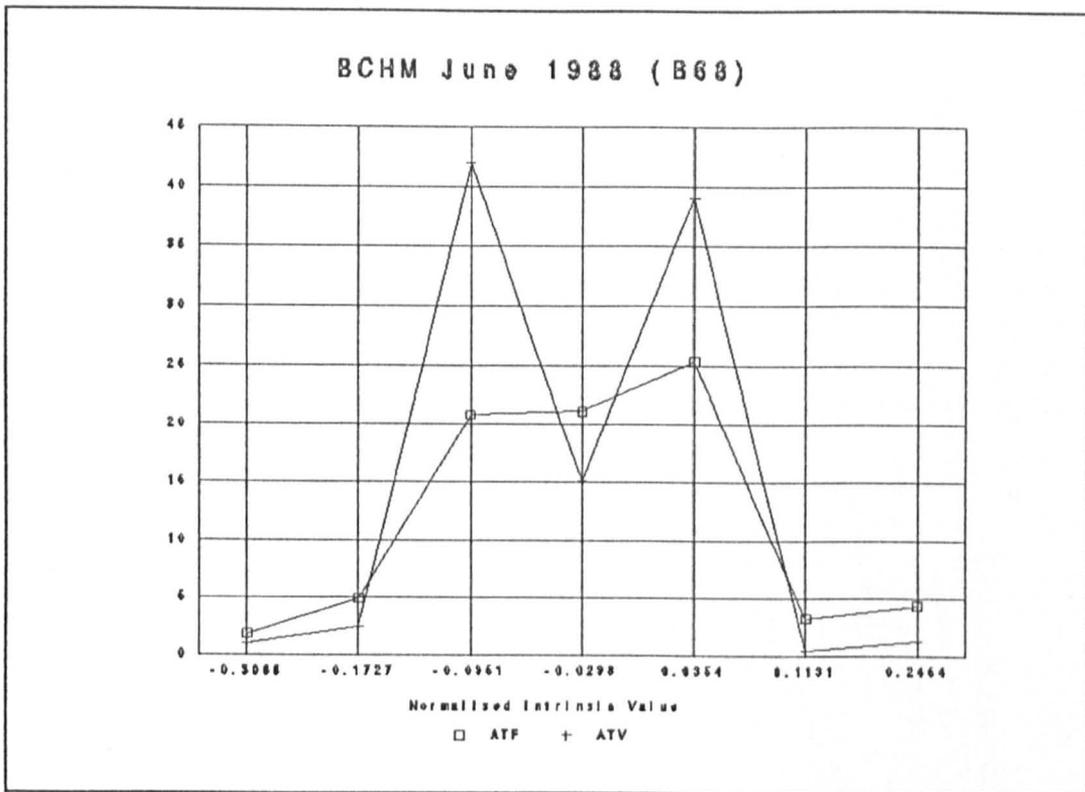


Figure 5.4 (continued)

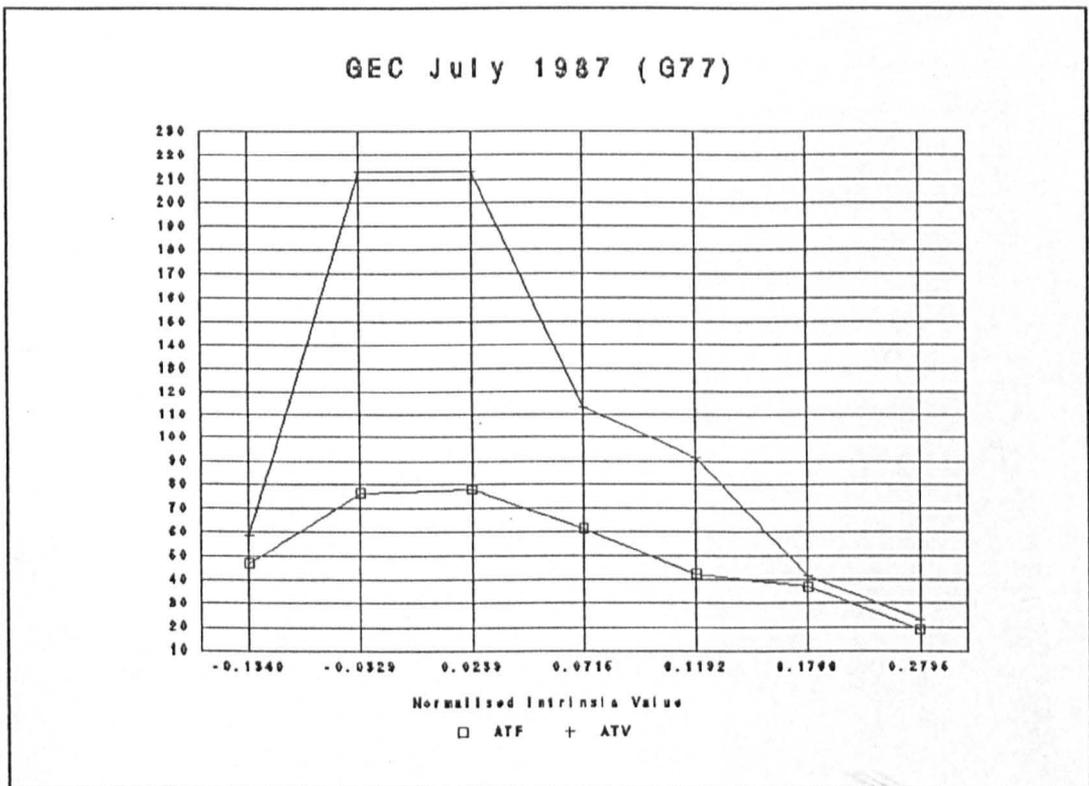
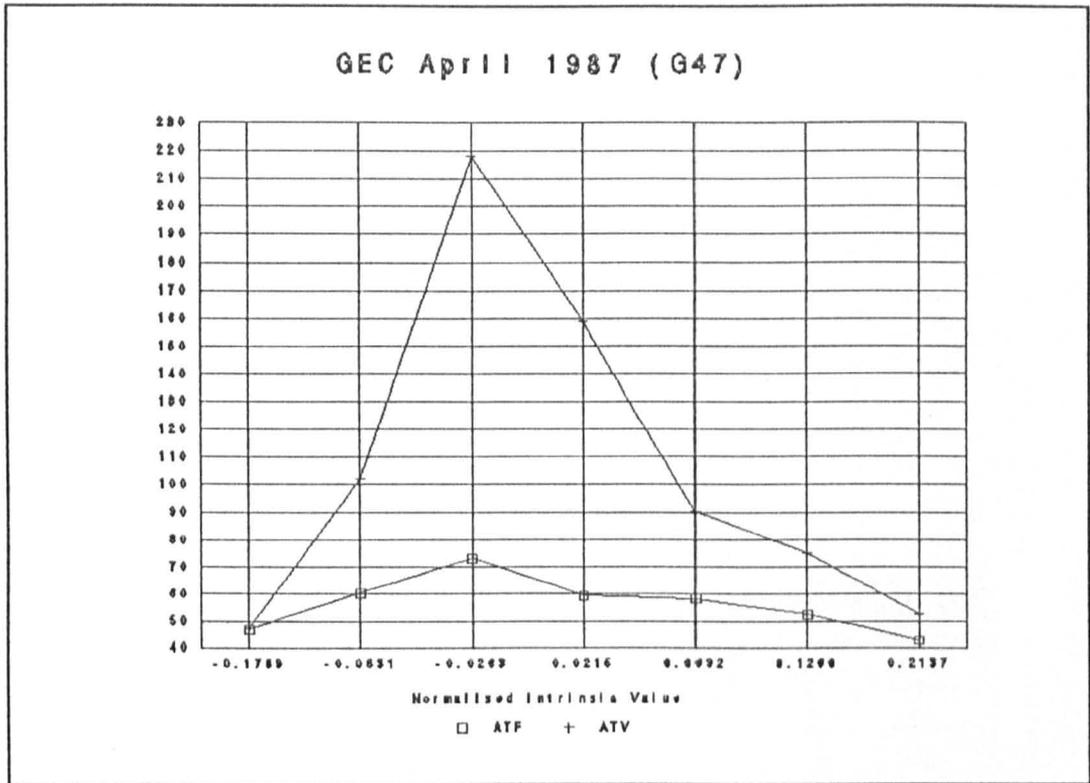


Figure 5.4 (continued)

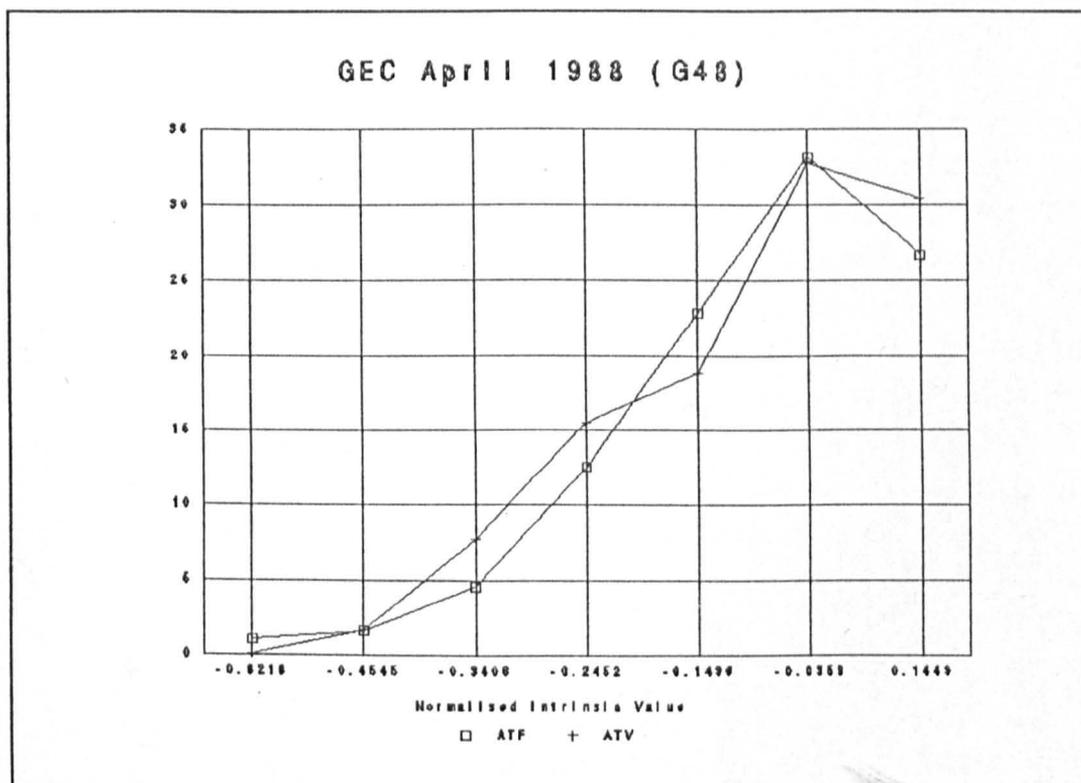
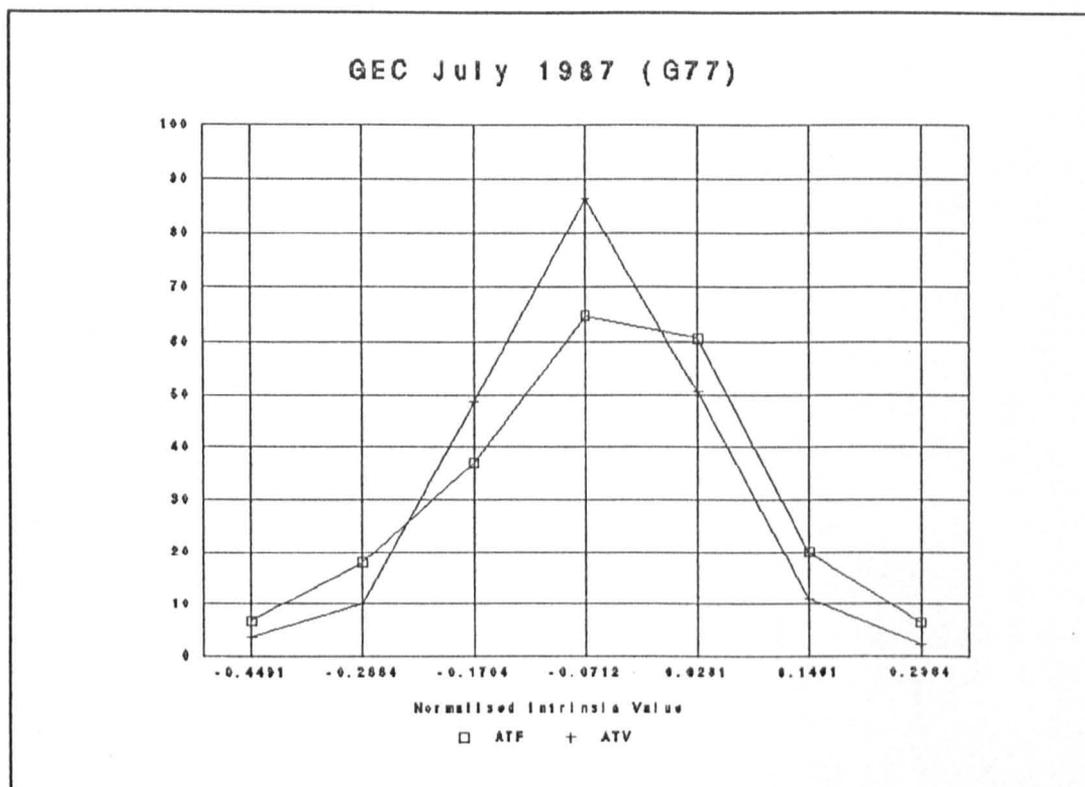


Figure 5.4 (continued)

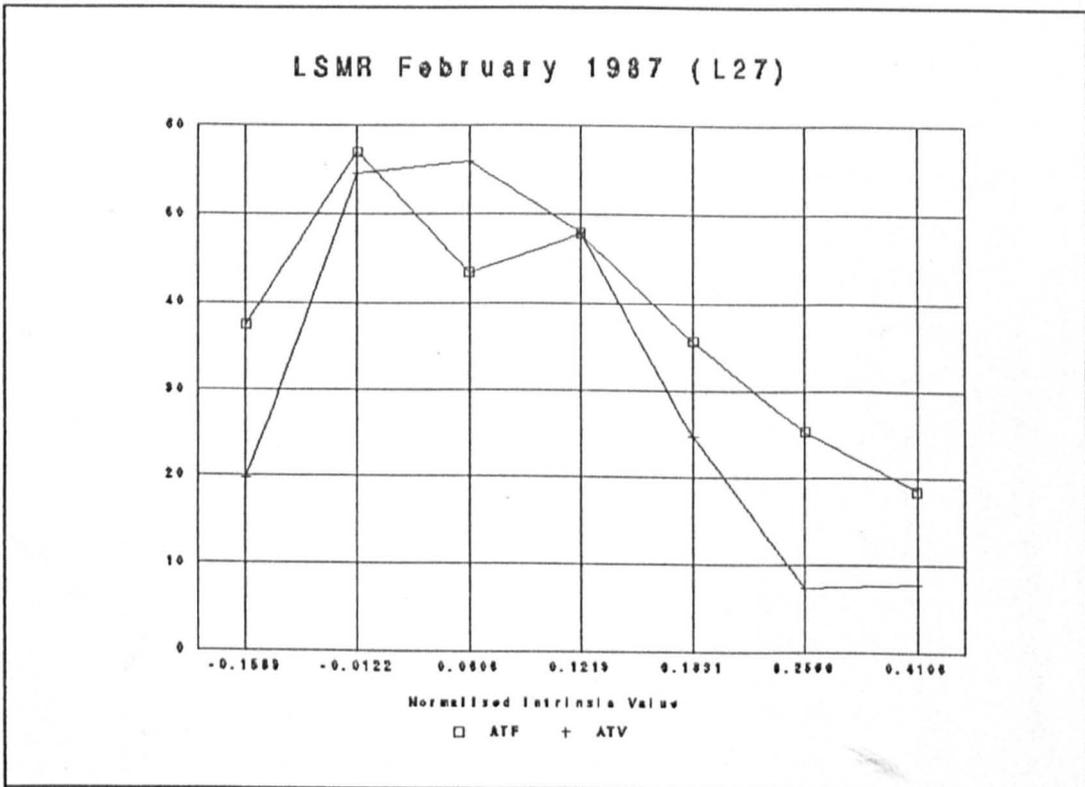
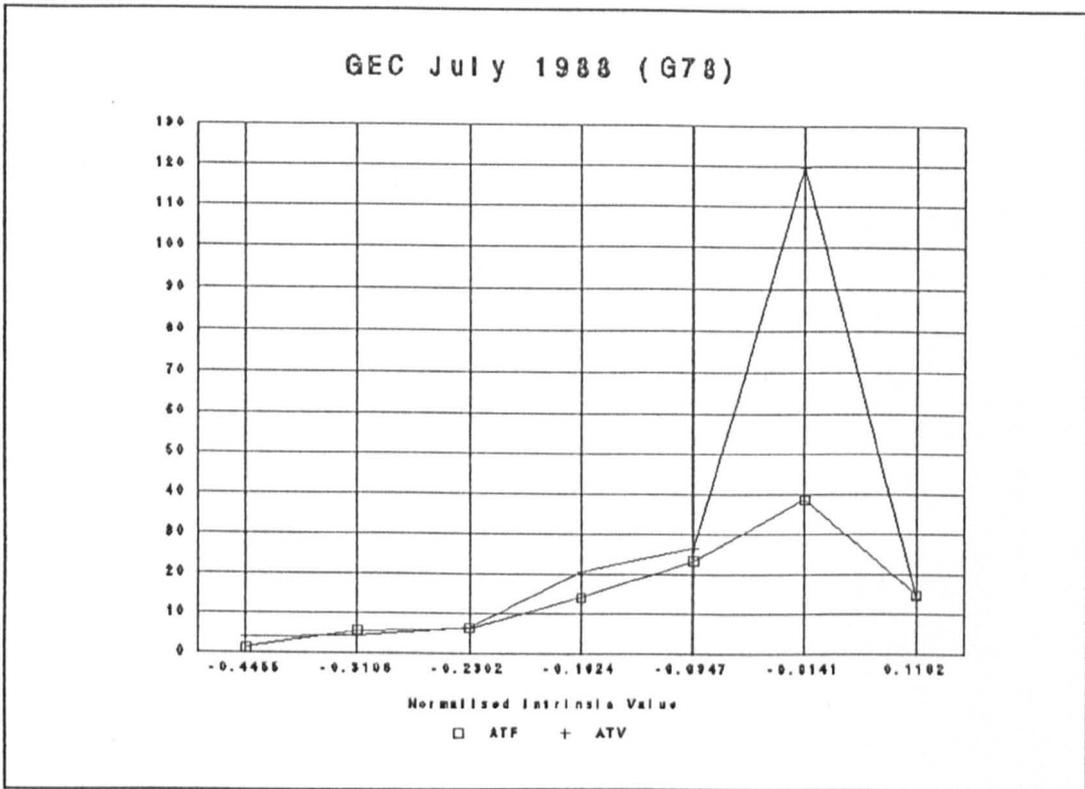


Figure 5.4 (continued)

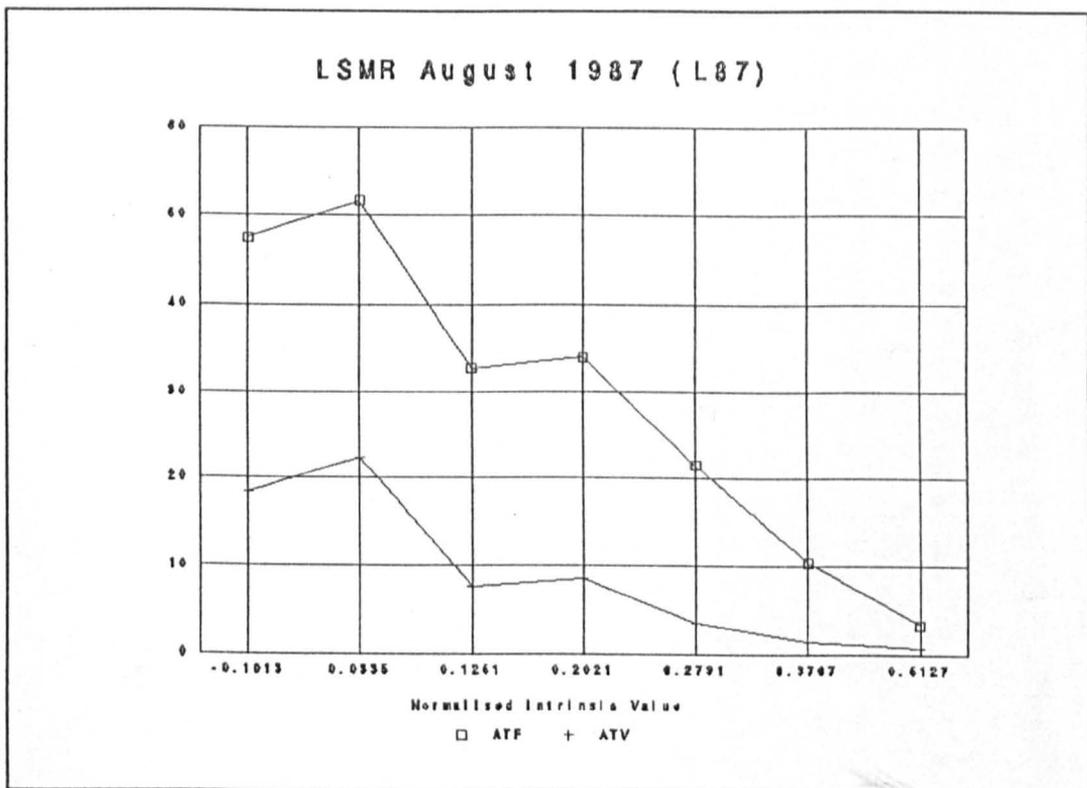
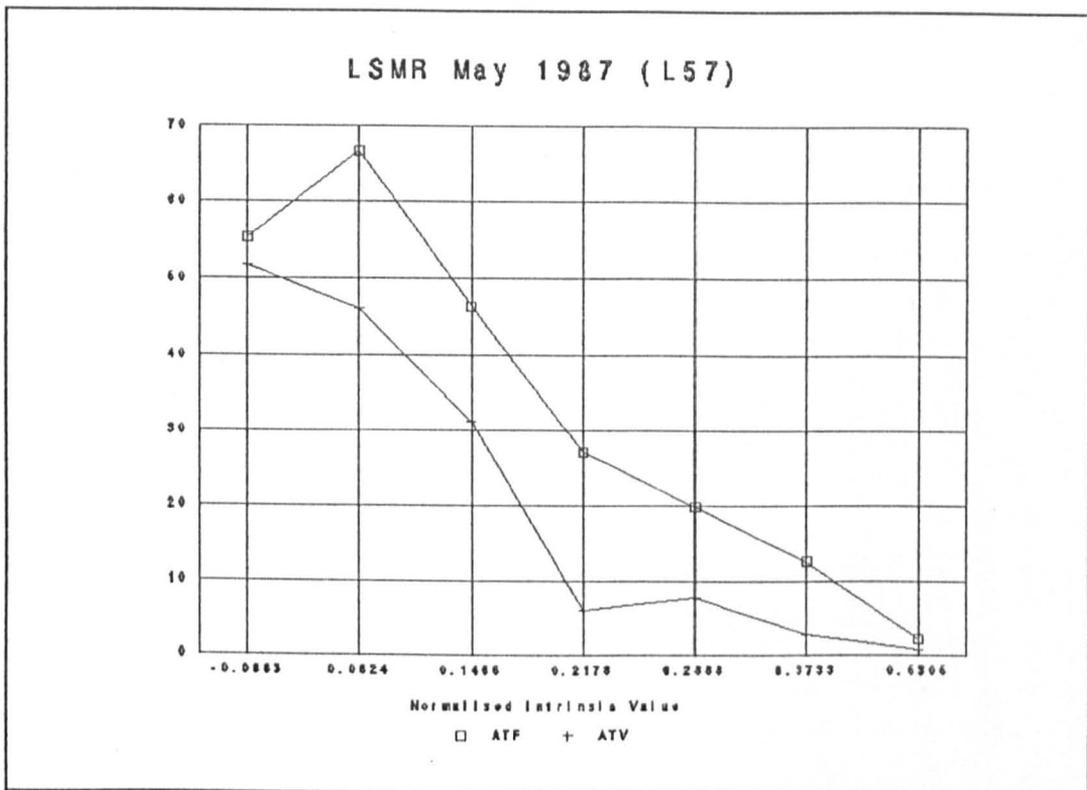


Figure 5.4 (continued)

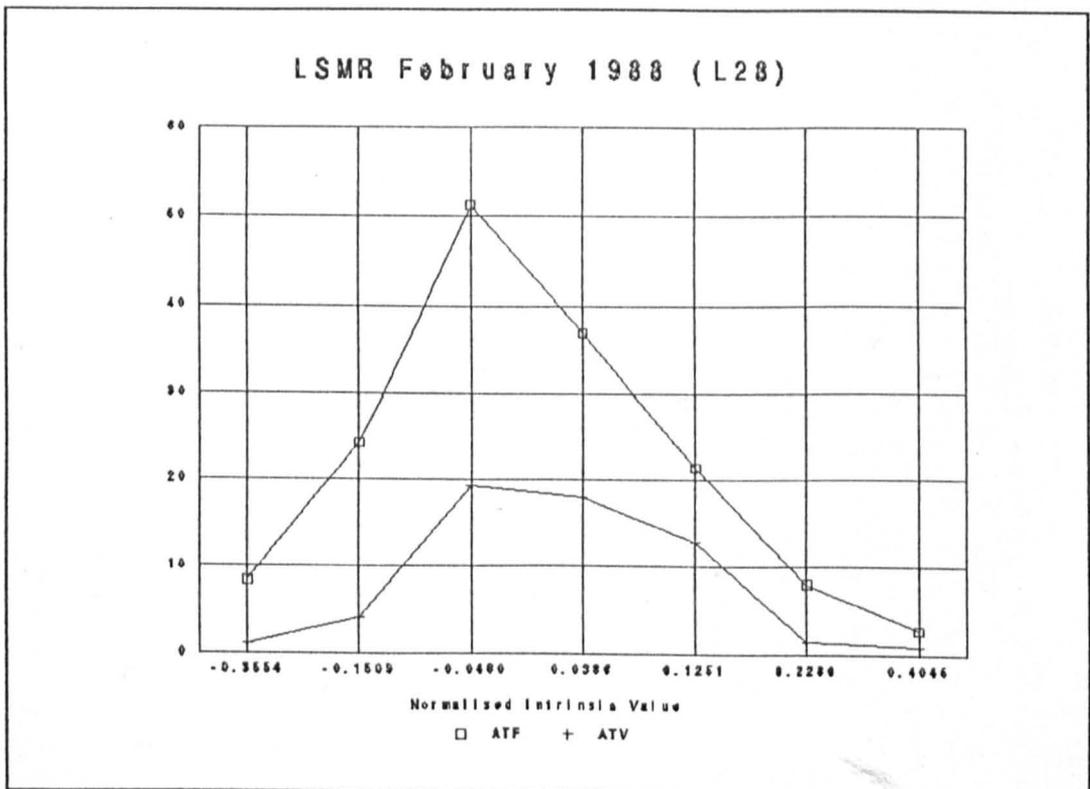
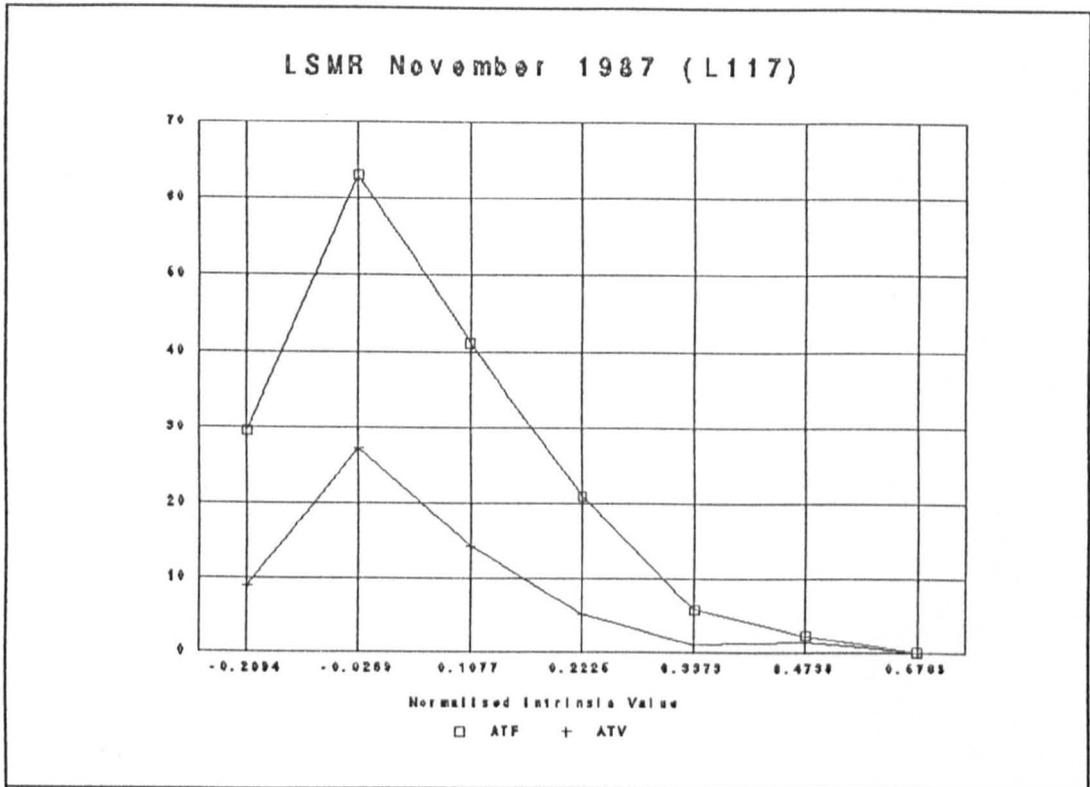


Figure 5.4 (continued)

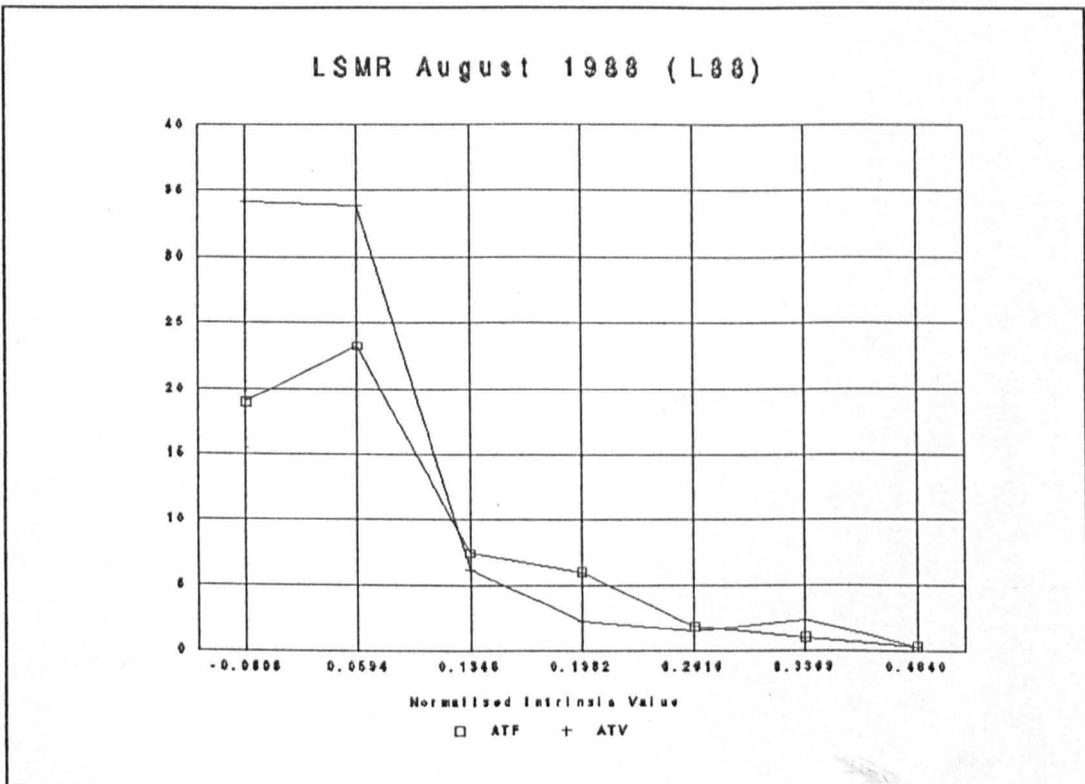
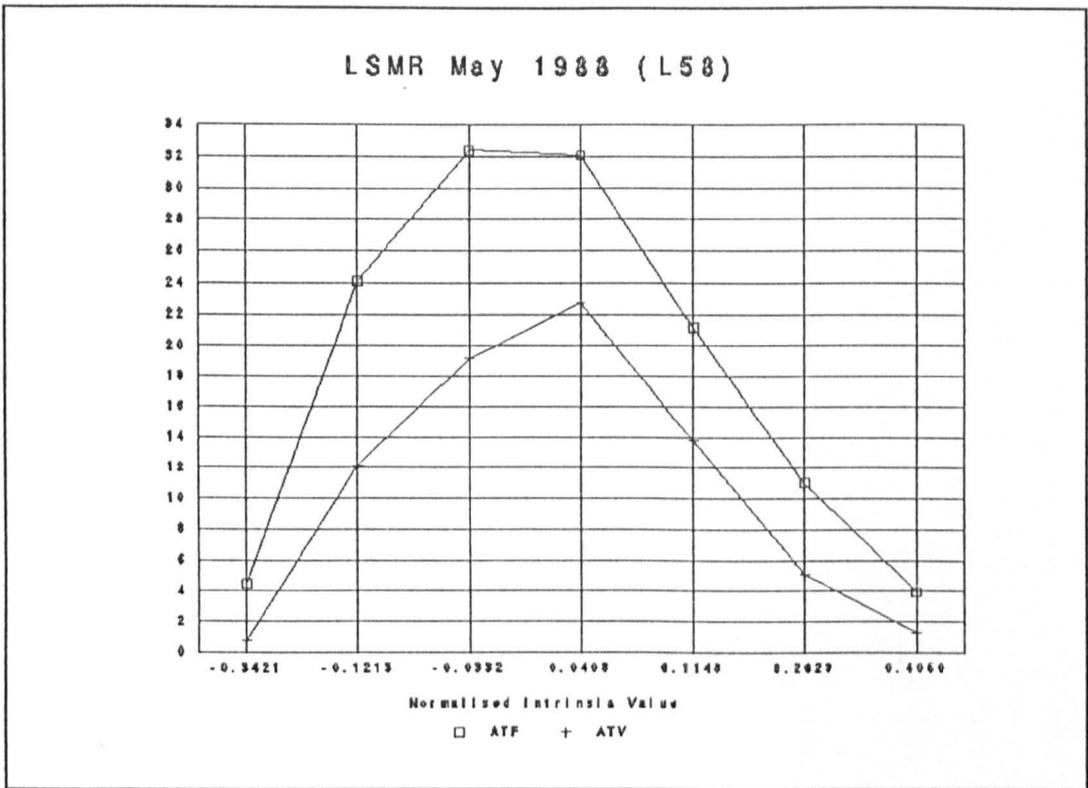


Figure 5.5 Beecham Group

The Trading Frequency Is An Increasing Function of  $\delta$  in

$$\left| \frac{S - X}{S} \right| \leq \delta\%$$

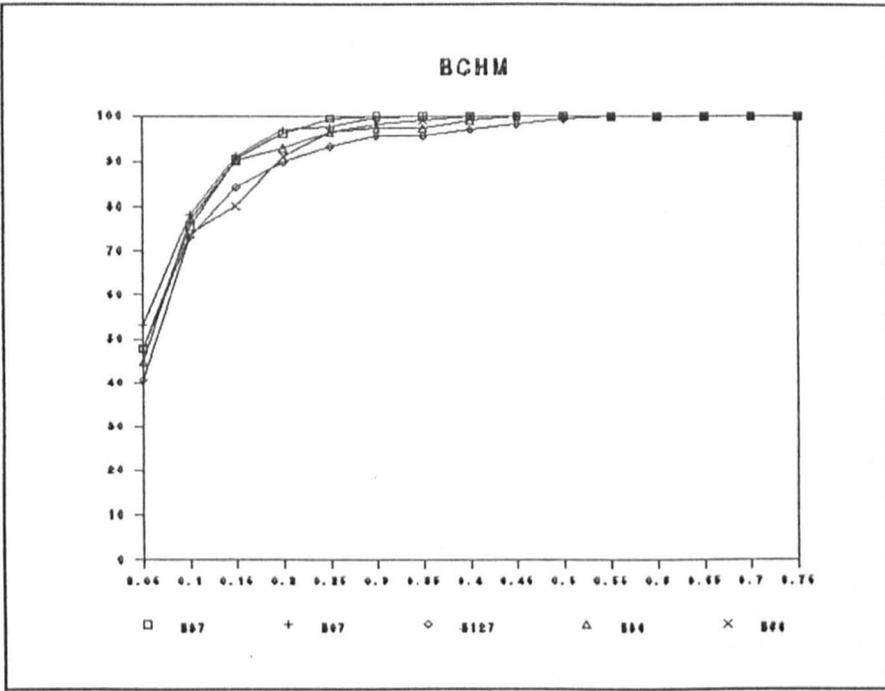


Figure 5.5 (continued)

General Electric Company

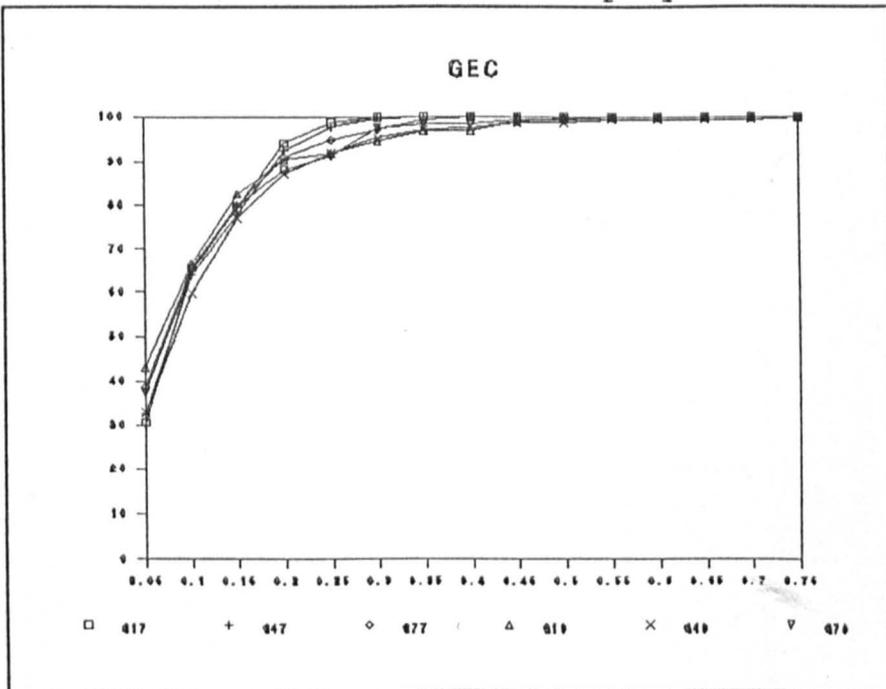


Figure 5.5 (continued)

London & Scottish Marine Oil

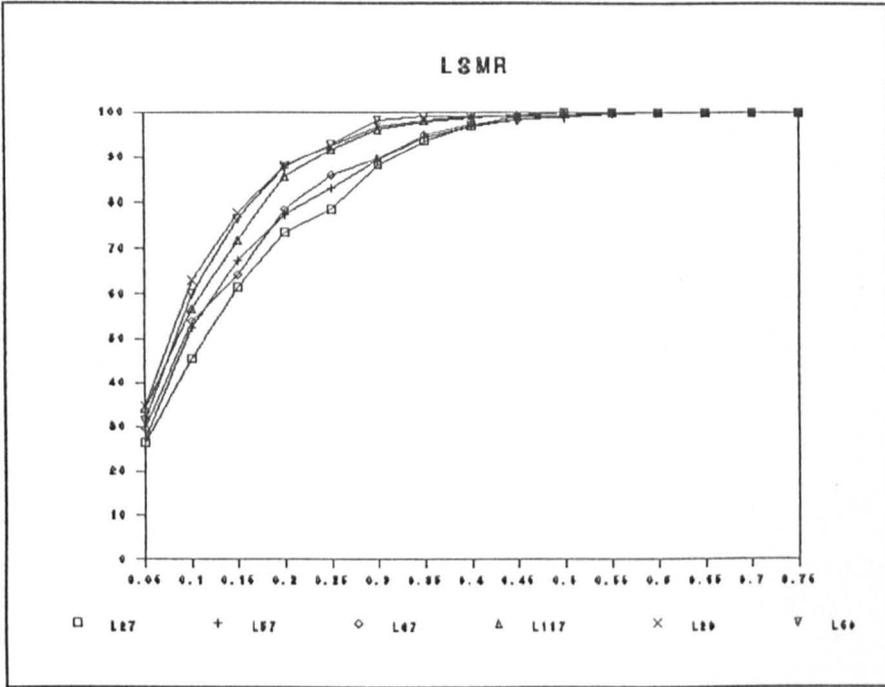
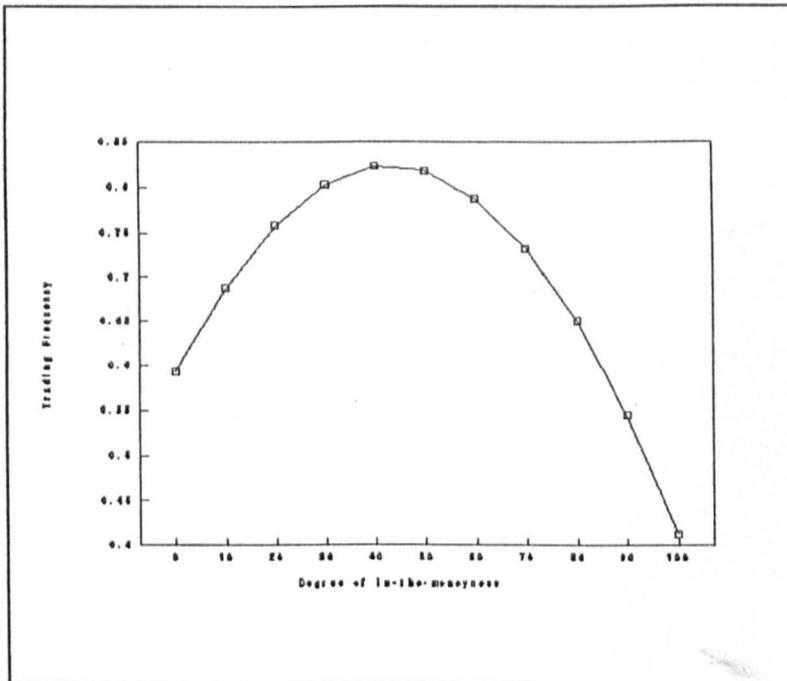


Figure 5.6

If  $r$  is held constant, TF attains its maximum when a call option is slightly out-of-the-money.



## Chapter 6

# The Implications of Ex-Dividend Day Share Price Behaviour upon Option Pricing

### 6.1 Introduction

When a share goes ex-dividend, the share becomes less valuable to a buyer because the owner of the share does not qualify for the receipt of the imminent dividend. The share price will be expected to fall to reflect this lower value. The implication of this to the valuation of options is that the option will be similarly affected. The question to be answered is, by how much should the share price fall for a given dividend? In the past, there are two competing schools of thought regarding the expected share price fall. The "tax clientele hypothesis" stipulates the proportionate fall should be less than one whilst the "short-term trading hypothesis" would predict that the proportionate expected fall to be exactly one. Recently, a third hypothesis, which we term the "riskfree arbitrage hypothesis", argues that although the share price may fall by an amount which differs from the dividend, any variant of the Black-Scholes option pricing model with dividend correction must adjust for the full dividend.

In this chapter, we use the "riskless arbitrage hypothesis" to derive the sensitivity of the call option with respect to the dividend. But, we note that the "riskfree arbitrage hypothesis" is subject to an empirical paradox posed

by Kaplanis (1986) and Barone-Adesi and Whaley (1986). It is concluded that, at present, whether the dividend should be adjusted by the full amount or a proportion of the dividend is an empirical issue. The dividend in Black's dividend correction model will therefore be adjusted in both ways and their implications on market efficiency will be discussed in Chapter 9.

## 6.2 The clientele hypothesis

Miller and Modigliani (1961) first propose the "tax clientele hypothesis". They show that, if capital markets are perfect, then the dividend policy of a firm, for a given investment policy, is irrelevant to its share price. However, in a world where the tax on dividends are higher than the tax on capital gains, investors may demand higher returns to hold shares with higher dividend yield. In equilibrium tax-induced "clientele" will form, with investors holding high yield shares having a lower marginal tax rate and investors having a higher marginal tax rate holding low yield shares. To see this, let a simplified economy be described by the following assumptions:

- (1) Investors are risk neutral.
- (2) There are no transaction costs.
- (3) The tax on short-term capital gain is equal to the tax on long-term capital gain.
- (4) The tax on dividend  $t_d$  and the tax on capital gains  $t_c$  are known with the relationship:  $t_d > t_c$ . All investors are subject to the same tax rates.
- (5) There are unrestricted short sale possibilities.

Further let

$P_B$  = Price on the day before the share goes ex-dividend.

$\bar{P}_A$  = The expected share price on the ex-dividend day.

$P_C$  = Price at which the share was purchased.

$D$  = The amount of the dividend per share.

In this economy, the irrelevance of dividend policy is equivalent to the shareholders' indifference to buy or sell the shares cum or ex-dividend, i.e.,

$$P_B - t_c(P_B - P_C) = \bar{P}_A - t_c(\bar{P}_A - P_C) + D(1 - t_d) \quad (6.1)$$

where the left hand side represents the after-tax receipt with the share sold cum-dividend and the right hand side represents the after-tax receipt with the share sold ex-dividend. Rearranging (6.1) leads to

$$\frac{P_B - \bar{P}_A}{D} = \frac{1 - t_d}{1 - t_c} \quad (6.2)$$

The proportional ex-dividend share price fall,  $(P_B - \bar{P}_A)/D$ , is regarded as a statistic representing the ex-dividend share price behaviour (Elton and Gruber 1970, Kalay 1982). Equation (6.2) has considerable intuitive appeal. First, as  $t_d > t_c$ ,  $(P_B - \bar{P}_A)/D < 1$ . Therefore, a tax clientele effect is only consistent with an ex-dividend share price fall of less than the full amount of the dividend. Second, dividend yields are positively related to  $(P_B - \bar{P}_A)/D$ . This follows because an investor holding high yielding shares will be in a lower tax bracket. This implies that  $1 - t_d$  is relatively high which in turn implies a relatively high  $(P_B - \bar{P}_A)/D$ . Third, assuming that  $t_c = \text{Min}(t_d/2, 25\%)$ , Elton and Gruber (1970) use equation (6.2) to infer the

shareholder marginal tax brackets. They also find a statistically significant positive correlation between the dividend yields and  $(P_B - \bar{P}_A)/D$ .

### 6.3 The short-term trading hypothesis

Kalay (1982) argues that the tax clientele hypothesis is inconsistent because it ignores short-term trading by members of the exchange (dealers) and tax exempt investors. Under some restrictive assumptions, Kalay shows that *the marginal tax brackets cannot be inferred from ex-dividend day share price fall*. A brief proof is given below:

The two revised assumptions from those made in the tax clientele hypothesis are:

(2') There are positive transaction costs.

(3') Short-term capital gains (less than twelve months) are taxed as the same as dividends (this is true in the United States), i.e.,  $t_d = t_c = t$ .

The proxy for the expected transaction costs of a "round trip" can be approximated by a proportion of the average of the cum and ex-dividend share prices, i.e.,

$$\lambda \bar{P}, \text{ where } \bar{P} = (P_B + \bar{P}_A)/2 \text{ and } \lambda > 0.$$

If the dividend is smaller than the expected fall in the ex-dividend price by more than transaction costs, then investors could sell short the cum dividend share and buy it back ex-dividend to gain an after-tax profit, i.e.,

$$D < (P_B - \bar{P}_A) - \lambda \bar{P} \Rightarrow \text{a profit of } (1-t) [(P_B - \bar{P}_A) - D - \lambda \bar{P}] > 0$$

which is equivalent to

$$P_B - \bar{P}_A - D > \lambda \bar{P}. \quad (6.3)$$

If the dividend is larger than the difference between the ex-dividend day share price fall and the transaction costs, then investors could buy the share cum dividend and sell it ex-dividend to gain a after-tax profit, i.e.,

$$D > (P_B - \bar{P}_A) + \lambda \bar{P} \Rightarrow \text{a profit of } (1-t) [(-P_B + \bar{P}_A) + D - \lambda \bar{P}] > 0$$

which is equivalent to

$$P_B - \bar{P}_A - D < -\lambda \bar{P} \quad (6.4)$$

The condition for no profit opportunities is therefore

$$\begin{aligned} -\lambda \bar{P} &\leq P_B - \bar{P}_A - D \leq \lambda \bar{P} \\ \Rightarrow D - \lambda \bar{P} &\leq P_B - \bar{P}_A \leq D + \lambda \bar{P} \\ \Rightarrow 1 - \frac{\lambda}{D/P} &\leq \frac{P_B - \bar{P}_A}{D} \leq 1 + \frac{\lambda}{D/P} \end{aligned} \quad (6.5)$$

If  $D/\bar{P}$  is regarded as a proxy for the dividend yield, then from (6.5) the proportional ex-dividend share price drop is *inversely* related to the dividend yield. This however implies that shareholder marginal tax brackets cannot be inferred from  $(P_B - \bar{P}_A)/D$  and therefore the tax clientele hypothesis does not hold.

Another implication of expression (6.5) is that the proportional ex-dividend share price fall could be either less than 1 or larger than 1 whilst the tax clientele effect is consistent only with  $(P_B - \bar{P}_A)/D$  being strictly less than 1.

For example, let the dividend yield be 1% and  $\lambda$  be .25%. Then the absence of profit opportunities will imply

$$.75 \leq \frac{P_B - \bar{P}_A}{D} \leq 1.25.$$

Kalay's (1982, p.1063, footnote 14) empirical study reported that  $(P_B - \bar{P}_A)/D$  ranges from 0.219 to 1.29. Kalay argues that a  $(P_B - \bar{P}_A)/D$  of greater than 1 could be interpreted as investors being risk averse.

Recent empirical studies support this hypothesis. Lakonishok and Vermaelen (1986) document that for taxable distributions (cash dividends), trading volume increases significantly around the ex-dividend day, particularly for high yield and actively traded shares. They also point out that this trading activity makes it difficult to infer the marginal tax brackets of the average trader, or the existence of clientele effects, from ex-dividend day share price behaviour (p.317). Karpoff and Walking (1988) find that after May 1975, there is a statistically significant correlation between transaction costs and ex-dividend day returns among high yield shares. This implies that positive ex-dividend day returns attract short-term traders who in turn eliminate the positive returns up to their marginal transaction costs.

#### 6.4 The riskless arbitrage hypothesis

In a frictionless economy (no transaction costs), Kalay's (1982) condition for the absence of arbitrage opportunities reduces to

$$1 - 0 \leq \frac{P_B - \bar{P}_A}{D} \leq 1 + 0$$

$$\Rightarrow P_B - \bar{P}_A \equiv D$$

which stipulates an ex-dividend share price change equal to the entire dividend. Heath and Jarrow (1988), however, argue that an arbitrage trading strategy should make positive profits at no risk. Therefore, as short-term trading involves risk (Kalay 1982, p.1063), the hypothesis is built on a wrong premise. Heath and Jarrow show that, with no arbitrage opportunities, the ex-dividend share price change can *differ* from the dividend. Since their argument emphasises riskless arbitrage, their theory is referred to as the "riskless arbitrage hypothesis". A brief proof is given below:

Let the share price process be denoted by  $\{S(t) : t \in [0, T]\}$ . The share is expected to pay a dividend of  $D$  at the ex-dividend day  $\tau \in (0, T)$ . The share  $S(T)$  is thus constructed to trade cum for  $t < \tau$  and ex for  $t \geq \tau$ . Let the share price an instant before the ex-dividend day be denoted by  $S(\tau-)$ . Then the condition

$$S(\tau-) - S(\tau) + (1-\alpha)D$$

denotes that the ex-dividend share price  $S(\tau)$  change by an amount more than, equal to, or less than the dividend when  $\alpha$  is larger than, equal to, or less than 1.

Adapting Harrison and Kreps' (1979) theory (which deals with some fundamental issues that arise in conjunction with the arbitrage theory of option pricing, p.381), Heath and Jarrow derive the sufficient and

necessary condition for the absence of arbitrage opportunities around the ex-dividend day that "either  $\alpha=1$  or, if  $\alpha$  is greater than 1 with positive probability then  $\alpha$  will also be less than 1 with positive probability".

In contrast to Kalay's (1982) short-term trading hypothesis, the share price change does not have to be the entire dividend for the absence of arbitrage opportunity. Empirical evidence does indicate that, although the ex-dividend share price change is usually less than the dividend, it is sometimes more (Campbell and Beranek 1955; Durand and May 1960; Elton and Gruber 1970; Kalay 1982; Barclay 1987).

The implication of Heath and Jarrow's proof is that short-term traders cannot make *riskless* arbitrage profits because of the uncertainty of the ex-dividend share price change. Kalay (1984, p.558. footnote 3) points out that the risk is in fact quite high. The Black-Scholes (1973) option pricing formula is built upon the premise that the hedged position is riskless. If the ex-dividend share price change is uncertain, the position is no longer riskless. A fall in the share price of an amount which differs from the dividend will therefore render the Black-Scholes formula inherently inconsistent. As a result, any variant of the Black-Scholes formula with dividend correction must be adjusted by the full amount of the dividend. In particular, the exact American call option pricing model derived by Roll (1977), Geske (1979), and Whaley (1981) will only be correct if the proportionate share price change specified in their formula is rectified to 1, i.e.,  $\alpha D$  should be strictly equal to  $D$ , or  $\alpha=1$ .

6.5. The sensitivity of a call option with respect to the dividend

Although the Black and Scholes formula has been adjusted for the effect of the dividend for nearly two decades, the sensitivities of the call option with respect to the dividend have not yet been examined. Heath and Jarrow's theorem that the ex-dividend share price change should be certain is applied to derive the sensitivity. Let the Black-Scholes be adjusted with  $n$  known, finite dividends:

$$C_t = (S_t - \sum_{i=1}^n D_i e^{-rr_i}) N(d_1) - XN(d_2) e^{-rT}$$

Then

$$\begin{aligned} \frac{\partial C_t}{\partial D_j} &= \hat{S}_t N'(d_1) \frac{\partial d_1}{\partial D_j} + N(d_1) \left[ \frac{\partial \hat{S}_t}{\partial D_j} - e^{-rr_j} \right] \\ &\quad - X e^{-rT} N'(d_2) \frac{\partial d_2}{\partial D_j} \\ &= \left( \hat{S}_t N'(d_1) - X N'(d_2) e^{-rT} \right) \frac{\partial d_1}{\partial D_j} + \left[ \frac{\partial \hat{S}_t}{\partial D_j} - e^{-rr_j} \right] N(d_1) \\ &= \left[ \frac{\partial \hat{S}_t}{\partial D_j} - e^{-rr_j} \right] N(d_1) \\ &= -e^{-rr_j} N(d_1) < 0 \end{aligned}$$

by applying the Kernel Lemma (Choi and Ward 1989)

$$\hat{S}_t N'(d_1) - X N'(d_2) e^{-rT} = 0.$$

According to the riskless arbitrage hypothesis, the expected share price fall, among other things, with respect to the dividend, is certain. Therefore,

$$\frac{\partial \hat{S}_t}{\partial D_j} = 0.$$

The sensitivity is negative and is therefore consistent with the expectation that a call option will be worth less when the dividend paid is larger. Note that when the time to the ex-dividend day is longer ( $\tau_j$  increases),  $\partial C_t / \partial D_j$  will become less negative which implies that the call option is more valuable.

#### 6.6. A conceptual difficulty

Recently, Kaplanis (1986) finds empirically that the average expected ex-dividend share price fall implicit in option prices is significantly less than the dividend. On the other hand, Barone-Adesi and Whaley (1986) find that the proportionate expected fall-off implied from option prices is not significantly different from one. Heath and Jarrow (1988) argue that these conflicting results are due to the fact that both papers adopt inconsistent option pricing models (i.e., allowing  $\alpha$  to be different from one) to infer the fall-off by equating the option's model price to the actual market price (p.105). However, Heath and Jarrow have misunderstood Kaplanis' research methodology. Kaplanis does not derive the ex-dividend share price from the Black-Scholes formula. She uses observed option and share prices, not values derived from the Black-Scholes model. Her methodology is to run the regression

$$\frac{C_x - C_c}{HD} = b_0 + b_1 \left( \frac{S_x - S_c}{D} \right) \quad (6.6)$$

where

$C_x, C_c$  = ex-dividend, cum-dividend option price

$S_x, S_c$  = ex-dividend, cum-dividend share price

$H = \partial C / \partial S$  (hedge ratio),  $D$  = dividend.

She finds that the proportionate expected ex-dividend share price fall  $b_0$  is significantly less than one (around 55%). (For a detailed derivation of equation (6.6) please refer to Kaplanis (1986, pp.413-414). As a result, Kaplanis' methodology is theoretically sound and her result is purely empirical.

It is thus seen that Kaplanis, based on Heath and Jarrow's riskless arbitrage hypothesis, finds a proportionate share price fall of less than one, whilst Barone-Adesi and Whaley, contradictory to Heath and Jarrow's argument, find that the expected proportionate fall is not significantly different from one. These results are somewhat paradoxical. From Heath and Jarrow's point of view, the research which is consistent with their theory contradicts their hypothesis whereas the research inconsistent with their theory supports their hypothesis!

## 6.7 Concluding remarks

This chapter has reviewed hypotheses on the expected ex-dividend share price fall. The "tax-clientele hypothesis" stipulates that the proportionate fall to be less strictly than one whereas the "short-term trading hypothesis" would predict that the proportionate fall to be exactly one. Recently, the "riskless arbitrage hypothesis" argues that although the expected share price may fall by an amount different from the

full dividend, any Black-Scholes option pricing model with dividend correction must adjust for the full dividend.

However, the conflicting empirical results of Kaplanis (1986) and Barone-Adesi and Whaley (1986) have posed a paradox for Heath and Jarrow's theory. If it is accepted that the usefulness of an option pricing model is judged by its ability to predict market prices (by its model price being close to market prices), then the above dilemma can only be resolved empirically. In this study, the Black-Scholes formula is therefore adjusted both by an amount less than the dividend and also by an amount equal to the dividend. The comparison between the impacts of both adjustments on efficiency tests is reported in chapter 9.

## Chapter 7

# Data Analysis and Boundary Conditions

### 7.1 Introduction

This chapter discusses the source of data input to the Black-Scholes model, data screening and classification, and the boundary conditions for call options.

### 7.2 The London Traded Options Market

The London Traded Options Market of the International Stock Exchange started on 21 April 1978 with the introduction of call options on ten leading equities. The market currently offers contracts with options available on more than sixty-five blue chip UK equity shares and the FTSE 100 Index. Traded options are of two types - call and puts.

A call option gives the right, but not the obligation, to buy shares at a fixed price on or before an agreed date. A put option gives the buyer the right, but not the obligation, to sell shares at a fixed price on or before a given date. Call options and put options can be traded in their own rights. A traded option's maximum life is nine months and its maturity date follows a predetermined three-month cycle. A cycle can be a January-April-July cycle, a February-May-August cycle, or a March-June-September cycle. All the call options of a particular share form a class. Within a class, there are a number of series with differing maturity dates and exercise prices.

### 7.3 Data source and period of study

The period studied spans 27 October 1986 to 30 June 1988. To study the implications of using bid-ask quotes and mid prices on market efficiency and in particular, to examine the thinness of the LTOM, data were collected from four sources:

#### 7.3.1 Share quotations

The share bid-ask quotations of eighteen UK companies were collected from Datastream. The first day that such quotations are available was 27 October 1986 - the "Big Bang" (Note 7.1).

The eighteen UK companies are categorised into the industries according to the FT-Actuaries grouping in Table 7.1. Most of the eighteen companies are constituents of the FTSE 100 Index (The FT-SE 100 Index, The Stock Exchange, November 1986) and are designated as "alpha" shares at the Big Bang. The mid share price is the average of the bid and ask prices.

#### 7.3.2 Call option quotations

Call option quotations of the eighteen UK companies were collected from the Daily Official List, published by the International Stock Exchange. This database includes the call option bid and ask prices, the exercise prices, and the dates of maturity. The maturity days have been double checked with those reported in the London Traded Option Users' Manual. (Table 7.2). The call option series of BP, CGLD, CTLD, CUAC, GEC, GMET, ICI, LAND, MKS, SHEL follow the January-April-July cycle, BARC, LSMR, P. & O., RCAL, RTZ follow the February-May-August cycle, and finally, BCHM, GKN, LRHO follow the March-June-

September cycle. There were 888 call option series which have the times to maturity in or before June 1988, the end of the sample period.

### 7.3.3 Interest rates

Finnerty (1978) points out that the interest rates should match the maturities of the options. In the UK, data are available for the London Treasury Bill only for one and three months maturities. On the other hand, the London Local Authority interest rates are available for one, three, six months and one year maturities and are therefore adopted in this study. Interest rates for other maturity months are obtained by interpolating the interest rates of the four available maturity months. The interest rate is converted into a continuously compounded rate by

$$r = \log_e(1 + r_{RF})$$

where  $r_{RF}$  is the Local Authority interest rate.

### 7.3.4 Dividend and ex-dividend dates

The dividends paid by the eighteen UK companies and the accompanying ex-dividend dates during the study period were collected from the Dividend and Interest Record, Extel Financial Limited, London, from the 86/87 to 88/89 issues.

The dividend data are consistent with Kaplanis' (1986) observation that in the UK companies usually pay a low interim and a high final dividend in a year. This suggests that the adoption of the Black's dividend correction model (equation 2.1) is appropriate to UK data.

Barone-Adesi and Whaley (1986) use one dividend in their call option database in studying the ex-dividend date share price drop. Some other empirical studies, such as those of Whaley (1982), Sterk (1982, 1983) all use one dividend in their call option models. In the UK, Ho (1990) also uses one dividend in testing the RGW call option pricing model. In this study, only those call options whose underlying shares paid at most one dividend over the call options' lives are included. Those call option series whose underlying shares paid more than one dividend during the call options' lives are thus excluded: the loss of data represented 16% (142 series) of the total sample size.

#### 7.4 The share price volatility

As has been shown in Chapter 3, the *ex ante* GARCH volatility process has the best forecasting accuracy of share price volatilities. It also embeds the property of variance persistence. The GARCH model is therefore adopted to generate volatility estimates for call option values following the process outlined in Chapter 4. French (1984) finds that share returns are generated by a process operating closer to trading time rather than calendar time. He therefore suggests that the Black-Scholes model should account for this empirical evidence by using a trading time variance. This study adopts this suggestion. It is assumed that there are 252 trading days in a year and the annualised variance is calculated as 252 times the daily variance.

## 7.5 The Account Day

In the UK, transactions in ordinary shares are for settlement on the Account Day relating to a particular account period, which has a standard duration of ten working days. This system allows considerable time for the investor to get funds ready or arrange for the delivery of securities to the broker. The existence of the Account period also provides opportunities for reversing a transaction and is regarded as a useful facility for short-term speculation. In Chapter 8, share-option hedges are set up in the context of the Account Day system.

## 7.6 Data screening

The call option database is filtered by considering the minimum trading days, the effect of the October crash in 1987 on option prices, the trading frequency of a call option, and the rational boundary conditions. The boundary conditions are stated in terms of bid-ask prices.

### 7.6.1 Minimum trading days

Manaster and Rendleman (1982, p.1046) note that for short term to maturity options, the Black-Scholes model is very sensitive to violations of its basic assumptions. They therefore require that an option must have at least 30 days to maturity. In this study, a call option is required to have at least 25 trading days to maturity. There were 92 call option series, or 10% of the total sample size, which violate this requirement and were excluded from the sample.

### 7.6.2 The October crash in 1987

The October crash in 1987 has had a big impact on the UK market. The effect of the crash is outlined in the Quality of Markets Quarterly Review (1990, p.19):

*"Price spreads and touches were also very competitive after Big Bang: the average best touch in alphas was 0.8%, ... market makers responded to the market crash of October 1987 by almost doubling their price spreads. By August 1988, spreads and touches in alphas had returned to the post Big Bang level."* and in Quality of Markets Quarterly (1988, p.9): *"Traded options have shown a high growth rate over recent years, but the number of contracts traded has fallen back since October."*

The sample period of this study (October 1986 to June 1988) includes October 1987. In response to the large drop of their underlying share prices during the crash, call option prices also had large drops at and after the crash. To separate from the crash effect, the sample data were divided into two sub-samples:

The 88 data. To avoid abnormal pricing of call options, 171 call option series, or 19% of the total sample size, which began their lives before the crash and matured after the crash are eliminated from the sample. Call option series which began their lives after the crash are still kept in this sub-sample.

The 87 data. This sub-sample includes only those call option series which matured before the crash.

In Chapters 8 and 9, empirical results will be discussed separately and contrasted for the 88 data and the 87 data.

### 7.6.3 Trading activity

In Chapter 5, an analytical criterion has been derived for measuring trading frequency in terms of the number of trading days  $\tau$  and the percentage of positive intrinsic values over the life of a call option (P):

$$TF = f(\tau, P).$$

In this study, frequently traded call options are defined as those series with  $TF \geq 20\%$  and infrequently traded call options as those series with  $TF \leq 10\%$ . Call option series with  $10\% < TF < 20\%$  are deliberately excluded to ensure that the two classes studied are distinct. There are 83 call option series, or 9% of the total sample size, with TF falling in this interval and are thus eliminated.

### 7.6.4 Classification of data according to trading frequency

After the call option series have been filtered by the minimum trading days, the consideration of the crash and trading activity, there are 400 call option series in the 88 data and 286 series in the 87 data (Table 7.3).

For the 88 data, of the 400 call option series 288 are frequently traded and 112 are infrequently traded. The average trading days of the frequently traded call options are 74 days and the average percentage of positive intrinsic values during the call options' lives is 53%. For the infrequently traded call option series the corresponding figures are 139 days and

95%. Of the frequently traded call options, about half of their underlying shares pay one dividend over the life of the call options. For the infrequently traded call options, almost all the underlying shares (97%) pay one dividend over the life of the call options.

For the 87 data, of the 286 call option series 201 are frequently traded and 85 are infrequently traded. The average trading days of the frequently traded call options are 72 days and the average percentage of positive intrinsic values during the call options' lives is 54%. For the infrequently traded call option series the corresponding figures are 144 days and 96%. Of the frequently traded call options, about half of their underlying shares pay one dividend over the life of the call options. For the infrequently traded call options, almost all the underlying shares (98%) pay one dividend over the life of the call options.

In summary, infrequently traded call options have larger average time to maturity and larger average percentage of positive intrinsic values than those of frequently traded call options.

#### 7.6.5 Rational lower boundary conditions

In the LTOM, the smallest price fraction allowed is  $1/4$  pence. All call option prices are thus required to be worth at least  $1/4$  pence, i.e., all very low value out-of-the-money call options are excluded. The data is further filtered so that every call option price satisfies the rational lower boundary conditions to eliminate riskfree arbitrage opportunities. Call

options are required not to be exercised early so that the Black-Scholes model is used in conditions which are conducive to its appropriateness.

Previous empirical studies in rational boundary conditions for call options with dividend adjustment are examined by Galai (1978), Bhattacharya (1983), and Halpern and Turnbull (1985), among others. The three boundary conditions are outlined as follows:

(a) Immediate exercise value

Initially, call option values are non-negative because buyers can exercise the call option any time at his/her discretion. A call option must be worth at least its intrinsic value. Otherwise, an arbitrage profit can be made by buying the call option and immediately exercising it.

$$C_t \geq S_t - X \quad (7.1)$$

(b) No early exercise during the call option's life

Since a call option is not dividend protected, there is a possibility that the call option might be exercised before maturity. Let there be one certain dividend  $D$  to be paid at time  $\tau$ . Then the lower boundary condition for the unprotected call option to be held until maturity is that its current value must be greater than its present value at maturity, with the dividend foregone. i.e.,

$$\begin{aligned} C_t &\geq (S_T - X) e^{-r(T-t)} - De^{-r(\tau-t)} \\ \Rightarrow C_t &\geq S_t - De^{-r(\tau-t)} - Xe^{-r(T-t)} \end{aligned} \quad (7.2)$$

(c) Black's (1975) condition for no early exercise

Black (1975) amends the Black-Scholes model for the possibility of early exercise by suggesting that the call option can be considered to expire an instant before the last ex-dividend date. This implies that if the call option is held until maturity, then its current value must be greater than its present value at the ex-dividend instant. i.e.,

$$\begin{aligned} C_t &\geq (S_\tau - X) e^{-r(\tau-t)} \\ \Rightarrow C_t &\geq S_t - X e^{-r(\tau-t)} \end{aligned} \tag{7.3}$$

#### 7.6.6 Further discussion on conditions for no early exercise

Roll (1977) and Manaster and Rendleman (1982) also suggested conditions for no early exercise which are different from Black's (1975). The following discussion points out the difference between their two conditions and further proves that Roll's condition and condition (7.2) together will imply the redundancy of Black's condition (7.3). Let  $t$  denote current time,  $\tau$  denote the ex-dividend instant, and  $T$  denote the maturity date in the following analysis.

##### (1) Roll's (1977) condition

In deriving the American call option formula, Roll (1977) identifies a condition for the possibility of early exercise. The condition assumes that the call option will only be exercised an instant before the ex-dividend date. This implies that the call value at the ex-dividend instance must be greater than its value at maturity, i.e.,

$$(S_T - X) + D > (S_T - X)e^{-r(T-t)} = S_T - Xe^{-r(T-t)}$$

$$\Rightarrow D > X[1 - e^{-r(T-t)}]$$

The condition for no early exercise is therefore

$$D \leq X[1 - e^{-r(T-t)}] \quad (7.4)$$

or

$$De^{-r(r-t)} \leq X[e^{-r(r-t)} - e^{-r(T-t)}] \quad (7.5)$$

### (2) Manaster and Rendleman's (1982) condition

Manaster and Rendleman (1982) consider the possibility of early exercising the call option any time before the ex-dividend date. In this case, the present value of exercising the call option early must be greater than its present value at maturity, i.e.,

$$(S_t - X) + De^{-r(r-t)} > (S_T - X)e^{-r(T-t)} = S_t - Xe^{-r(T-t)}$$

$$\Rightarrow De^{-r(r-t)} > X[1 - e^{-r(T-t)}]$$

The condition for no early exercise is therefore

$$De^{-r(r-t)} \leq X[1 - e^{-r(T-t)}] \quad (7.6)$$

### (3) Comparison of Roll's and Manaster and Rendleman's boundary conditions

Roll's condition (7.5) and Manaster and Rendleman's condition (7.6) are different from Black's condition (7.3) for no early exercise in that they are both independent of the share price. Manaster and Rendleman's condition is more restrictive than Roll's because they consider the possibility of early exercising the call option at any time, including the

instant before the ex-dividend date. The implication is that by applying their condition more call options would be eliminated. However, Merton (1973), points out that it will never be optimal to exercise a call option prematurely except an instant before the ex-dividend date. Roll's condition is therefore preferred to Manaster and Rendleman's condition for not exercising the call option early.

(4) Roll's condition (7.5) and condition (7.2) imply the redundancy of Black's condition (7.3):

$$\begin{aligned}
 C_t &\geq S_t - De^{-r(r-t)} - Xe^{-r(T-t)} \\
 &\geq S_t - [Xe^{-r(r-t)} - Xe^{-r(T-t)}] - Xe^{-r(T-t)} \\
 &= S_t - Xe^{-r(r-t)}
 \end{aligned}$$

Therefore, the independent conditions (7.1, 7.2, 7.4) will be used as the rational lower boundary conditions in this study. If the bid-ask quotes are taken into account, then the three conditions become

$$\begin{aligned}
 C_{t,ask} &\geq S_{t,bid} - X \\
 C_{t,ask} &\geq S_{t,bid} - De^{-r(r-t)} - Xe^{-r(T-t)} \\
 D &\leq [1 - e^{-r(T-t)}]
 \end{aligned}$$

#### 7.7 Filtered frequently traded call options data

After screening the call option data for minimum trading days, the crash effect, trading frequencies and the rational boundary conditions, there is more than seventy percent of the frequently traded call option prices in both the 87 and 88 data sub-samples which satisfied all boundary conditions.

Specifically, for the 87 data, there are 13,214 prices in the 201 call option series (Table 7.4a). Of these prices, 192 prices (1.5%) violate the immediate exercise condition and 3,243 prices (24.5%) violate the early exercise condition. Thus, 3,435 prices are discarded, which represents 26% of the original 13,214 prices. There are 9,779 prices (74%) to be used in the market efficiency tests. For the 88 data, there are 19,127 call option prices in the 288 call option series (Table 7.4b). Of these prices, 219 prices (1.1%) violate the immediate exercise condition and 4,936 prices (25.8%) violate the early exercise conditions. Thus 5,155 prices, or 27% of the original 19,127 prices, are eliminated. There are 13,972 prices (73%) to be used in the market efficiency tests.

The 87 and 88 data sub-samples are each composed of very different price data and are therefore worth to be examined independently in the efficiency tests. First, the 88 data exceeds the 87 data by more than five thousand prices. However, the 88 data has marginally more prices (1%) violated the boundary conditions than the 87 data. Second, the percentage of prices satisfying the boundary conditions changes drastically for many companies between the two data sub-samples. For instance, 99.3% of GKN prices satisfies the boundary conditions in the 87 data sub-sample but decreases to 88.1% in the 88 data sub-sample; 57.3% of RTZ prices satisfies the boundary conditions in the 87 data sub-sample and increases to 72.9% in the 88 data sub-sample. The empirical results in the following chapters are therefore reported for both data sub-samples.

## 7.8 Comparison of call option prices before and after satisfying the boundary conditions

A close examination of the time to maturity, the intrinsic values, and the percentage spreads of the data before and after satisfying the boundary conditions reveals that, in general, the set of call options which satisfies the boundary conditions have a smaller average time to maturity (i.e., traded later in their lives), are more in-the-money, and have larger percentage spreads than those which violate the boundary conditions. These observations are summarised in Table 7.5. This result is reasonable because before the Black-Scholes model is useful in identifying mispriced call options, the call options which violate the boundary conditions are likely to be slightly out-of-the-money, have smaller percentage spreads and are identified earlier in their lives in an efficient market.

## 7.9 Concluding remarks

This chapter discusses two main points: data analysis and rational boundary conditions for call options.

The period of study spans from 27 October 1986 to 30 June 1988. Call option quotations were collected from the Daily Official List. The underlying share price quotations of the options and the riskfree interest rates were collected from Datastream. Dividends and the accompanying ex-dividend dates were collected from Extel Financial Limited. The *ex ante* GARCH volatility estimates developed in chapter 4 was adopted as the share price variability.

A call option series is required to have at least 25 trading days. To separate the crash effect, the database was divided into the 87 data and 88 data sub-samples. To examine the thin trading issue, call option series are classified into frequently and infrequently traded call options series by using the analytical model developed in chapter 5. Before carrying out the efficiency tests, call option data are required to be screened of the immediate exercise condition and the early exercise conditions. Thus the filtered data are European call options which are purged of the crash effect. Frequently traded call option prices which satisfied the boundary conditions are summarised in Table 7.4.

## Note

### Note 7.1

The following note explains the term "Big Bang" (cf. An Introduction to the Stock Market, The Stock Exchange, December 1986, pp.26-27):

"Big Bang was a term used to describe the changes which brought the Stock Exchange into line with international practice. Starting on that day, the jobbing and broking system was abolished. Member firms are now able to act as principal, ie, deal directly with their clients on the basis of quotations they themselves make in shares, and as agent, acting on behalf of a client and dealing with a market maker who fixes the price of the shares. The SEAQ, the Stock Exchange Automated Quotations System, was introduced in the same period. SEAQ is an electronic information service and has become the primary means of disseminating information on trade in UK securities".

Table 7.1

## The Underlying Shares of Call Options

Company	Industry	DM'
Barclays	Banks	BARC
British Petroleum Co.	Oil & Gas	BCHM
Beecham Group	Health & Household	BP
Consolidated Gold Fields	Mining Finance	CGLD
Courtaulds	Textiles	CTLD
Commercial Union Assurance	Insurance	CUAC
General Electric Co.	Electronics	GEC
GKN	Motor Components	GKN
Grand Metropolitan	Brewers & Distillers	GMET
Imperial Chemical Industries	Chemicals	ICI
Land Securities	Property	LAND
Lonrho	Overseas Traders	LRHO
London & Scottish Marine Oil	Oil & Gas	LSMR
Marks and Spencer	Stores	MKS
P & O Steam Navigation Co.	Shipping & Transport	PO
Racal Electronics	Electronics	RCAL
RTZ Corp.	Mining Finance	RTZ
Shell Transport & Trading Co.	Oil & Gas	SHEL

\*DM denotes Datastream Mnemonic

Table 7.2

## Call Option Maturity Dates

1986	Oct 22	Nov 19	Dec 17
1987	Jan 21	Feb 18	Mar 18
	Apr 22	May 27	Jun 24
	Jul 22	Aug 19	Sep 23
	Oct 21	Nov 18	Dec 16
1988	Jan 20	Feb 17	Mar 16
	Apr 20	May 18	Jun 15

Table 7.3

## Classification of Filtered Data (Number of series)

Trading Activity	88 Data			87 Data		
	Number	$\tau$	P(%)	Number	$\tau$	P(%)
Frequent	288	74	53	201	72	54
Infrequent	112	139	95	85	144	96
Total	400			286		

Table 7.4a (87 data)

Frequently Traded Call Option Series (number of prices)  
After Being Filtered by the Boundary Conditions

Series	Frequently Traded Call Option Prices	Prices Violated the Condition		Prices Satisfying Boundary Conditions
		Immediate Exercise	Early Exercise	
BARC	351	0.0%	30.2%	69.8%
BCHM	829	0.6	40.3	59.1
BP	856	3.5	20.1	76.4
CGLD	890	1.8	21.3	76.9
CTLD	621	1.1	25.0	73.9
CUAC	799	0.9	36.0	63.1
GEC	641	0.2	0.5	99.4
GKN	687	0.4	0.3	99.3
GMET	550	0.4	0.5	99.1
ICI	1151	1.5	27.8	70.7
LAND	575	1.6	26.4	72.0
LRHO	491	1.0	49.9	49.1
LSMR	834	2.5	34.7	62.8
MKS	526	0.2	0.4	99.4
PO	702	0.7	27.5	71.8
RCAL	816	1.0	22.1	77.0
RTZ	931	2.8	40.0	57.3
SHEL	964	3.0	24.6	72.4
Total	13,214	1.5%	24.5%	74.0%

Table 7.4b (88 data)

Frequently Traded Call Option Series (number of prices)  
After Being Filtered by the Boundary Conditions

Series	Frequently Traded Call Option Prices	Prices Violated the Condition		Prices Satisfying Boundary Conditions
		Immediate Exercise	Early Exercise	
BARC	569	0.2%	35.0%	64.9%
BCHM	1302	1.0	30.2	68.8
BP	1138	2.6	22.1	75.2
CGLD	1507	1.1	24.4	74.6
CTLD	897	0.9	17.3	81.8
CUAC	848	0.8	34.3	64.9
GEC	924	0.1	0.3	99.6
GKN	918	0.7	11.2	88.1
GMET	900	0.4	19.1	80.4
ICI	1766	1.0	33.3	65.7
LAND	695	1.6	21.9	76.5
LRHO	1059	0.9	46.6	52.5
LSMR	1039	2.3	40.0	57.7
MKS	823	0.1	0.2	99.6
PO	1003	0.6	30.8	68.6
RCAL	891	0.9	28.1	71.0
RTZ	1470	1.8	25.3	72.9
SHEL	1378	2.1	30.4	67.5
Total	19,127	1.1%	25.8%	73.0%

Table 7.5  
Comparison of Call Option Prices  
Before and After Satisfying Boundary Conditions

Period	Number of companies having larger		
	Time to maturity	Percentage of intrinsic values	Call option percentage spread
	after satisfying all boundary conditions		
88 data	4	11	11
87 data	7	9	10

## Chapter 8

# The Implications of Bid-Ask Quotes and Trading Volume on Market Efficiency Tests

### 8.1 Introduction

In this chapter, call option and share bid-ask quotes are used to test the efficiency of the LTOM, with an emphasis on using actively traded call option data. In the efficiency tests, a call option spread is regarded as an implicit trading cost. It is thus necessary to first discuss the determination of call option spreads, the importance of using bid-ask data in empirical tests and their impacts on a thinly traded market such as the LTOM.

The tests in this chapter are novel in that previous research has not used a Black and Scholes hedge portfolio with bid-ask quotes to test market efficiency. In the context of bid-ask data, the hedge ratio, the rules for identifying mispriced call options, the abnormal returns and market efficiency must be redefined.

The empirical results show that when bid-ask quotes are used, the hypothesis that the LTOM is efficient cannot be rejected. However, when mid prices are used, the LTOM is shown to have some residual inefficiency. To examine whether the anomaly will be corrected, efficiency tests are further examined by increasing the holding period from one to 2 through 9 holding days.

The empirical results are then contrasted by comparing ex ante and ex post results and the trading frequency of call option data. Finally, the call and share spreads, without entering into any hedge portfolios, are checked independently against whether they can explain the variations in the abnormal returns through regression analysis.

## 8.2 Call option bid-ask spread

The call option bid-ask spread is the difference between the highest quote to buy and the lowest offer to sell registered in the LTOM. The bid-ask spread represents a major component of the transaction costs faced by an investor who participated in the options markets (Demsetz 1968). Phillips and Smith (1980) point out that an investor who actively seeks to establish a hedge will inevitably incur the expense of the bid-ask spread. This section discusses the determination of call option spreads, the importance of call option spreads in empirical studies and their impacts in the thinly traded LTOM.

The determination of option spreads. Before examining the effect of the bid-ask spread on market efficiency, it is important first to consider the theoretical determination of the option spread. Dawson and Gemmill (1990) argue that as option prices are a derivative of their underlying share prices by frictionless arbitrage, option spreads should also be a derivative of the share spreads and they should be related by:

$$C_{ask} - C_{bid} = N(d_1) (S_{ask} - S_{bid})$$

where  $C_{bid}$  and  $C_{ask}$  are the option market bid and ask prices,  $S_{bid}$  and  $S_{ask}$  are the share market bid and ask prices and  $N(d_1)$  is the hedge ratio. A concise proof for the above equation follows:

Proof.

Let a market-maker sell a call at  $C_{ask}$ . This call can be hedged by buying  $N(d_1)$  shares at  $S_{ask}$  so that the position

$$-C_{ask} + N(d_1) S_{ask}$$

is riskless. Suppose an investor immediately comes in and sells the call. The market-maker buys the call at  $C_{bid}$  earning the option spread of

$$C_{ask} - C_{bid}$$

and at the same time removes the hedge by selling the  $N(d_1)$  shares at  $S_{bid}$ , making a loss of

$$N(d_1) (S_{ask} - S_{bid}).$$

To avoid arbitrage, the option spread should therefore equal a fraction  $N(d_1)$  of the share spread.

As a hedge ratio is a positive number between zero and one, a call option spread is a positive number between zero and the underlying share spread. However, call option spreads in the LTOM are often larger than the "frictionless" size. Dawson and Gemmill respond to this fact by pointing out that: (1) the market-maker has to cover fixed costs and some order-processing costs. These include membership costs

of the exchange and the cost of using a clearing agent; (2) informed investors may prefer to use options rather than shares due to the gearing advantage. As the market-maker may face the adverse selection problem, they therefore widen the option spread.

The importance of bid-ask spreads. In previous studies on market efficiency using arbitrage portfolios, the bid-ask spread was generally represented implicitly by a transaction cost estimate. Most of the published research on the efficiency of the options market have used closing prices and not bid-ask quoted prices for both options and their underlying shares. Galai (1977, 1978), Chiras and Manaster (1978), and Klemkosky and Resnick (1979) all reported that their trading rules earned positive average profits. However, Phillips and Smith (1980), after subtracting the average spread from the average profits reported in each of these studies, found that the adjustments appear to eliminate all average profits and the hypothesis that the options market is efficient cannot be rejected. But, Phillips and Smith's simple adjustment has the problem that it assumes that the bid-ask spread on the share and call option of each company is identical.

In the UK, Gemmill and Dickens (1986) note that because trading on the LTOM is thin, the bid-ask spread could be as high as 40% of the mid price. In this study, it is found that the average call option percentage spread varies over time and from one share to another. For instance, the

average percentage spread of call options of Courtauld and GEC were 17% and 22% for the 87 data but changed to 19% and 23% for the 88 data. The average call option percentage spread of the eighteen shares ranges from 7% to as high as 34% of the mid price. Therefore, Phillips and Smith's method of using the absolute spread to test market efficiency was very crude.

In this study, the variable bid-ask spread is incorporated into the data used in the tests. The market call option bid-ask quotes are used to compare with the hypothesised bid and ask values so as to identify mispriced call options. Market quotations of the call option and its underlying share are then used to form hedge portfolios and thereby calculate the abnormal returns. In this way, the transaction cost implicit in the spread is accounted for in the return from the hedges. If the abnormal returns are negative, the hypothesis that the LTOM is efficient cannot be rejected.

The impact of call option spreads in the LTOM. Demsetz (1968), Copeland and Galai (1983), Nisbet and Dickinson (1987), and Neal (1987) have suggested that bid-ask spreads tend to be wider for thinly traded securities. Given this association it is natural to examine whether any observed inefficiency or efficiency in the LTOM may be influenced by the greater bid-ask spread on options which are thinly traded. One way of examining this is to partition call options on the basis of trading frequency.

### 8.3 Methodology

This study uses call option and share bid-ask quotes directly in forming the Black-Scholes hedge portfolio. The spreads are regarded as implicit trading costs. The definitions of the hedge ratio, the rules for identifying mispriced call options, abnormal returns and market efficiency must be defined to take account of the bid-ask quotes. The efficiency tests are based on ex ante trading rules which are discussed first as follows.

#### 8.3.1 Ex ante test

In testing market efficiency, Galai (1977) distinguishes between ex post and ex ante trading rules. An ex post rule assumes that trades can be carried out at the same prices which generate the mispricing signal. However, Galai argues that this does not imply market inefficiency. He points out that the dealer has to place orders in both the option and share markets to exploit the short-lived opportunity. However, the prices at the next available transaction might have become unfavourable for the previously indicated opportunity. Therefore an ex ante rule assumes that trades can use prices only after some time has elapsed, e.g., one day later.

Phillips and Smith (1980, p.186) point out another problem that ex post tests are also more likely to use non-contemporaneous data of the option and the share to generate abnormal returns. In using prices at which investors might not actually have been able to trade, error and bias are

introduced. The results of the bias might suggest a degree of market inefficiency that could not in practice be corrected by appropriate trading. They suggest that the source of such bias can be circumvented by employing an ex ante test and be reduced by using bid-ask quotes as signals for the trading rule.

### 8.3.2 Definition of the hedge ratio

In this study, the Black-Scholes model adjusted for one dividend is adopted to identify the mispricing of call options:

$$C_t = \hat{S}_t N(d_1) - XN(d_2)e^{-r\tau} \quad (2.1)$$

where

$$\begin{aligned} \hat{S}_t &= S_t - De^{r\tau}, \\ d_1 &= \frac{\ln\left(\frac{\hat{S}_t}{X}\right) + (r + \sigma^2)T}{\sigma\sqrt{T}}, \\ d_2 &= d_1 - \sigma\sqrt{T}, \end{aligned}$$

and  $t < t_1 < T$ ,  $\tau = t_1 - t$ ,  $t_1$  is the ex-dividend date. Therefore, the following tests of market efficiency are jointly tests of the validity of this model.

When share bid-ask quotes are used to derive call values, there is a dilemma on using the appropriate hedge ratio in a hedged position. Bhattacharya (1983, pp.166, 173) shows that if the model call option bid value is derived from the share market bid price, i.e.,

$$C_{bid}^M = \hat{S}_{bid} N(d_1) - X N(d_2) e^{-rT} \quad (8.1)$$

then the hedge ratio in equation (8.1) defined by

$$N(d_1)_{bid} = \frac{\partial C_{bid}^A}{\partial S_{bid}^A}$$

represents the amount of a change in the call market bid price caused by a unit change in the market bid price of the share. The above relationship holds also for ask prices.

Now, suppose a call option is underpriced, the appropriate hedged position is to long the call option and to short a number of shares determined by the hedge ratio:

$$C_{ask,t+1}^A - N(d_1) S_{bid,t+1}. \quad (8.2)$$

However, the short position in the share bid price is only a perfect hedge for a long position in the call bid price. This implies that the hedged position (8.2) may not be riskfree. Similarly, the hedged position is not riskfree if the hedge ratio is derived from the share ask price.

To solve the dilemma, it is proposed that the hedge ratio be derived from the mid price of the share for two reasons. Firstly, the mid price of the share is hypothetically purged of the bid-ask spread and is an increasing function of the share, i.e.,

$$\frac{\partial N(d_1)}{\partial S} = \frac{1}{\sqrt{2\pi}} \frac{\partial}{\partial S} \int_{-\infty}^{d_1} e^{-x^2/2} dx = \frac{1}{\sqrt{2\pi}} e^{-d_1^2/2} \frac{\partial d_1}{\partial S} > 0$$

implying that

$$N(d_1)_{\text{bid}} < N(d_1)_{\text{mid}} < N(d_1)_{\text{ask}}.$$

The hedge ratio  $N(d_1)_{\text{mid}}$  will closely simulate the response of the call option price to a change in either the bid or the ask price of the share. In addition, Roll (1977, p.1128) suggests that the centre of the spread could be inferred to as the equilibrium value. He points out that:

*"When news arrives, both the bid and the ask prices move to different levels such that their average is the new equilibrium value."*

Secondly, in the hedge portfolio (8.2), the bid-ask spreads have already been accounted for in the buying and selling of the call option and the share and should not be double counted in the hedge ratio.

### 8.3.3 Rules for identifying mispriced call options

The rules for identifying mispriced call options are defined for both mid prices and bid-ask quotes data.

Mid prices. A call option is considered underpriced if the actual (i.e., market) mid price is less than the hypothetical value and considered as overpriced if the actual mid price is larger than the hypothetical value, i.e., if

$$C_{\text{mid}}^A < C_{\text{mid}}^M, \quad \text{or} \quad C_{\text{mid}}^A > C_{\text{mid}}^M.$$

Since a call option model value will rarely equal the market price, this implies that virtually all call options will be mispriced to a greater or lesser extent.

Bid-ask quotes. A call option is considered underpriced if the actual ask price is less than the model bid price and overpriced if the actual bid price is higher than the model ask price, i.e.,

$$C_{ask}^A < C_{bid}^M \quad \text{and} \quad C_{bid}^A > C_{ask}^M$$

respectively. If the ranges of actual bid and ask prices are not either completely less than or completely more than the range of model bid and ask values, then they represent cases in which arbitrage is not possible (Bhattacharya 1987, p.6). If the actual and model price ranges overlap, it is inferred that the market is efficient with respect to the option pricing model. The mispricing criteria for bid-ask quotes are more stringent than those for mid prices in that the mispricing instances will be greatly decreased.

#### 8.3.4 Definitions of abnormal returns

Abnormal returns are defined for both mid prices and bid-ask data.

Mid prices. If a call option is considered as underpriced on day  $t$ , a buy strategy is indicated. The call option is bought and  $N(d_t)$  shares are sold short on day  $t+1$ .

The position is liquidated on day t+2. The initial position is defined as

$$H = C_{mid,t+1}^A - N(d_{1mt}) S_{bid,t+1}$$

For US markets, H is taken as the initial investment of the hedge. However, in the UK, most of the transactions in shares are for settlement on the Account Day (Thomas 1989). The implication of the Account period for the hedged position is that because the position is held for only one day, if it is assumed that all hedges can be formed on the day not immediately preceding the Account Day, then the share price can be ignored in calculating the initial investment, i.e., the initial investment equals the mid price of the call on day t+1. Let

$$IR = C_{mid,t+1}^A r / 365$$

represent the investment of the call price on the riskfree interest rate for one day, then the excess abnormal ex ante return is equal to

$$ER = (C_{mid,t+2}^A - C_{mid,t+1}^A) - N(d_{1mt}) (S_{mid,t+2} - S_{mid,t+1}) - IR$$

and the excess abnormal percentage return (abnormal return) is defined as ER divided by the mid price of the call on day t+1.

If a call option is considered overpriced on day t, this suggests a sell strategy. The call option is written and  $N(d_1)$  shares are bought on day t+1. The position is liquidated on t+2. The initial position is defined as

$$- C_{\text{mid},t+1}^A + N(d_{1\text{mt}}) S_{\text{mid},t+1}.$$

As argued above, the share price can be ignored in calculating the initial investment. However, the LTOM stipulates that the short position in the call option must be margined to ensure that the contract would be honoured on exercise. The margin required by the LTOM is calculated by taking 20% of the share price and adjusting the resulting figure by reference to the intrinsic value of the call option, i.e., either adding on the amount by which the option is in-the-money, or subtracting the amount by which it is out-of-the-money:

$$\begin{aligned} \text{Margin} &= 20\%S + (S - X) \\ &= 1.2S - X. \end{aligned}$$

For the purpose of calculating the margin, the price of the underlying share will be the closing mid price. The Council of the LTOM has ruled that a member firm should not pay interest on cash margin received from a short call option contract. The minimum margin is three percent of the underlying mid share price. A compact expression for the margin is therefore

$$\text{Margin} = \text{Max} (1.2S_{\text{mid},t+1} - X, 3\%S_{\text{mid},t+1})$$

This is thus taken as the initial investment. From this it follows that

$$\begin{aligned} \text{IR} &= \text{Max}(1.2S_{\text{mid},t+1} - X, 3\%S_{\text{mid},t+1}) r/365, \quad \text{and} \\ \text{ER} &= - (C_{\text{mid},t+2}^A - C_{\text{mid},t+1}^A) + N(d_{1\text{mt}}) (S_{\text{mid},t+2} - S_{\text{mid},t+1}) - \text{IR}. \end{aligned}$$

The abnormal return therefore equals ER/Margin.

Bid-ask quotes. The definitions for hedges using bid-ask quotes are similar to those above except that the ask price is used for a buy strategy and the bid price is used for a selling strategy.

If a call option is considered underpriced, then

$$H = C_{ask,t+1}^A - N(d_{1mt}) S_{bid,t+1}, \quad IR = C_{ask,t+1}^A r/365, \quad \text{and}$$

$$ER = (C_{bid,t+2}^A - C_{ask,t+1}^A) - N(d_{1mt}) (S_{ask,t+2} - S_{bid,t+1}) - IR,$$

and the abnormal return is defined as ER divided by the actual ask price of the call on day t+1. If a call option is considered overpriced, then

$$H = -C_{bid,t+1}^A + N(d_{1mt}) S_{ask,t+1},$$

$$\text{Margin} = \text{Max}(1.2S_{mid,t+1} - X, 3\%S_{mid,t+1}),$$

$$IR = \text{Margin}(r/365), \quad \text{and}$$

$$ER = - (C_{ask,t+2}^A - C_{bid,t+1}^A) + N(d_{1mt}) (S_{bid,t+2} - S_{ask,t+1}) - IR,$$

and the abnormal return = ER/Margin.

### 8.3.5 Definition of market efficiency

The efficiency of the LTOM is tested by examining the distributions of the abnormal returns generated by share-option hedges. According to Jensen (1978), market efficiency implies that the economic profits from trading are zero, where economic profits are risk-adjusted returns net of all

costs. A more intuitive definition by Gemmill and Dickens (1986) is that efficiency implies above normal risk-adjusted profits cannot be made. If the abnormal return are not larger than the riskfree rate, then the LTOM cannot be rejected as efficient.

#### 8.4 Results

The empirical results of market efficiency tests are discussed with reference to the implications of bid-ask quotes, the trading volume and the two data sub-samples.

The Kolmogorov-Smirnov tests (Note 8.1) show that the distributions of abnormal returns for the 88 data are significantly different from those for the 87 data for eight out of the eighteen companies (Table 8.1). Thus it is necessary to discuss the empirical results for each sub-sample.

##### 8.4.1 Results with frequently traded calls and mid prices

Using mid prices, the empirical results show some residual market inefficiency in the LTOM.

For the 87 data, the mean abnormal returns for fourteen companies are, although positive, insignificantly different from zero. The prices of these fourteen companies thus appear to be priced efficiently. The four exceptions (not efficient) are those of GEC and LAND which are significantly positive, and those of LRHO and PO which are negative but not significant. The value of the mean abnormal returns for all eighteen companies lies within the interval [-0.88%, 1.76%].

For the 88 data, the mean abnormal returns for fourteen companies are, although positive, insignificantly different from zero. Thus the prices of these fourteen companies appear to be priced efficiently. However, the other four (GEC, LAND, MKS and RTZ) are all significantly positive (not efficient). The value of the mean abnormal returns lies within the interval [0%, 1.28%]. Therefore the prices of four out of the eighteen companies are not efficient.

The results of both data sub-samples (Table 8.2) therefore suggest that the market was relatively inefficient when mid prices are used in the tests.

#### 8.4.2 Results with frequently traded calls and bid-ask quotes

When bid-ask quotes are used, no hedge portfolio can earn more than the riskfree interest rate and therefore the hypothesis that the LTOM is efficient cannot be rejected.

For the 87 data, the mean abnormal returns for all eighteen companies are all significantly negative and lie within the interval [-33.01%, -6.28%]. The mean abnormal returns for the 88 data are also all significantly negative and lie within the interval [-32.14%, -10.36%]. These results (Table 8.3) strongly indicate that, for both data sub-samples, the hypothesis that the LTOM is efficient cannot be rejected when bid-ask quotes are accounted for even when transaction costs are negligible. This implies that a trader could not earn riskfree profits by forming Black-Scholes type hedges.

#### 8.4.3 Results with infrequently traded calls and mid prices

Some of the hedge portfolios earn more than the riskfree interest rate and therefore the market has some residual inefficiency.

For both the 87 and 88 data, the mean abnormal returns have various signs and are not significantly different from zero for seventeen companies. The only exceptions are that of SHEL for the 87 data (significantly negative) and that of GEC for the 88 data (significantly positive) (Table 8.4). This implies that the LTOM has some inefficiency before the bid-ask spread is used with infrequently traded call options.

#### 8.4.4 Results with infrequently traded calls and bid-ask quotes

The results show that the mean abnormal returns are all significantly negative for all eighteen companies and for both data sub-samples (Table 8.5), implying that the hypothesis that the LTOM is efficient cannot be rejected.

#### 8.4.5 Concluding remarks for efficiency tests

The implications of using bid-ask quotes and mid prices on efficiency tests are distinct in two aspects.

Market efficiency. Using mid prices, the empirical results show that the LTOM has some residual inefficiency. On the other hand, using bid-ask quotes, the LTOM is shown to be efficient, even when transaction costs are negligible.

Distributions of abnormal returns. The Kolmogorov-Smirnov tests show that the distributions of abnormal returns generated by the mid prices are significantly different from those generated by the bid-ask quotes for all eighteen companies (Tables 8.6a, 8.6b).

Most of the distributions generated by mid prices have a mean centred around zero and are skewed to the right. On the other hand, the distributions of abnormal returns generated by bid-ask quotes have negative means and are skewed to the left, i.e., a hedge portfolio with bid-ask quotes will likely earn less than the riskfree interest rate (Figures 8.1a, 8.1b, 8.2a, 8.2b).

The above two aspects hold true for both frequently and infrequently traded call options, and for both 87 data and 88 data sub-samples, i.e., the results are independent of the trading activity of call options and the crash effect. The data therefore strongly suggest that the mid prices and the bid-ask quotes have completely different implications on market efficiency tests.

#### 8.5 Market efficiency tests over increasing holding periods

It was shown in the last section that the market is relatively inefficient when mid prices are used in the hedge portfolios for one holding day. It might be because it was not possible to correct the inefficiency immediately. As time passed, investors and traders will spot the anomaly and trade in such a way as to correct the situation. Thus it is expected that the market will become more efficient as the

holding period of a hedge portfolio increases.

Let a call option be identified as mispriced on day  $t$ . Based on this, let a hedge portfolio using *mid* prices be formed on day  $t+1$  and held for 2 to 9 days.

Empirical results show that the prices of most companies are efficient or become efficient at a later holding period. Most of the companies are efficiently priced when analysed over all 9 holding periods. The mean abnormal returns of LRHO and PO are significantly negative and the those of BP, CTLD, CUAC, GKN and RCAL are all not significantly different from zero at every holding lag. The previous price inefficiencies of the companies BARC, CGLD, GEC, GMET are corrected starting at the ninth, fifth, eighth and third holding periods respectively.

However, because bid-ask spreads were not accounted for, a third of the companies has price inefficiencies over the nine holding periods (Table 8.7a). The mean abnormal returns of ICI, LAND, LSMR, MKS, RTZ and SHEL are significantly positive. But, when bid-ask quotes are used to form the hedge portfolios, the mean abnormal returns of all eighteen companies become all significantly negative at every holding lag (Table 8.7b). Thus, the apparent inefficiency must have been caused by not considering the bid-ask quotes.

## 8.6 Discussions on the empirical design and results

The empirical tests on efficiency tests are conducted ex ante and emphasise using bid-ask quotes and trading volume. This section contrasts between ex ante and ex post results, results generated by bid-ask quotes and mid prices, and frequently and infrequently traded call options data. It is pointed out that hedge portfolios for one holding day need not be adjusted for risk. Finally, it is found that the call option spreads significantly explain the variations in the abnormal returns.

### 8.6.1 Comparison between ex post and ex ante results

The above empirical results are conducted ex ante. It is useful to compare these results with ex post results for both mid prices and bid-ask quotes data.

Mid prices. If the hedge is changed from an ex ante to an ex post basis, the mean abnormal returns of 10 companies change from insignificantly positive to significantly positive (Table 8.8a). This strong evidence indicates that the LTOM cannot be accepted as being efficient when mid prices are used in the ex post tests. This result is not surprising for if a call option is identified as mispriced and a hedged position is executed immediately, the abnormal return likely will be more positive, particularly when the bid-ask spread is not considered. However, Phillips and Smith (1980, p.186) point out that these results are spurious because ex post tests do not represent an

implementable trading rule and thus are subject to selection bias. An ante test better ensures that observed prices are synchronous and implementable.

Bid-ask quotes. The ex post mean abnormal returns generated by bid-ask quotes are all significantly negative for all 18 companies, i.e., same as the ex ante results (Table 8.8b). Thus, the hypothesis that the LTOM is efficient cannot be rejected.

#### 8.6.2 Comparison between results generated by frequently and infrequently traded call options data

The distributions of abnormal returns generated by frequently and infrequently traded call options bid-ask quotes are shown by the Kolmogorov-Smirnov tests to be significantly different (Table 8.9). Frequently traded call option data have a shorter time to maturity and/or close to at-the-moneyness. Infrequently traded call options might not have been traded and have a higher likelihood that the call and share prices are not synchronous. Therefore, only the frequently traded call option bid-ask quotes will be used in the subsequent tests. (The only exception is in the examination of the thin trading issue in the next chapter.)

#### 8.6.3 Diversification of hedge portfolios

The hedge portfolios do not need to be adjusted by any risk factor. It is found that the correlations of abnormal returns with the FTSE 100 Index returns are insignificant

for all eighteen companies. All correlation coefficients lie within the interval  $[-0.268, 0.145]$ . Equivalently, the regression between the abnormal returns and the FTSE 100 Index returns are in general characterised by negative and insignificant  $\beta$ 's for which

$$\text{Abnormal return} = \alpha + \beta(\text{FTSE 100 Index return})$$

with the R-squareds lying in the interval  $[0\%, 7.2\%]$ .

The above results hold for both the 87 and 88 data sets (Tables 8.10a, 8.10b). Therefore the abnormal returns have almost no association with the UK market. As a result, the risk of all hedges can be diversified away by forming a large number of such hedges in a portfolio.

#### 8.6.4 The call option spreads significantly explain the abnormal returns

The efficiency tests show that the mean abnormal returns of all eighteen companies are significantly negative when bid-ask data are built into the hedge portfolios. Another way to affirm the impact of the bid-ask data is to examine the linear relationship between the abnormal returns and the call and share spreads.

Call spreads and abnormal returns. The relation between the abnormal returns and the call option spreads is first examined by the regression:

$$\text{Abnormal return} = \alpha + \beta(\text{call percentage spread}).$$

For the 87 data, the results (Table 8.11a) show that  $\beta$  is negative for seventeen out of eighteen companies. Of the seventeen negative  $\beta$ 's, fourteen are significantly different

from zero. Because the abnormal returns are negative, a wider call option spread will lead to a more negative abnormal return. This is justifiable as spreads are like transaction costs. Thus a larger spread will cause a hedge to be less profitable. However, the R-squareds of the regression lie between a very wide interval of [0%, 53.7%] indicating that a linear model might not have adequately described the relationship between them or that there are some other explanatory variables. The results for the 88 data (Table 8.11b) show that  $\beta$  is significantly negative for 15 companies and the range of the R-squareds is [0.3%, 50.1%]. Thus, for both data sub-samples, a larger call spread will induce a more negative abnormal return.

Call and share spreads and abnormal returns. Secondly, the share spread is added in as an explanatory variable in the regression equation, i.e.,

$$\text{Abnormal return} = \alpha + \beta(\text{call \% spread}) + \gamma(\text{share \% spread}).$$

For the 87 data, the empirical results show only a slight improvement of the R-squareds from the original range of [0%, 53.7%] (contributed by call option spreads alone) to [1.4%, 54.4%] with only ten companies having simultaneous significantly negative  $\beta$ 's and  $\gamma$ 's (Table 8.12a). For the 88 data, only four companies have simultaneous significantly negative  $\beta$ 's and  $\gamma$ 's (Table 8.12b). Therefore, the abnormal returns are mainly explained by call option spreads. However, it can be accepted that, in general, the abnormal returns are inversely related to both the call and share spreads.

## 8.7 Concluding remarks

In this chapter, call option and share bid-ask quotes have been used to identify mispriced call options. This study argues that *ex ante* tests be adopted in efficiency tests. The empirical results show that using mid prices, there is some residual market inefficiency in the LTOM. On the other hand, using bid-ask quotes, the hypothesis that the LTOM is efficient cannot be rejected, even when transaction costs are negligible. Most of the price inefficiencies with mid prices are corrected as the holding periods of the hedge portfolios lengthens. It is further found that persistent inefficiencies over holding periods are accounted for by the fact that bid-ask quotes are not used.

If the hedge portfolios using mid prices are changed from an *ex ante* to an *ex post* basis, the mean abnormal returns of 10 companies change from insignificantly positive to significantly positive. This strong evidence indicates that the LTOM cannot be accepted as efficient when mid prices are used in the *ex post* tests. The *ex post* mean abnormal returns generated by bid-ask quotes are all significantly negative for all 18 companies, i.e., same as the *ex ante* results. Thus, using bid-ask data, the hypothesis that the LTOM is efficient cannot be rejected, either the test is conducted *ex ante* or *ex post*.

Frequently traded call option data have a shorter time to maturity and/or close to at-the-moneyness. Infrequently traded call options might not have been traded and have a

higher likelihood that the call and share prices are not synchronous. Therefore, only the frequently traded call option bid-ask quotes were used in the subsequent tests.

It is found that, in general, the abnormal returns are inversely related to both the call and share percentage spreads but are significantly explained only by the call option percentage spreads.

In analysing these data, it would be useful to consider how, in practice, an investor would have identified the anomalies and how he might have tried to exploit them. The investor can choose call options which are near-the-money and use the Black-Scholes model with dividend correction to derive a 'fair' value for the call options. A call option can be identified as underpriced if the market ask price is less than the model bid value and as overpriced if the market bid price is larger than the model ask value. A hedge portfolio can then be set up and held until the call option is correctly priced. However, the empirical results in this study show that, because of the large bid-ask spreads observed in the LTOM, it would be very difficult to exploit any price inefficiencies.

## Note

### Note 8.1 The Kolmogorov-Smirnov test (The KS test)

The Kolmogorov-Smirnov two-sample test is adopted to distinguish two distributions of abnormal returns. This non-parametric statistical test is used because abnormal returns are not normally distributed. A brief outline of the test is as follows.

Assume that the data for analysis consist of two independent random samples of sizes  $m$  and  $n$ . Let the observations be measured on an ordinal scale and denoted by  $X_1, X_2, \dots, X_m$  and  $Y_1, Y_2, \dots, Y_n$  respectively. Let  $F_1(x)$  and  $F_2(x)$  denote the unknown distribution functions of the  $X$ 's and  $Y$ 's respectively. The two-sided hypothesis is to test:

$$H_0: F_1(x) = F_2(x) \text{ for all } x,$$

$$H_1: F_1(x) \neq F_2(x) \text{ for at least one value of } x.$$

The test statistic is

$$D = \text{Max } |S_1(x) - S_2(x)|$$

where  $S_1(x) = (\text{number of observed } X\text{'s} \leq x) / m,$

$S_2(x) = (\text{number of observed } Y\text{'s} \leq y) / n.$

If the two samples have been drawn from identical populations,  $S_1(x)$  and  $S_2(x)$  should be fairly close for all values of  $x$ .  $D$  is a measure of the extent to which  $S_1(x)$  and  $S_2(x)$  fail to agree. If  $D$  is sufficiently small, then differences at all other values of  $X$  are also small and  $H_0$  is supported. Otherwise,  $H_0$  is rejected. The function which enters into the calculation of the significance ( $\alpha$ ) can be

written as the following sum (Press et al 1986, p.473)

$$\alpha = Q_{ks}(\lambda) = 2 \sum_{j=1}^{\infty} (-1)^{j-1} e^{-2j^2\lambda^2} \text{ where } \lambda = D \sqrt{\frac{mn}{m+n}}$$

which is a monotonic function with the limiting values

$$Q_{ks}(0) = 1, \quad Q_{ks}(\infty) = 0.$$

The null hypothesis is rejected for  $\alpha = Q_{ks}(\lambda) < 0.05$ .

Table 8.1

The Distributions of Abnormal Returns for the (87 data and 88 data sub-samples) are Significantly Different

Share	Number of Prices		The KS Test	
	1988	1987	$\lambda$	$Q_{KS}(\lambda)$
BARC	85	55	0.90	0.39
BCHM	400	149	1.60	0.01*
BP	369	268	1.39	0.04*
CGLD	640	386	4.05	0.00*
CTLD	364	201	1.52	0.02*
CUAC	179	164	0.28	1.00
GEC	389	233	0.88	0.43
GKN	320	243	1.02	0.25
GMET	220	156	0.33	1.00
ICI	482	271	2.00	0.00*
LAND	181	111	0.82	0.50
LRHO	198	70	2.39	0.00*
LSMR	212	180	0.19	1.00
MKS	313	125	2.00	0.00*
P. & O.	304	264	0.31	1.00
RCAL	204	204	0.05	1.00
RTZ	595	225	6.70	0.00*
SHEL	482	356	0.99	0.28

\*significantly different at the indicated level.

Table 8.2

Mean Abnormal Returns Generated by Mid Prices  
Frequently Traded Call Options

Share	87 Data			88 Data		
	m	Mean	t-ratio	n	Mean	t-ratio
BARC	355	0.0096	1.16	550	0.0041	0.68
BCHM	717	0.0010	0.20	1155	0.0003	0.08
BP	620	0.0008	0.18	889	0.0016	0.44
CGLD	663	0.0130	1.22	1247	0.0116	1.95
CTLD	543	0.0005	0.07	811	0.0011	0.21
CUAC	830	0.0040	0.73	865	0.0037	0.71
GEC	672	0.0128	3.30 <sup>*</sup>	980	0.0126	4.02 <sup>*</sup>
GKN	762	0.0004	0.06	970	0.0000	0.00
GMET	546	0.0065	0.62	847	0.0038	0.54
ICI	962	0.0061	1.03	1544	0.0025	0.61
LAND	453	0.0150	2.51 <sup>*</sup>	556	0.0123	2.47 <sup>*</sup>
LRHO	397	-0.0088	-1.14	918	0.0068	0.66
LSMR	639	0.0176	1.92	798	0.0128	1.58
MKS	569	0.0096	1.95	879	0.0084	2.16 <sup>*</sup>
P. & O.	544	-0.0032	-0.45	820	0.0018	0.34
RCAL	774	0.0070	0.46	850	0.0059	0.42
RTZ	629	0.0033	1.39	1679	0.0068	2.42 <sup>*</sup>
SHEL	671	0.0011	0.24	1068	0.0055	1.38

Note: m,n denote number of prices.

<sup>\*</sup>significantly different from zero at 0.05 level.

Table 8.3

Mean Abnormal Returns Generated by Bid-ask Quotes  
Frequently Traded Call Options

Share	87 Data			88 Data		
	m	Mean	t-ratio	n	Mean	t-ratio
BARC	55	-0.2877	-9.22 <sup>*</sup>	85	-0.2422	-10.24 <sup>*</sup>
BCHM	149	-0.2687	-11.28 <sup>*</sup>	400	-0.2131	-15.95 <sup>*</sup>
BP	268	-0.1037	-10.61 <sup>*</sup>	369	-0.1036	-14.16 <sup>*</sup>
CGLD	386	-0.0760	-14.98 <sup>*</sup>	640	-0.1350	-21.98 <sup>*</sup>
CTLD	201	-0.2423	-13.97 <sup>*</sup>	364	-0.1961	-18.33 <sup>*</sup>
CUAC	164	-0.1532	-10.64 <sup>*</sup>	179	-0.1520	-11.47 <sup>*</sup>
GEC	233	-0.1648	-19.11 <sup>*</sup>	389	-0.1676	-27.40 <sup>*</sup>
GKN	243	-0.2909	-17.29 <sup>*</sup>	320	-0.2573	-18.69 <sup>*</sup>
GMET	156	-0.1315	-11.98 <sup>*</sup>	220	-0.1271	-15.08 <sup>*</sup>
ICI	271	-0.2166	-15.10 <sup>*</sup>	482	-0.1656	-17.68 <sup>*</sup>
LAND	111	-0.1373	-8.10 <sup>*</sup>	181	-0.1351	-12.12 <sup>*</sup>
LRHO	70	-0.0828	-4.61 <sup>*</sup>	198	-0.1130	-3.42 <sup>*</sup>
LSMR	180	-0.1169	-7.84 <sup>*</sup>	212	-0.1205	-9.30 <sup>*</sup>
MKS	125	-0.1570	-9.28 <sup>*</sup>	313	-0.1818	-17.65 <sup>*</sup>
P. & O.	264	-0.3301	-18.49 <sup>*</sup>	304	-0.3214	-19.89 <sup>*</sup>
RCAL	204	-0.2821	-16.93 <sup>*</sup>	204	-0.2821	-16.93 <sup>*</sup>
RTZ	225	-0.0628	-17.41 <sup>*</sup>	595	-0.2909	-20.31 <sup>*</sup>
SHEL	356	-0.1150	-12.98 <sup>*</sup>	482	-0.1127	-15.46 <sup>*</sup>

Note: m,n denote number of prices.

<sup>\*</sup>significantly different from zero at 0.05 level.

Table 8.4

Mean Abnormal Returns Generated by Mid Prices  
Infrequently Traded Call Options

Share	87 Data			88 Data		
	m	Mean	t-ratio	n	Mean	t-ratio
BARC	68	0.0025	0.46	155	0.0031	0.95
BCHM	501	-0.0010	-0.44	1181	-0.0014	-0.54
BP	102	-0.0013	-0.32	263	0.0027	1.00
CGLD	654	-0.0026	-1.95	834	-0.0014	-1.09
CTLD	462	0.0034	0.98	568	0.0033	1.13
CUAC	544	-0.0025	-0.95	706	-0.0014	-0.61
GEC	394	0.0043	1.70	481	0.0052	2.33*
GKN	200	0.0013	0.48	285	0.0016	0.72
GMET	380	0.0022	0.91	430	0.0018	0.79
ICI	619	-0.0018	-0.90	691	-0.0011	-0.56
LAND	139	0.0038	1.24	321	0.0024	1.30
LRHO	264	0.0004	0.19	318	0.0010	0.50
LSMR	711	0.0146	1.76	877	0.0123	1.82
MKS	166	0.0025	0.88	251	0.0035	1.49
P. & O.	104	-0.0020	-0.43	176	0.0025	0.72
RCAL	207	0.0059	1.70	335	0.0022	0.79
RTZ	411	-0.0004	-0.23	411	-0.0004	-0.23
SHEL	397	-0.0039	-2.21*	477	-0.0021	-1.00

Note: m, n denote number of prices.

\*significantly different from zero at 0.05 level.

Table 8.5

Mean Abnormal Returns Generated by Bid-ask Quotes  
Infrequently Traded Call Options

Share	87 Data			88 Data		
	m	Mean	t-ratio	n	Mean	t-ratio
BARC	3	-0.1559	-2.23*	4	-0.1435	-2.82*
BCHM	198	-0.0996	-19.91*	522	-0.1553	-16.38*
BP	7	-0.0448	-3.18*	13	-0.0713	-4.20*
CGLD	129	-0.0718	-14.82*	154	-0.0892	-16.30*
CTLD	143	-0.0881	-15.11*	203	-0.0916	-19.44*
CUAC	175	-0.0882	-15.54*	178	-0.0884	-15.82*
GEC	59	-0.0988	-10.91*	90	-0.0954	-14.87*
GKN	30	-0.0634	-8.66*	31	-0.0621	-8.63*
GMET	118	-0.0782	-16.44*	120	-0.0774	-16.47*
ICI	339	-0.0984	-26.35*	342	-0.0980	-26.40*
LAND	38	-0.0841	-8.61*	135	-0.0955	-19.32*
LRHO	7	-0.0222	-10.01*	9	-0.0297	-5.14*
LSMR	52	-0.0410	-11.64*	54	-0.0413	-12.15*
MKS	67	-0.0675	-15.28*	109	-0.0815	-20.29*
P. & O.	4	-0.0274	-3.98*	5	-0.0466	-2.33*
RCAL	45	-0.1045	-16.70*	131	-0.1294	-26.38*
RTZ	133	-0.0574	-15.04*	133	-0.0574	-15.04*
SHEL	336	-0.0706	-18.62*	338	-0.0709	-18.77*

Note: m, n denote number of prices.

\*significant at the 0.05 level.

Table 8.6a

Results Generated by Mid Prices and Bid-ask Quotes

Frequently Traded Call Options

Kolmogorov-Smirnov Tests

Share	87 Data		88 Data	
	$\lambda$	$Q_{KS}(\lambda)$	$\lambda$	$Q_{KS}(\lambda)$
BARC	5.69	0.00	6.83	0.00
BCHM	7.56	0.00	11.67	0.00
BP	8.19	0.00	10.03	0.00
CGLD	9.36	0.00	13.15	0.00
CTLD	8.22	0.00	11.09	0.00
CUAC	7.45	0.00	7.81	0.00
GEC	10.54	0.00	13.44	0.00
GKN	9.62	0.00	10.88	0.00
GMET	7.58	0.00	9.09	0.00
ICI	8.81	0.00	10.84	0.00
LAND	6.43	0.00	8.04	0.00
LRHO	4.73	0.00	7.75	0.00
LSMR	7.31	0.00	8.21	0.00
MKS	7.03	0.00	11.48	0.00
P.&O.	9.22	0.00	10.32	0.00
RCAL	8.72	0.00	8.87	0.00
RTZ	7.99	0.00	15.73	0.00
SHEL	7.57	0.00	9.14	0.00

Note

\*For frequently traded call options, the distributions of abnormal returns generated by mid prices are all significantly different from those generated by bid-ask quotes at the 0.05 level.

Table 8.6b

## Results Generated by Mid Prices and Bid-ask Quotes

## Infrequently Traded Call Options

## Kolmogorov-Smirnov Tests

Share	87 Data		88 Data	
	$\lambda$	$Q_{KS}(\lambda)$	$\lambda$	$Q_{KS}(\lambda)$
BARC	1.45	0.03	1.75	0.00
BCHM	8.35	0.00	13.34	0.00
BP	1.58	0.01	2.40	0.00
CGLD	7.24	0.00	7.97	0.00
CTLD	7.04	0.00	8.51	0.00
CUAC	6.79	0.00	7.21	0.00
GEC	4.94	0.00	6.51	0.00
GKN	3.61	0.00	3.67	0.00
GMET	7.17	0.00	7.16	0.00
ICI	10.66	0.00	10.86	0.00
LAND	4.52	0.00	7.85	0.00
LRHO	2.03	0.00	2.25	0.00
LSMR	4.05	0.00	4.20	0.00
MKS	5.33	0.00	6.91	0.00
P.&O.	1.36	0.05	1.57	0.01
RCAL	0.50	0.96	8.18	0.00
RTZ	6.58	0.00	6.58	0.00
SHEL	6.88	0.00	7.07	0.00

\*significant at the level indicated by  $Q_{KS}(\lambda)$ .

Table 8.7a

## Market Efficiency Tests over Holding Periods (Mid Prices)

Holding lag	n	Mean	t-ratio	n	Mean	t-ratio
	BARC			CGLD		
1	355	0.0096	1.16	663	0.0130	1.22
2	350	0.0091	1.19	656	0.0103	1.82
3	347	0.0089	1.76	652	0.0089	2.44*
4	343	0.0076	1.97	652	0.0086	2.30*
5	338	0.0077	2.00*	649	0.0121	1.82
6	336	0.0089	2.14*	646	0.0146	1.67
7	332	0.0092	2.10*	643	0.0127	1.69
8	329	0.0077	2.07*	641	0.0097	1.77
9	325	0.0067	1.94	638	0.0087	1.51
	BCHM			CTLD		
1	717	0.0010	0.20	543	0.0005	0.07
2	709	0.0002	0.07	534	0.0001	0.02
3	700	-0.0004	-0.20	529	0.0030	0.62
4	689	-0.0017	-0.96	527	0.0046	0.87
5	683	-0.0022	-1.44	525	0.0103	1.52
6	674	-0.0021	-1.52	521	0.0108	1.59
7	666	-0.0015	-1.20	519	0.0110	1.67
8	657	-0.0012	-1.04	515	0.0104	1.70
9	648	-0.0006	-0.59	512	0.0086	1.67
	BP			CUAC		
1	620	0.0008	0.18	830	0.0040	0.73
2	615	0.0009	0.31	820	0.0004	0.13
3	614	0.0005	0.24	813	0.0014	0.57
4	613	0.0010	0.62	804	0.0005	0.26
5	602	0.0018	1.25	793	0.0001	0.04
6	596	0.0018	1.41	781	0.0002	0.12
7	588	0.0015	1.46	775	0.0010	0.57
8	578	0.0011	1.10	769	0.0015	0.90
9	578	0.0011	1.18	760	0.0015	1.02

Note: n denotes number of prices.  
\*significant at 5% level.

Table 8.7a (continued)

## Market Efficiency Tests over Holding Periods (Mid Prices)

Holding lag	n	Mean	t-ratio	n	Mean	t-ratio
	GEC			ICI		
1	672	0.0128	3.34*	962	0.0061	1.03
2	664	0.0093	4.36*	946	0.0062	1.26
3	658	0.0064	4.03*	939	0.0107	2.35*
4	650	0.0031	2.39*	931	0.0128	2.38*
5	642	0.0024	1.85	922	0.0140	2.40*
6	633	0.0022	2.05*	906	0.0153	2.36*
7	625	0.0024	2.30*	899	0.0165	2.55*
8	618	0.0015	1.54	889	0.0193	2.52*
9	611	0.0011	1.21	881	0.0192	2.42*
	GKN			LAND		
1	762	0.0010	0.17	453	0.0150	2.51*
2	753	0.0034	0.95	445	0.0150	3.24*
3	745	0.0026	0.84	438	0.0116	3.25*
4	734	0.0005	0.18	431	0.0085	2.84*
5	727	0.0008	0.40	428	0.0054	2.34*
6	720	0.0017	0.86	423	0.0028	1.52
7	713	0.0010	0.62	415	0.0023	1.48
8	702	0.0007	0.45	408	0.0027	1.89
9	694	0.0008	0.47	406	0.0029	2.02*
	GMET			LRHO		
1	546	0.0065	0.62	397	-0.0088	-1.14
2	541	0.0092	2.21*	393	-0.0042	-1.38
3	536	-0.0002	-0.07	389	-0.0035	-1.65
4	531	-0.0027	-0.76	382	-0.0028	-1.50
5	526	0.0002	0.04	376	-0.0032	-2.12*
6	519	-0.0027	-1.23	368	-0.0029	-1.91
7	513	-0.0037	-2.49*	364	-0.0035	-2.69*
8	506	-0.0041	-3.04*	357	-0.0044	-3.43*
9	499	-0.0039	-2.98*	350	-0.0051	-4.50*

Note: n denotes number of prices.  
\*significant at 5% level.

Table 8.7a (continued)

## Market Efficiency Tests over Holding Periods (Mid Prices)

Holding lag	n	Mean	t-ratio	n	Mean	t-ratio
	LSMR			RCAL		
1	639	0.0176	1.92	774	0.0070	0.46
2	628	0.0138	2.80*	767	0.0088	0.62
3	617	0.0141	3.76*	764	0.0082	0.64
4	611	0.0123	3.46*	756	0.0071	0.60
5	600	0.0113	4.20*	751	0.0038	0.48
6	598	0.0124	5.00*	747	0.0009	0.15
7	589	0.0129	5.43*	743	-0.0003	-0.08
8	579	0.0120	5.31*	738	0.0103	0.77
9	579	0.0110	5.11*	735	0.0116	0.92
	MKS			RTZ		
1	569	0.0096	1.95	629	0.0033	1.39
2	562	0.0071	2.51*	621	0.0038	2.22*
3	556	0.0061	2.63*	614	0.0051	3.30*
4	550	0.0036	1.89	612	0.0051	3.05*
5	543	0.0043	2.85*	608	0.0060	3.67*
6	536	0.0026	1.90	606	0.0059	3.69*
7	530	0.0025	2.26*	599	0.0054	3.80*
8	523	0.0032	3.16*	596	0.0051	4.00*
9	514	0.0029	3.08*	588	0.0049	4.28*
	PO			SHEL		
1	544	-0.0032	-0.45	671	0.0011	0.24
2	536	-0.0035	-0.72	662	-0.0001	-0.04
3	533	-0.0031	-0.79	660	-0.0007	-0.28
4	525	-0.0041	-1.17	657	0.0007	0.31
5	521	-0.0042	-1.37	655	0.0031	1.53
6	517	-0.0047	-1.71	646	0.0047	2.45*
7	511	-0.0047	-1.78	643	0.0047	2.57*
8	505	-0.0043	-1.80	638	0.0057	2.81*
9	499	-0.0043	-2.02*	632	0.0065	3.16*

Note: n denotes number of prices.  
\*significant at 5% level.

Table 8.7b

## Market Efficiency Tests over Holding Periods (Bid-Ask Quotes)

Holding lag	n	Mean	t-ratio	n	Mean	t-ratio
	BARC			CGLD		
1	55	-0.2877	-9.22	386	-0.0760	-14.98
2	53	-0.1359	-8.30	379	-0.0390	-14.06
3	53	-0.0802	-7.01	382	-0.0239	-13.52
4	52	-0.0487	-5.67	379	-0.0159	-10.14
5	51	-0.0395	-5.12	376	-0.0107	-6.72
6	53	-0.0339	-5.24	375	-0.0084	-6.23
7	51	-0.0309	-4.66	371	-0.0066	-5.85
8	51	-0.0310	-4.45	370	-0.0055	-5.33
9	49	-0.0327	-4.83	368	-0.0053	-7.03
	BCHM			CTLD		
1	149	-0.2687	-11.28	201	-0.2423	-13.97
2	148	-0.1337	-11.06	201	-0.1127	-12.19
3	149	-0.0897	-11.52	203	-0.0710	-8.82
4	148	-0.0695	-11.30	205	-0.0522	-6.28
5	153	-0.0576	-10.65	206	-0.0337	-3.20
6	148	-0.0507	-11.03	209	-0.0305	-3.77
7	145	-0.0428	-10.44	207	-0.0268	-3.85
8	141	-0.0384	-10.72	206	-0.0240	-3.97
9	141	-0.0319	-11.06	206	-0.0223	-4.69
	BP			CUAC		
1	268	-0.1037	-10.61	164	-0.1532	-10.64
2	270	-0.0512	-9.13	163	-0.0706	-11.43
3	272	-0.0358	-8.53	161	-0.0438	-9.07
4	274	-0.0252	-7.80	160	-0.0276	-9.52
5	276	-0.0188	-7.12	153	-0.0226	-8.59
6	274	-0.0154	-6.78	154	-0.0167	-7.65
7	276	-0.0133	-7.49	152	-0.0133	-6.97
8	277	-0.0124	-7.42	152	-0.0106	-6.07
9	279	-0.0108	-7.17	154	-0.0112	-5.43

Note: n denotes number of prices.

All mean abnormal returns are significantly negative.

Table 8.7b (continued)

## Market Efficiency Tests over Holding Periods (Bid-ask Quotes)

Holding lag	n	Mean	t-ratio	n	Mean	t-ratio
	GEC			ICI		
1	233	-0.1648	-19.11	271	-0.2166	-15.10
2	232	-0.0769	-18.34	274	-0.1055	-12.43
3	229	-0.0488	-16.64	278	-0.0663	-9.23
4	227	-0.0350	-14.96	283	-0.0452	-7.30
5	226	-0.0287	-13.87	280	-0.0337	-5.35
6	224	-0.0229	-13.63	283	-0.0225	-3.66
7	222	-0.0187	-12.20	283	-0.0153	-2.48
8	221	-0.0166	-11.85	283	-0.0088	-1.34
9	217	-0.0141	-10.99	288	-0.0061	-1.03
	GKN			LAND		
1	243	-0.2909	-17.29	111	-0.1373	-8.10
2	240	-0.1405	-16.76	111	-0.0651	-7.62
3	240	-0.0921	-16.23	110	-0.0412	-7.97
4	235	-0.0690	-16.18	110	-0.0275	-6.94
5	232	-0.0556	-16.59	106	-0.0236	-6.41
6	229	-0.0461	-16.84	107	-0.0236	-6.70
7	227	-0.0405	-16.55	109	-0.0200	-7.17
8	225	-0.0363	-16.99	109	-0.0174	-6.49
9	223	-0.0326	-16.52	113	-0.0170	-6.47
	GMET			LRHO		
1	156	-0.1315	-11.98	70	-0.0828	-4.61
2	155	-0.0616	-9.32	72	-0.0331	-3.63
3	154	-0.0344	-7.50	73	-0.0211	-4.41
4	154	-0.0285	-7.60	70	-0.0163	-2.91
5	152	-0.0206	-5.99	71	-0.0139	-3.41
6	151	-0.0165	-6.00	73	-0.0126	-3.13
7	149	-0.0131	-5.43	74	-0.0102	-3.37
8	150	-0.0122	-5.91	71	-0.0110	-2.88
9	149	-0.0092	-6.72	69	-0.0111	-3.39

Note: n denotes number of prices.

All mean abnormal returns are significantly negative.

Table 8.7b (continued)

## Market Efficiency Tests over Holding Periods (Bid-ask Quotes)

Holding lag	n	Mean	t-ratio	n	Mean	t-ratio
	LSMR			RCAL		
1	180	-0.1169	-7.84	204	-0.2821	-16.93
2	182	-0.0577	-7.70	205	-0.1459	-16.48
3	176	-0.0366	-6.60	206	-0.0934	-15.38
4	180	-0.0252	-5.84	208	-0.0667	-14.72
5	175	-0.0194	-5.26	207	-0.0527	-15.49
6	175	-0.0146	-4.79	205	-0.0452	-14.86
7	174	-0.0130	-4.43	206	-0.0382	-12.16
8	173	-0.0101	-3.62	206	-0.0322	-11.67
9	170	-0.0077	-2.81	205	-0.0276	-11.60
	MKS			RTZ		
1	125	-0.1570	-9.28	225	-0.0628	-17.41
2	123	-0.0768	-9.73	224	-0.0287	-15.38
3	122	-0.0501	-10.44	219	-0.0163	-10.11
4	123	-0.0366	-10.42	227	-0.0126	-10.74
5	123	-0.0303	-9.38	225	-0.0093	-8.33
6	120	-0.0248	-9.12	224	-0.0066	-4.77
7	117	-0.0216	-8.49	222	-0.0060	-5.42
8	111	-0.0175	-8.44	221	-0.0047	-4.33
9	111	-0.0158	-8.19	217	-0.0043	-3.64
	PO			SHEL		
1	264	-0.3301	-18.49	356	-0.1150	-12.98
2	263	-0.1685	-16.14	349	-0.0576	-1.59
3	262	-0.1133	-14.90	346	-0.0385	-10.47
4	259	-0.0869	-13.48	345	-0.0280	-9.11
5	259	-0.0697	-12.55	344	-0.0200	-7.57
6	257	-0.0587	-11.87	340	-0.0156	-6.62
7	254	-0.0509	-11.06	339	-0.0134	-6.11
8	252	-0.0452	-10.71	337	-0.0111	-5.17
9	249	-0.0414	-10.70	337	-0.0095	-4.82

Note: n denotes number of prices.

All mean abnormal returns are significantly negative.

Figure 8.8a

## Comparison of Ex Ante and Ex Post Results

87 Data, Frequently Traded Call Options and Mid Prices

Share	Ex Post Tests			Ex Ante Tests		
	m	Mean	t-ratio	n	Mean	t-ratio
BARC	359	0.0138	2.36*	355	0.0096	1.16
BCHM	726	0.0120	2.37*	717	0.0010	0.20
BP	620	0.0011	0.23	620	0.0008	0.18
CGLD	663	0.0133	2.07*	663	0.0130	1.22
CTLD	547	-0.0012	-0.22	543	0.0005	0.07
CUAC	837	0.0114	2.00*	830	0.0040	0.73
GEC	677	0.0212	4.84*	672	0.0128	3.34*
GKN	770	0.0102	1.87	762	0.0010	0.17
GMET	551	0.0243	3.45*	546	0.0065	0.62
ICI	969	0.0158	3.22*	962	0.0061	1.03
LAND	455	0.0178	2.40*	453	0.0150	2.51*
LRHO	400	-0.0005	-0.09	397	-0.0088	-1.14
LSMR	646	0.0253	4.51*	639	0.0176	1.92
MKS	578	0.0263	4.70*	569	0.0096	1.95
PO	550	-0.0042	-0.66	544	-0.0032	-0.45
RCAL	785	0.0238	1.59	774	0.0070	0.46
RTZ	639	0.0119	4.74*	629	0.0033	1.39
SHEL	673	-0.0023	-0.52	671	0.0011	0.24

Note: m,n denote number of prices.

The subscripts 1, 2 and 3 denote not significant but negative, not significant but positive, and significantly positive respectively.

Table 8.8b

## Comparison of Ex Post and Ex Ante Results

87 Data, Frequently Traded Call Options and Bid-Ask Quotes

Share	Ex Post Tests			Ex Ante Tests		
	m	Mean	t-ratio	n	Mean	t-ratio
BARC	55	-0.2891	-8.36	55	-0.2877	-9.22
BCHM	141	-0.2346	-10.32	149	-0.2687	-11.28
BP	266	-0.1005	-9.64	268	-0.1037	-10.61
CGLD	386	-0.0668	-9.76	386	-0.0760	-14.98
CTLD	200	-0.2521	-13.28	201	-0.2423	-13.97
CUAC	163	-0.1332	-9.53	164	-0.1532	-10.64
GEC	233	-0.1416	-16.69	233	-0.1648	-19.11
GKN	249	-0.2842	-16.00	243	-0.2909	-17.29
GMET	159	-0.0958	-8.36	156	-0.1315	-11.98
ICI	269	-0.1627	-11.52	271	-0.2166	-15.10
LAND	111	-0.1093	-4.95	111	-0.1373	-8.10
LRHO	69	-0.0554	-6.18	70	-0.0828	-4.61
LSMR	179	-0.0990	-7.76	180	-0.1169	-7.84
MKS	129	-0.1200	-8.07	125	-0.1570	-9.28
PO	265	-0.3241	-19.10	264	-0.3301	-18.49
RCAL	209	-0.2595	-15.53	204	-0.2821	-16.93
RTZ	225	-0.0586	-15.52	225	-0.0628	-17.41
SHEL	356	-0.1090	-12.27	356	-0.1150	-12.98

Note: m, n denote number of prices.

All abnormal returns are significantly negative.

Table 8.9

Comparison between Results Generated by  
Frequently and Infrequently Traded Call Options  
The Kolmogorov-Smirnov Test

Share	87 Data				88 Data			
	m	n	$\lambda$	$Q_{KS}(\lambda)$	m	n	$\lambda$	$Q_{KS}(\lambda)$
BARC	55	3	0.71	0.70	85	4	0.62	0.84
BCHM	198	149	3.66	0.00*	522	400	2.86	0.00*
BP	268	7	0.84	0.49	369	13	0.90	0.40
CGLD	386	129	1.85	0.00*	640	154	1.73	0.01*
CTLD	201	143	3.87	0.00*	364	203	3.43	0.00*
CUAC	175	164	1.96	0.00*	179	178	2.09	0.00*
GEC	233	59	1.60	0.01*	389	90	2.42	0.00*
GKN	243	30	3.23	0.00*	320	31	3.09	0.00*
GMET	156	118	1.95	0.00*	220	120	2.08	0.00*
ICI	339	271	4.47	0.00*	482	342	3.16	0.00*
LAND	111	38	1.01	0.26	181	135	1.70	0.01*
LRHO	70	7	1.80	0.00*	198	9	1.90	0.00*
LSMR	180	52	3.73	0.00*	212	54	3.87	0.00*
MKS	125	67	2.43	0.00*	313	109	3.74	0.00*
P. & O.	264	4	1.79	0.00*	304	5	1.68	0.01*
RCAL	204	45	3.44	0.00*	204	131	4.07	0.00*
RTZ	225	133	0.97	0.30	595	133	5.86	0.00*
SHEL	356	336	2.32	0.00*	482	338	2.26	0.00*

m = number of prices of frequently traded call options.

n = number of prices of infrequently traded call options.

\*The distributions of abnormal returns generated by frequently traded call options are significantly different from those generated by infrequently traded call options.

Table 8.10a (87 data)

## Diversification of Hedge Portfolios

$$\text{Abnormal return} = \alpha + \beta (\text{FTSE 100 Index return})$$

Share	m	r	$\alpha$	$t(\alpha)$	$\beta$	$t(\beta)$	$R^2(\%)$
BARC	55	-0.107	-0.28	-8.87	-2.87	-0.78	1.1
BCHM	149	-0.042	-0.27	-10.92	-1.59	-0.51	0.2
BP	268	-0.036	-0.10	-10.22	-0.66	-0.60	0.1
CGLD	386	-0.098	-0.07	-14.04	-1.10	-1.93	1.0
CTLD	201	-0.028	-0.24	-13.50	-0.85	-0.39	0.1
CUAC	164	0.050	-0.15	-10.57	1.03	0.64	0.2
GEC	233	-0.147	-0.16	-18.58	-2.48	-2.26	2.2
GKN	243	-0.013	-0.29	-16.59	-0.45	-0.20	0.0
GMET	156	-0.135	-0.13	-10.85	-2.22	-1.69	1.8
ICI	271	-0.014	-0.22	-14.55	-0.40	-0.22	0.0
LAND	111	0.133	-0.14	-8.21	3.21	1.40	1.8
LRHO	70	-0.032	-0.08	-4.36	-0.48	-0.27	0.1
LSMR	180	0.015	-0.12	-7.48	0.36	0.20	0.0
MKS	125	0.145	-0.16	-9.48	3.14	1.63	2.1
P.&O.	264	-0.263	-0.32	-18.53	-8.64	-4.41	6.9
RCAL	204	-0.113	-0.28	-16.08	-3.11	-1.61	1.3
RTZ	225	-0.038	-0.06	-17.08	-0.22	-0.58	0.1
SHEL	356	-0.006	-0.11	-12.78	-0.11	-0.11	0.0

Notation:

r denotes the correlation coefficient between abnormal returns and the FTSE 100 Index return.

$t(\alpha)$  and  $t(\beta)$  denote the t-ratios for the parameters  $\alpha$  and  $\beta$  respectively.

Table 8.10b (88 data)

## Diversification of Hedge Portfolios

$$\text{Abnormal return} = \alpha + \beta (\text{FTSE 100 Index return})$$

Share	m	r	$\alpha$	$t(\alpha)$	$\beta$	$t(\beta)$	$R^2(\%)$
BARC	85	-0.109	-0.24	-10.14	-1.76	-1.00	1.2
BCHM	400	-0.150	-0.21	-16.20	-2.61	-3.03	2.3
BP	369	-0.031	-0.10	-13.97	-0.36	-0.60	0.1
CGLD	640	-0.080	-0.13	-21.56	-0.96	-2.03	0.6
CTLD	364	-0.093	-0.19	-18.23	-1.38	-1.78	0.9
CUAC	179	0.055	-0.15	-11.45	0.91	0.74	0.3
GEC	389	-0.121	-0.17	-27.38	-1.13	-2.39	1.5
GKN	320	-0.036	-0.26	-18.34	-0.76	-0.64	0.1
GMET	220	-0.180	-0.12	-14.58	-1.91	-2.70	3.2
ICI	482	-0.159	-0.16	-17.53	-2.38	-3.53	2.5
LAND	181	0.069	-0.14	-12.15	0.88	0.93	0.5
LRHO	198	-0.050	-0.11	-3.29	-1.72	-0.70	0.2
LSMR	212	-0.022	-0.12	-8.88	-0.40	-0.31	0.0
MKS	313	-0.052	-0.18	-17.52	-0.66	-0.91	0.3
P.&O.	304	-0.268	-0.31	-19.85	-7.98	-4.84	7.2
RCAL	204	-0.113	-0.28	-16.08	-3.11	-1.61	1.3
RTZ	595	-0.024	-0.29	-20.26	-0.52	-0.58	0.1
SHEL	482	-0.018	-0.11	-15.41	-0.25	-0.40	0.0

Notation:

$r$  denotes the correlation coefficient between abnormal returns and the FTSE 100 Index return.

$t(\alpha)$  and  $t(\beta)$  denote the t-ratios for the parameters  $\alpha$  and  $\beta$  respectively.

Table 8.11a (87 data)

The Call Option Percentage Spread as an  
Explanatory Variable for the Abnormal Returns  
 $Abnormal\ return = \alpha + \beta(\text{call percentage spread})$ .

Share	m	$\alpha$	$t(\alpha)$	$\beta$	$t(\beta)$	$R^2(\%)$
BARC	55	-0.30	-4.62	0.03	0.23	0.1
BCHM	149	-0.01	-0.41	-1.19	-8.70*	34.0
BP	268	-0.01	-0.41	-0.99	-8.96*	23.2
CGLD	386	-0.04	-5.00	-0.27	-4.24*	4.5
CTLD	201	-0.05	-2.61	-0.85	-13.68*	48.5
CUAC	164	-0.04	-1.91	-0.39	-7.47*	25.6
GEC	233	-0.15	-9.49	-0.08	-1.41	0.9
GKN	243	-0.10	-5.06	-0.55	-12.26*	38.4
GMET	156	-0.10	-6.42	-0.15	-2.59*	4.2
ICI	271	-0.20	-8.95	-0.07	-0.84	0.3
LAND	111	-0.03	-1.26	-0.42	-4.72*	16.9
LRHO	70	-0.06	-2.23	-0.15	-1.28	2.4
LSMR	180	-0.07	-2.46	-0.38	-2.10*	2.4
MKS	125	-0.08	-2.55	-0.33	-3.35*	8.3
P.&O.	264	-0.07	-3.60	-1.01	-17.43*	53.7
RCAL	204	-0.06	-2.96	-0.96	-14.25*	50.1
RTZ	225	-0.03	-3.76	-0.26	-4.06*	6.9
SHEL	356	0.01	0.89	-1.10	-13.45*	33.8

Notation:

\*significant at 0.05 level.

m = number of prices of frequently traded call options.

$t(\alpha)$  and  $t(\beta)$  denote the t-ratios for the parameters  $\alpha$  and  $\beta$  respectively.

Table 8.11b (88 data)

The Call Option Percentage Spread as an Explanatory Variable for the Abnormal Returns  
 $Abnormal\ return = \alpha + \beta(\text{call percentage spread})$ .

Share	m	$\alpha$	$t(\alpha)$	$\beta$	$t(\beta)$	$R^2(\%)$
BARC	85	-0.21	-4.71	-0.09	-0.79*	0.8
BCHM	400	0.02	0.99	-1.31	-12.98	29.7
BP	369	-0.04	-3.66	-0.48	-6.76	11.1
CGLD	640	-0.02	-1.59	-0.77	-12.08	18.6
CTLD	364	-0.05	-3.78	-0.66	-13.07	32.1
CUAC	179	-0.04	-2.19	-0.39	-7.74	25.3
GEC	389	-0.15	-12.54	-0.09	-1.89*	0.9
GKN	320	-0.08	-4.99	-0.58	-14.48	39.7
GMET	220	-0.09	-7.05	-0.19	-3.65	5.8
ICI	482	-0.15	-9.42	-0.08	-1.27*	0.3
LAND	181	-0.05	-2.59	-0.42	-6.28	18.1
LRHO	198	-0.03	-0.67	-0.38	-2.13	2.3
LSMR	212	-0.06	-2.59	-0.43	-2.69	3.3
MKS	313	-0.07	-3.77	-0.47	-7.41	15.0
P.&O.	304	-0.07	-3.70	-0.93	-15.78	45.2
RCAL	204	-0.06	-2.96	-0.96	-14.25	50.1
RTZ	595	-0.03	-1.40	-0.96	-12.32	20.4
SHEL	482	-0.01	-1.24	-0.75	-10.91	19.9

m = number of prices of frequently traded call options.

$t(\alpha)$  and  $t(\beta)$  denote the t-ratios for the parameters  $\alpha$  and  $\beta$  respectively.

Table 8.12a (87 data)

The Call Option and Share Percentage Spreads  
as Explanatory Variables for the Abnormal Returns

$$\text{Abnormal return} = \alpha + \beta(\text{call \% spread}) + \gamma(\text{share \% spread}).$$

Share	$\alpha$	$t(\alpha)$	$\beta$	$t(\beta)$	$\gamma$	$t(\gamma)$	$R^2$
BARC	-0.180	-1.07	0.07	0.45	-11.01	-0.82	1.4
BCHM	0.170	2.74	-1.12	-8.44	-22.55	-3.56	39.3
BP	0.080	2.61	-0.92	-8.35	-15.20	-3.13	25.9
CGLD	0.010	0.96	-0.25	-4.00	-9.33	-5.25	10.9
CTLD	0.210	3.81	-0.79	-13.16	-33.68	-4.97	54.2
CUAC	0.005	0.09	-0.38	-7.07	-5.32	-0.96	26.1
GEC	-0.040	-1.63	-0.05	-0.91	-6.27	-4.40	8.6
GKN	0.030	0.43	-0.54	-11.89	-13.91	-2.03	39.5
GMET	-0.180	-3.86	-0.13	-2.18	8.46	1.84	6.3
ICI	-0.080	-1.47	-0.02	-0.27	-21.53	-2.40	2.4
LAND	0.100	1.42	-0.37	-4.15	-18.91	-2.07	21.0
LRHO	0.040	0.57	-0.17	-1.42	-16.43	-1.40	5.1
LSMR	-0.070	-2.19	-0.39	-2.11	0.53	0.25	2.5
MKS	0.020	0.36	-0.35	-3.52	-8.02	-1.90	11.0
P.&O.	-0.001	-0.02	-0.98	-16.76	-10.28	-1.99	54.4
RCAL	0.040	1.27	-0.94	-14.30	-7.78	-3.79	53.5
RTZ	0.001	0.06	-0.19	-2.94	-6.44	-4.85	15.8
SHEL	0.140	6.64	-0.91	-11.18	-42.23	-7.15	42.2

Notation:

$m$  = number of prices of frequently traded call options.

$t(\alpha)$ ,  $t(\beta)$  and  $t(\tau)$  denote the t-ratios for the parameters  $\alpha$ ,  $\beta$  and  $\tau$  respectively.

Table 8.12b (88 data)

The Call Option and Share Percentage Spreads  
as Explanatory Variables for the Abnormal Returns

$$\text{Abnormal return} = \alpha + \beta(\text{call \% spread}) + \gamma(\text{share \% spread}).$$

Share	$\alpha$	$t(\alpha)$	$\beta$	$t(\beta)$	$\gamma$	$t(\gamma)$	$R^2$
BARC	-0.23	-2.38	-0.09	-0.80	1.65	0.24	0.8
BCHM	0.07	2.05	-1.26	-12.06	-6.76	-1.82	30.3
BP	-0.06	-3.17	-0.52	-6.54	2.48	1.02	11.3
CGLD	0.01	1.31	-0.33	-4.81	-7.09	-11.80	33.2
CTLD	-0.07	-3.03	-0.66	-13.05	1.63	0.99	32.2
CUAC	-0.004	-0.11	-0.39	-7.65	-3.99	-1.20	25.9
GEC	-0.09	-4.57	-0.08	-1.62	-3.68	-3.58	4.1
GKN	-0.08	-1.89	-0.58	-14.43	0.12	0.03	39.7
GMET	-0.08	-3.17	-0.19	-3.65	-0.70	-0.29	5.8
ICI	-0.19	-6.86	-0.08	-1.35	5.20	1.68	0.9
LAND	0.01	0.59	-0.41	-6.27	-6.49	-3.32	22.8
LRHO	-0.02	-0.20	-0.37	-2.06	-1.94	-0.27	2.3
LSMR	-0.07	-2.18	-0.43	-2.66	0.13	0.07	3.3
MKS	-0.02	-0.47	-0.47	-7.46	-3.76	-1.83	15.9
P. & O.	-0.08	-2.78	-0.93	-15.59	0.87	0.31	45.2
RCAL	0.04	1.27	-0.94	-14.30	-7.78	-3.79	53.5
RTZ	0.04	2.05	-0.19	-2.30	-10.23	-16.07	44.5
SHEL	-0.03	-2.25	-0.79	-11.17	3.89	2.27	20.7

Notation:

$m$  = number of prices of frequently traded call options.

$t(\alpha)$ ,  $t(\beta)$  and  $t(\tau)$  denote the  $t$ -ratios for the parameters  $\alpha$ ,  $\beta$  and  $\tau$  respectively.

Figure 8.1a (87 data)

Comparison of the Distributions of Abnormal Returns Generated from Mid Prices and Bid-ask Quotes Frequently Traded Call Options

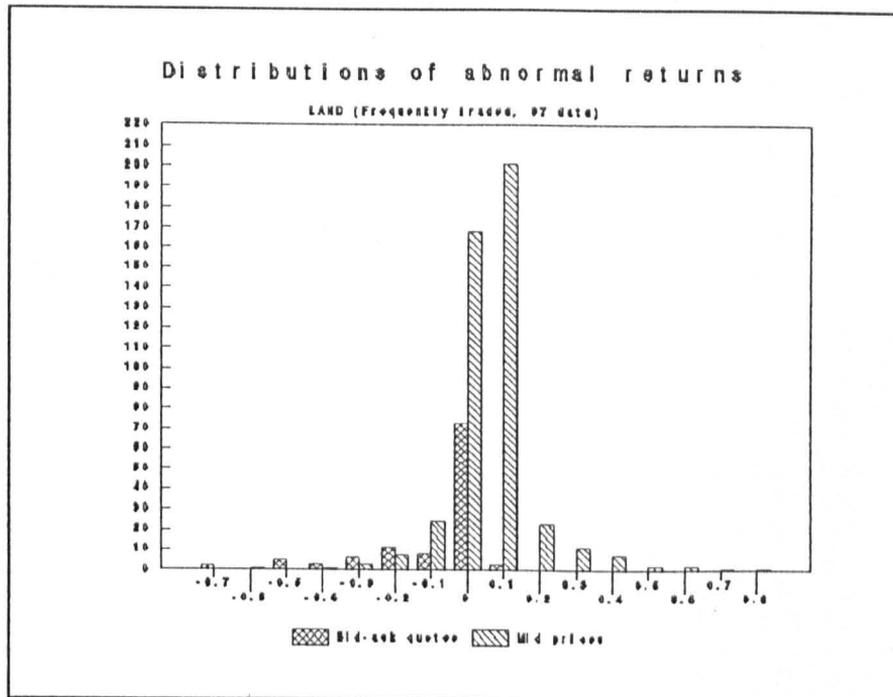


Figure 8.1b (88 data)

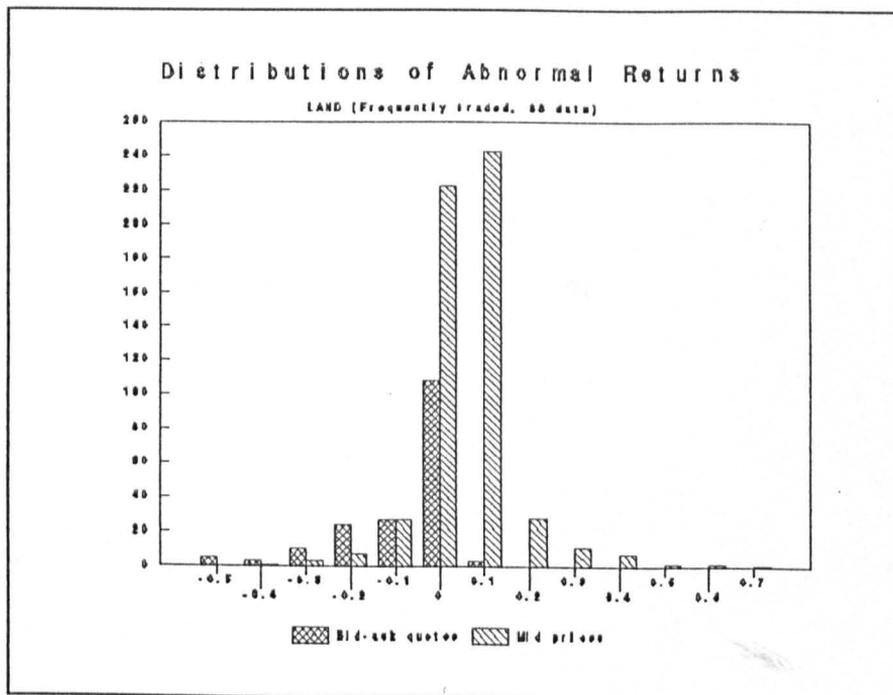


Figure 8.2a (87 data)

Comparison of the Distributions of Abnormal Returns Generated from Mid Prices and Bid-ask Quotes Infrequently Traded Call Options

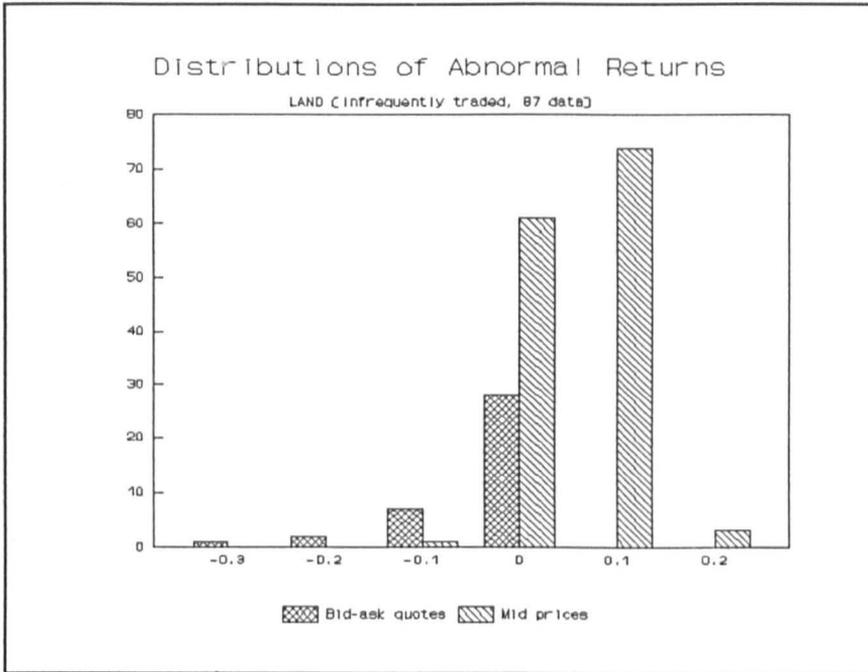
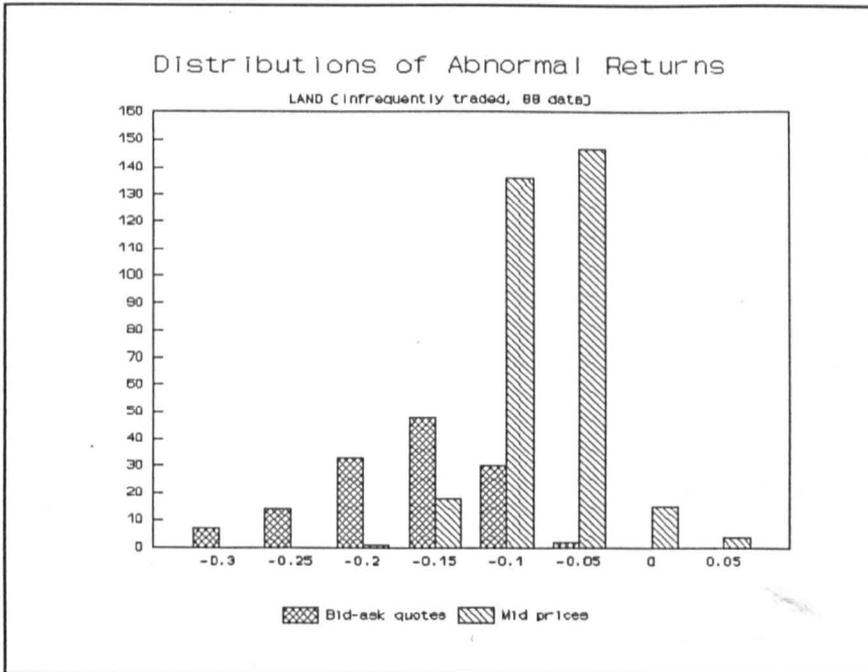


Figure 8.2b (88 data)



## Chapter 9

# Further Discussions on Market Efficiency Tests Using Bid-Ask Quotes

### 9.1 Introduction

It was shown in the last chapter that the use of mid prices and bid-ask quotes data have very different implications for market efficiency. In particular, the LTOM cannot be rejected as efficient when bid-ask spreads are taken into account in the trading strategies.

In this chapter, three major areas of contemporary finance issues which are related to market efficiency tests involving bid-ask quotes are examined. The first area examines the implications of the bid-ask spread on a thinly traded market: these include the spread-induced implicit trading cost, the thin trading issue, and the persistence of market efficiency. The second area examines three empirical issues on using the Black-Scholes model: the ex-dividend share price decline, the share price volatility, and the indivisibility of a call option contract. The third area examines special attributes of mispriced call options in terms of the magnitude of their bid-ask spreads, intrinsic values and times to maturity. In addition, it is found that call option percentage spreads can be explained by the reciprocal of the call option price and the intrinsic value.

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Tables and figures which are not put within the text can be found at the back of this chapter.

## 9.2 The implications of the bid-ask spread in a thin market

### 9.2.1 Spread-induced implicit trading cost

The absolute spread of a call option is defined as the difference between its ask price and bid price. A call option percentage spread is defined as the absolute spread as a percentage of the mid price:

$$\frac{ask - bid}{\left(\frac{bid + ask}{2}\right)}$$

The percentage spread is a more appropriate measure of comparison across companies than the absolute spread (cf. Choi, Salandro and Shastri 1988, p.223). Therefore the call option percentage spread is adopted in the following discussions.

In market efficiency tests using both mid prices and bid-ask quotes, the spread-induced trading cost can be examined by comparing the two measures  $r_{quotes}$  and  $r_{mid}$  where  $r_{quotes}$  = mean abnormal return generated by bid-ask quotes and,  $r_{mid}$  = the difference between the mean abnormal return generated by mid prices and the sum of the average call and share percentage spreads. For each company, if  $r_{mid}$  is negative, then the market is efficient in Phillips and Smith's context. However, if  $r_{mid}$  is greater (less) than  $r_{quotes}$ , then the implicit trading costs embedded in a hedge using bid-ask quotes is greater (less) than the sum of the average call option and share spreads. That is, if  $r_{mid} > r_{quotes}$  then the sum of the average call option and share percentage spreads has underestimated the implicit trading cost in a hedge involving bid-ask quotes and vice versa.

In this study, it was found that the implicit trading costs were overstated for fourteen of the companies and understated for four of the companies for the 87 data (Table 9.1a). The result is similar for the 88 data with some swaps of companies (Table 9.1b). In fact, the implicit trading costs involved in a hedge varies from company to company. Even for the same company, the implicit trading costs could be overstated or understated, because the call and share percentage spreads are changing over time. Therefore, the use of an overall sum of average call and share percentage spreads as a proxy for trading costs is not an infallible guide. The bid-ask quotes must be used directly in testing market efficiency.

### 9.2.2 The thin trading issue

An additional test of the thin trading effect is to see whether a simple correction (adjustment) to the bid-ask spreads to reflect thin trading will affect the results of the efficiency tests. This correction is carried out by setting actual bid (ask) price of the call option equal to the actual mid price of the call option minus (plus) half the spread of the model call bid and ask values, i.e.,

$$\hat{C}_{bid}^A = C_{mid}^A - \frac{1}{2} (C_{ask}^M - C_{bid}^M)$$

$$\hat{C}_{ask}^A = C_{mid}^A + \frac{1}{2} (C_{ask}^M - C_{bid}^M)$$

respectively.

Because the model call option values are theoretically fairly priced, it is expected that the model spreads would be

less than the actual spreads. Since the effect of the thin trading correction would be to reduce the bid-ask spread of the options, the mean abnormal returns are expected to be more positive or less negative.

Empirical results show that, for the frequently traded call options, the effect of the correction is as expected, i.e., the mean abnormal returns are greater although still significantly negative. In addition, the Kolmogorov-Smirnov tests show that the distributions of abnormal returns without and with the thin trading correction are significantly different for all eighteen companies for both the 87 and 88 data (Tables 9.2a, 9.2b). Thus, the thin trading correction is effective. For each company, the distribution of abnormal returns after thin trading correction shifts to the right of that without correction. Examples are taken from Land Securities and are given in Figures 9.1a and 9.1b for the 87 data and 88 data respectively.

For infrequently traded call options, the abnormal returns with the correction are less negative than those without the correction and the distributions of abnormal returns with the correction shifts to the right of those without the correction (Figures 9.2a, 9.2b). However, for the 87 data, only four companies have distributions of abnormal returns without the correction significantly different from those with the correction (Table 9.3a). For the 88 data, there are only seven differences (Table 9.3b). The thin trading correction on infrequently traded call options is therefore inconclusive.

It is important to examine whether the infrequently traded call options after adjusting for the thin trading correction would generate distributions of abnormal returns which are not different from those of the frequently traded call options. The Kolmogorov-Smirnov tests show on the contrary that for the 87 data, fourteen companies have such different distributions (Table 9.4a) and for the 88 data, there are sixteen such different distributions (Table 9.4b). These results suggest that the data quality of the infrequently traded call options cannot be upgraded to the quality of the frequently traded options by a thin trading correction.

### 9.2.3 The persistence of market efficiency

A market cannot be accepted as efficient if abnormal returns exist and are persistent over time (Galai 1977). To pursue this aspect of market efficiency, the persistence of a mispricing signal is examined over the next following three days. If an option is mispriced on day  $t$ , then hedges are established on days  $t+1$ ,  $t+2$ , and  $t+3$  (or, lag 1, lag 2 and lag 3) and are liquidated on days  $t+2$ ,  $t+3$ ,  $t+4$  respectively.

If the market is efficient it is expected that abnormal returns will be predominantly negative. Furthermore, the market would be expected to react quickly to eliminate possible profitable opportunities and this could lead to abnormal returns which were progressively less profitable (more negative) as  $t$  increases. Empirical results show that for the 87 data, thirteen companies have progressively more negative mean abnormal returns from lag 1 to lag 2. However this pattern

is not continued. Only three companies have progressively more negative abnormal returns from lag 1 to lag 3 (Table 9.5a). For the 88 data, the results are quite similar (Table 9.5b). If by chance, the cost of trading is overestimated in our tests, this might imply some residual inefficiency in the behaviour of the LTOM.

The thin trading correction is further applied to examine the pattern of abnormal returns over different lags. It is found that the thin trading correction is most effective for lag 1 hedges. The Kolmogorov-Smirnov tests show that the distributions of abnormal returns without and with the correction are significantly different for lag 1 for all the eighteen companies for both 87 and 88 data (Tables 9.6a, 9.6b). With the correction, there is a clear pattern of progressively more negative mean abnormal returns from lag 1 to lag 3 for both the 87 and 88 data (Tables 9.7a, 9.7b). The only exception is GMET whose data has already been shown in Chapter 3 to be problematic over the sample period. However, for lag 2 and lag 3 hedges the distributions of abnormal returns with correction are progressively not significantly different from those without the correction. These results confirm the comment in the preceding paragraph and is consistent with the view that the market quickly absorbs all available information.

The rationale for applying this correction is simply an illustration of the "noise" introduced by the traded options market operating with greater spreads than might be expected. It is concluded that the efficiency of the market is blurred by the wide spreads of call options, and that the thin trading

correction uncovers the basic efficiency of the market. This conclusion is however restrictive in the sense that the observed abnormal returns are all significantly negative. The persistence of market efficiency would become clearer if a positive abnormal return becomes progressively less positive or becomes negative over time. However, as large bid-ask spreads are observed in the LTOM, abnormal returns from hedge portfolios will tend to be negative. The thin trading correction thus remains a suitable research design in examining the persistence of market efficiency.

### 9.3 Empirical issues on using the Black-Scholes model

Three issues on using the Black-Scholes model are examined: different adjustments of the ex-dividend share price drop, different estimates of the share price volatility, and the indivisibility of a call option contract.

#### 9.3.1 Different adjustments of ex-dividend share price decline

In Chapter 6, it was pointed out that empirical findings on the amount of ex-dividend share price fall are paradoxical. Kaplanis (1986) finds that the expected ex-dividend share price decline is significantly less than the dividend. On the other hand, Barone-Adesi and Whaley (1986) find that the expected decline is not significantly different from the dividend. It is important to examine whether different adjustments of the share price drop will imply different conclusions for market efficiency tests.

In testing market efficiency, the Black-Scholes formula is therefore adjusted for the full dividend and also a fraction (55%, according to Kaplanis) of the dividend. Empirical results show that, for the 87 data, the distributions of abnormal returns for an adjustment of the full dividend are not significantly different from those adjusted for 55% of the dividend for all 18 companies (Table 9.8a); for the 88 data, the distribution of abnormal returns of fifteen out of the eighteen companies are not significantly different between the two adjustment methods (Table 9.8b). Moreover, the mean abnormal returns of all eighteen companies and for both data sub-samples are all significantly negative, implying that the hypothesis that the LTOM is efficient cannot be rejected. It is therefore concluded that market efficiency tests are not sensitive to the amount of dividend adjustments. To be consistent with the riskless arbitrage hypothesis inherent in the Black-Scholes model, it is appropriate to adjust the share price for the full dividend.

### 9.3.2 Different estimates of the share price volatility

It was found in chapter 4 that 78% of the GARCH volatility estimates lies within the 95% confidence interval of the "true" volatility. However, the Kolmogorov-Smirnov tests show that the distributions of abnormal returns using the GARCH volatility estimates are in general different from those using the actual volatilities. There are thirteen companies having significantly different distributions of abnormal returns for both the 87 and 88 data (Tables 9.9a, 9.9b). In particular, the mean abnormal

returns generated by the actual volatilities are all negative but in general larger than those generated by using the GARCH volatilities (Tables 9.9a, 9.9b). This result is reasonable as the actual volatilities are *ex post* measures while the GARCH volatilities are *ex ante* estimates. A trader is likely to be able to form more profitable hedges if the *ex post* actual volatility is known.

### 9.3.3 The indivisibility of a call option contract

Contrary to Black and Scholes' assumption that it is possible to buy any fraction of a share, an option contract is indivisible and orders cannot be executed in fractions of a contract (Traded Option Users' Reference Manual 1984). Indivisibility implies that a hedge ratio will be inexact so that a hedged position may be always slightly under or over hedged. If an option contract is on  $N$  units of its underlying share, the hedge ratio can only assume values that are an integral multiple of  $1/N$  (Figlewski 1989). Since a call option contract in the LTOM normally represents the options on 1,000 shares of its underlying shares, a hedge ratio can take up at most three decimal places. For instance, a long position in a call option and a short position of  $N(d_1)=0.57143$  shares will be:

$$\begin{aligned} & (C - 0.57143S) \times 1000 \\ & - 1000C - 571.43S \\ & - 1000C - 571S \quad (\text{rounded}) \end{aligned}$$

This hedged position consists of whole numbers of options and shares and is therefore meaningful.

It is found by the Kolmogorov-Smirnov tests that the distributions of abnormal returns generated by the exact hedge ratios are not significantly different from those generated by the rounded first three decimal places of the hedge ratios (Tables 9.10a, 9.10b). These results agree with Figlewski's (1989) simulated results that indivisibilities do not have a large impact on expected returns. The results not only show that the problem of indivisibility is not significant but also reconcile the choice of the appropriate hedge ratio in a hedge involving bid-ask quotes.

#### 9.4 The special attributes of mispriced call options

The subset of mispriced call option prices is found to have different characteristics from the original call option prices before any mispricing signal is identified.

##### 9.4.1 The empirical evidences

The Kolmogorov-Smirnov tests show that the subset of mispriced call option prices has distinct attributes from the original call option prices before any mispricing signal is generated. The call option attributes are defined as the call option percentage spread, the intrinsic value, the normalised absolute intrinsic value and the time to maturity (Tables 9.11a-9.11d). Detailed contrasts of these four attributes for individual companies are given in Tables 9.12a-9.12b for the 87 data and Tables 9.12c-9.12d for the 88 data. The overall

results across companies are summarised below:

Table 9.13

The Special Attributes of Mispriced Call Options

Attributes	87 Data		88 Data	
	Original	Mispriced	Original	Mispriced
Call option percentage spread (%)	18	20	19	21
Intrinsic value (£)	22.87	-0.32	12.26	-5.96
Normalised absolute intrinsic value (%)	9.33	7.18	8.65	7.50
Time to maturity (days)	60	67	59	71

9.4.2 The interpretation for the special attributes of mispriced call option prices

The percentage spread. For the 87 data, it is found that the average percentage spread across all companies for the original call option prices is 18% but widens to 20% for all mispriced call option prices. For the 88 data, the corresponding figures are 19% and 21% respectively. This implies that mispriced call options are generally characterised by wider percentage spreads.

The intrinsic value. For the 87 data, the average intrinsic value is positive (£22.87) for the original call option prices but changes to negative (-£0.32) for mispriced call option prices. For the 88 data, the average intrinsic value changes from £12.26 to -£5.96. This suggests that mispriced call options tend to be slight out-of-the-money options.

The normalised absolute intrinsic value. The normalised absolute intrinsic values is a measure for near-the-moneyness. The average normalised absolute intrinsic values shrink from 9.33% for the original call option prices to 7.18% for mispriced call option prices for the 87 data. For the 88 data, the corresponding figures narrowed down from 8.65% to 7.50%. This means that mispriced call option prices tend to be near-the-money calls. More importantly, the average normalised intrinsic values for both the 87 and 88 data satisfy the condition

$$4.39\% < \left| \frac{S-X}{S} \right| < 13.7\%$$

for each individual company and the overall mean for all companies satisfies

$$\left| \frac{S-X}{S} \right| < 7.50\%$$

which are below the earlier definition of

$$\left| \frac{S-X}{S} \right| < 15\%$$

for near-the-money calls. Thus mispriced call options are verified ex post to possess the quality of frequently traded call options and thus are more synchronous with their underlying share prices.

Time to maturity. For the 87 data, the average time to maturity for the original call options is 60 trading days and becomes 67 trading days for mispriced call options. The larger the time to maturity implies that the earlier the call options are in their lives. Thus mispriced call options are likely to be identified as mispriced and traded early in their lives. The same observation holds also for the 88 data. Moreover, the time

to maturity for mispriced call options lengthens from 59 days for the 87 data to 71 days for the 88 data. This implies that after the 1987 crash call options are identified as mispriced much earlier and traded in their lives.

Explanatory variables for the call percentage spread. It is important to pursue the variables which best explain the variations in call option percentages spreads. Nisbet (1990) finds that for the LTOM, about half of the variation in the percentage spread can be explained by the call option price. This study finds that the call percentage spreads are well explained by the call option prices but it is even better explained by the reciprocal of the call option price on the evidence of higher R-squareds. In addition, this study finds that the intrinsic value also explains the percentage spreads significantly.

Let the explanatory variables for the call option percentage spread be the reciprocal of the call option price and the intrinsic value, i.e.,

$$\text{Call option percentage spread} = \alpha + \beta \frac{1}{C} + \gamma (S-X)$$

Empirical results show that for the original call option prices the  $\beta$  coefficients are positive and the  $\gamma$  coefficients are negative and are both significant for all eighteen companies. For both 87 and 88 data (Tables 9.13a, 9.13b), the R-squareds lie between the interval

$$47.5\% < R^2 < 90.6\%.$$

For mispriced call option prices, the  $\beta$  coefficients are significantly positive for all eighteen companies, but four  $\gamma$  coefficients for the 87 data and one  $\gamma$  coefficient for the 88 data are insignificant (Tables 9.13c, 9.13d). The ranges for  $R^2$  lie between the interval:

$$43.3\% < R^2 < 88.7\%.$$

This slightly less efficient result for the mispriced call option prices is acceptable because they are a subset of the original call option prices.

The regression equation above can be interpreted as follows: from the positive beta, the absolute call option spread increases as the call option price rises (Copeland and Galai 1983) implying that the percentage spread will increase with the *reciprocal* of the call price. From the negative gamma, a larger intrinsic value implies that the call option is more in-the-money and is therefore more valuable. As the call option price becomes larger its percentage spread becomes smaller. Furthermore, the R-squareds for each regression for the 87 data is larger than those of the 88 data. This result holds for the original call option prices and for the subset of mispriced call option prices. This might suggest that pre-crash period call option percentage spreads are better explained by both the call option price and the intrinsic value of the call options.

## 9.5 Summary and conclusions

Three major areas of contemporary finance issues have been discussed in this chapter.

The first area concerns the implications of the bid-ask spread on a thinly traded call options market, i.e., the LTOM. It is pointed out that the spread-induced implicit trading cost is not unique across companies. This implies that an overall average percentage spread is not an appropriate measure in testing market efficiency. In a thinly traded market, the quality of infrequently traded call option prices with thin trading correction does not appear to be upgraded to the quality of frequently traded call option prices. The persistence of market efficiency which is blurred by the large bid-ask spreads is shown to be uncovered by applying the thin trading correction.

The second area examines empirical issues on using the Black-Scholes model. It is found that the different estimates of the ex-dividend share price drop, either the full dividend amount or a fraction of it, did not alter the results of the efficiency tests. The distributions of abnormal returns generated by the GARCH volatility estimates were found to be significantly different from those generated by the actual volatility estimates. This result is reasonable because the GARCH volatility is an ex ante measure while the actual volatility is an ex post measure. Given that more than 75% of the GARCH volatilities lies within 95% confidence intervals of the actual volatilities and that the aim of this study is to test market efficiency, the adoption of the GARCH volatility

is appropriate. It is also found that the distributions of abnormal returns was relatively not sensitive to the precision of the hedge ratio.

The third area examines the special attributes of mispriced call option prices. It is found that mispriced call option prices tend to have wider percentage spreads, are slightly out-of-the-money, are more actively traded and are traded earlier in their lives than the original call option prices. Finally, the call option percentage spread is found to be significantly explained by the reciprocal of the call option price and a new explanatory variable, the intrinsic value.

Table 9.1a (87 data)

## Spread-induced Implicit Trading Cost

		Spread (%)		Comparison		
Share	$r^*$	call	share	$r_{mid}^*$		$r_{quotes}^*$
BARC	0.0096	0.3789	0.0126	-0.3819	<	-0.2877
BCHM	0.0010	0.2135	0.0087	-0.2212	>	-0.2687
BP	0.0008	0.0991	0.0060	-0.1043	<	-0.1036
CGLD	0.0130	0.1161	0.0065	-0.1096	<	-0.0760
CTLD	0.0005	0.2261	0.0081	-0.2337	>	-0.2423
CUAC	0.0040	0.2920	0.0086	-0.2966	<	-0.1532
GEC	0.0128	0.2254	0.0173	-0.2299	<	-0.1648
GKN	0.0004	0.3423	0.0098	-0.3517	<	-0.2909
GMET	0.0065	0.1936	0.0093	-0.1964	<	-0.1315
ICI	0.0061	0.2185	0.0061	-0.2185	<	-0.2166
LAND	0.0150	0.2482	0.0076	-0.2408	<	-0.1373
LRHO	-0.0088	0.1598	0.0061	-0.1747	<	-0.0828
LSMR	0.0176	0.1292	0.0104	-0.1220	<	-0.1169
MKS	0.0095	0.2463	0.0115	-0.2483	<	-0.1570
P.O.	-0.0032	0.2593	0.0073	-0.2698	>	-0.3301
RCAL	0.0070	0.2326	0.0135	-0.2391	>	-0.2821
RTZ	0.0033	0.1188	0.0064	-0.1219	<	-0.0628
SHEL	0.0011	0.1143	0.0037	-0.1169	<	-0.1150

## Notes:

1. A "<" (>") sign denotes that the sum of call and share percentage spreads has overstated (understated) the implicit trading cost induced by bid-ask quotes.
2.  $r^*$  = mean abnormal return (mid prices),  $r_{mid}^*$  = difference between  $r$  and the sum of average call and share percentage spreads,  $r_{quotes}^*$  = mean abnormal return (bid-ask quotes).

Table 9.1b (88 data)

## Spread-induced Implicit Trading Cost

		Spread (%)		Comparison		
Share	$r^*$	call	share	$r_{mid}^*$		$r_{quotes}^*$
BARC	0.0041	0.3249	0.0128	-0.3336	<	-0.2422
BCHM	0.0003	0.1792	0.0086	-0.1875	>	-0.2131
BP	0.0016	0.1276	0.0072	-0.1332	<	-0.1036
CGLD	0.0116	0.1529	0.0139	-0.1552	<	-0.1350
CTLD	0.0011	0.2164	0.0116	-0.2269	<	-0.1961
CUAC	0.0037	0.2849	0.0094	-0.2906	<	-0.1520
GEC	0.0126	0.2101	0.0164	-0.2139	<	-0.1676
GKN	0.0000	0.3062	0.0107	-0.3169	<	-0.2573
GMET	0.0038	0.1946	0.0095	-0.2003	<	-0.1271
ICI	0.0024	0.2136	0.0073	-0.2185	<	-0.1656
LAND	0.0123	0.2129	0.0096	-0.2102	<	-0.1351
LRHO	0.0068	0.2109	0.0097	-0.2138	<	-0.1130
LSMR	0.0128	0.1312	0.0110	-0.1294	<	-0.1205
MKS	0.0084	0.2426	0.0138	-0.2480	<	-0.1818
P.O.	0.0018	0.2682	0.0084	-0.2748	>	-0.3214
RCAL	0.0059	0.2326	0.0135	-0.2402	>	-0.2821
RTZ	0.0068	0.2679	0.0278	-0.2889	>	-0.2909
SHEL	0.0054	0.1310	0.0054	-0.1310	<	-0.1127

## Notes:

1. A "<" (">") sign denotes that the sum of call and share percentage spreads has overstated (understated) the implicit trading cost induced by bid-ask quotes.
2.  $r^*$  = mean abnormal return (mid prices),  $r_{mid}^*$  = difference between  $r$  and the sum of average call and share percentage spreads,  $r_{quotes}^*$  = mean abnormal return (bid-ask quotes).

Table 9.2a (87 data)

## Thin Trading Correction

## Frequently Traded Call Options

Share	Mean abnormal return (#)	t-ratio		Mean abnormal return	$\lambda$	$Q_{ks}(\lambda)$
BARC	-0.1916	-8.24	>	-0.2877	1.97	0.00*
BCHM	-0.2020	-9.93	>	-0.2687	1.58	0.01*
BP	-0.0797	-9.24	>	-0.1037	2.26	0.00*
CGLD	-0.0455	-3.43	>	-0.0760	3.08	0.00*
CTLD	-0.1889	-10.92	>	-0.2423	1.98	0.00*
CUAC	-0.1302	-8.92	>	-0.1532	3.07	0.00*
GEC	-0.1279	-15.52	>	-0.1648	2.46	0.00*
GKN	-0.1948	-17.42	>	-0.2909	3.15	0.00*
GMET	-0.0939	-9.67	>	-0.1315	2.54	0.00*
ICI	-0.1536	-12.83	>	-0.2166	2.65	0.00*
LAND	-0.0919	-8.20	>	-0.1373	2.04	0.00*
LRHO	-0.0527	-4.17	>	-0.0828	1.86	0.00*
LSMR	-0.0727	-5.36	>	-0.1169	1.86	0.00*
MKS	-0.1308	-11.39	>	-0.1570	1.41	0.04*
P.&O.	-0.2429	-15.96	>	-0.3301	2.35	0.00*
RCAL	-0.2334	-15.94	>	-0.2821	1.60	0.01*
RTZ	-0.0434	-13.66	>	-0.0628	2.01	0.00*
SHEL	-0.0805	-10.73	>	-0.1150	2.37	0.00*

Note: \*significant at 0.05 level.

#:with thin trading correction.

The Kolmogorov-Smirnov tests show that with the thin trading correction, the distributions of abnormal returns are significantly different from those without the correction.

Table 9.2b (88 data)

## Thin Trading Correction

## Frequently Traded Call Options

Share	Mean abnormal return (#)	t-ratio		Mean abnormal return	$\lambda$	$Q_{KS}(\lambda)$
BARC	-0.1741	-8.69	>	-0.2422	2.44	0.00*
BCHM	-0.1357	-13.16	>	-0.2131	3.63	0.00*
BP	-0.0758	-12.13	>	-0.1036	2.60	0.00*
CGLD	-0.0912	-10.05	>	-0.1350	2.72	0.00*
CTLD	-0.1504	-14.04	>	-0.1961	2.68	0.00*
CUAC	-0.1267	-9.31	>	-0.1520	3.16	0.00*
GEC	-0.1201	-20.49	>	-0.1676	4.15	0.00*
GKN	-0.1777	-18.40	>	-0.2573	3.13	0.00*
GMET	-0.0875	-11.52	>	-0.1271	3.16	0.00*
ICI	-0.1170	-14.17	>	-0.1656	3.51	0.00*
LAND	-0.0896	-11.08	>	-0.1351	2.64	0.00*
LRHO	-0.0657	-2.54	>	-0.1130	3.04	0.00*
LSMR	-0.0749	-6.40	>	-0.1205	2.00	0.00*
MKS	-0.1243	-17.04	>	-0.1818	3.61	0.00*
P.&O.	-0.2277	-17.74	>	-0.3214	2.72	0.00*
RCAL	-0.2334	-15.94	>	-0.2821	1.60	0.01*
RTZ	-0.1583	-18.06	>	-0.2909	4.16	0.00*
SHEL	-0.0814	-12.94	>	-0.1127	2.43	0.00*

Note: \*significant at 0.05 level.  
#:with thin trading correction.

The Kolmogorov-Smirnov tests show that with the thin trading correction, the distributions of abnormal returns are significantly different from those without the correction.

Table 9.3a (87 data)

## Thin Trading Correction

## Infrequently Traded Call Options

Share	Mean abnormal return (#)	t-ratio		Mean abnormal return	$\lambda$	$Q_{KS}(\lambda)$
BARC	-0.1625	-2.26	<	-0.1559	0.41	1.00
BCHM	-0.0762	-19.43	>	-0.0996	1.72	0.01*
BP	-0.0435	-3.14	>	-0.0448	0.27	1.00
CGLD	-0.0602	-17.06	>	-0.0718	1.14	0.15
CTLD	-0.0806	-12.48	>	-0.0881	0.96	0.31
CUAC	-0.0762	-17.53	>	-0.0882	0.96	0.32
GEC	-0.0942	-11.80	>	-0.0988	0.53	0.94
GKN	-0.0572	-8.68	>	-0.0634	0.79	0.57
GMET	-0.0668	-18.32	>	-0.0782	1.33	0.06
ICI	-0.0806	-25.41	>	-0.0984	1.75	0.00*
LAND	-0.0716	-10.05	>	-0.0841	0.70	0.72
LRHO	-0.0238	-11.18	<	-0.0222	0.41	1.00
LSMR	-0.0313	-8.00	>	-0.0410	1.51	0.02*
MKS	-0.0662	-13.64	>	-0.0675	0.47	0.98
P. & O.	-0.0251	-6.16	>	-0.0274	0.52	0.95
RCAL	-0.0999	-17.75	>	-0.1045	0.50	0.96
RTZ	-0.0518	-15.38	>	-0.0574	0.87	0.44
SHEL	-0.0556	-17.47	>	-0.0706	1.53	0.02*

Note: \*significant at 0.05 level.

#:with thin trading correction.

The thin trading correction is not very effective for infrequently traded call options.

Table 9.3b (88 data)

## Thin Trading Correction

## Infrequently Traded Call Options

Share	Mean abnormal return*	t-ratio		Mean abnormal return	$\lambda$	$Q_{KS}(\lambda)$
BARC	-0.1473	-2.78	<	-0.1435	0.35	1.00
BCHM	-0.0890	-16.82	>	-0.1553	4.16	0.00*
BP	-0.0800	-5.19	<	-0.0713	0.52	0.95
CGLD	-0.0736	-18.23	>	-0.0892	1.23	0.10
CTLD	-0.0806	-16.84	>	-0.0916	1.16	0.14
CUAC	-0.0766	-17.87	>	-0.0884	0.95	0.33
GEC	-0.0873	-14.46	>	-0.0954	0.69	0.73
GKN	-0.0561	-8.62	>	-0.0621	0.76	0.61
GMET	-0.0661	-18.50	>	-0.0774	1.36	0.05*
ICI	-0.0801	-25.55	>	-0.0980	1.76	0.00*
LAND	-0.0701	-18.95	>	-0.0955	1.70	0.01*
LRHO	-0.0288	-6.85	>	-0.0297	0.34	1.00
LSMR	-0.0318	-8.33	>	-0.0413	1.51	0.02*
MKS	-0.0734	-19.22	>	-0.0815	0.84	0.48
P.&O.	-0.0374	-2.92	>	-0.0466	0.54	0.94
RCAL	-0.1008	-23.10	>	-0.1294	1.97	0.00*
RTZ	-0.0518	-15.38	>	-0.0574	0.87	0.44
SHEL	-0.0561	-17.70	>	-0.0709	1.49	0.02*

Note: \*significant at 0.05 level.

#:with thin trading correction.

The thin trading correction is not very effective for infrequently traded call options.

Table 9.4a (87 data)

## Thin Trading Correction

Share	F	IT	The KS Test	
	m	n	$\lambda$	$Q_{KS}(\lambda)$
BARC	55	3	0.77	0.60
BCHM	260	149	4.33	0.00*
BP	268	7	0.89	0.40
CGLD	386	168	1.15	0.14
CTLD	201	147	4.19	0.00*
CUAC	235	164	2.79	0.00*
GEC	233	75	1.76	0.00*
GKN	243	36	3.55	0.00*
GMET	156	156	2.55	0.00*
ICI	393	271	5.19	0.00*
LAND	111	51	1.32	0.06
LRHO	70	9	1.73	0.00*
LSMR	180	69	4.39	0.00*
MKS	125	73	2.41	0.00*
P. & O.	264	6	2.19	0.00*
RCAL	204	49	3.57	0.00*
RTZ	225	169	1.44	0.03*
SHEL	363	356	2.92	0.00*

The distributions of abnormal returns generated by frequently traded call options (F) are generally different from those generated by infrequently traded call options with thin trading correction (IT).

Table 9.4b (88 data)

## Thin Trading Correction

Share	F	IT	The KS Test	
	m	n	$\lambda$	$Q_{KS}(\lambda)$
BARC	85	4	0.67	0.77
BCHM	682	400	5.10	0.00*
BP	369	16	0.59	0.87
CGLD	640	200	2.77	0.00*
CTLD	364	213	4.24	0.00*
CUAC	239	179	2.88	0.00*
GEC	389	107	3.00	0.00*
GKN	320	37	3.46	0.00*
GMET	220	160	3.05	0.00*
ICI	482	399	4.07	0.00*
LAND	181	165	2.29	0.00*
LRHO	198	11	2.22	0.00*
LSMR	212	71	4.55	0.00*
MKS	313	128	4.41	0.00*
P. & O.	304	7	2.07	0.00*
RCAL	204	150	4.73	0.00*
RTZ	595	169	6.48	0.00*
SHEL	482	370	3.17	0.00*

The distributions of abnormal returns generated by frequently traded call options (F) are generally different from those generated by infrequently traded call options with thin trading correction (IT).

Table 9.5a (87 data)

## Persistence of Market Efficiency

Share	Abnormal returns				
	Lag 1		Lag 2		Lag 3
BARC	-0.2877	<	-0.2721	<	-0.2580
BCHM	-0.2687	>	-0.2773	<	-0.2685
BP	-0.1036	>	-0.1108	<	-0.1105
CGLD	-0.0760	>	-0.0816	<	-0.0704
CTLD	-0.2423	<	-0.2401	>	-0.2617
CUAC	-0.1532	>	-0.1553	>	-0.1651
GEC	-0.1648	>	-0.1654	>	-0.1674
GKN	-0.2909	<	-0.2863	<	-0.2755
GMET	-0.1315	>	-0.1397	<	-0.1228
ICI	-0.2166	<	-0.2099	>	-0.2120
LAND	-0.1373	>	-0.1501	>	-0.1544
LRHO	-0.0828	<	-0.0804	>	-0.0842
LSMR	-0.1169	>	-0.1403	<	-0.1228
MKS	-0.1570	>	-0.1623	<	-0.1588
P. & O.	-0.3301	>	-0.3355	<	-0.3245
RCAL	-0.2821	>	-0.3033	<	-0.2856
RTZ	-0.0628	>	-0.0650	<	-0.0627
SHEL	-0.1150	>	-0.1166	<	-0.1133

Table 9.5b (88 data)

## Persistence of Market Efficiency

Share	Abnormal Returns				
	Lag 1		Lag 2		Lag 3
BARC	-0.2422	<	-0.2342	<	-0.2306
BCHM	-0.2131	>	-0.2141	<	-0.2040
BP	-0.1036	>	-0.1094	<	-0.1087
CGLD	-0.1350	>	-0.1412	<	-0.1348
CTLD	-0.1961	>	-0.1974	>	-0.2070
CUAC	-0.1520	>	-0.1526	>	-0.1629
GEC	-0.1676	>	-0.1682	>	-0.1689
GKN	-0.2573	<	-0.2551	<	-0.2474
GMET	-0.1271	>	-0.1347	<	-0.1214
ICI	-0.1656	<	-0.1584	<	-0.1573
LAND	-0.1351	>	-0.1430	>	-0.1450
LRHO	-0.1130	<	-0.1127	<	-0.1003
LSMR	-0.1205	>	-0.1402	<	-0.1261
MKS	-0.1818	<	-0.1772	>	-0.1864
P.&O.	-0.3214	>	-0.3236	<	-0.3195
RCAL	-0.2821	>	-0.3033	<	-0.2856
RTZ	-0.2909	>	-0.2929	<	-0.2825
SHEL	-0.1127	>	-0.1191	<	-0.1183

Table 9.6a (87 data)

## Effectiveness of the Thin Trading Correction

Share	Lag 1		Lag 2		Lag 3	
	$\alpha$	$Q_{KS}(\lambda)$	$\alpha$	$Q_{KS}(\lambda)$	$\alpha$	$Q_{KS}(\lambda)$
BARC	1.97	0.00*	1.05	0.22	0.72	0.67
BCHM	1.57	0.01*	0.93	0.35	0.45	0.99
BP	2.26	0.00*	1.31	0.06	0.68	0.75
CGLD	3.08	0.00*	1.73	0.01*	0.44	0.99
CTLD	1.98	0.00*	1.08	0.19	0.69	0.72
CUAC	3.07	0.00*	1.24	0.09	0.81	0.53
GEC	2.46	0.00*	1.57	0.01*	0.49	0.97
GKN	3.15	0.00*	1.34	0.06	0.88	0.43
GMET	2.54	0.00*	1.21	0.11	0.35	1.00
ICI	2.65	0.00*	1.36	0.05	0.92	0.36
LAND	2.04	0.00*	1.28	0.08	0.28	1.00
LRHO	1.86	0.00*	1.12	0.17	0.48	0.98
LSMR	1.86	0.00*	1.08	0.20	0.31	1.00
MKS	1.41	0.04*	0.57	0.90	0.82	0.51
P.O.	2.35	0.00*	0.85	0.47	0.34	1.00
RCAL	1.60	0.01*	0.70	0.72	0.43	0.99
RTZ	2.01	0.00*	1.09	0.19	0.28	1.00
SHEL	2.37	0.00*	1.63	0.01*	0.28	1.00

Note: The thin trading correction is most effective for lag 1.  
\*significantly different from zero.

Table 9.6b (88 data)

## Effectiveness of the Thin Trading Correction

Share	Lag 1		Lag 2		Lag 3	
	$\alpha$	$Q_{KS}(\lambda)$	$\alpha$	$Q_{KS}(\lambda)$	$\alpha$	$Q_{KS}(\lambda)$
BARC	2.44	0.00*	1.28	0.07	0.64	0.81
BCHM	3.63	0.00*	1.64	0.01*	0.23	1.00
BP	2.60	0.00*	1.27	0.01*	0.49	0.97
CGLD	2.72	0.00*	1.42	0.04*	0.16	1.00
CTLD	2.68	0.00*	1.25	0.09	0.27	1.00
CUAC	3.16	0.00*	1.47	0.03*	0.81	0.53
GEC	4.15	0.00*	2.26	0.00*	0.19	1.00
GKN	3.13	0.00*	3.04	0.00*	3.15	0.00*
GMET	3.16	0.00*	1.81	0.00*	0.37	1.00
ICI	3.51	0.00*	1.74	0.00*	0.46	0.98
LAND	2.64	0.00*	1.57	0.01*	0.35	1.00
LRHO	3.04	0.00*	1.44	0.03*	0.50	0.96
LSMR	2.00	0.00*	1.13	0.15	0.32	1.00
MKS	3.61	0.00*	1.59	0.01*	0.59	0.88
P.O.	2.72	0.00*	1.05	0.22	0.31	1.00
RCAL	1.60	0.01*	0.70	0.72	0.47	0.98
RTZ	4.16	0.00*	2.21	0.00*	0.34	1.00
SHEL	2.43	0.00*	1.48	0.03*	0.29	1.00

Note: The thin trading correction is most effective for lag 1.  
 \*significantly different from zero.

Table 9.7a (87 data)

Persistence of Market Efficiency after  
Applying the Thin Trading Correction

Share	Mean Abnormal Returns				
	Lag 1		Lag 2		Lag 3
BARC	-0.1916	>	-0.2153	>	-0.2343
BCHM	-0.2020	>	-0.2443	>	-0.2659
BP	-0.0797	>	-0.0959	>	-0.0996
CGLD	-0.0455	>	-0.0714	>	-0.0741
CTLD	-0.1889	>	-0.2447	>	-0.2661
CUAC	-0.1302	>	-0.1892	>	-0.2141
GEC	-0.1279	>	-0.1421	>	-0.1562
GKN	-0.1948	>	-0.2917	>	-0.3286
GMET	-0.0939	>	-0.1421	<	-0.1368
ICI	-0.1536	>	-0.1850	>	-0.1919
LAND	-0.0919	>	-0.1263	>	-0.1651
LRHO	-0.0527	>	-0.0663	>	-0.0794
LSMR	-0.0727	>	-0.1219	>	-0.1263
MKS	-0.1308	>	-0.1623	>	-0.1835
P.O.	-0.2429	>	-0.3077	>	-0.3307
RCAL	-0.2334	>	-0.2959	>	-0.3048
RTZ	-0.0434	>	-0.0582	>	-0.0650
SHEL	-0.0805	>	-0.0999	>	-0.1105

Table 9.7b (88 data)

Persistence of Market Efficiency after  
Applying the Thin Trading Correction

Share	Mean Abnormal Returns				
	Lag 1		Lag 2		Lag 3
BARC	-0.1741	>	-0.1898	>	-0.2117
BCHM	-0.1357	>	-0.1753	>	-0.2034
BP	-0.0758	>	-0.0931	>	-0.1010
CGLD	-0.0912	>	-0.1215	>	-0.1337
CTLD	-0.1504	>	-0.1997	>	-0.2184
CUAC	-0.1267	>	-0.1824	>	-0.2084
GEC	-0.1201	>	-0.1502	>	-0.1719
GKN	-0.1777	>	-0.2649	>	-0.3013
GMET	-0.0875	>	-0.1288	>	-0.1333
ICI	-0.1170	>	-0.1428	>	-0.1528
LAND	-0.0896	>	-0.1211	>	-0.1530
LRHO	-0.0657	>	-0.0911	>	-0.0973
LSMR	-0.0749	>	-0.1198	>	-0.1278
MKS	-0.1243	>	-0.1668	>	-0.1930
P.O.	-0.2277	>	-0.2891	>	-0.3204
RCAL	-0.2334	>	-0.2959	>	-0.3048
RTZ	-0.1583	>	-0.2251	>	-0.2725
SHEL	-0.0814	>	-0.1037	>	-0.1183

Table 9.8a (87 data)

## Empirical Issues on Using the Black-Scholes Model

## Ex-dividend Share Price Decline

Share	m	ar55	t-ratio		ar1	$\lambda$	$Q_{KS}(\lambda)$
BARC	84	-0.2906	-13.45	<	-0.2877	0.91	0.38
BCHM	159	-0.2704	-12.15	<	-0.2687	0.54	0.93
BP	294	-0.1208	-11.89	<	-0.1037	0.68	0.74
CGLD	436	-0.0813	-17.32	<	-0.0760	0.83	0.49
CTLD	315	-0.2225	-18.51	>	-0.2423	0.66	0.78
CUAC	181	-0.1499	-11.36	>	-0.1532	0.20	1.00
GEC	225	-0.1642	-18.47	>	-0.1648	0.15	1.00
GKN	249	-0.2906	-17.63	>	-0.2909	0.18	1.00
GMET	166	-0.1533	-13.43	<	-0.1315	0.98	0.29
ICI	489	-0.1988	-22.20	>	-0.2166	1.14	0.15
LAND	165	-0.1446	-11.44	<	-0.1373	0.97	0.30
LRHO	107	-0.0759	-6.34	>	-0.0828	0.44	0.99
LSMR	187	-0.1181	-8.20	<	-0.1169	0.16	1.00
MKS	120	-0.1658	-9.37	<	-0.1570	0.32	1.00
P.O.	443	-0.3013	-25.66	>	-0.3301	0.90	0.40
RCAL	255	-0.3409	-19.47	<	-0.2821	1.13	0.16
RTZ	218	-0.0661	-15.83	<	-0.0628	0.36	1.00
SHEL	353	-0.1155	-12.85	<	-0.1150	0.16	1.00

## Notation:

ar55 = mean abnormal returns generated by 55% of the share price decline.

ar1 = mean abnormal returns generated by 100% of the share price decline.

The distributions of abnormal return generated by adjustments on the share price from the full dividend or a fraction of the dividend are not significantly different.

Table 9.8b (88 data)

## Empirical Issues on Using the Black-Scholes Model

## Ex-dividend Share Price Decline

Share	m	ar55	t-ratio		ar1	$\lambda$	$Q_{KS}(\lambda)$
BARC	113	-0.2570	-13.79	<	-0.2422	1.04	0.23
BCHM	409	-0.2155	-16.56	<	-0.2131	0.39	1.00
BP	396	-0.1166	-15.06	<	-0.1036	0.64	0.81
CGLD	806	-0.1443	-27.37	<	-0.1350	1.50	0.02*
CTLD	474	-0.1950	-22.35	<	-0.1961	0.68	0.75
CUAC	196	-0.1491	-12.17	>	-0.1520	0.18	1.00
GEC	376	-0.1682	-26.66	<	-0.1676	0.13	1.00
GKN	326	-0.2577	-19.00	<	-0.2573	0.16	1.00
GMET	342	-0.1916	-17.69	>	-0.1271	1.80	0.00*
ICI	701	-0.1686	-23.92	<	-0.1656	1.43	0.03*
LAND	234	-0.1405	-14.89	<	-0.1351	0.63	0.82
LRHO	302	-0.1372	-5.92	<	-0.1130	0.66	0.77
LSMR	218	-0.1217	-9.63	<	-0.1205	0.18	1.00
MKS	308	-0.1857	-17.76	<	-0.1818	0.25	1.00
P.O.	484	-0.2971	-26.62	>	-0.3214	0.81	0.53
RCAL	255	-0.3409	-19.47	<	-0.2821	1.13	0.16
RTZ	590	-0.2950	-20.39	<	-0.2909	0.29	1.00
SHEL	480	-0.1122	-15.17	>	-0.1127	0.13	1.00

## Notation:

ar55 = mean abnormal returns generated by 55% of the share price decline.

ar1 = mean abnormal returns generated by 100% of the share price decline.

The distributions of abnormal return generated by adjustments on the share price from the full dividend or a fraction of the dividend are not significantly different.

Table 9.9a (87 data)

## Empirical Issues on Using the Black-Scholes Model

Comparison of the Distributions of Abnormal Returns Generated by Actual versus GARCH Volatility

Share	Mean Abnormal Returns				The KS Test		
	m	Actual	t-ratio		GARCH	$\lambda$	$Q_{KS}(\lambda)$
BARC	121	-0.1285	-14.08	>	-0.2877	2.51	0.00*
BCHM	130	-0.1198	-10.60	>	-0.2687	3.09	0.00*
BP	239	-0.0850	-8.61	>	-0.1037	0.98	0.30
CGLD	439	-0.0968	-13.40	<	-0.0760	1.86	0.00*
CTLD	176	-0.0995	-11.67	>	-0.2423	4.30	0.00*
CUAC	245	-0.1057	-19.91	>	-0.1532	1.32	0.06
GEC	213	-0.2182	-20.20	<	-0.1648	2.36	0.00*
GKN	214	-0.1207	-16.36	>	-0.2909	4.40	0.00*
GMET	280	-0.0849	-17.11	>	-0.1315	2.29	0.00*
ICI	262	-0.0806	-9.21	>	-0.2166	4.96	0.00*
LAND	125	-0.1748	-9.24	<	-0.1373	0.96	0.32
LRHO	71	-0.0792	-5.78	>	-0.0828	0.56	0.91
LSMR	306	-0.1927	-16.85	<	-0.1169	2.41	0.00*
MKS	176	-0.2306	-15.56	<	-0.1570	2.15	0.00*
P.O.	139	-0.0781	-8.70	>	-0.3301	6.42	0.00*
RCAL	233	-0.2446	-19.51	>	-0.2821	0.67	0.76
RTZ	260	-0.0970	-15.58	<	-0.0628	2.17	0.00*
SHEL	353	-0.0529	-11.41	>	-0.1150	3.05	0.00*

\*significantly different from zero.

Table 9.9b (88 data)

## Empirical Issues on Using the Black-Scholes Model

Comparison of the Distributions of Abnormal Returns Generated  
by Actual versus GARCH Volatility

Share	Mean Abnormal Returns				The KS Test		
	m	Actual	t-ratio		GARCH	$\lambda$	$Q_{KS}(\lambda)$
BARC	165	-0.1175	-16.61	>	-0.2422	2.37	0.00*
BCHM	325	-0.1879	-14.71	>	-0.2131	0.78	0.59
BP	324	-0.0895	-11.81	>	-0.1036	1.07	0.20
CGLD	548	-0.1377	-16.51	<	-0.1350	0.87	0.43
CTLD	316	-0.1896	-15.59	>	-0.1961	2.20	0.00*
CUAC	257	-0.1088	-20.69	>	-0.1520	1.37	0.05*
GEC	378	-0.1926	-27.69	<	-0.1676	1.51	0.02*
GKN	269	-0.1360	-17.49	>	-0.2573	3.56	0.00*
GMET	331	-0.0986	-18.88	>	-0.1271	1.71	0.01*
ICI	430	-0.1397	-14.55	>	-0.1656	1.95	0.00*
LAND	165	-0.1711	-11.57	<	-0.1351	1.15	0.14
LRHO	298	-0.2250	-8.65	<	-0.1130	2.72	0.00*
LSMR	320	-0.1900	-17.24	<	-0.1205	2.43	0.00*
MKS	349	-0.2405	-21.75	<	-0.1818	2.18	0.00*
P.O.	174	-0.1391	-10.34	>	-0.3214	5.31	0.00*
RCAL	235	-0.2505	-18.94	>	-0.2821	0.63	0.83
RTZ	449	-0.3584	-18.07	<	-0.2909	1.89	0.00*
SHEL	464	-0.0752	-12.50	>	-0.1127	2.52	0.00*

\*significantly different from zero.

Table 9.10a (87 data)

Empirical Issues on Using the Black-Scholes Model  
Indivisibility of a Call Option Contract

Share	Mean Abnormal Return				The KS Test	
	m	H3	t-ratio	HF	$\lambda$	$Q_{KS}(\lambda)$
BARC	55	-0.2877	-9.22	-0.2877	0.10	1.00
BCHM	149	-0.2687	-11.28	-0.2687	0.12	1.00
BP	268	-0.1036	-10.62	-0.1036	0.13	1.00
CGLD	386	-0.0760	-14.97	-0.0760	0.07	1.00
CTLD	201	-0.2423	-13.97	-0.2423	0.15	1.00
CUAC	164	-0.1532	-10.64	-0.1532	0.06	1.00
GEC	233	-0.1648	-19.11	-0.1648	0.09	1.00
GKN	243	-0.2910	-17.28	-0.2909	0.09	1.00
GMET	156	-0.1315	-11.98	-0.1315	0.11	1.00
ICI	271	-0.2166	-15.10	-0.2166	0.09	1.00
LAND	111	-0.1373	-8.10	-0.1373	0.13	1.00
LRHO	70	-0.0828	-4.61	-0.0828	0.17	1.00
LSMR	180	-0.1169	-7.85	-0.1169	0.11	1.00
MKS	125	-0.1570	-9.28	-0.1570	0.06	1.00
P.O.	264	-0.3301	-18.05	-0.3301	0.09	1.00
RCAL	204	-0.2821	-16.93	-0.2821	0.10	1.00
RTZ	225	-0.0628	-17.40	-0.0628	0.09	1.00
SHEL	356	-0.1150	-12.98	-0.1150	0.07	1.00

Notation:

H3 = mean abnormal return generated by three decimal places of the hedge ratio  $N(d_1)$ .

HF = mean abnormal returns generated by the full hedge ratio.

The distributions of abnormal returns generated by either H3 or HF are not significantly different.

Table 9.10b (88 data)

## Empirical Issues on Using the Black-Scholes Model

## Indivisibility of a Call Option Contract

Share	Mean Abnormal Return				The KS Test	
	m	H3	t-ratio	HF	$\lambda$	$Q_{KS}(\lambda)$
BARC	85	-0.2422	-10.24	-0.2422	0.08	1.00
BCHM	400	-0.2131	-15.95	-0.2131	0.07	1.00
BP	369	-0.1036	-14.16	-0.1036	0.04	1.00
CGLD	640	-0.1350	-21.98	-0.1350	0.03	1.00
CTLD	364	-0.1961	-18.33	-0.1961	0.04	1.00
CUAC	179	-0.1520	-11.47	-0.1520	0.05	1.00
GEC	389	-0.1676	-27.40	-0.1676	0.07	1.00
GKN	320	-0.2573	-18.69	-0.2573	0.08	1.00
GMET	220	-0.1271	-15.08	-0.1271	0.05	1.00
ICI	482	-0.1656	-17.68	-0.1656	0.03	1.00
LAND	181	-0.1351	-12.12	-0.1351	0.05	1.00
LRHO	198	-0.1130	-3.42	-0.1130	0.05	1.00
LSMR	212	-0.1205	-9.30	-0.1205	0.05	1.00
MKS	313	-0.1818	-17.65	-0.1818	0.04	1.00
P.O.	304	-0.3214	-19.89	-0.3214	0.08	1.00
RCAL	204	-0.2821	-16.93	-0.2821	0.05	1.00
RTZ	595	-0.2909	-20.31	-0.2909	0.06	1.00
SHEL	482	-0.1127	-15.46	-0.1127	0.03	1.00

## Notation:

H3 = mean abnormal return generated by three decimal places of the hedge ratio  $N(d_1)$ .

HF = mean abnormal returns generated by the full hedge ratio.

The distributions of abnormal returns generated by either H3 or HF are not significantly different.

Table 9.11a (87 data)

## Special Attributes of Mipriced Call Option Prices

Share	Number of prices		Percentage spread		T	
	Original	Mispriced	$\lambda$	$Q_{KS}(\lambda)$	$\lambda$	$Q_{KS}(\lambda)$
BARC	245	55	6.70	0.00*	0.86	0.46
BCHM	490	149	10.69	0.00*	1.21	0.11
BP	654	268	13.79	0.00*	2.88	0.00*
CGLD	684	386	15.71	0.00*	1.76	0.00*
CTLD	459	201	11.82	0.00*	2.05	0.00*
CUAC	504	164	11.12	0.00*	1.72	0.01*
GEC	637	233	13.06	0.00*	1.72	0.01*
GKN	681	243	13.38	0.00*	2.74	0.00*
GMET	545	156	11.01	0.00*	1.31	0.06
ICI	814	271	14.26	0.00*	1.26	0.08
LAND	414	111	9.36	0.00*	1.64	0.01*
LRHO	241	70	7.37	0.00*	0.85	0.47
LSMR	524	180	11.57	0.00*	2.29	0.00*
MKS	523	125	10.04	0.00*	2.00	0.00*
P.O.	504	264	13.16	0.00*	1.72	0.01*
RCAL	628	204	12.41	0.00*	2.42	0.00*
RTZ	533	225	12.58	0.00*	2.81	0.00*
SHEL	698	356	15.35	0.00*	2.46	0.00*

\*significantly different from zero.

Table 9.11b (87 data)

## Special Attributes of Mipriced Call Option Prices

Share	Number of prices		S - X		(S - X)/S	
	Original	Mispriced	$\lambda$	$Q_{KS}(\lambda)$	$\lambda$	$Q_{KS}(\lambda)$
BARC	245	55	4.38	0.00*	1.04	0.23
BCHM	490	149	6.09	0.00*	1.78	0.00*
BP	654	268	12.16	0.00*	3.91	0.00*
CGLD	684	386	9.48	0.00*	2.56	0.00*
CTLD	459	201	7.26	0.00*	2.48	0.00*
CUAC	504	164	6.49	0.00*	1.06	0.21
GEC	637	233	7.79	0.00*	0.68	0.75
GKN	681	243	8.25	0.00*	1.03	0.24
GMET	545	156	7.52	0.00*	1.02	0.24
ICI	814	271	7.92	0.00*	2.17	0.00*
LAND	414	111	5.31	0.00*	1.87	0.00*
LRHO	241	70	6.23	0.00*	1.50	0.02*
LSMR	524	180	8.73	0.00*	2.29	0.00*
MKS	523	125	6.61	0.00*	1.23	0.09
P.O.	504	264	9.64	0.00*	3.55	0.00*
RCAL	628	204	6.46	0.00*	1.17	0.13
RTZ	533	225	8.19	0.00*	3.26	0.00*
SHEL	698	356	10.62	0.00*	3.90	0.00*

\*significantly different from zero.

Table 9.11c (88 data)

## Special Attributes of Mipriced Call Option Prices

Share	Number of prices		Percentage spread		T	
	Original	Mispriced	$\lambda$	$Q_{KS}(\lambda)$	$\lambda$	$Q_{KS}(\lambda)$
BARC	369	85	0.99	0.28	1.34	0.05
BCHM	896	400	1.52	0.02*	2.99	0.00*
BP	856	369	3.38	0.00*	2.77	0.00*
CGLD	1124	640	1.65	0.01*	3.19	0.00*
CTLD	734	364	2.29	0.00*	3.26	0.00*
CUAC	550	179	1.55	0.02*	2.01	0.00*
GEC	920	389	1.83	0.00*	2.41	0.00*
GKN	808	320	1.64	0.01*	2.98	0.00*
GMET	724	220	0.85	0.46	1.90	0.00*
ICI	1160	482	1.94	0.00*	2.12	0.00*
LAND	532	181	2.04	0.00*	2.71	0.00*
LRHO	556	198	2.23	0.00*	1.16	0.14
LSMR	599	212	1.93	0.00*	2.75	0.00*
MKS	820	313	3.12	0.00*	1.85	0.00*
P.O.	688	304	2.79	0.00*	1.71	0.01*
RCAL	633	204	2.02	0.00*	2.48	0.00*
RTZ	1072	595	2.61	0.00*	4.29	0.00*
SHEL	930	482	3.15	0.00*	2.87	0.00*

\*significantly different from zero.

Table 9.11d (88 data)

## Special Attributes of Mipriced Call Option Prices

Share	Number of prices		S - X		(S - X)/S	
	Original	Mispriced	$\lambda$	$Q_{KS}(\lambda)$	$\lambda$	$Q_{KS}(\lambda)$
BARC	369	85	1.20	0.11	0.86	0.45
BCHM	896	400	3.15	0.00*	1.91	0.00*
BP	856	369	3.98	0.00*	3.99	0.00*
CGLD	1124	640	4.27	0.00*	2.74	0.00*
CTLD	734	364	3.07	0.00*	2.23	0.00*
CUAC	550	179	2.42	0.00*	0.81	0.53
GEC	920	389	2.48	0.00*	0.58	0.89
GKN	808	320	2.15	0.00*	1.43	0.03*
GMET	742	220	1.30	0.07	1.22	0.10
ICI	1160	482	2.74	0.00*	1.40	0.04*
LAND	532	181	2.88	0.00*	2.46	0.00*
LRHO	556	198	3.01	0.00*	2.16	0.00*
LSMR	599	212	2.77	0.00*	2.63	0.00*
MKS	820	313	1.14	0.15	1.64	0.01*
P.O.	688	304	3.36	0.00*	2.53	0.00*
RCAL	633	204	3.24	0.00*	1.14	0.15
RTZ	1072	595	5.13	0.00*	1.28	0.08
SHEL	930	482	3.73	0.00*	3.48	0.00*

\*significantly different from zero.

Table 9.12a (87 data)

## Special Attributes of Mipriced Call Option Prices

Share	S - X			(S - X)/S		
	Original		Mispriced	Original		Mispriced
BARC	-21.96	>	-39.88	0.076	<	0.096
BCHM	12.80	>	-3.54	0.066	>	0.046
BP	71.82	>	37.23	0.180	>	0.117
CGLD	45.99	>	4.09	0.087	>	0.061
CTLD	17.88	>	-0.66	0.076	>	0.054
CUAC	-5.29	>	-19.13	0.075	<	0.085
GEC	-4.16	>	-13.64	0.083	<	0.089
GKN	-10.35	>	-18.12	0.108	>	0.095
GMET	-11.20	>	-15.84	0.058	>	0.055
ICI	-11.59	>	-32.15	0.061	>	0.048
LAND	26.35	>	0.26	0.089	>	0.060
LRHO	26.13	>	10.56	0.114	>	0.098
LSMR	24.79	>	8.61	0.125	>	0.094
MKS	-6.53	<	-5.62	0.075	>	0.062
P.O.	81.34	>	30.30	0.139	>	0.088
RCAL	2.50	>	-7.95	0.081	>	0.074
RTZ	92.95	>	3.51	0.110	>	0.058
SHEL	64.64	>	9.07	0.079	>	0.045

Table 9.12b (87 data)

## Special Attributes of Mipriced Call Option Prices

Share	Call Option Percentage Spread			T		
	Original		Mispriced	Original		Mispriced
BARC	0.3112	<	0.3789	59.3	>	59.1
BCHM	0.1736	<	0.2135	70.1	<	80.3
BP	0.0660	<	0.0991	42.5	<	52.3
CGLD	0.1078	<	0.1161	57.7	<	61.9
CTLD	0.1687	<	0.2261	52.5	<	68.4
CUAC	0.2569	<	0.2920	60.4	<	72.7
GEC	0.2224	<	0.2254	77.0	<	83.7
GKN	0.3448	>	0.3423	65.2	<	77.1
GMET	0.2139	>	0.1936	81.5	<	84.6
ICI	0.1954	<	0.2185	59.3	>	58.3
LAND	0.2192	<	0.2482	52.5	<	58.4
LRHO	0.1146	<	0.1598	27.5	<	27.6
LSMR	0.1224	<	0.1292	48.2	<	58.4
MKS	0.3110	>	0.2463	71.8	>	58.8
P.O.	0.1692	<	0.2593	42.1	<	51.0
RCAL	0.1898	<	0.2326	81.2	<	97.8
RTZ	0.0990	<	0.1188	58.7	<	77.0
SHEL	0.0877	<	0.1143	54.1	<	67.6

Table 9.12c (88 data)

## Special Attributes of Mipriced Call Option Prices

Share	S - X			(S - X)/S		
	Original		Mispriced	Original		Mispriced
BARC	-11.54	>	-20.06	0.072	<	0.084
BCHM	9.79	>	-4.64	0.060	>	0.047
BP	57.00	>	28.18	0.153	>	0.100
CGLD	38.83	>	-7.94	0.091	>	0.068
CTLD	11.63	>	-1.06	0.068	>	0.053
CUAC	-1.31	>	-15.83	0.079	<	0.084
GEC	-3.78	>	-9.28	0.081		0.081
GKN	-7.50	>	-14.43	0.102	>	0.086
GMET	0.16	>	-5.79	0.065	>	0.056
ICI	-8.29	>	-30.52	0.062	>	0.056
LAND	24.17	>	-2.21	0.083	>	0.052
LRHO	17.02	>	4.63	0.097	>	0.070
LSMR	28.22	>	10.56	0.132	>	0.095
MKS	-5.63	<	-5.14	0.071	>	0.059
P.O.	56.42	>	21.65	0.120	>	0.089
RCAL	2.38	>	-7.95	0.081	>	0.074
RTZ	32.81	>	-31.24	0.138	>	0.137
SHEL	50.48	>	5.72	0.070	>	0.044

Table 9.12d (88 data)

## Special Attributes of Mipriced Call Option Prices

Share	Call Option Percentage Spread			T		
	Original		Mispriced	Original		Mispriced
BARC	0.3057	<	0.3249	47.8	<	49.4
BCHM	0.1646	<	0.1792	78.5	<	100.
BP	0.0947	<	0.1276	42.6	<	51.4
CGLD	0.1429	<	0.1529	56.5	<	65.8
CTLD	0.1918	<	0.2164	56.6	<	75.5
CUAC	0.2480	<	0.2849	58.9	<	71.9
GEC	0.2349	>	0.2101	79.3	<	95.5
GKN	0.3172	>	0.3062	61.7	<	71.8
GMET	0.1994	>	0.1946	71.5	<	75.8
ICI	0.1942	<	0.2136	54.0	<	55.7
LAND	0.2007	<	0.2129	62.3	<	84.8
LRHO	0.1677	<	0.2109	31.5	<	33.8
LSMR	0.1198	<	0.1312	48.6	<	60.8
MKS	0.3051	>	0.2426	74.5	<	84.1
P.O.	0.2038	<	0.2682	46.1	<	52.7
RCAL	0.1926	<	0.2326	80.6	<	97.8
RTZ	0.2238	<	0.2679	61.2	<	76.3
SHEL	0.1136	<	0.1310	51.0	<	63.0

Table 9.13a (87 data)

$$\text{Call percentage spread} = \alpha + \beta \frac{1}{C} + \gamma (S-x)$$

## Original Call Option Prices

Share	$\alpha$	$t(\alpha)$	$\beta$	$t(\beta)$	$\gamma$	$t(\gamma)$	$R^2(\%)$
BARC	0.15	16.01	1.34	14.59	-0.0006	-2.91	75.1
BCHM	0.12	28.98	1.18	27.79	-0.0010	-12.17	74.5
BP	0.04	12.65	2.01	27.72	-0.0003	-9.47	81.5
CGLD	0.06	23.72	2.38	37.84	-0.0002	-10.00	78.5
CTLD	0.10	20.97	1.32	40.12	-0.0012	-11.88	85.7
CUAC	0.12	17.38	0.96	23.48	-0.0018	-7.30	71.3
GEC	0.10	18.01	0.67	28.43	-0.0007	-2.67	67.9
GKN	0.17	26.45	0.64	27.25	-0.0033	-16.69	83.9
GMET	0.10	20.85	1.23	28.80	-0.0010	-8.18	71.4
ICI	0.12	39.84	1.66	40.35	-0.0005	-19.02	81.1
LAND	0.12	22.36	1.34	43.17	-0.0007	-10.82	88.4
LRHO	0.12	16.42	0.62	14.63	-0.0020	-12.10	80.0
LSMR	0.06	14.79	1.59	37.04	-0.0005	-8.47	83.5
MKS	0.16	20.73	0.66	20.42	-0.0035	-8.89	66.8
P.O.	0.11	21.05	1.99	42.65	-0.0005	-14.10	89.1
RCAL	0.12	29.65	0.73	27.04	-0.0020	-12.81	72.7
RTZ	0.05	17.33	3.62	36.84	-0.0001	-5.85	83.6
SHEL	0.04	22.30	2.89	54.94	-0.0002	-15.55	90.6

$\beta$  and  $\gamma$  are significantly positive and negative respectively.

Table 9.13b (88 data)

$$\text{Call percentage spread} = \alpha + \beta \frac{1}{C} + \gamma (S-X)$$

## Original Call Option Prices

Share	$\alpha$	$t(\alpha)$	$\beta$	$t(\beta)$	$\gamma$	$t(\gamma)$	$R^2(\%)$
BARC	0.19	30.38	0.76	23.55	-0.0016	-11.64	78.1
BCHM	0.12	42.67	1.18	30.73	-0.0010	-17.02	66.4
BP	0.09	23.98	1.28	25.25	-0.0006	-16.82	74.4
CGLD	0.12	36.32	1.84	19.48	-0.0003	-12.92	47.5
CTLD	0.12	30.31	1.34	40.22	-0.0015	-15.55	80.4
CUAC	0.13	19.12	0.97	24.83	-0.0015	-7.09	71.0
GEC	0.11	22.49	0.74	36.30	-0.0005	-2.25	69.8
GKN	0.17	30.86	0.65	30.47	-0.0032	-18.72	84.4
GMET	0.12	27.97	1.20	29.23	-0.0010	-10.09	68.7
ICI	0.13	54.23	1.37	44.27	-0.0006	-25.94	79.1
LAND	0.12	28.87	1.32	47.10	-0.0008	-13.65	87.2
LRHO	0.14	25.01	0.65	29.96	-0.0022	-14.79	82.7
LSMR	0.06	17.49	1.55	36.38	-0.0005	-9.59	81.8
MKS	0.16	27.21	0.69	27.00	-0.0031	-9.45	66.4
P.O.	0.16	32.24	1.85	35.86	-0.0007	-21.36	83.6
RCAL	0.14	33.15	0.60	25.95	-0.0023	-14.72	71.1
RTZ	0.17	42.44	1.36	32.89	-0.0004	-17.00	68.5
SHEL	0.06	19.17	3.02	29.99	-0.0003	-12.43	71.0

$\beta$  and  $\gamma$  are significantly positive and negative respectively.

Table 9.13c (87 data)

$$\text{Call percentage spread} = \alpha + \beta \frac{1}{C} + \gamma (S-X)$$

## Mispriced Call Option Prices

Share	$\alpha$	$t(\alpha)$	$\beta$	$t(\beta)$	$\gamma$	$t(\gamma)$	$R^2(\%)$
BARC	0.16	6.41	1.22	6.99	-0.0004	-0.67 <sup>#</sup>	72.2
BCHM	0.13	14.35	1.09	14.28	-0.0010	-4.71	66.9
BP	0.06	8.50	1.79	14.38	-0.0004	-6.07	77.7
CGLD	0.07	21.73	1.93	22.42	-0.0003	-8.54	70.9
CTLD	0.10	12.80	1.22	24.85	-0.0014	-5.69	81.3
CUAC	0.11	8.67	0.97	16.32	-0.0014	-3.29	73.8
GEC	0.11	13.43	0.55	13.88	-0.0007	-1.33 <sup>#</sup>	64.5
GKN	0.14	14.24	0.63	15.84	-0.0034	-8.50	82.6
GMET	0.07	6.62	1.44	14.90	-0.0008	-2.64	70.1
ICI	0.10	18.41	1.53	25.80	-0.0005	-6.95	82.2
LAND	0.11	9.94	1.27	16.14	-0.0007	-3.22	78.6
LRHO	0.07	4.13	1.01	7.86	-0.0014	-3.29	85.0
LSMR	0.07	11.72	1.24	15.56	-0.0008	-6.76	75.5
MKS	0.10	6.61	0.92	9.31	-0.0005	-0.61 <sup>#</sup>	59.3
P.O.	0.13	16.99	1.82	28.53	-0.0006	-9.10	86.1
RCAL	0.11	11.35	0.83	14.71	-0.0025	-5.85	68.7
RTZ	0.04	6.82	4.03	17.21	0.0000	0.10 <sup>#</sup>	70.6
SHEL	0.04	17.95	2.66	40.03	-0.0003	-10.29	88.7

Notation:  $\beta$  and  $\gamma$  are significantly positive and negative respectively except for those indicated by # (not significantly different from zero).

Table 9.13d (88 data)

$$\text{Call percentage spread} = \alpha + \beta \frac{1}{C} + \gamma (S-X)$$

## Mispriced Call Option Prices

Share	$\alpha$	$t(\alpha)$	$\beta$	$t(\beta)$	$\gamma$	$t(\gamma)$	$R^2(\%)$
BARC	0.19	11.02	0.80	7.04	-0.0013	-3.55	71.8
BCHM	0.12	27.95	1.08	18.91	-0.0015	-12.08	60.6
BP	0.08	12.57	1.58	17.53	-0.0006	-8.15	73.8
CGLD	0.12	31.61	1.31	10.60	-0.0005	-13.17	43.3
CTLD	0.12	21.59	1.21	28.15	-0.0017	-8.58	76.6
CUAC	0.13	10.97	0.97	16.72	-0.0010	-2.50	72.7
GEC	0.13	24.13	0.53	17.47	-0.0006	-1.86	61.9
GKN	0.14	19.20	0.63	18.70	-0.0034	-10.45	83.0
GMET	0.11	11.85	1.35	13.99	-0.0006	-2.39	60.0
ICI	0.13	31.08	1.19	26.51	-0.0007	-14.07	75.7
LAND	0.12	17.27	1.23	20.36	-0.0010	-5.85	77.1
LRHO	0.15	16.45	0.58	13.78	-0.0029	-7.90	75.7
LSMR	0.08	14.10	1.15	13.84	-0.0008	-7.48	70.3
MKS	0.15	19.65	0.67	14.00	-0.0026	-5.45	57.3
P.O.	0.15	20.31	1.78	28.48	-0.0006	-10.90	84.1
RCAL	0.11	11.35	0.83	14.71	-0.0025	-5.85	68.7
RTZ	0.17	30.34	1.13	19.10	-0.0007	-9.70	59.0
SHEL	0.07	18.33	2.57	24.13	-0.0004	-9.58	70.9

$\beta$  and  $\gamma$  are significantly positive and negative respectively.

Figure 9.1a (87 data)  
Frequently Traded Call Options

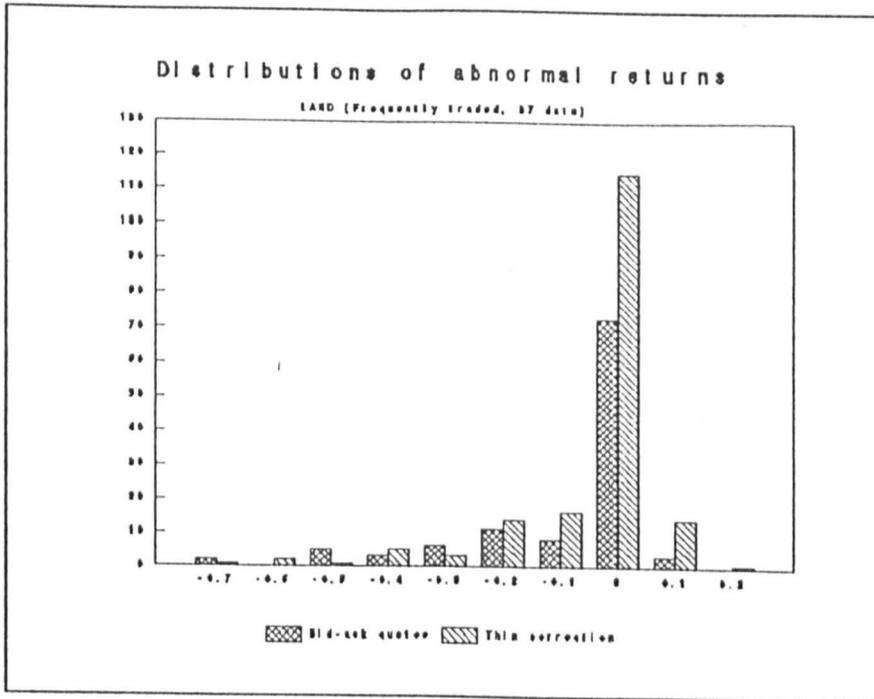


Figure 9.1b (88 data)  
Frequently Traded Call Options

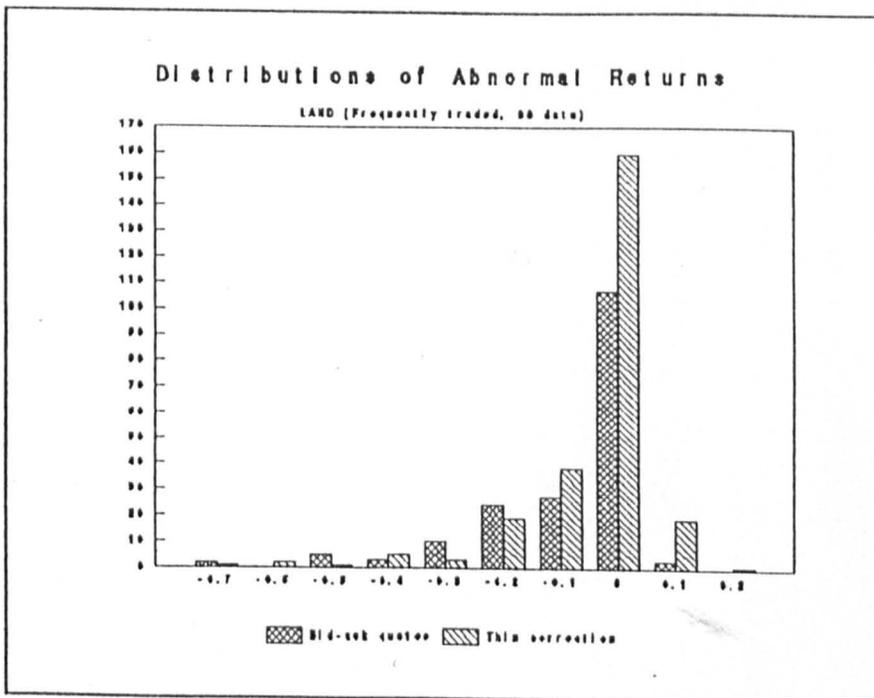
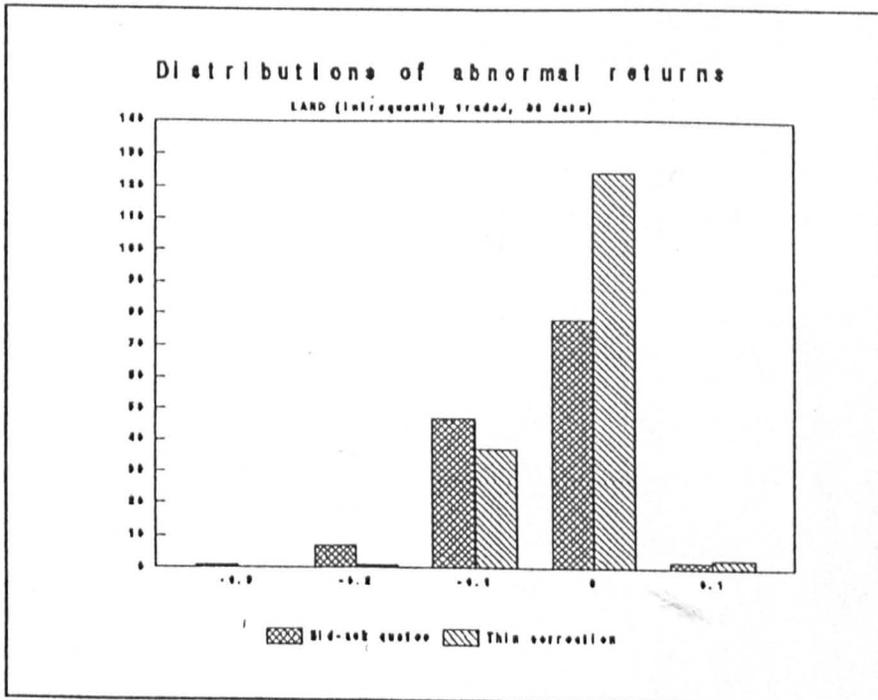


Figure 9.2a (87 data)  
 Infrequently Traded Call Options



Figure 9.2b (88 data)  
 Infrequently Traded Call Options



## Chapter 10

### Conclusions

This chapter concludes the study. It points out the major contributions made in this study to the research literature, identifies the limitations, and suggests several approaches to extend the present study.

#### 11.1 The contributions made in this study

This study is a test of the efficiency on the London Traded Options Market. It contributes to the research literature in three aspects.

First, it investigates the effect of a non-constant share price volatility. Hitherto, there has been no agreed procedure on modelling or forecasting the future share price volatility. This study shows that the GARCH process has the best forecasting accuracy. The out-of-the-sample forecasting accuracy of the GARCH process is shown to be superior to the moving average method. This implies that Dimson and March's (1990) claim that a simple moving average volatility estimate might outperform the more sophisticated ones is not generally true. The ex ante Garch volatility is then incorporated in the Black-Scholes model. Because the volatility is assumed constant in the Black-Scholes model, the consideration of adapting the GARCH volatility into the model sheds insight on bridging empirical results and theoretical requirements.

Second, because the LTOM is thinly traded the quoted prices may not reflect prices at which trade did or could take place. However, information on call option trading volume may not be available. This study develops and implements an analytical model to select the most actively traded call option series. The prices of the call option series selected by this model bear the basic characteristics of those frequently traded call option prices where trading volume is available.

Third, the tests in this study are novel in that previous research has not used a Black-Scholes hedge portfolio with bid-ask data. By incorporating the bid-ask spread directly in the establishment of arbitrage portfolios, an accurate assessment of transactions data can be made. The empirical results show that the hypothesis that the LTOM is efficient cannot be rejected when bid-ask spreads are considered, even when the transaction costs are negligible. The implications of this study for an investor is that the investor can correctly identify a mispriced call option and try to exploit the anomaly. However, because of the large bid-ask spreads observed in the LTOM, exploiting the inefficiencies would be very difficult.

## 10.2 Limitations

There are two constraints in this study which limit the strength of the conclusions. First, information was not available on the actual call option trading volume for all the securities examined in this study. Some of the mispriced call options might not have been traded and some of the prices might

therefore not have reflected the market's valuation. Second, intra-day data are not available in this study implying that share and call option prices may not be synchronous.

### 10.3 Extensions and implications for future research

We plan to further examine if the conclusions obtained in this study will hold also for an option-option hedge and put-call parity, on behalf of the share-option hedge. We shall also examine how sensitive are our results to choice of other interest rate estimates such as the London Interbank Offer Rate.

Beyond this study at least four extensions seem clear:

First, in this study the set of call options tested were constrained to those for which it was not optimal to be exercised early. Future research could relax this restriction and use the RGW call option model with call and share quotations to test market efficiency. This implies that the boundary conditions analyzed in this study would be altered to allow for early exercise. The effect of relaxing this constraint would have been to increase the data set in this study by 25%. It is arguable whether the results would have been materially effected by such a change but conceptually the study would be impressed by using the larger sample set.

Another perspective is to pursue the possibility of modelling the implicit cost of trading as an analytical function of the transaction costs and the combined effects of call option and share percentage spreads.

Third, this study is conducted in the context of the UK market with its special features in the Account Day, the margin requirements and a thinly traded options market. It would be insightful to apply the same methodology in testing the efficiency of other major call options market.

Finally, this study can also be extended to different option pricing models such as Cox's (1975) constant elasticity of variance diffusion formula and Merton's (1976) jump-diffusion formula. The empirical results will be greatly enhanced if daily transaction, time-stamped data, or at least intraday data are available.

## Major Algorithms in This Study

## Chapter 3

ARCH Programme {RATS programme}

```
CAL(DAILY) 79 7 31
ALL 0 88:6:30
OPE DAT C:\RET\CTLD.
DAT / Y
SET TREND = T
SET Y = Y(T)/100.
*


---


PROC ARCH SERIES START END
TYPE SERIES SERIES
TYPE PARAM START END
OPTIONS ORDER INTEGER 1
LOCAL INTEGER STARTL ENDL
*
IF START.AND.END {
  IEVAL STARTL=START ; IEVAL ENDL=END
}
ELSE
  INQUIRE(SERIES=SERIES) STARTL ENDL
*
NON MU A0 A1
FRM REGRESID = SERIES(T) - MU
FRM ARCHVAR = A0 + A1 * REGRESID(T-1) ** 2
FRM ARCHLNL = -.5 * (LOG(ARCHVAR(T)) + REGRESID(T) ** 2 / ARCHVAR(T))
*
SMPL STARTL ENDL
LIN SERIES
# CONSTANT
*
EVA MU = BETA(1); EVA A0 = SEESQ; EVA A1 = .05
SMPL STARTL+ORDER ENDL
MAX(METHOD=BHHH, RECURSIVE) ARCHLNL
DIS 'Sum of coefs. =' #.##### A0+A1
*


---


SET RESSQR STARTL ENDL = SERIES(T) ** 2
LINREG(NOPRINT) RESSQR STARTL+ORDER ENDL
# CONSTANT RESSQR{ORDER}
FETCH RSQUARED=RSQUARED
EVAL CHISTAT=NOBS*RSQUARED
CDF(NOPRINT) CHISQR CHISTAT ORDER
FETCH SIGNIF=SIGNIF
DISPLAY 'Test for ARCH of Order' ### ORDER $
      ' TR**2 = ' #####.#### CHISTAT 'SIGNIF. LEVEL' #.##### SIGNIF
END
@ARCH Y 83:6:6 88:6:3
END
```

GARCH Model {RATS Programme}

```

CAL(DAILY) 79 7 31
ALL 0 88:6:30
OPE DAT C:\RET\BCHM.
DAT / Y
SET TREND = T
SET Y = Y(T)/100.
*
PROC GARCH SERIES START END
TYPE SERIES SERIES
TYPE PARAM START END
OPTIONS ORDER INTEGER 3
LOCAL INTEGER STARTL ENDL
*
IF START.AND.END {
  IEVAL STARTL=START ; IEVAL ENDL=END
}
ELSE
  INQUIRE(SERIES=SERIES) STARTL ENDL
*
SET U = 0.0
SET V = 0.0
NON MU A0 A1 A2 A3 B1 B2 B3
FRM REGRESID = SERIES(T)-MU
FRM GARCHVAR = A0+A1*U(T-1)**2+A2*U(T-2)**2+A3*U(T-3)**2+$
B1*V(T-1)+B2*V(T-2)+B3*V(T-3)
FRM GARCHLNL = -.5*(LOG(V(T)=GARCHVAR(T))+$
(U(T)=REGRESID(T))**2/V(T))
*
SMPL STARTL ENDL
LIN SERIES
# CONSTANT
*
EVA MU=BETA(1); EVA A0=SEESQ; EVA A1=.05; EVA A2=.2; EVA A3=.3
EVA B1=.3; EVA B2=.2; EVA B3=.3
SMPL STARTL+ORDER ENDL
MAX(METHOD=BHHH,RECURSIVE) GARCHLNL
DIS 'Sum of coefs. =' #.##### A1+A2+A3+B1+B2+B3
END
@GARCH Y 83:6:6 88:6:3
END

```

Determination of the EWMA Forecasting Constant  
 {Pascal Programme}

```

program f_constant;
const
  nobs          = 1305; {t1 nobs before forecasting}
  days          = 20;
  max           = 65;
var
  minfile,infile,outfile : text;
  fname             : string[8];
procedure Min_w(var p:text);
var
  temp,w          : real;
  n,k,k1,k2,t,period,
  shift          : integer;
  y               : array[1..2089] of real;
  x               : array[1..nobs] of real;
  msum,mean,ssum,pstd
  f               : array[1..65] of real;
  msesum,mse      : array[1..20] of real;
begin
  {max number of pstd}
  n:=nobs div days;

  {self recognition of data range}
  for k:=1 to 2089 do
  begin
    read(p,y[k]);
    case k of
      45..1709: x[k-44]:=y[k]/100.0;
    end;
  end;

  for period:=1 to 4 do
  begin
    shift:=120*(period-1)+5;

  {means}
  for k1:=1 to n do
  begin
    msum[k1]:=0.0;
    for k2:=(1+20*(k1-1)+shift) to (days+20*(k1-1)+shift) do
      msum[k1]:=msum[k1]+x[k2];
    mean[k1]:=msum[k1]/days;
  end;

  {historical standard deviations}
  for k1:=1 to n do
  begin
    ssum[k1]:=0.0;
    for k2:=(1+20*(k1-1)+shift) to (days+20*(k1-1)+shift) do
      ssum[k1] := ssum[k1]+sqr(x[k2]-mean[k1]);
    pstd[k1] := sqrt(ssum[k1]/(days-1));
  end;

  {EWMA process, w=1 ~ naive, comparison of historical & EWMA}
  for k1:=1 to 20 do
  begin
    msesum[k1]:=0;
    w:=0.05*k1;
    f[k1,2]:=pstd[1];
    for k2:=3 to n do
    begin
      f[k1,k2] := w*pstd[k2-1] + (1-w)*f[k1,k2-1];
      msesum[k1] := msesum[k1] + sqr(pstd[k2-1]-f[k1,k2-1]);
    end;
  end;
end;

```

```

end;

{last forecast interval, nth}
for k:=1 to 20 do
mse[k] := (msum[k]+sqr(pstd[n]-f[k,n]))/(n-1);
{there are totally n-1 comparisons, the first interval is not used}

{sorting w corresponding to least mse}
temp:=mse[1]; t:=1;
for k:=2 to 20 do
begin
if mse[k] < temp then begin temp:=mse[k]; t:=k; end;
end;

{printing pstd, mse, and w~least mse}
{* for k:=1 to n do writeln(outfile,pstd[k]:8:6); writeln(outfile);
for k:=1 to 20 do writeln(outfile,mse[k]*10000:8:6); writeln(outfile);*}
writeln(outfile,t*0.05);
append(outfile);
end;
end;

{0 data input & output}
begin
assign(minfile,'series');
reset(minfile);
assign(outfile,'report');
rewrite(outfile);
while not eof(minfile) do
begin
readln(minfile,fname);
write(fname,'      wait.....');
assign(infile,fname);
reset(infile);
Min_w(infile);
close(infile);
append(outfile);
end;
writeln; writeln('now, look at report!');
close(outfile); close(minfile);
end.

```

Comparing the Forecasting Accuracy of Competing Models  
 {Pascal Programme}

```

program comparison;
var
  sfile,gfile,afile,wfile,ptfile,
  hfile,ffile,stdfile,dtdfile      : text;
type
  v                                  = array[1..22] of real;

{$I mse} {rmse,mae,mape}

procedure compare
  (varpgfile,pafile,psfile,pwfile,pptfile:text);
const
  days                                  = 20;
var
  y                                      : array[1..2328] of real;
  x,epsilon                             : array[1..439] of real; {20x21+19}
  h,arch                                 : array[1..421] of real;
                                       {1,2-421 <-> x[1] - x[420]}
  k,period,shift,adjust                 : integer;
  fc,ab0,aa0,aa1,gb0,ga0,ga1,gb,
  nrmse,nmae,nmape,crmse,cmae,cmape,
  grmse,gmae,gmape,armse,amae,amape,
  trmse,tmae,tmape,gsr,gsr2            : real;
  nstd,cstd,hstd,gstd,astd,tstd,ts     : v;

begin
  {self-recognition of data range}
  for k:=1 to 2328 do
    begin
      read(psfile,y[k]);
      case k of
        1890..2328:x[k-1889]:=y[k]/100.0;
      end;
    end;

  gstd[1]:=0.0; astd[1]:=0.0; h[1]:=0;
  {automatic shifting 20 observations forward}
  for period:=1 to 22 do
    begin{period}
      shift:=20*(period-1);

      {out-of-sample GARCH estimated parameters}
      read(pgfile,gb0,ga0,ga1,gb); read(pafile,ab0,aa0,aa1);

      {defining GARCH h(t), arch(t)}
      if period < 22 then
        begin
          for k:=1+shift to days+shift do
            begin
              epsilon[k]:=x[k]-gb0;
              h[k+1]:=ga0+ga1*sqr(epsilon[k])+gb*h[k];
              if k > 1 then
                begin
                  gsr:=epsilon[k]/sqr(h[k]); gsr2:=sqr(gsr);
                  writeln(dtdfile,gsr:11:7,gsr2:14:7);
                end;
              epsilon[k]:=x[k]-ab0;
              arch[k+1]:=aa0+aa1*sqr(epsilon[k]);
            end;
            gstd[period+1]:=sqr(h[days+shift+1]);
            astd[period+1]:=sqr(arch[days+shift+1]);
          end;
        end;
      end; {period}
    end;
  end;

```

```

{G[1] <-> h[2] - h[21] <-> x[1] - x[20] ->  $\sigma$ [2]}
for k:=1 to 22 do read(pptfile,hstd[k],ts[k]);

{generating naive, EWMA standard deviations}
read(pwfile,fc);
cstd[1]:=hstd[1]; cstd[2]:=hstd[1];
nstd[1]:=hstd[1]; nstd[2]:=hstd[1];
for k:=3 to 22 do
begin
  cstd[k]:=fc*hstd[k-1]+(1-fc)*cstd[k-1];
  nstd[k]:=hstd[k-1];
end;

{printing historical deviation, EWMA, GARCH}
for k:=2 to 22 do
writeln(stdfile,k:3,hstd[k]:10:6,gstd[k]:10:6,
astd[k]:10:6,cstd[k]:10:6,nstd[k]:10:6);

{reporting accuracy of all five estimated  $\delta$ }
grmse:=rmse(hstd,gstd); gmae:=mae(hstd,gstd);
gmape:=mape(hstd,gstd); armse:=rmse(hstd,astd);
amae:=mae(hstd,astd); amape:=mape(hstd,astd);
crmse:=rmse(hstd,cstd); cmae:=mae(hstd,cstd);
cmape:=mape(hstd,cstd); nrmse:=rmse(hstd,nstd);
nmae:=mae(hstd,nstd); nmape:=mape(hstd,nstd);

writeln(ffile,grmse:10:7,gmae:10:7,gmape:10:7);
writeln(ffile,armse:10:7,amae:10:7,amape:10:7);
writeln(ffile,crmse:10:7,cmae:10:7,cmape:10:7);
writeln(ffile,nrmse:10:7,nmae:10:7,nmape:10:7);

end;{procedure}

begin
  assign(sfile,'c:\ret\LAND');
  reset(sfile);
  assign(gfile,'LAND79.g');
  reset(gfile);
  assign(afile,'LAND79.a');
  reset(afile);
  assign(wfile,'LAND.w');
  reset(wfile);
  assign(ptfile,'LAND.PT');
  reset(ptfile);
  assign(ffile,'LAND79.f');
  rewrite(ffile);
  assign(stdfile,'LAND79.d');
  rewrite(stdfile);
  assign(dtfile,'LAND.dt');
  rewrite(dtfile);
  write('LAND          wait.....');
  writeln(ffile,'LAND':8);
  compare(gfile,afile,sfile,wfile,ptfile);
  writeln;
  writeln('now, look at the report!');
  close(sfile);close(gfile);close(afile);close(wfile);
  close(dtfile);close(ffile);close(stdfile);close(ptfile);
end.

```

## Chapter 5 Call Option Trading Volume

Modelling the Call Option Trading Volume by  
Past Trading History  
{Pascal Programme}

```
program history;
const
  max                = 275;
type
  intarray           = array[0..max] of integer;
  realarray          = array[0..max] of real;
var
  cvfile,infile,outfile : text;
  k,row                : integer;
  trade                : intarray;
  filename              : string[8];

procedure transform(var f: text);
var
  sum,s,sx,call,p      : real;
  cv,t                 : intarray;
  d,j,pv               : integer;
  cp                   : realarray;

begin
  j := 0; sum := 0;
  read(f,s,sx,cv[j]);
  if cv[j] > 0 then t[j] := 1 else t[j] := 0;
  while not eof(f) do
    begin
      j := j + 1;
      read(f,s,sx,cv[j]);
      if cv[j] > 0 then t[j] := 1 else t[j] := 0;
      sum := sum + t[j - 1];
      cp[j] := sum/j;
      append(outfile);
      writeln(outfile,s:8:2,sx:8:2,cv[j]:8,' ',cp[j]:8:5);
    end;
  end;

begin
  assign(cvfile,'s');
  reset(cvfile);
  assign(outfile,'h');
  rewrite(outfile);

  while not eof(cvfile) do
    begin
      readln(cvfile,filename);
      write(filename,'transforming...':18); writeln;
      assign(infile,filename);
      reset(infile);
      writeln(outfile,filename);
      transform(infile);
      close(infile);
      writeln(outfile);
    end;
  close(outfile);
  close(cvfile);
end.
```

# Classification of Intrinsic Values into 3, 5, 7 or 9 Classes {Pascal Programme}

```

program cm;
const
  max =1500; max1 =1500; max2 =4; max3 =9;
var
  infile,outfile,statfile,
  minfile           :text;
  fname             :string[8];

procedure transform(var p:text);
var
  k,j               :array[1..max1] of integer;
  pv                :array[1..max1] of real;
  sx,nu,stan,r2,r3,r4,r5,r6,
  r7,r8,r9,sum1,x2sum,s,cv,
  call,p1,p2,d,pr   :real;
  n,pn,i1,i2,i3,i4,i5,i6,m,
  sum,n1,k1         :integer;
  z                 :array[1..max2] of real;
  c                 :array[0..max3] of real;
  x                 :array[1..max] of real;
  label            1;

begin
  n1:=1;
  while not eof(p) do
  begin
    read(p,s,x[n1],cv,call,p1,p2,d,pr);
    n1:=n1+1;
  end; n1:=n1-1; sum1:=0;
  for k1:=1 to n1 do sum1:=sum1+x[k1];
  nu:=sum1/n1;
  x2sum:=0;
  for k1:=1 to n1 do
  begin
    x[k1]:=x[k1]-nu;
    x2sum:=x2sum+sqr(x[k1]);
  end;
  stan :=sqrt(x2sum/n1);
  append(outfile);
  writeln(outfile,n1:10,' ',nu:8:2,stan:8:2);

  for m:=3 to 9 do
  begin
    if m/2-m div 2 =0 then goto 1;
    for i1:=1 to m do
    begin
      k[i1]:=1; j[i1]:=0; pv[i1]:=0;
    end;
    case m of
      3: z[1]:=0.430729;
      5: begin z[1]:=0.841623; z[2]:=0.253349; end;
      7: begin z[1]:=1.06757; z[2]:=0.565951; z[3]:=0.180011; end;
      9: begin z[1]:=1.22064; z[2]:=0.764712; z[3]:=0.430729;
          z[4]:=0.139712; end;
    end;

    i3:=1; while i3 < m do
    begin
      if i3=1 then i4:=i3;
      c[i3] :=nu-stan*z[i4];
      c[i3+1]:=nu+stan*z[i4];
      i3:=i3+2; i4:=(i3 div 2)+1;
    end;
  end;
end;

```

```

case m of
  5 :begin
    r2:=c[2]; r3:=c[3]; r4:=c[4];
    c[2]:=r3; c[3]:=r4; c[4]:=r2;
    end;
  7 :begin
    r2:=c[2]; r3:=c[3]; r4:=c[4]; r5:=c[5];
    r6:=c[6]; c[2]:=r3; c[3]:=r5; c[4]:=r6;
    c[5]:=r4; c[6]:=r2;
    end;
  9 :begin
    r2:=c[2]; r3:=c[3]; r4:=c[4]; r5:=c[5];
    r6:=c[6]; r7:=c[7]; r8:=c[8]; r9:= c[9];
    c[2]:=r3; c[3]:=r5; c[4]:=r7; c[5]:=r8;
    c[7]:=r4; c[8]:=r2;
    end;
end;
reset(p);
while not eof(p) do
begin
  c[0]:=-300; c[m]:=300;
  read(p,s,sx,cv,call,p1,p2,d,pr); i5:=1;
  while i5 <= m do
  begin
    if sx >= c[i5-1] then if sx < c[i5] then
    begin
      k[i5]:=k[i5]+1;
      if cv > 0 then
        begin j[i5] :=j[i5]+1; pv[i5]:=pv[i5]+cv; end;
    end; i5:=i5+1;
  end;
end;
append(outfile);
reset(p);
writeln(outfile,fname,m);
c[m]:=0; sum:=0;
for i6:=1 to m do
begin
  k[i6]:=k[i6]-1; sum:=sum+k[i6]; writeln(k[i6]:4);
  if k[i6] <> 0 then
    writeln(outfile,j[i6]:10,k[i6]:10,j[i6]/k[i6]*100:10:2,
    pv[i6]/k[i6]:10:2,c[i6]:10:2) else
    writeln(outfile,j[i6]:10,k[i6]:10);
end;
writeln(outfile,sum:10);
1:end;
end;

begin
assign(minfile,'series');
reset(minfile);
assign(outfile,'cm');
rewrite(outfile);
while not eof(minfile) do
begin
  readln(minfile,fname); write(fname,'      wait.....'); writeln;
  assign(infile,fname);
  reset(infile);
  transform(infile);
  writeln('_____');
  append(outfile);
  writeln(outfile);
  close(outfile);
  close(infile);
end;
writeln('Now, look at cm.!');
end.

```

Ordinary Least Square Regression  
{SHAZAM Programme}

```
FILE 4 C:\WORK\B6836
FILE 6 C:\WORK\BO6836
SAMPLE 1 154
READ (4) S SX CV CALL PV P B6836D PR
GENR ASX = ABS(SX)
GENR R = LOG(S) - LOG (LAG(S))
GENR CR = LOG(CALL) - LOG (LAG(CALL))
GENR AR = ABS(R)
GENR ACR = ABS(CR)
OLS B6836D S
OLS B6836D SX
OLS B6836D CALL
OLS B6836D PV
OLS B6836D P
OLS B6836D PR
OLS B6836D ASX
OLS B6836D R
OLS B6836D CR
OLS B6836D AR
OLS B6836D ACR
STOP
```

Logit Model {SHAZAM Programme}

```
file 4 C:\WORK\G18
file 6 C:\WORK\G18N
sample 1 878
read (4) s sx g18d
genr sx=abs(sx/s)
LOGIT G18D SX
SAMPLE 1 28
LOGIT G18D SX
SAMPLE 29 83
LOGIT G18D SX
SAMPLE 84 267
LOGIT G18D SX
SAMPLE 268 420
LOGIT G18D SX
SAMPLE 421 563
LOGIT G18D SX
SAMPLE 564 734
LOGIT G18D SX
SAMPLE 735 878
LOGIT G18D SX
STOP
```

## Chapter 6 Selection of Frequently Traded Call Options

Selection of Call Option Series by Their  
Trading Activity  
{Pascal Programme}

```
program select;
uses Dos,crt;
var
  sfile,infile,outfile      : text;
  fname                    : string[8];
  n,sum,sum0,sum1          : integer;
procedure transform(var pinfile:text;var nd:integer);
var
  mat,code,t,td,trades,plus: integer;
  x,d,sb,sa,cb,ca,p,tf     : real;
begin
  trades:=0; plus:=0; read(pinfile,nd);
  if nd < 2 then
  begin
    case nd of
      1: begin
          read(pinfile,mat,x,d);
          while not eof(pinfile) do
          begin
            read(pinfile,code,t,sb,sa,cb,ca,td);
            trades:=trades+1;
            if ((sb+sa)/2)-x > 0 then plus:=plus+1;
          end;
        end;
      0: begin
          read(pinfile,mat,x);
          while not eof(pinfile) do
          begin
            read(pinfile,code,t,sb,sa,cb,ca); trades:=trades+1;
            if ((sb+sa)/2)-x > 0 then plus:=plus+1;
          end;
        end;
      end;
    p:= (plus/trades)*100;
    tf:= 0.59266-0.00248*mat+0.0108*p-0.000126*sqr(p);
    case nd of
      0:writeln(outfile,fname:8,nd:3,tf:10:3,mat:5,p:8:2,plus:4,trades:5);
      1:writeln(outfile,fname:8,nd:6,tf:3,mat:5,p:8:2,plus:4,trades:5);
    end;
  end;
end;
begin
  Window(1,1,80,60); ClrScr;
  assign(sfile,'hs');      reset(sfile);
  assign(outfile,'tf.pas'); rewrite(outfile);
  sum:=0; sum0:=0; sum1:=0;
  while not eof(sfile) do
  begin
    readln(sfile,fname);
    writeln(fname:25,'wait.....':15);
    assign(infile,fname); reset(infile);
    transform(infile,n);
    case n of
      0:sum0:=sum0+1;
      1:sum1:=sum1+1;
    end;
    close(infile);
  end;
end;
```

```
sum:=sum0+sum1;
writeln(outfile);
writeln(outfile,sum0:11,sum1:3,'There are ':13,sum:2,' files. ');
close(outfile); close(sfile);
writeln; writeln('Now, look at the report!':42);
{repeat Sound(240); until KeyPressed; NoSound;}
end.
```

## Chapter 8, 9 and 10

### Market Efficiency Tests {Pascal Programme}

- Bid-ask Spread, Mid Prices
- Boundary Conditions
- GARCH Volatility and Actual Volatility
- Thin Trading Correction
- Dividend Adjustment
- Diversification of Hedge Portfolios
- Indivisibilities

```
program HedgeReturn;
uses Crt,Dos;
const total=439; max=195; bid=0.25; lag=1;
var
  clsfile,cfile,rfile,lfile,supfile,
  hfile,gfile,pfile,ftfile,sedfile,
  sepfile,nd1file           : text;
  fname                     : string[8];
  nd,mat,k,k1,k2,first,last,sign,c,i,
  period,remainder,separate : integer;
  d,r,v,x,sigma2,sbx,sx,d1,sum,mu,
  actual,hv0,hv1,hv2,margin,es,ps : real;
  lotus,four                : array[1..total] of integer;
  date,code                  : array[1..max] of integer;
  sb,sa,sm,sr,ab,aa,am,mb,ma,mm,t,td : array[1..max] of real;
  int                         : array[1..9,0..total] of real;
  h                            : array[1887..2309] of real;
  g                            : array[1..22,1..4] of real;
  p                            : array[1..22,1..2] of integer;
  ft                           : array[0..total] of real;
  ftr                         : array[1..total] of real;
```

```
{ $I \call\pas\day } { $I \call\pas\nd1 } { $I \call\pas\call }
```

```
procedure
cal(sign:integer;vars1,s2,c1,c2,excess,percent:real);
var return,nd1: real;
begin
  nd1:=(n(d1)+0.0005)*1000;
  nd1:=(nd1-frac(nd1))/1000;
  writeln(nd1file,nd1:7:3);
  case sign of
    -1:begin
      return:= -((c2-c1)-nd1*(s2-s1));
      excess:=(return-margin*r*lag/365)*10;
      percent:=excess/(margin*10);
    end;

    1:begin
      return:= ((c2-c1)-nd1*(s2-s1));
      excess:=(return-c1*r*lag/365)*10;
      percent:=excess/(c1*10);
    end;
  end;
end;
```

```

begin {1}
  Window(1,1,80,60); ClrScr;
  { assign(sedfile,'ned37'); rewrite(sedfile);}
  assign(sepfile,'nep37'); rewrite(sepfile);
  assign(ndlfile,'ndl37'); rewrite(ndlfile);

  {input of common data - v,h,g}
  assign(supfile,'v'); reset(supfile);
  read(supfile,hv0,hv1,hv2);
  h[1889]:=hv0; h[1888]:=hv1; h[1887]:=hv2;
  close(supfile);

  assign(hfile,'h'); reset(hfile);
  for k:= 1890 to 2309 do read(hfile,h[k]);
  close(hfile);

  assign(gfile,'g'); reset(gfile);
  for k1:=1 to 22 do for k2:=1 to 4 do read(gfile,g[k1,k2]);
  close(gfile);

  assign(pfile,'\call\period'); reset(pfile);
  for k1:=1 to 22 do for k2:=1 to 2 do read(pfile,p[k1,k2]);
  close(pfile);

  {diversification}
  assign(ftfile,'\call\ftse'); reset(ftfile);
  for k:=0 to total do
  begin
    read(ftfile,ft[k]);
    if k > 0 then ftr[k]:=ln(ft[k]/ft[k-1]);
  end;
  close(ftfile);

  {input of call option data}
  assign(clsfile,'tf127'); reset(clsfile);
  while not eof(clsfile) do
  begin {2}
    readln(clsfile,fname); writeln(fname:30,'wait.....':15);
    assign(cfile,fname); reset(cfile);
    read(cfile,nd,mat,x); if nd=1 then read(cfile,d);
    {if nd=1 then begin read(cfile,d); d:=d*0.55; end;}

    c:=0; sum:=0.0;
    while not eof(cfile) do
    begin
      c:=c+1;
      case nd of
        1:read(cfile,code[c],t[c],sb[c],sa[c],ab[c],aa[c],td[c]);
        0:read(cfile,code[c],t[c],sb[c],sa[c],ab[c],aa[c]);
      end;
      t[c]:=t[c]/365; td[c]:=td[c]/365;
      date[c]:=order(code[c]);
      sm[c]:=(sb[c]+sa[c])/2; am[c]:=(ab[c]+aa[c])/2;
      if c > 1 then
        begin sr[c-1]:=ln(sm[c]/sm[c-1]); sum:=sum+sr[c-1]; end;
    end;
    close(cfile);

  { actual volatility
  mu:=sum/(c-1);
  sum:=0.0;
  for k:=1 to c-1 do sum:=sum+sqr(sr[k]-mu);
  actual:=sqrt(sum/(c-2)*252);
  v:=actual;
  }
}

```

```

{input of interest}
assign(rfile, '\call\la_int'); reset(rfile);
for k:=0 to total do
read(rfile, int[1,k], int[2,k], int[3,k], int[4,k],
int[5,k], int[6,k], int[7,k], int[8,k], int[9,k]);
close(rfile);
remainder:=mat mod 30; mat:=mat div 30;
if remainder > 15 then mat:=mat+1;
r:=ln(1+int[mat,date[1]-1]*0.01);

{matching data range}
assign(lfile, '\call\four.prn'); reset(lfile);
for k:=1 to total do
begin
  read(lfile, lotus[k], four[k]);
  if code[1]=lotus[k] then first:=four[k];
  if code[c]=lotus[k] then last:=four[k];
end;
close(lfile);

{calculating GARCH volatility}
for k:=1 to 22 do if last > p[k,2]
then if last < p[k+1,2] then period:=k;
sigma2:=g[period,2]/(1-g[period,3]-g[period,4]);

sum:=0.0;
for k:=first to last do
begin
  h[k]:=g[period,2]+g[period,3]*sigma2+g[period,4]*h[k-1];
  sum:=sum+h[k];
end;
v:=sqrt(sum/(last-first+1)*252);

{processing call option prices}
for k:=1 to (c-1)-(1+lag) do
begin
  {Thin trading correction}
  for i:=0 to 1+lag do
  begin
    if nd=0 then td[k+i]:=0;
    if td[k+i]=0.0 then d:=0;
    mb[k+i]:=call(sb[k+i], x, r, v, t[k+i], d, td[k+i]);
    ma[k+i]:=call(sa[k+i], x, r, v, t[k+i], d, td[k+i]);
    ab[k+i]:=am[k+i]-(ma[k+i]-mb[k+i])/2;
    aa[k+i]:=am[k+i]+(ma[k+i]-mb[k+i])/2;
  end;
  {call premium >= bid}
  if ab[k] >= bid then if (ab[k+1] >= bid) then if
  (ab[k+1+lag] >= bid) then
  begin {3}
    if mb[k] >= bid then
    begin {4}
      {no immediate exercise}
      sbx:=sb[k]-x; if sbx < 0 then sbx:=0.0;
      if aa[k] >= sbx then
      begin {5}
        {no early exercise}
        if d <= x * (1-exp(-r*(t[k]-td[k]))) then
        if aa[k] >= sb[k]-d*exp(-r*td[k])-x*exp(-r*t[k]) then
        {redundancy}
        if >= aa[k] >= sb[k] - x * exp(-r*td[k]) then
        begin {6}
          {mid N(d1mt)}
          mm[k]:=call(sm[k], x, r, v, t[k], d, td[k]);
          margin:=1.2*sm[k+1]-x;
          if margin < 0.03*sm[k+1] then
          margin:=0.03*sm[k+1];
        end;
      end;
    end;
  end;
end;

```

```

{checking mispricing}
if ab[k] > ma[k] then
begin
  cal(-1,sa[k+1],sb[k+1+lag],ab[k+1],aa[k+1+lag],es,ps);
  {writeln(sedfile,es:12:8);} writeln(sepfile,ps:12:8);
end else
if mb[k] > aa[k] then
begin
  cal(1,sb[k+1],sa[k+1+lag],aa[k+1],ab[k+1+lag],es,ps);
  {writeln(sedfile,es:12:8);} writeln(sepfile,ps:12:8);
end;
end; {6}
end; {5}
end; {4}
end; {3}
end; {money, T, spread}
end; {2}
close(clsfile); {close(sedfile);} close(sepfile); close(ndlfile);
end. {1}

```

Black-Scholes Model {Pascal sub-programme}

```
function call(var fs,fx,fr,fv,ft,fd,ftd:real) :real;
var
  {d1,}d2,fsd :real;
begin
  fsd:=fs - fd * exp(-fr * ftd);
  d1:=(ln(fsd/fx) + (fr + sqr(fv)/2)*ft) / (fv * sqrt(ft));
  d2:=d1 - fv * sqrt(ft);
  call:=fsd * n(d1) - fx * n(d2) * exp(-fr*ft);
end;
```

Error Measurement Functions  
{Pascal sub-programme}

```
function rmse(var v1,v2:v) : real;
var
  sum:real; k:integer;
begin
  sum:=0.0;
  for k:=2 to 22 do sum:=sum+sqr(v1[k]-v2[k]);
  rmse:=sqr(sum/21);
end;
function mae(var v1,v2:v) : real;
var
  sum:real; k:integer;
begin
  sum:=0.0;
  for k:=2 to 22 do sum:=sum+abs(v1[k]-v2[k]);
  mae:=sum/21;
end;
function mape(var v1,v2:v) : real;
var
  sum:real; k:integer;
begin
  sum:=0.0;
  for k:=2 to 22 do sum:=sum+abs((v1[k]-v2[k])/v1[k]);
  mape:=sum/21;
end;
```

Transforming LOTUS Date Codes into Order of Observations  
{Pascal sub-programme}

```
function order(var fx:integer):integer;
begin
  case fx of
    31712:order:= 1;
    31713:order:= 2;
    .
    etc.
    .
    32323:order:=438;
    32324:order:=439;
  end;
end;
```

Calculation of the Hedge Ratio  
{Pascal sub-programme}

```
function n(var fd:real) :real;
  var
    z,t,f,polynomial,m      :real;
    i                        :integer;
    label                    1;
  begin
    z := abs(fd); if z >= 5 then
      begin
        m := 1;
        if fd < 0 then m := 0;
        n := m;
        goto 1;
      end;
    t := 1/(1 + 0.2316419 * z);
    polynomial := (((1.330274429 * t - 1.821255978) * t
      + 1.781477937) * t - 0.356563782) * t
      + 0.31938153) * t;
    f := 0.3989423 * exp(-sqr(fd)/2);
    m := 1 - f * polynomial;
    if fd < 0 then m := 1 - m;
    n := m;
  1:end;
```

The Kolmogorov-Smirnov Test  
 {Pascal Programme}

```

program Kolmogorov_Smirnov;
uses Dos,crt;
const max = 5000;
type dataarray      = array[1..max] of real;
var
  data1,data2,outfile      : text;
  x1,x2                    : dataarray;
  n1,n2,k,k1,k2            : integer;
  f01,f02,fn1,fn2,d,dt,prob,lam,nn: real;
label                      6,7;

function probks(var flam:real): real;
const eps1=0.001; eps2=1.E-8;
var c,fac,term,termbf,ks: real; k: integer; label 1;
begin
  c:=-2*sqr(flam); fac:=2; ks:=0; termbf:=0;
  for k:=1 to 100 do
  begin
    {writeln(exp(-2*sqr(flam)):10:7);}
    if c < -15 then begin probks:=0; goto 1; end;
    term:=fac*exp(c*sqr(k));
    {writeln('term=',term);}
    ks:=ks+term;
    if ((abs(term) < eps1*termbf) or (abs(term) < eps2*ks))
    then begin probks:=ks; goto 1; end;
    fac:=-fac; termbf:=abs(term);
  end;
  probks:=1;
1:end;

procedure sort(n: integer; var RA: dataarray);
var L,IR,I,J: integer; RRA: real; label 2,3,4;
begin
  writeln('= sorting =');
  L:=n div 2+1; IR:=n;
2:if L>1 then begin L:=L-1; RRA:=RA[L]; end else
  begin
    RRA:=RA[IR]; RA[IR]:=RA[1]; IR:=IR-1;
    if IR=1 then begin RA[1]:=RRA; goto 4; end;
  end;
  I:=L; J:=L+L;
3:if J<=IR then
  begin
    if J<IR then if RA[J] < RA[J+1] then J:=J+1;
    if RRA < RA[J] then begin RA[I]:=RA[J]; I:=J; J:=J+J; end
    else J:=IR+1;
    goto 3;
  end;
  RA[I]:=RRA; goto 2;
4:end;

begin
  Window(1,1,80,60); ClrScr;
  assign(outfile,'ks7');
  rewrite(outfile);
  writeln(outfile,'KS7');
  assign(data1,'sep7');
  reset(data1);
  k:=1;
  while not eof(data1) do
  begin read(data1,x1[k]); k:=k+1; end;
  close(data1); n1:=k-1; if x1[n1]=0 then n1:=n1-1;

```

```

assign(data2,'mep7');
reset(data2);
k:=1;
while not eof(data2) do
begin read(data2,x2[k]); k:=k+1; end;
close(data2); n2:=k-1; if x2[n2]=0 then n2:=n2-1;

sort(n1,x1); sort(n2,x2);

{Kolmogorov_Smirnov}
f01:=0; f02:=0; d:=0; k1:=1; k2:=1;
while ((k1 <= n1) and (k2 <=n2)) do
begin
  if x1[k1] < x2[k2] then
  begin
    fn1:=k1/n1;
    if abs(fn1-f02) > abs(f01-f02) then dt:=abs(fn1-f02)
    else dt:=abs(f01-f02);
    if dt > d then d:=dt;
    f01:=fn1; k1:=k1+1;
  end else
  begin
    fn2:=k2/n2;
    if abs(fn2-f01) > abs(f02-f01) then dt:=abs(fn2-f01)
    else dt:=abs(f02-f01);
    if dt > d then d:=dt;
    f02:=fn2; k2:=k2+1;
  end;
end;

writeln('d=',d:8:3);
lam:=sqrt((n1*exp(ln(n2)))/(n1+n2))*d;
writeln(n1:4,n2:5,lam:8:5);
prob:=probks(lam);
writeln(outfile,'(n1,n2)=':8,n1:4,n2:5);
writeln(outfile,'d=':8,d:8:5);
writeln(outfile,'lambda=':8,lam:8:5);
writeln(outfile,'alpha=':8,prob:8:5);
writeln('alpha=':8,prob:8:5);
close(outfile);
{Repeat Sound(240); until KeyPressed; Delay(10); NoSound;}
end.

```

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