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David Greasley

Nick Hanley

Eoin McLaughlin

Les Oxley

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## ***The Emperor Has New Clothes: Empirical Tests of Mainstream Theories of Economic Growth***<sup>1</sup>.

David Greasley<sup>\*</sup>, Nick Hanley<sup>+</sup>, Eoin McLaughlin<sup>+</sup>, and Les Oxley<sup>#</sup>.

<sup>\*</sup>School of History and Classics, University of Edinburgh, Scotland; <sup>+</sup> Department of Geography and Sustainable Development, University of St. Andrews, Scotland; <sup>#</sup>Department of Economics, University of Waikato, New Zealand and Honorary Professor, University of Stirling.

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*“It is a capital mistake to theorize before one has data. Insensibly one begins to twist facts to suit theories, instead of theories to fit facts.”*  
*Sir Arthur Conan Doyle.*

**Abstract:** Modern macroeconomic theory utilises optimal control techniques to model the maximisation of individual well-being using a lifetime utility function. Agents face choices over current and future consumption (with resultant implied savings decisions) seeking to maximise the present value of current plus future well-being. However, such inter-temporal welfare-maximising assumptions remain empirically untested. In the work presented here we test whether welfare was in (historical) fact maximised in the US between 1870-2000 and find empirical support for the optimising basis of growth theory, but only once a comprehensive view of what constitutes a country’s wealth or capital is taken into account.

**Keywords:** inter-temporal utility maximisation; modern growth theory; US; comprehensive wealth

**JEL classifications:** E21, E22, C61.

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## 1. Introduction.

The popularity of modern growth theory is likely linked to its rigour associated with the widespread adoption of optimal control-type techniques, which utilise a traditional micro-economic theory of inter-temporal welfare maximisation. Armed with this toolkit, modern macro-theorists are able to consider a range of issues within an optimising framework. This approach to macroeconomics utilises the technical tools of Ramsey (1928), Weitzman (1976), Arrow (1968) and Arrow and Kurz (1970) among others, to produce an elegant, mainstream, model of economic growth that is now the basis of modern macroeconomic teaching and research as exemplified by the popular works of Acemoglu (2009), Barro and Sala-i-Martin (2004) and Aghion and Howitt (2008). Therein, one finds that the 'baseline assumption'<sup>2</sup> to the building of modern macroeconomics is that the representative infinitely lived individual maximises their well-being using a lifetime utility function of the form:

$$W = \int_0^{\infty} e^{-\rho t} u[c(t)]dt \quad (1)$$

where  $c(t)$  is the time path of consumption (this could be an extended consumption vector),  $u(\cdot)$  is an instantaneous utility function with positive, yet diminishing marginal utility, and  $\rho$  represents a positive rate of time preference.

Foreshadowing the work of Stiglitz, Sen and Fitoussi (2010), this version of macroeconomic theory sees consumption rather than production as the focus of attention, where economic agents face choices over current and future consumption paths (with resultant implied savings decisions), seeking to maximise the present value of current plus future well-being.

The success and elegance of the theory is not, however, matched by empirical testing and support. Empirical research into the drivers of economic growth (why country growth rates differ for example), remains backward rather than forward looking, focussing upon large panels of countries where data are typically constrained to recent, typically post-1945 periods (see for example, Durlauf, Kourtellos, and Tan (2008), Brock and Durlauf (2001) and Durlauf, Johnson, and Temple (2005)).

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<sup>2</sup> Acemoglu (2009, p.288)

The lack of empirical support for neoclassical growth models, which assume exogenous population processes, does not surprise Galor and Weil (2000)<sup>3</sup>, where they see demographic transition, technological change and standard of living as inexorably linked. In their ‘unified growth model’, the pace of technological progress separates Malthusian and Post-Malthusian regimes, and demographic transition is the mechanism that shifts economies into a “Modern Growth” era where technology and output per capita increase rapidly as population growth moderates. The demographic transition, however, is in part, influenced by the pace of technological change leading to decisions to invest more in human capital – inducing a substitution of quality (more educated) children for quantity.

A clear implication in their unified model is an increasing proportion of human capital in the (broadly defined) capital stock as we progress through the Modern era. That is, as economies grow, the balance between different types of capital within total wealth is changing (World Bank, 2006). Furthermore, ‘children with high levels of human capital are, in turn, more likely to advance the technological frontier or to adopt advanced technologies,’ (op cit., p. 810). Fertility and income are also important in their model in relation to the possibility of ‘wealth dilution’, where growing populations make higher demands on the availability of (exhaustible and fixed) capital availability to future populations. In contrast to the simple Neo-classical model of growth, key elements in the Galor and Weil (2000) approach are thus non-constant population growth (with demographic transition as endogenous); the pace of technological change (which is affected by the stock of human capital); and the potential effects of wealth dilution (of population growth on fixed or exhaustible capital). We incorporate all of these features in the empirical work reported below.

Although important differences exist between e.g., Galor and Weil (2000) and the simpler Neo-classical underpinnings of the benchmark models of e.g., Barro and Sala-i-Martin (2004), Acemoglu (2009) and Aghion and Howitt (2008), there is an inter-temporal welfare-maximising assumption which underlies both approaches. This remains an untested assumption. The fact that no such tests are reported or referred to in any of these seminal texts reflects a striking dearth of empirical testing in this crucial area of macroeconomics.

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<sup>3</sup> “Neoclassical growth models with exogenous population are unable to capture this intricate (evolution of population, technology and growth throughout human history) transition process.” Galor and Weil, 2000, p.809.

The work presented here is, we believe, the first to explicitly test the present value optimizing assumptions expressed as equation (1) above, the importance of which cannot be over-stated in the field of modern macroeconomics. Utilising the modelling framework of Ferreira and Vincent (2005) and Ferreira, Vincent and Hamilton (2008), which is based on a theoretical result from Weitzman (1976), we will test whether welfare was in (historical) fact maximised in the US over the period 1870-2000. We focus on the USA over this particular time period as it is the longest interval over which consistent macro data on investment (broadly defined) and consumption can be assembled.

In detail, Section 2 will briefly describe the theoretical model presented by Ferreira and Vincent (2005) and Ferreira, Vincent and Hamilton (2008), and demonstrate how its testable implications relate to the *untested* assumptions of for example, Acemoglu et. al. This involves us looking at how changes in net investment in multiple forms of capital are related to changes in the present value of future consumption (Sala-I-Martin, 1997) over the long run, as well as the effects of including technological advances as measured by changes in Total Factor Productivity, and changes in the population growth rate (Easterly and Levine 2001; Arrow et al, 2003). In Section 3 we describe the data, followed in Section 4 by the econometric testing implications. Section 5 presents the empirical results and Section 6 concludes.

## **2. The theoretical model and its testable implications.**

We start with the model of Weitzman (1976). He studies an economy which produces a single consumption good (or multiple consumption goods representable by an index number) using multiple types of capital over infinite time, with a constant discount rate equal to the consumption rate of interest,  $r$ . Weitzman states that this total stock of capital includes produced capital (“..equipment, structures and inventories”..), but also human capital, technology and natural resources. He assumes that all sources of growth can be attributed to one of these capital stocks. Prices for consumption goods and for investment goods are determined competitively. Setting the price of consumption goods equal to one and using these as the numeraire, then the national accounting identity states that NNP in period  $t$  is by definition equal to the sum of consumption and net investment, the latter evaluated at a vector of prices  $\mathbf{p}$ :

$$Y(t) \equiv C(t) + \mathbf{p}(t)\mathbf{I}(t) \quad (2)$$

His most important result is then that Net National Product at time  $t$ ,  $Y(t)$ , is equal to a weighted average of future consumption,  $\bar{C}(t)$ :

$$Y(t) = (\bar{C}(t)) \quad (3)$$

where the inter-temporal weights used to calculate  $\bar{C}(t)$  depend on  $r$ . He shows that NNP in any period is identical to the Hamiltonian of the optimal control problem which maximises social welfare  $W$  (the discounted value of future consumption):

$$W(t) = \int_t^{\infty} C(s) e^{-r(s-t)} ds \quad (4)$$

subject to the production possibilities of the economy. NNP, in this optimising economy, is thus a forward-looking measure of future well-being as measured by the discounted value of future consumption streams. For (3) to correctly describe a dynamic economy, a number of assumptions must hold, noted by Weitzman as (i) perfectly competitive markets, including a perfect capital market (implying prices used to measure NNP being equal to marginal rates of transformation) and (ii) perfect foresight<sup>4</sup>.

Ferreira and Vincent (2005) adapt (3) by deducting the value of present period consumption from each side of (2) to obtain:

$$\bar{C}(t) - C(t) = \mathbf{p}(t)\mathbf{I}(t) \quad (5)$$

They then use (5) to derive a reduced form econometric model which enables them to test the predictive ability of increasingly-comprehensive measures of net investment (that is, as one considers an increasingly wide set of capital stocks) in relation to changes in future consumption. This provides the key insight for what we do in the empirical part of the paper.

Based on Hamilton and Hartwick (2005), Ferreira, Hamilton and Vincent (2008) amend (5) to the following, to show how current-period net investment<sup>5</sup> in all forms of capital ( $I_t$ ) is related to future well-being:

$$\int_t^{\infty} \frac{dC(s)}{ds} e^{-\int_t^s r(\tau)d\tau} ds = I(t) \quad (6)$$

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<sup>4</sup> Asheim and Weitzman (2001) extend Weitzman (1976) for the case of multiple consumption goods whose relative values are expressed using a Divisia index.

<sup>5</sup> They refer to net investment in all forms of capital as “genuine savings”.

where  $C$  is consumption and  $r$  is the consumption rate of discount. Equation (6) states that the present value of changes in consumption will be equal to the value of net investment in period  $t$ , assuming the economy is indeed on a PV-optimal path. Changes in a country's total capital – its comprehensive wealth – are thus related to changes in future consumption relative to the present (Weitzman, 2003; Arrow et al, 2012). If population is growing over time at some constant rate  $\gamma$ , then Dasgupta (2001) shows that the relationship in (6) can be re-stated in per-capita terms as:

$$\int_t^{\infty} \frac{dc(s)}{ds} e^{-\int_t^s -r(\tau)-\gamma d\tau} ds = g(t) \quad (7)$$

where  $c$  is now per-capita consumption, the discount rate is reduced by the rate of population growth  $\gamma$ , and  $g$  is per capita net investment in all forms of capital (genuine savings per capita, in the terminology of Ferreira et al).

As noted in section 1, Galor and Weil (2000) point to the importance of technological progress in modelling growth. Arrow et al (2012) include the effects of technological change over time in their measure of comprehensive wealth (the value of all capital stocks in the economy), arguing that this is “...an increment to knowledge capital beyond what is captured in (changes in) human capital” (p 321). Arrow et al accomplish this by using changes in TFP as their measure of technological change, adding the TFP growth rate to the year-on-year change in comprehensive wealth. In the empirical work reported below, we also include the value of TFP growth in our estimates of changes in total capital, albeit using a different approach to Arrow et al.

The specification of the link between changings in inter-temporal welfare (the left hand side of (6) and changes in comprehensive wealth (the right hand side of (6) leads to an econometric model which can be tested. With population growing at a constant rate, the relationship between net investment and future consumption implied by (6) is given in per capita terms by:

$$PV\Delta C_t = \beta_0 + \beta_1 g_t + \epsilon_t \quad (8)$$

where  $\{PV\Delta C_t\}$  is the present value of changes in consumption in years  $(t+1, t+2, \dots, t+T)$  relative to consumption in period  $t$ . Net investment per capita is defined as:

$$g = \frac{\dot{K}}{N} - \gamma\omega \quad (9)$$

where  $-\gamma\omega$  is a wealth dilution effect determined by wealth per capita  $\omega$  and the population growth rate  $\gamma$  for a population of size  $N$  in time  $t$ . If the population growth rate varies over time, then (8) becomes:

$$PV\Delta C_t + PV(\Delta \gamma_t \omega_t) = \beta_0 + \beta_1 g_t + \epsilon_t \quad (10)$$

If (6) or (7) describes reality (and therefore that economic agents are present-value maximizing), then a testable hypothesis is that  $\beta_1 = 1$  in (8) or (10). We now describe the data used to test this hypothesis.

### 3. The Data

#### 3.1 Measures of net changes in capital stocks over time

This section defines and outlines the components of increasingly comprehensive measures of annual investment in the USA, 1869-2000. Further information can be found in the Data Appendix. Aggregate wealth comprises produced, human, natural and knowledge capital (Arrow et al, 2012). In summary, we have constructed a sequence of net changes in capital stocks:

- NETINV: changes in net produced and net foreign capital.
- GREENINV: NETINV plus changes in farmland, renewable and non-renewable resources, and the disinvestment associated with CO2 emissions.
- CI: GREENINV plus net changes in human capital.
- GREENTFP and CITFP: GREEN and CI augmented with the value of changes in the knowledge stock.
- CITFPW: CITFP less wealth dilution.

This increasingly comprehensive view of what constitutes “capital” follows Ferreira et al (2008) and Greasley et al (2014) and allows for scrutiny of the hypotheses  $\beta_1 = 1$  with the alternative net investment measures.

##### *3.1.1 Changes in produced and net overseas capital (NETINV)*

Net produced investment comprises net fixed capital formation, changes in inventories and net overseas investment (Figure A1) where these data are shown relative to GDP. Produced capital formation fell from around 15-20% of GDP 1870-1900 to around 5% at the start of the 21<sup>st</sup>

century. The initial down-step in the produced investment ratio occurred in the 1920s, and the USA experienced a long period of negative investment, which spanned the Great Depression and World War 2. There was an upturn in the produced investment after 1945 but the earlier highs were not regained. Net overseas investment was generally positive from the 1890s to the 1970s, while inventory changes gradually diminished relative to GDP. In per capita terms NETINV falls after 1929, then it is generally negative until 1945, but the levels of the early 20<sup>th</sup> century were restored in the 1970s (Figure 1).

### *3.1.2 Adding changes in natural capital (GREENINV)*

The chief elements of natural capital included here are forestry, mining (metals and minerals) and agricultural land. Forest area fell to the 1920s but rose over the next half-century to peak at around 300 million hectares in the early 1970s. The standing value of the trees also fell to the 1920s but rose thereafter, partly reflecting the higher timber volumes per hectare after 1945. The rental value of forest depletion (valued using the difference between harvest price and marginal cost) averaged around 1% of GDP each year in the period 1870-1900; whereas afforestation took place during the twentieth century (Table A1).

Increases in the area of farmland or its' per hectare value are treated as net additions to the natural capital stock. The farmland area of the USA more than doubled in size 1870-2000, despite a gradual decline from around 1950. Changes in the rental value of farmland generally augmented the US natural capital stock before 1950, although there was a brief decline during the rural financial crisis after the post-World War 1 boom (Table A2). However, the annual changes in the rental value of farmland are small, and peaked at around 0.42% of GDP in the 1890s.

Over the period 1869-2000 mining output valued at market prices averaged 3.9% of GDP while the value of extracted mining rents, which deduct marginal extraction costs from prices, averaged 2.8% of GDP. Fuels, including coal, oil and gas account for most of the extracted rents (Figure A2). The market value of extracted metals, including iron ore, copper and bauxite peaked relative to GDP during World War 1, and fell to below 1% thereafter. Other minerals' output, including, gypsum, stone and salt, had a market value over 1% of GDP in the 1920s, but this ratio fell thereafter. Overall, the extraction of mining rents rose above 5% of GDP during World War 1, and hit 6% around 1980. Extracted mineral rents never fell below 1% of GDP,

and when the produced investment ratio collapsed during the 1930s, the depletion of minerals accentuated the marked fall in the US capital stock.

Next we consider the extent to which pollution depletes natural capital. Emissions of greenhouse gases add to the stock in the atmosphere, and many authors have included estimates of the shadow cost of carbon emissions in comprehensive investment-type calculations (World Bank, 2011; Pezzey and Burke, 2013). This value is a deduction from natural capital since it represents a using-up of scarce global assimilative capacity. The estimates here, following Kunnas et al (2014), suggest the disinvestment associated with carbon pollution averaged around 0.3% of GDP during the 20<sup>th</sup> century (Figure A3), but pollution costs rose sharply in the period to 1920 when energy-GDP ratios were also rising (Devine, 1983).

### *3.1.3 Adding changes in human capital (CI)*

Like the World Bank (2006, 2011), we use annual investment in public education as a measure of the change in the stock of human capital. Whilst one could use an alternative approach, based on lifetime earnings and changes in worker productivity (Arrow et al, 2012), the expenditure approach fits naturally with measures of comprehensive investment. A measure of such expenditures would ideally include private spending on education and spending by firms on worker training, but consistent, continuous data are not available on either of these. Public education investment rose to around 6.5% of GDP by the 1960s, but the ratio levelled thereafter (Figure A4). The earlier spike and trough in the ratio reflect that education spending was maintained both when GDP collapsed at the onset of the Great Depression and surged during World War 2.

Putting together these individual changes in capital stocks for the USA, we see that real CI per capita rises by around four times 1869-2000 (Figure 1). Within these years CI per capita shows no discernible trend from around 1880-1925. Net produced investment was above CI during these years since public education investment was insufficient to offset the effects of natural resource depletion. From 1925-1945 the USA witnessed a major slump in CI associated with the Great Depression and World War 2, which included spells when the capital stocks included in CI fell. After 1945 net produced investment per capita was typically no higher than it had been before 1925. The major change after 1945 was that higher public investment in education

more than offsets natural capital depletion, hence CI per capita rises, and exceeds the earlier 1906 peak for the first time in 1965.

#### *3.1.4 Adding changes in the value of exogenous technological progress (CITFP)*

Trend growth TFP estimates can be used to value exogenous technological progress. Arrow et al (2012) simply augment their measure of comprehensive investment with the current value of TFP, to show how technological progress increases current income. Strictly, however, treating time as an uncontrolled capital stock means TFP's contribution to the change in wealth in any year should be included in the measure of CI. Our approach to gauging how TFP contributes to changes in the value of wealth follows Pezzey et al (2006, Equation 14) but calculates the present value of future changes in TFP over a 20 years horizon. Trend growth TFP is illustrated as Figure A5. Adding the value of TFP to CI results in the more comprehensive measure, CITFP per capita (Figure 2), which is always positive. In 2000 CITFP is above \$8000 per capita or around three-times higher than CI.

Treating technological progress as an uncontrolled stock of capital associated with the 'passing of time' which can be measured by TFP assumes that all technological progress is exogenous. This is clearly not the case empirically, and part of the TFP might arise from, for example, R&D spending. A particular issue for the CITFP measure is its inclusion of public education investment, which might be associated with endogenous technological change. This potentially introduces an element of double counting into the measure. Accordingly, our empirical tests also consider an alternative formulation of technology-augmented investment, GREENTFP, which adds the technological progress premium to GREENINV, also shown in Figure 2.

#### *3.1.5 Allowing for wealth dilution (CITFPW)*

Finally, wealth dilution is included in the measure of investment defined as CITFPW, shown in Figure 4. The effect arises from the sharing of a given level of total capital across a higher population. The wealth dilution effect, measured by the product of the population growth rate and wealth per capita, is at its strongest during the baby boom of 1940-60, and led to negative rates of CITFPW during these years.

### 3.2 Measuring Changes in Well-Being.

Based on the approach in Ferreira, Hamilton and Vincent (2008) as shown in Equation 7, we use the present value of future changes in consumption per capita as a measure of changes in well-being. The present value of the change in consumption is calculated over four time horizons; 20, 30, 50, 100 years initially using a 3.5% year discount rate. This discount rate is the difference between the mean long-term interest rate on US government bonds and the mean inflation rate, interpreted here as the real return on risk-free assets (Officer, 2014). Figure 3 illustrates the present value of changes in future consumption. Finally, to correspond with Equation 10, we have incorporated a wealth dilution effect where the values of the future well-being changes are adjusted, including by subtracting the average population growth rate from the discount rate.

## **4. Econometric testing**

The long spans of the univariate macroeconomic time series data used in the estimation and testing of the various models have the potential to exhibit non-stationary properties.<sup>6</sup> Thus, without appropriate methods, estimates may be inefficient or spurious and the usual significance tests may be invalid. Engle and Granger (1987) show that a linear combination of two or more series that are integrated of order 1 may be stationary. The linear combination, if it exists, defines a cointegrating relationship where the resulting vector characterises the long-run relationship between the variables. A cointegration estimation approach: (i) resolves the problem of non-stationary time series data and the inference issues of its neglect, (ii) has the interpretation that the cointegrating relationship (if it exists) can be regarded as a (potentially) unique long-run economic equilibrium relationship, (iii) has the properties that the estimates are 'super-consistent' i.e. they are consistent with much smaller sample sizes, (iv) 'washes-out' in the long-run random errors that may exist in one or both series and, (v) means inferences can be made on the levels of the series. If cointegration exists, the power of its long-run properties dominates short-run variations, which by definition are going to be stationary. Cointegrating relationships, however, and their benefits and properties, do not exist with all combinations of non-stationary series – there is a need to test for their existence. Furthermore, not all estimators are efficient in the presence of strong endogeneity, although they are typically super-consistent.

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<sup>6</sup> This contrasts with the samples of e.g., Ferreira, Hamilton and Vincent (2008) where they explicitly rule-out any investigation of cointegration given the small  $t$  component of their panel dataset.

There are a range of methods available to test for the existence of cointegration ranging from the simple and popular Engle-Granger (1987) 'two-step' approach which appraises the time series properties of the residuals in a levels OLS regression and where the null hypothesis is of no-cointegration; to the maximum likelihood-based tests of Johansen (1995) and the adjustments made by Phillips and Hansen's (1990) Fully Modified OLS (FMOLS). The time series properties of the residuals are investigated using the unit root test of Elliott, Rothenberg and Stock (1996).

When translated into a (potentially) cointegrating regression environment, the approaches of Engle and Granger (1987), Phillips and Ouliaris (1988) and Johansen (1995) are not the best to use if there are breaks in the cointegrating vector as the methods fail to reject the null hypothesis of no cointegration less often than they should. In an attempt to counteract this potential problem we consider the cointegration test associated with Hansen (1992) which explicitly involves testing the cointegrating relationship for parameter stability. In contrast to the residual based tests underpinning Engle-Granger, etc., Hansen's test does not rely on estimates from the original equation.

One important issue that arises in the 'wealth dilution' versions of the tests is the issue of endogeneity (see Ferreira, Hamilton and Vincent (2008) pp. 241-42 for a discussion). Their response was to use a generalized two-stage least squares (2SLS) estimator.<sup>7</sup> We also report 2SLS-based results, where appropriate, but in addition provide both OLS and FMOLS<sup>8</sup>, cointegration-based results given the long spans of the data. As is well known, ordinary 2SLS estimates will be less efficient than OLS, but will account for any endogeneity-based bias. We utilise the Durbin-Wu-Hausman test to evaluate whether 2SLS is appropriate, where we use instruments similar to those of Ferreira, Hamilton and Vincent (2008)<sup>9</sup>. In addition, however, given the efficiency loss of 2SLS, we also report FMOLS results. Although estimates from a cointegrated model will be superconsistent, in small samples they may be biased, although this will disappear asymptotically. In order to eliminate second-order bias in small samples, Phillips and Hansen (1990) correct the single-

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<sup>7</sup> The set of instruments they used included lagged values of green savings, produced capital, the percentage of the population of working age, the population growth rate, and a time trend.

<sup>8</sup> We also considered Dynamic OLS (DOLS) and Canonical Cointegrating Regression (CCR) approaches, which provided results that were qualitatively similar in all cases and are not reported here.

<sup>9</sup> The list of instruments included (one period lags of) long run and short run interest rates; population growth rates; the relevant measure of savings and a time trend.

equation estimates non-parametrically to obtain median-unbiased and asymptotically normal estimates.

It should be stressed however, that a finding of non-cointegration does not invalidate the results, but they are potentially less robust. As will be seen in what is presented below, coefficient estimates in (statistically) non-cointegrated models, and the inferences made, are generally very similar to cases where cointegration (in a similar model) has been established. At this point we also reiterate that in the results presented below we are specifically and solely concerned with consideration of the results as tests of the size and sign of  $\beta_1$ . Estimation and testing is restricted to testing this implication, and as such the results should not to be construed as structural models of the growth process, which would clearly entail much richer models with additional variables drawn from a wide range of candidates.

## 5. Results

Firstly, the welfare-maximising assumption of the standard neo-classical growth model is investigated. This model includes a constant population growth rate and implies a relationship between changes in future consumption and changes in capital as defined in Equation 7. The increasingly broad measures of net investment  $g$  discussed in section 3 are introduced sequentially in the estimation of Equation 8, with the results for net produced and comprehensive investment shown in Table 1 for consumption horizons spanning from 20-100 years. The estimates of  $\beta_1$  for the narrower measure of produced investment NETINV offer scant support for the neo-classical model with a maximum value of 0.401. Given the imprecision (the high standard errors) of the estimates, the hypothesis  $\beta_1 = 1$  is not rejected over the 20 and 30 years horizons, but, in all cases, the tests for no cointegration fail to reject the null. The broader investment measure CI, which adjusts for changes in natural and human capital, offers more support for the standard theory, with the non-cointegration null rejected over 20 or 30 years, and the estimates of  $\beta_1$  falling in the range 1.24-1.34. However, the theoretical model incorporates an infinite consumption horizon and the  $\beta_1$  estimates over the 50 and 100 years horizons do not support the standard neo-classical model. One possibility for the lack of support for the theoretical model over the longer horizons is that the measures NETINV and CI define net investment too narrowly. One missing element from the changes in wealth assessed so far is the value of technological progress (Weitzman, 1997).

The adjustment for changes in TFP on net investment is undertaken using the method outlined in Greasley et al (2014). We focus just on the more inclusive measures of changes in total capital, namely CI and GREENINV. Augmenting these measures of net investment with a value for changes in the stock of knowledge, measured here by the discounted value of TFP, yields estimates of  $\beta_1$  that are closer to unity, especially over the 30 years consumption horizon, with coefficients of 0.86 and 1.04 (Table 2). The difference depends on whether or not education investment is included, as in the case of CITFP, or not, as for GREENTFP, where human capital formation will be simply reflected in TFP. The null of non-cointegration is not rejected over the 20-50 years horizons for any of the technology-augmented measures, further highlighting the case for including changes in knowledge's value in net investment. Generally, the estimated coefficients for technology-augmented investment have lower standard errors, compared to those for NETINV and CI, and the hypothesis that  $\beta_1 = 1$  is rejected in all cases except for GREENTFP over the 30 years horizon.

Next, the implications of extending the standard neo-classical model to allow varying population growth are examined. Relaxing the assumption of a constant population growth leads to the relationship between investment and future consumption being defined as Equation 10, where net investment is reduced by wealth dilution if the population is growing. Since Equation 10 also embeds wealth in the dependent variable, a potential endogeneity issue arises. The results in Table 3 focus on CITFP, given the demonstrated utility of including changes in the value of knowledge in net investment, but the estimated coefficients are from OLS, 2SLS and FMOLS methods, to allow investigation of the possible bias from endogeneity.

The OLS results (Estimates A-D) in Table 3 modify the findings of Tables 1 and 2 to allow for the effects of wealth dilution associated with varying population growth. The estimates of  $\beta_1$  are generally closer to unity with the adjustment for wealth dilution, and in all cases (A-D) the hypothesis that  $\beta_1 = 1$  is not rejected. The tests for the null of no-cointegration are less clear-cut<sup>10</sup> with the ERS adjusted Dickey-Fuller statistics rejecting the null only over 30 and 100 years horizons. The utility of Estimates A-D also need to be judged in relation to the choice of

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<sup>10</sup> See fn. 11.

discount rate, which is not adjusted for population growth, and possible OLS estimation bias from endogeneity.

To investigate the possible bias of the OLS estimates 2SLS estimates are also reported (Estimates E-H). An additional modification in the 2SLS estimates is the reduction of the consumption discounting factor by the population growth rate, which has averaged 1.48%/year since 1870. Ferreira, Hamilton and Vincent (2008, p.236) articulate the consequences for Equation 8 above of relaxing the restriction of constant population growth, including for the consumption discount rate. One distinctive feature of the 2SLS results presented here is the support they offer to the extended, varying population, neo-classical growth model over the 100 years horizon, where the null of non-cointegration is rejected by the DF-ERS test, but the hypothesis  $\beta_1 = 1$  is not rejected. Further, the W-D-U test null for an exogenous independent variable is only rejected in the case of the 30 years consumption horizon.

Accordingly, further, more efficient OLS estimates, are reported (Estimates I-L), and these only differ from the OLS Estimates A-D in their use of the population growth adjusted discount rate of 1.98%/year for all the variables. The use of the lower discount rate adds to the support for the population varying neo-classical model over the 100 years horizon, with estimated  $\beta_1 = 1.1$  and non-cointegration rejected by the DF-ERS statistic. These results may be contrasted with those of Tables 1 and 2 where the estimated coefficients over the 100 years horizon for CI and CITFP were 0.24 and 0.36. Collectively the results highlight the importance of including a value for changes in technology and population-related wealth dilution in tests of the welfare maximizing assumption of the neo-classical growth model. The findings here are supportive of the premises of the neoclassical model, but there are a few caveats.

First, the results are sensitive to the choice of discount rate, especially over the longer 50 and 100 years horizons. And while adjusting the standard neo-classical model for varying population is important, it introduces a possible bias in the empirical estimates of  $\beta_1$  arising from endogeneity. To an extent the findings of cointegration and their support for the existence of long-run equilibrium relationships lessen the concerns surrounding endogeneity. Further, the W-D-U tests only rejected an exogenous independent variable for the 30 years horizon. FMOLS estimation, which is robust where endogeneity exists, over the 30 years horizon (Estimate N) gives a  $\beta_1 = 0.91$ , although the P-O statistic does not reject no-cointegration. The P-O test is known to bias in favour of non-rejection of a unit root where there are structural breaks in the

sample period. This provides a salutary reminder that the post-1870 sample used here contains two world wars and a great depression, with inevitable consequences for the shorter-run variations in the measures of net investment and future consumption. Hence our preference for the results over the longer 100 years horizon, including Estimate L, which do not reject cointegration and provide an estimate of  $\beta_1 = 1.11$ , which accords closely with the assumptions of the neo-classical model adjusted for varying population growth.

## 6. Conclusions

The idea that (macro) economic agents are rational, welfare optimisers who base their savings and consumption decisions on present value maximisation is a powerful one in economics. Modern theories of economic growth revolve around this basic assumption. However, the extent to which it has been empirically tested over the long run is very limited to date. In this paper, we make use of a framework for testing the predictions of the “new economics of wealth and well-being” (Hamilton and Hepburn, 2014) to examine the properties of a data set from the USA which traces year-on-year changes in produced, human and natural capital from 1869 to 2000. Deriving a number of increasingly inclusive indicators of changes in total capital (comprehensive wealth: Arrow et al, 2012), we examine the relationship between these indicators and the present value of changes in future consumption, up to 100 years ahead. The Weitzman/Ferreira/Hamilton/Vincent framework generates testable hypotheses on the parameter relating changes in total capital to this present value of changes in future consumption, conditional on the assumption of an economy which is competitive in the sense of Dixit et al (1980). This testing procedure has previously been applied to the examination of the forward-looking properties of sustainable development indicators such as Genuine Savings (Arrow et al, 2003).

The main results that emerge are that once the measures of changes in total capital are extended to include human and natural capital, then the  $\beta_1$  parameter is found to be close to unity (as predicted by the theory). Cointegration (and thus a long-run equilibrium relationship) also exists between these indicators of changes in total capital and future consumption up to 30 years ahead. Once a measure of the value of technological progress is included in net investment,  $\beta_1$  moves even closer to unity, and evidence of cointegration is found up to 50 years ahead. Adding in the effects of varying population growth rates over time with consequent wealth dilution effects improves the fit between theory and reality even more.

Overall, then, 130 years of data from the USA provides support for the basic building block of modern growth theory, but only once a sufficiently comprehensive view of what constitutes a country's wealth or capital is taken. Caveats are many: we have very partial measures of changes in natural and human capital; no account is taken of changes in social capital; our measure of technological progress is easy to criticise. Nevertheless, we argue that the paper provides interesting and important findings which are genuinely novel.

## Data Appendix

**GDP, GDP deflator, population:** Johnston and Williamson (2013).

**Consumption:** 1869-1900 from Rhode (2002), 1901-1962 from Carter et al (2006) and 1963-2012 from ERP (2012). Nominal series are deflated with CPI from Johnston and Williamson (2013). The present value of the change in consumption adopts 3.5%/year (or 1.98%/year in the varying population growth model) discount rate based on Officer (2014).

**Net Investment:** Net investment consists of produced capital, inventories and overseas investment. Gross fixed capital formation, inventories and net overseas investment for 1869-1909 are from Rhode (2002), for 1909-1929 from Kuznets (1961), for 1929-1992 from Carter et al (2006) and for 1992-2000 from the ERP (2011). Capital consumption from Kuznets (1961) for 1869-1929, from ERP (1963, 1995, 2011) for 1929-2000.

### Green Investment:

**Forestry:** Changes in forestry stock are estimated by the product of the area of forests and the standing volume of timber (m<sup>3</sup>). Forest area is from Carter et al (2006, series CF101-118 and Cf135-144) and standing volume from (Zon (1910), Zon & Sparhawk (1923), Clawson (1979), Oswald et al. (2007), USDA (1997), Smith & Darr (2002), Smith, et al. (1997), USDA (1997) and Carter et al (2006). The earliest estimate of standing volume of 94.59 cubic metres per hectare in 1920 is adopted for 1850-1920. The change in the standing volume of timber is valued at market prices minus average costs. For the period 1869-1904 forestry prices are derived from Warren & Pearson (1932) and stumpage prices for 1905-2000 from Carter et al (2006). Employment and annual lumbering estimates are derived from the Carter et al (2006) and Lebedys (2004), the wage cost per m<sup>3</sup> use unskilled wages from Officer (2012) and David and Solar (1977).

**Land:** Changes in the volume of farmland are valued using the present value of rents looking forward 30 years. Land values are from Carter et al (2006), DA17. Lindert (1988) shows that rental values average 15% of the land values, a ratio used here to estimate annual rental values. Rents to 2030 are forecasted using an ARIMA (5,1,1).

**Non-renewables:** 1880-2000 mining (fuel, metals and minerals) data are from Carter et al (2006). Fuel comprises Coal Bituminous, Coal Subbituminous, Coal Lignite, Coal Pennsylvania Anthracite, Crude Petroleum, Natural Gasoline and Cycle Products, and Liquefied Petroleum Gases, Natural Gas Marketed, Uranium Concentrate. Metals included are Iron Ore, Copper, Zinc, Manganese Ore, Chromite, Tungsten Concentrates, Molybdenum Ores and Concentrates, Vanadium Ores and Concentrates, Nickel, Bauxite, Aluminum Primary, Magnesium Primary, Gold, and Silver. Minerals are comprised of Crude Gypsum Mined, Lime, Sand and Gravel, Stone, Sulfur Production from Frasch Mines, Pyrites Production, Salt, Potash sold by producers, and Phosphate Rock. 1869-1880 mining production estimates are from Herfindahl (1996), (Gallman 1960) and Carter (2006) and valued at international prices. Commodities included are iron ore, copper, lead, zinc, gold, silver, coal and crude petroleum. Mining wage costs per tonne are based on coal wages and the relative productivity of coal and other mining. Over the period 1869-2000 the mean relative labor productivity difference between coal and all forms of mining was 1.06. Employment and wage data are from Carter et al (2006).

**Carbon Emissions:** US carbon pollution estimates are taken from Andres et al (1999) and Boden et al (1995) and the price series from is derived from Tol (2012). The 2015 price of \$29 per tonne of carbon is discounted by 1.99%/year to 1869.

**TFP:** The present value of future changes in TFP is measured over 20 horizons using a 3.5%/year discount rate (or 1.98%/year in the case of varying population growth model). Trend TFP growth rates are estimated for the period 1870 to 2013 with the Kalman filter (Figure A5). The TFP estimates include the real capital stock data of Gallman (1992), Kendrick (1963) and BEA (2014). Labour and hours worked are taken from Margo (2000), Greasley & Madsen (2006), and BLS (2014). Real GDP are from Johnston and Williamson (2013) and factor shares used from Greasley and Madsen (2006).

**Wealth dilution:** Wealth is calculated from the above estimates of private and public consumption. The net present value of consumption is calculated over a 25 years horizon discounted at 3.5%/year (or 1.98%/year in the case of varying population growth model). For measures of wealth from 1988-2010 it is necessary to estimate future consumption for  $t+25$ . This was done using an ARIMA (3,1,2). Wealth dilution is calculated as the product of wealth per capita and the population growth rate.

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Figure 1: Comprehensive investment (CI) per capita

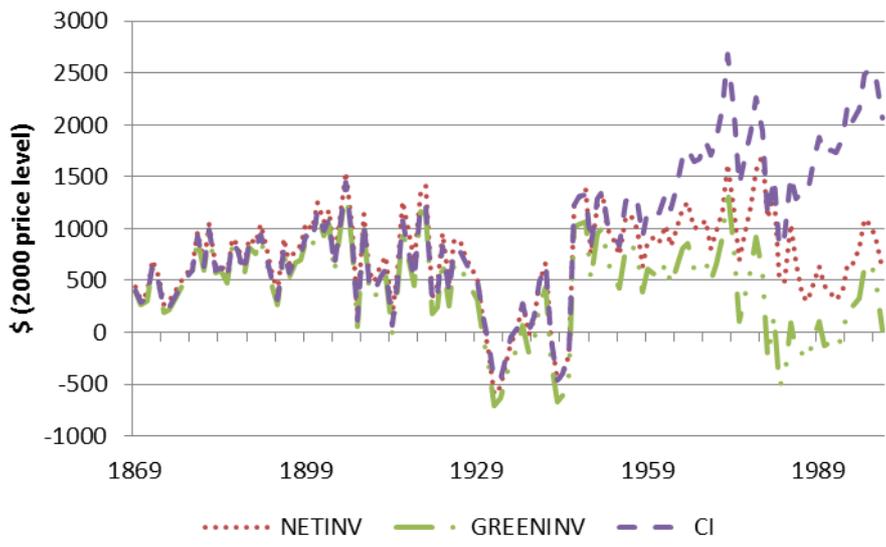


Figure 2: CITFP and GREENTFP per capita

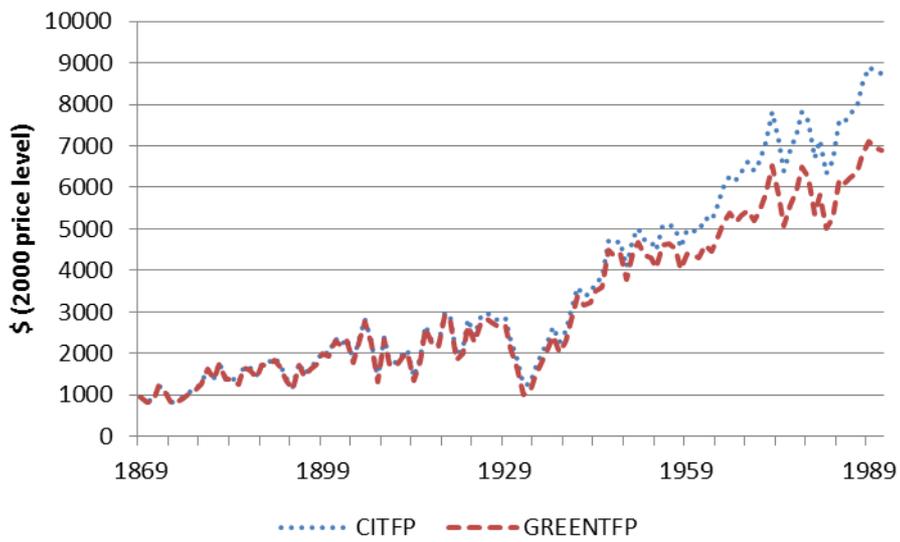


Figure 3 Present value consumption \$(2000 price level, discounted at 3.5%)

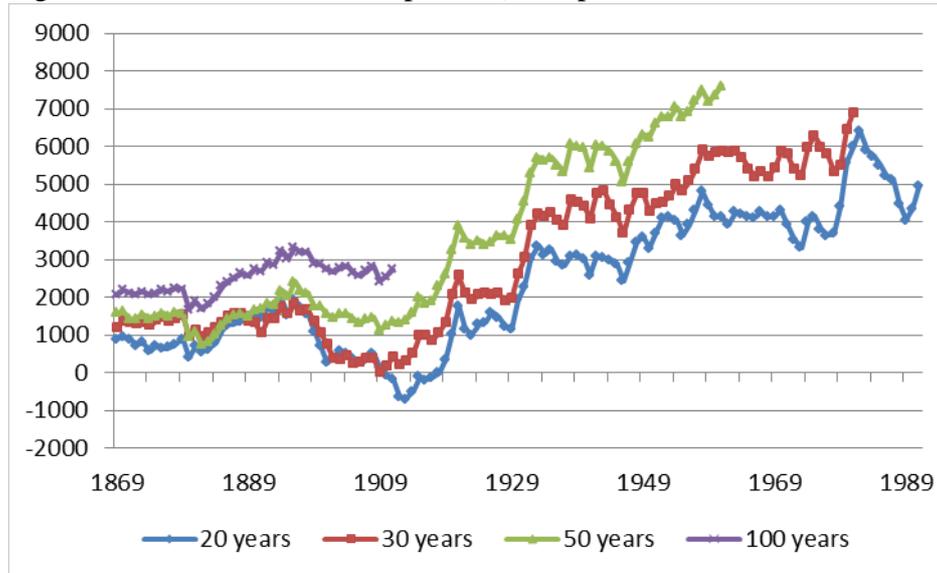


Figure 4: Wealth dilution effect

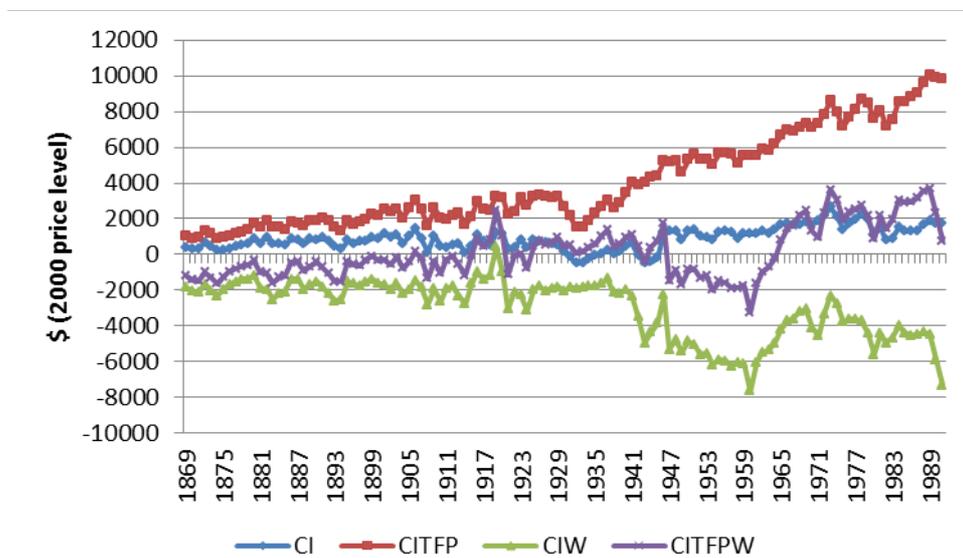


Table 1. OLS Estimates of $\beta_0$ and $\beta_1$ for Two Investment Series (3.5% per annum discount rate)					
Dependent	Independent	$\beta_0$	$\beta_1$	$\beta_1=1$	DF-ERS
CONS 20	NETINV	2267.4*	0.154	3.39	-2.47
		(412.1)	(0.459)	(0.06)	
CONS 30		2660.2*	0.401	2.16	-1.84
		(553.1)	(0.684)	(0.14)	
CONS 50		3916.6*	-0.854	14.5*	-1.22
		(390.2)	(0.487)	(0.00)	
CONS 100		2270.6*	0.308	10.5*	-2.68
		(172.1)	(0.214)	(0.00)	
CONS 20	CI	1289.8*	1.242*	0.38	-3.25*
		(496.3)	(0.392)	(0.54)	
CONS 30		1845.2*	1.343*	1.44	-2.87**
		(293.8)	(0.285)	(0.23)	
CONS50		3399.8*	-0.068	1.21	-0.99
		(663.6)	(0.971)	(0.27)	
CONS 100		2335.5*	0.243	10.1*	-2.51
		(212.3)	(0.238)	(0.00)	

**Notes:** CONS=Net Present Value of Consumption per capita for 20-100 year horizons, \* and \*\* denote significant at the 5 and 10% level respectively. DF-ERS = Elliott, Rothenberg and Stock DF statistic<sup>11</sup>.

<sup>11</sup> The cvs for the DF-ERS statistic have not been corrected for the fact that residuals from equations 8 or 10 have been used in the calculation. This is likely to mean that the null hypothesis of no cointegration is less likely to be rejected than if true standard errors had been used representing a standard generated regressor problem.

<b>Table 2. OLS Estimates of <math>\beta_0</math> and <math>\beta_1</math> for technology-augmented investment series (3.5% per annum discount rate)</b>					
<b>Dependent</b>	<b>Independent</b>	<b><math>\beta_0</math></b>	<b><math>\beta_1</math></b>	<b><math>\beta_1=1</math></b>	<b>DF-ERS</b>
CONS 20	CITFP	123.4 (330.7)	0.628* (0.070)	28.0* (0.00)	-2.92**
CONS 30		208.1 (200.9)	0.856* (0.053)	7.26* (0.00)	-2.82**
CONS50		-127.4 (301.0)	1.405* (0.108)	13.9* (0.00)	-2.87**
CONS 100		1922.9* (215.3)	0.360* (0.104)	37.6* (0.00)	-2.75
CONS 20	GREENTFP	-122.7 (367.0)	0.791* (0.087)	5.72* (0.02)	-3.19*
CONS 30		-53.7 (426.9)	1.043* (0.092)	0.22 (0.64)	-3.06*
CONS50		-140.5 (551.9)	1.500* (0.163)	9.39* (0.00)	-2.88**
CONS 100		1941.8* (217.4)	0.357* (0.107)	35.5* (0.00)	-2.75

**Notes:** As for Figure 3

Table 3: Wealth Adjusted Estimates of $\beta_0$ and $\beta_1$ for technology-augmented investment (Estimates A-D 3.5%/year and Estimates E-Q 1.98%/year discount rates)							
OLS	Dependent	Independent	$\beta_0$	$\beta_1$	$\beta_1=1$	D-W-H	DF-ERS
A	CONSWP 20	CITFPW	1744.7* (149.0)	1.064* (0.110)	0.340 (0.56)		-2.59
B	CONSWP 30		2397.2* (160.1)	1.191* (0.139)	1.88 (0.17)		-2.81**
C	CONSWP50		3120.2* (213.9)	1.080* (0.268)	0.08 (0.77)		-2.48
D	CONSWP 100		2327.2* (118.7)	0.700* (0.182)	2.73 (0.09)		-2.89**
2SLS							
E	CONSWPAD 20	CITFPWAD	2679.1* (168.3)	0.630* (0.132)	7.80* (0.00)	0.20 (0.88)	-3.18*
F	CONSWPAD 30		3746.5* (219.2)	0.753* (0.204)	1.46 (0.23)	4.53** (0.10)	-2.67
G	CONSWPAD50		4825.6* (366.1)	0.286 (0.443)	2.60 (0.11)	3.66 (0.16)	-0.92
H	CONSWPAD 100		6018.0* (332.7)	1.430* (0.392)	1.20 (0.27)	1.89 (0.17)	-3.64*
OLS							
I	CONSWPAD 20	CITFPWAD	2646.4* (166.2)	0.646* (0.114)	9.59* (0.00)		-3.14*
J	CONSWPAD 30		3727.6* (217.3)	0.756* (0.169)	2.09 (0.15)		-2.76**
K	CONSWPAD50		4810.2* (338.2)	0.267 (0.319)	5.26* (0.02)		-1.38
L	CONSWPAD 100		5756.8* (242.0)	1.114* (0.263)	0.19 (0.66)		-3.52*
FMOLS							
M	CONSWPAD 20	CITFPWAD	2630.4* (348.2)	0.767* (0.239)	1.00 (0.32)		-2.28
N	CONSWPAD 30		3735.1* (466.0)	0.909* (0.363)	0.06 (0.80)		-1.32
P	CONSWPAD50		4879.9* (645.2)	0.376 (0.611)	1.04 (0.31)	0.85	
Q	CONSWPAD 100		6143.2* (409.7)	1.590* (0.449)	1.73 (0.19)		-9.22

**Notes:** CONSWP =  $PV\Delta C_{it} + PV(\Delta Y_{it}\omega_{it})$  from Equation 8, with 3.5%/year discount rate, for 20-100 years horizons. CONSWPAD = CONSWP with 3.5%/year – population growth discount rate. CITFPW = CITFP less wealth dilution with 3.5%/year discount rate. CITFPWAD = CITFPW with population growth adjusted discount rate, 1.98%/year. W-D-H = Durbin-Wu-Hausman J test. DF-ERS = Elliott, Rothenberg and Stock DF statistic. P-O = Phillips and Ouliaris cointegration test.

Figure A1: US net fixed capital formation, inventories and net overseas investments.

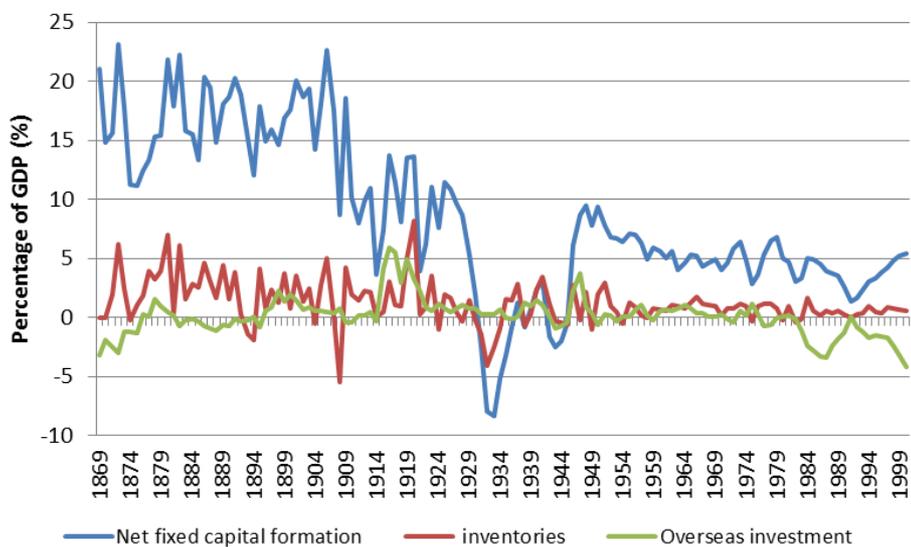


Figure A2: Depletion of minerals.

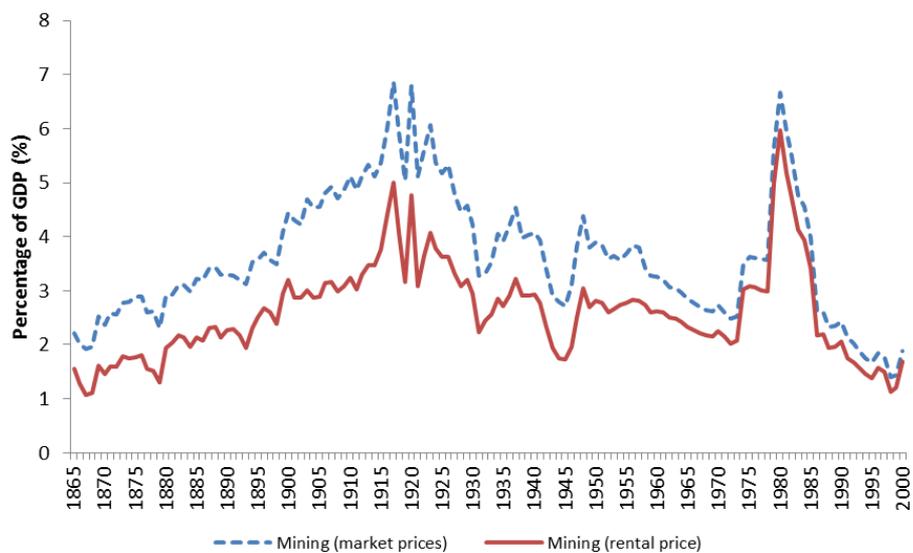


Figure A3: Carbon emissions and carbon share of GDP, 1869-2000

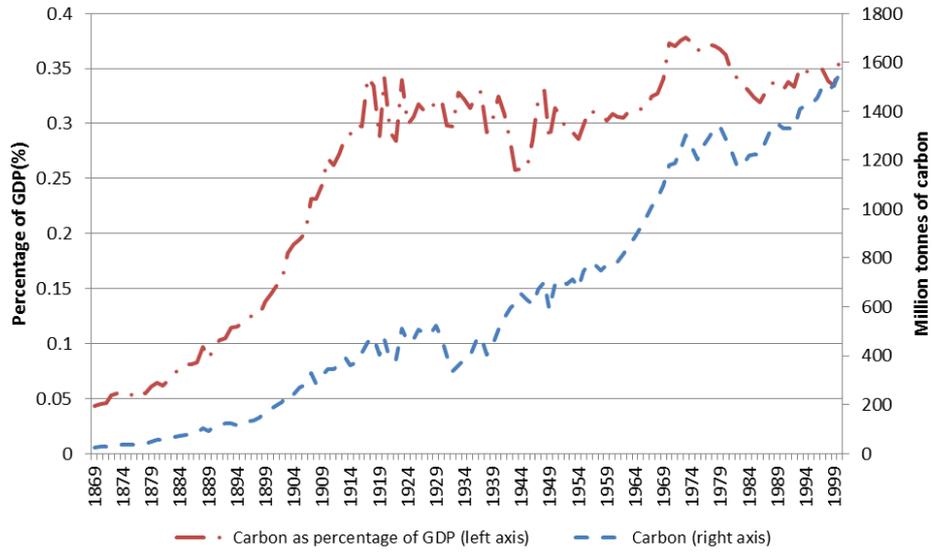


Figure A4: Public investment in education

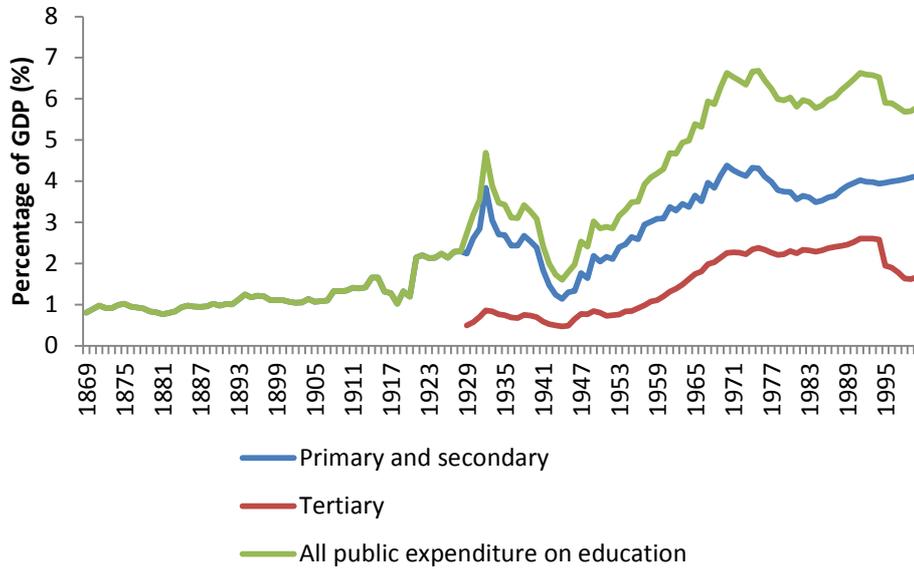


Figure A5: Trend Total Factor Productivity (%)

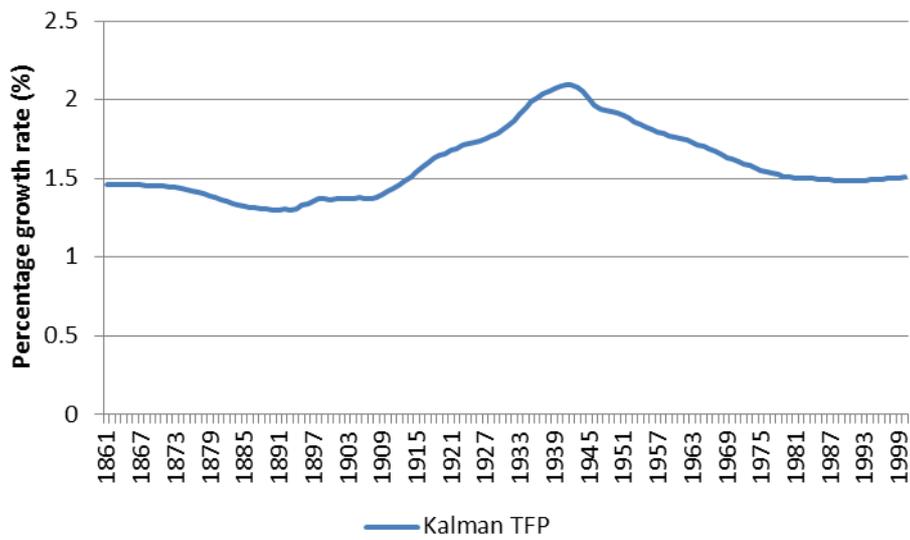


Figure A6 US consumption

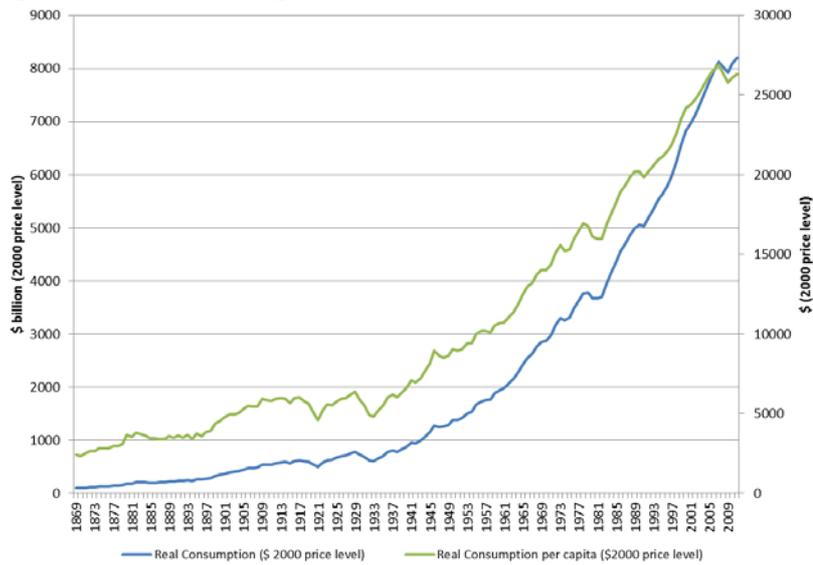


Table A1: The value of US forest stocks (decade average)

	Area	Volume per hectare	Standing volume	Change in Standing volume	Price - cost	Value of change in volume	Value of change in volume /GDP
	Million hectares	M <sup>3</sup> per hectare	Million M <sup>3</sup>	Million M <sup>3</sup>	\$ M <sup>3</sup>	\$ million	%
1861-1870	256.06	94.86	24289.05	-97.00	0.25	-24.18	-0.31
1871-1880	237.14	94.86	22494.33	-243.59	0.24	-58.04	-0.68
1881-1890	220.44	94.86	20909.75	-92.13	0.23	-21.46	-0.17
1891-1900	176.60	94.86	16751.08	-644.90	0.21	-137.30	-0.84
1901-1910	145.21	94.86	13773.59	-49.90	0.48	-23.61	-0.08
1911-1920	137.23	94.86	13016.88	-95.97	0.74	-71.45	-0.16
1921-1930	137.23	94.86	13016.88	-95.97	0.74	-71.45	-0.16
1931-1940	147.34	89.94	13251.84	-37.67	0.44	-24.15	-0.02
1941-1950	151.27	86.90	13139.04	167.66	0.88	232.23	0.08
1951-1960	201.09	86.35	17400.32	689.77	3.05	2043.01	0.45
1961-1970	239.80	92.87	22271.17	272.42	2.63	981.51	0.10
1971-1980	291.88	102.50	29941.73	724.68	12.64	5912.80	0.40
1981-1990	294.74	110.91	32691.59	231.54	11.79	2477.30	0.05
1991-2000	298.21	115.55	34458.02	119.52	29.33	3436.64	0.05

Sources: see Data Appendix

Table A2: Changes in farmland rental value, decade averages

	Present value of rent per acre	Total farmland	Change in farmland	Change in rental value	Change in rents/GDP
	\$	Million acres	Million acres	\$ m	%
1861-1870	1.27	407.74	0.52	0.66	0.00
1871-1880	3.27	536.08	128.35	419.64	0.40
1881-1890	5.73	623.22	87.14	499.15	0.33
1891-1900	4.00	841.20	217.98	872.93	0.42
1901-1910	2.18	881.43	40.23	105.22	0.03
1911-1920	4.17	923.38	7.72	32.37	0.06
1921-1930	6.94	950.70	3.14	31.58	0.03
1931-1940	12.14	1044.65	7.50	81.38	0.12
1941-1950	35.57	1132.03	9.63	270.04	0.13
1951-1960	52.82	1144.49	-4.06	-198.25	-0.04
1961-1970	65.90	1091.85	-6.73	-468.95	-0.06
1971-1980	94.49	1019.34	-5.29	-492.57	-0.03
1981-1990	101.58	971.79	-4.76	-481.00	-0.01
1991-2000	111.40	936.03	-2.94	-326.91	0.00
Sources: see Data Appendix					