The Effects of Rent Seeking over Tradable Pollution Permits

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Abstract

The establishment of a tradable permit market requires the regulator to select a level of aggregate emissions and then distribute the associated permits to specific groups. Both these decisions create opportunities for rent seeking. In this paper, we use a contest model to analyse the incentives to rent seek for pollution permits and to analyse the consequences for social welfare. We find differences in firms’ rent-seeking choices compared to a conventional rent-seeking contest. We see that a fundamental aspect of firms’ incentives to rent seek depends on the market value of the permits, that is, the value of the ex post reallocated rents. This impact depends on the responsiveness of the regulator to aggregate rent-seeking effort. The responsiveness, in some cases, may improve welfare by reducing the per-unit value of permits, which may lower the rent-seeking effort more than it increases the damages experienced from the additional emissions.

KEYWORDS: tradable permit market, rent seeking, initial allocation

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1 Introduction

 Tradable pollution markets have become an increasingly mainstream regulatory tool for controlling pollution. Since Montgomery (1972), economists have known that under certain conditions, such markets can achieve an efficient (that is, cost-minimising) allocation of pollution control efforts across polluters, irrespective of the initial allocation of permits by a regulator. This is because post-allocation trading will allow all potential gains from trades to be realised. However, the initial allocation of permits has become a matter of political debate and academic interest, since firms’ gains and losses in the real world may depend on this initial allocation. Moreover, since permits are valuable, allocation creates rents over which firms can be expected to compete ex ante by rent seeking. In many existing tradable permit markets, regulators’ decisions over the distribution and absolute level of emissions have been influenced by interested parties (see, for example, Svendsen 2005).\(^1\) Rent seeking is typically seen as socially unproductive and often as a significant and sustained problem.

 Important questions for economists include how rent-seeking strategies are determined in tradable permit markets, how this influences social welfare, and whether these effects depend on the degree to which a regulator allows rent seeking to determine both the distribution and absolute level of these rents. This paper seeks to answer these questions, using a contest model. Our conclusion is that when regulators are responsive to aggregate rent seeking (i.e., are willing to change the total supply of permits), then rent-seeking strategies for tradable permits differ significantly from standard (non-tradable) rents. When the regulator’s responsiveness to aggregate rent seeking increases, we find this results in an ambiguous change in welfare that depends on the trade-off between the increase in aggregate emissions and any decrease in rent seeking from a reduction in the permit value. This suggests that no simple answers are available in an actual policy context about how the degree of responsiveness to rent seeking will affect social welfare.

 In this paper, we use a contest framework to analyse rent seeking. Starting from the seminal works of Krueger (1974), Posner (1975) and Tullock (1980), a substantial body of work has focused on rent dissipation issues, where the total rents available for capture across all agents are taken as fixed (for surveys of the literature, see Nitzan 1994; Hillman and Riley 1989; Congleton et al. 2008; Konrad 2009). Chung (1996) extends a Tullock-style rent-seeking contest model to include a rent that is endogenously determined by aggregate efforts and finds that the contest

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\(^1\)Anecdotal evidence exists for the existence of lobbying (from individual senators) in the US SO2 trading scheme (Ellerman et al. 2000) and under the ‘Waxman-Markey’ Bill (HR. 2454). In the set up of the European Union Emissions Trading Scheme (EU-ETS), industry was heavily involved in lobbying behaviour (Svendsen 2005; Ellerman et al. 2007).
generates excessive effort levels that are socially wasteful. More recently, Shaffer (2006) finds that effort levels tend to adjust in the direction of the change in the rent. For example, when the lobbying is ‘productive’ (where the rent is increasing in aggregate efforts), agents tend to invest more effort in rent seeking. However, this literature typically assumes that rents are non-tradable, which limits the insights one can draw when considering tradable pollution permit markets. In our analysis, we allow the total value of rents to be endogenously determined by aggregate rent-seeking effort, extending this to both the quantity (total supply of permits) and price (ex post value of traded permits) dimensions.

A tradable permit market involves an ex post reallocation of rents through the buying and selling of permits. It has long been understood that such an ex post reallocation of emission rights is key to an efficient allocation of emission reductions among polluters (Montgomery 1972). Few authors, however, have considered rent-seeking contests when ex post reallocation is not just possible but essential to the operation of the policy instrument. Dari-Mattiacci et al. (2009) and Sui (2009) find that although contests are allocatively efficient, effort levels tend to increase when ex post reallocation is permitted. However, it is not clear how these rents differ from standard rents in terms of agents’ rent seeking strategies and regulatory responses. In the context of tradable pollution permit markets, Lai (2008) has investigated the social welfare consequences of firms and environmental groups lobbying over the determination of an aggregate emissions cap and finds that allocating freely may be more efficient than auctioning. However, the incentives for firms to rent seek for their own private benefit (to increase their own individual share of permits at the expense of rival firms) are not considered by Lai.

In the contest modelled herein, polluting firms have the option to invest in rent-seeking effort that has the potential to increase their own initial permit allocation within a tradable permit market and, simultaneously, the aggregate supply of permits from the regulator. We allow the regulator to select a provisional aggregate emissions target (for example, by announcing draft legislation) which can be subsequently influenced by firms’ rent-seeking effort. We provide cases where the regulator views rent seeking as ‘purely’ socially wasteful and, alternatively, where the regulator obtains political contributions from rent seekers. Our focus is on how the market value of the ex post reallocated rent, which is endogenously determined by the marginal costs of participating firms and the aggregate supply of permits, alters rent-seeking behaviour and social welfare. We find differences in firms’ rent-seeking choices compared to a conventional contest. We see that a fundamental aspect of firms’ incentives to rent seek depends on the market value of the permits, that is, the value of the ex post reallocated rents.

The paper is organised as follows. Section 2 introduces the model. In Section 3, the firm’s optimal choice of emissions is determined. In Section 4, the
firm’s equilibrium rent-seeking strategy is discussed, and aggregate rent-seeking effort is then derived. Section 5 investigates the regulator’s optimal choice of aggregate emissions and discusses whether alternative responses to rent seeking can be welfare improving. Section 6 discusses modifications and extensions of the basic model. Section 7 provides a discussion of policy implications and Section 8 concludes.

2 The model

Consider a set of firms \( \{1, 2, \ldots, n\} \) that participate in a competitive tradable pollution permit market. In this market, permits are initially allocated freely but each firm has the ability to alter the amount of permits it receives from the regulator by investing in rent-seeking effort denoted by \( s_i \) for \( i = 1, 2, \ldots, n \). A unilateral increase in firm \( i \)'s rent-seeking investment will result in that firm obtaining relatively more permits prior to the beginning of the market.\(^2\) Additionally, we allow for the possibility that aggregate rent-seeking effort influences the regulator’s final decision when selecting an absolute level for the aggregate emissions cap. That is, market participants can apply political pressure on the regulator to increase the aggregate emissions level.\(^3\) Therefore, an increase in rent seeking by firm \( i \) increases their share of the ‘pie’ and provides pressure to increase the absolute size of the ‘pie.’ After the initial distribution of the rents, firms are free to trade and reallocate these permits.

Our model is split into three stages. In stage one, the regulator selects a provisional level of aggregate emissions for the trading permit market denoted by \( \bar{A} \) (such as draft legislation). In stage two, given this information, each firm invests in rent-seeking effort \( s_i \) \( \forall i \) to obtain a share of the aggregate emissions which results in a ‘final’ aggregate allocation for the permit market denoted by \( A \). In stage three,

\(^2\)Our results are qualitatively similar when one considers a hybrid allocation approach where both auctioning and grandfathering can be used (where the rent now available for rent seekers is simply the total allocation minus the permits allocated from the auction). This approach has been advocated by energy companies for a US wide cap-and-trade program (Point Carbon 2009). Furthermore, our results may provide analysis on how firms rent seek for permits where allocation mechanisms use ‘reserves’, energy intensity targets and ‘safety valves’ prior to the beginning of the scheme (Pizer 2002; Newell et al. 2005). To introduce full auctioning of permits, the distribution of permits can be modelled as a multi-unit auction. In this case, rent seeking influences the aggregate level of emissions but not the distribution of permits.

\(^3\)Importantly, this does not require cooperation between market participants. Each participant rent-seeks in order to obtain a permit allocation for themselves. Accumulated rent seeking provides pressure on the regulator to increase the aggregate emissions.
the market commences, and each firm selects a level of pollution to emit in the market.⁴

To model how the provisional aggregate emissions level in stage one differs from the final emissions level in stage two, we introduce an *exogenous* political ‘responsiveness’ parameter \( \mu \in [0, \bar{\mu}] \) which is common knowledge among all firms and the regulator. The political responsiveness parameter \( \mu \) represents the political, cultural and governance relationships between the regulator and regulated firms. When \( \mu = 0 \), the regulator is unresponsive to aggregate rent seeking, and the resulting aggregate emissions cap is simply the provisional aggregate emissions chosen by the regulator \( \tilde{A} \). An upper bound on \( \mu \) will exist where the responsiveness is sufficiently large to reduce the equilibrium permit price to zero. For \( \mu > 0 \), the regulator is responsive to firms’ rent-seeking efforts. Formally, the final aggregate emissions cap \( A \) set by the regulator is determined by

\[
A = \tilde{A} \cdot \left( 1 + \mu \sum_{i=1}^{n} s_i \right)
\]  

(1)

where the final rent available in the contest is endogenously determined by the regulator’s initial draft legislation \( \tilde{A} \) and aggregate rent seeking effort \( \sum_{i=1}^{n} s_i \). We initially assume that firms’ rent seeking has a positive effect on the level of aggregate emissions. We later relax this assumption and instead investigate the choice to rent seek for a more stringent level of aggregate emissions. To solve the subgame perfect Nash equilibrium, we solve the model using backward induction and as a result outline and first solve stage three.

3 Stage three: firms’ choice of equilibrium emissions

In stage three, the tradable permit market commences, and firm \( i \) selects a level of emissions. Assuming the equilibrium permit price \( p^* \) and the level of allocation

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⁴As our focus is on the distributional impact of permit allocation and the subsequent effect on the tradable permit market and social welfare, we assume that regulated firms are the only rent-seeking agents. This assumption fits where legislation has been drafted and the associated permit allocation is contestable. For example, one could interpret the political activity under the ‘Waxman-Markey’ Bill (H.R. 2454) or the rent seeking surrounding the National Allocation Plans (NAPs) in the EU-ETS as similar to our three-stage game. Environmental groups may also invest in rent seeking in order to influence the aggregate target, but they would not participate in rent seeking for permit distribution. The determination of environmental policy under political influence from interest groups has been widely analysed. For surveys, see, for example, Keohane et al. (1998), Oates and Portney (2003) and Stavins (2004).
\( a^0_i \) obtained in stage two is taken as given (and hence the aggregate allocation \( A = \sum_{i=1}^{n} a^0_i \) finalised in stage two), firm \( i \)'s payoff from the tradable permit market is:

\[
\max_{e_i} \Pi_i = p^*(a^0_i - e_i) - c_i(e_i) \quad \forall \ i = 1, 2, ..., n
\]  

(2)

where \( e_i \) is the level of net emissions (inclusive of abatement choices) and \( c_i(e_i) \) is the abatement cost function where \( c'_i(e_i) < 0, c''_i(e_i) > 0 \). The term \( (a^0_i - e_i) \) shows firm \( i \)'s net supply of permits to the market (which can also be negative). Given the allocation to each firm (and the subsequent equilibrium permit price determined by aggregate emissions), differentiating (2) with respect to \( e_i \) yields the following first-order condition:

\[
-p^* - c'_i(e_i) = 0 \quad \forall \ i = 1, 2, ..., n,
\]  

(3)

which is solved for \( e^*_i \) and the market clearing condition is given by:

\[
\sum_{i=1}^{n} e_i = A.
\]  

(4)

The first-order condition (3) states the familiar result that each firm will choose a level of emissions to equate their marginal abatement costs with the equilibrium permit price. Condition (4) is the market clearing condition where, in equilibrium, the aggregate emissions must equate to the aggregate supply of permits.

Differentiating (3) with respect to \( p^* \) and (4) with respect to \( A \) we obtain:

\[
-1 - c''_i(e_i) \frac{\partial e_i}{\partial p^*} = 0 \quad \forall \ i = 1, 2, ..., n
\]  

(5)

\[
\sum_{i=1}^{n} \frac{\partial e_i}{\partial p^*} \frac{\partial p^*}{\partial A} = 1 \quad \forall \ i = 1, 2, ..., n
\]  

(6)

where substitution yields:

\[
\frac{\partial p^*}{\partial A} = -\left[ \frac{1}{\sum_{i=1}^{n} c'_i(e_i)} \right] < 0.
\]  

(7)

Expression (7) shows that as the aggregate allocation increases, the equilibrium permit price decreases. Note that the extent of this depreciation is based on the slope of firms’ marginal abatement costs, where steeper marginal abatement costs result in a larger change in the equilibrium permit price. As will be discussed later in the paper, the relationship in (7) is the key to understanding how rent seeking for \textit{ex post} reallocated rents (such as pollution permits) differs from standard rents.
4 Stage two: firms’ optimal rent-seeking effort

In this stage, firm $i$ selects a level of rent-seeking effort to obtain an initial allocation of permits for the beginning of the tradable permit market in stage three. Let us assume that in stage one $\hat{A}$ was chosen by the regulator where each firm knows that the final aggregate emissions cap for the tradable permit market is determined by $\hat{A} (1 + \mu \sum_{i=1}^{n} s_i)$. This rent seeking, from the viewpoint of society, is unproductive.\(^5\)

Formally, we represent the allocation of permits to firm $i$ by:

$$a_i = \begin{cases} \frac{f(s_i, s_{-i})A}{\hat{A}} & \text{if } \sum_{i=1}^{n} s_i > 0 \\ \text{otherwise} & \forall i = 1, 2, \ldots, n \end{cases}$$

(8)

where $s_{-i} = \sum_{j \neq i} s_j$, $A$ is the aggregate emissions level given in (1), and the contest success function is given by the conventional Tullock (1980) rent-seeking model with constant returns to rent seeking and linear costs:\(^6\)

$$f(s_i, s_{-i}) = \frac{s_i}{s_i + s_{-i}}.$$  

(9)

From (1), (8) and (9) observe that $a_i$ is increasing in $s_i$ and decreasing in $s_{-i}$. As shown above, rent seeking allows each firm to capture a bigger share of the ‘pie’ and simultaneously increase the aggregate emissions cap.

4.1 Equilibrium effort

We now consider the incentives to rent seek for a permit allocation that can be ex post reallocated. Firms may invest in rent-seeking effort to influence their own allowance of permits and, from (1), the aggregate allocation. This means that the permit price is endogenously determined by the level of aggregate allocation and hence the level of aggregate rent-seeking effort. We allow an individual firm’s influence on the aggregate emissions cap to be positive. Here the regulator is influenced by individual rent seeking. This occurs due to the specific actions undertaken in the rent-seeking contest. In particular, individual rent seeking is observable to the regulator. For example, the regulator may hold meetings with regulated firms to

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\(^{5}\)In some instances rent seeking may be appealing to a regulator that has limited information as it may reveal valuable information about firms’ activities and preferences (Montero et al. 2002). We assume throughout that the net effect from rent seeking is negative.

\(^{6}\)Throughout the paper we use the interpretation of a divisible prize among agents. However, provided risk neutrality of the agents, a non-divisible rent, where there is a non-zero probability of winning, is functionally equivalent. Therefore, the alternative interpretation of this model is where agents participate in a contest for a single prize which can then be ex post reallocated after initial distribution.
canvas opinions about what constitutes a ‘reasonable’ allocation of permits. Thus the regulator is aware of the concerns of individual firms which influence the determination of the aggregate emissions cap. As the regulator’s choice of aggregate emissions is influenced by the accumulation of observable individual rent-seeking efforts, it follows that, at the margin, the aggregate emissions cap will increase in an individual firm’s rent-seeking effort (however small).7

Firm $i$ now selects a level of rent seeking to maximise its payoff:

$$\max_{s_i} p^* (a_i - e_i^* (A(s_i))) - s_i - c_i (e_i^* (A(s_i)))$$

(10)

where $e_i^* (A)$ is the equilibrium level of emissions chosen in stage three, $a_i$ is given by (8) and the cost of rent seeking is given by $s_i$. Differentiating (10) with respect to $s_i$ and noting from (3) that in the perfectly competitive market $-\frac{\partial c_i}{\partial e_i} = p^*$, we obtain the following first-order condition:8

$$p^* \frac{\partial a_i}{\partial s_i} + \frac{\partial p^*}{\partial A} \frac{\partial A}{\partial s_i} (a_i - e_i^*) - 1 = 0 \quad \forall i = 1, 2, \ldots, n$$

(11)

where

$$\frac{\partial a_i}{\partial s_i} = f'(s_i, s_{-i}) A + f(s_i, s_{-i}) A'(s).$$

(12)

To begin our discussion on rent-seeking strategies under ex post reallocation, note that (11) illustrates two important marginal effects of firm $i$’s rent-seeking effort. The first term in (11) shows a positive marginal effect where a unilateral increase in firm $i$’s rent seeking will increase its permit allocation and wealth, given the permit price $p^*$. From (12) one can see, from the first term, that this positive marginal influence is based on the marginal increase in firm $i$’s share of permits (given a fixed allocation) and, from the second term, a marginal increase in permits from an increase in the aggregate cap (given a constant share of permits). The effect of the second term in (11) is ambiguous and is directly related to ex post reallocation. As will be discussed further below, when rent seeking increases this may increase the aggregate emissions cap and decrease the equilibrium permit price. This is a positive marginal effect when the firm is an ex post net buyer of permits (i.e., $a_i - e_i^* < 0$), as permits now become cheaper to purchase. However, if the firm is an ex post net seller of permits (i.e., $a_i - e_i^* > 0$) this marginal effect is negative.

7An alternative interpretation is that firms rent seek through interest groups which supply collective effort. As Nitzan (1991) shows, if the rents within the interest group are rewarded based on relative rent-seeking effort then rent dissipation is identical to the standard Tullock (1980) contest model.

8The second-order conditions are satisfied for sufficiently small (absolute) values of $\frac{\partial^2 p^*}{\partial A^2}$. We assume throughout that the second-order conditions are satisfied at the optimal levels.
as the additional permits sold are now sold at a lower price. It follows from (11) that net buyers of permits tend to invest more in rent seeking than net sellers of permits. This result shows that allowing ex post reallocation in the form of a tradable permit market for rents creates a situation where equilibrium rent-seeking effort is now dependent on equilibrium rents held.

When $\mu = 0$, this reduces the model to a special case where each firm believes their rent-seeking effort has no influence on the aggregate emissions target. In such a case, firm rent seeking is a matter of distributional conflict over a fixed aggregate emissions target. The second term in the first-order condition (11) is zero. This is the standard rent-seeking strategy for a Tullock (1980) contest where rent-seeking effort is chosen to equate the marginal costs and benefits of rent seeking. This situation may occur when the emissions target has been fixed via incumbent legislation. Under Title IV of the Clean Air Act Amendments 1990, a ‘Ratchet’ provision allowed the aggregate level of emissions to remain at the targeted level (Joskow and Schmalensee 1998). If any special interest group behavior increased the aggregate target, allowance allocations would be reduced pro rata to all emitters to allow them to reach the target.

### 4.2 Aggregate rent-seeking effort

To find aggregate rent seeking, (11) is summed over all $n$ firms. This is simplified due to the market clearing condition (4) where, in equilibrium, the aggregate supply of permits will equal the aggregate emissions. Thus the first-order conditions become:

$$p^* \sum_{i=1}^{n} \frac{\partial a_i}{\partial s_i} - n = 0.$$  \hspace{1cm} (13)

From (13), the interior equilibrium solution exists when the aggregate marginal cost is equal to the marginal aggregate benefit of rent seeking. Solving (13) yields the aggregate equilibrium rent-seeking effort for regulated firms:

$$S^* = p^* \frac{(n-1)}{n} \frac{\bar{A}}{1 - p^* \mu \bar{A}}$$  \hspace{1cm} (14)

for $p^* \mu \bar{A} < 1$ ($\bar{\mu} = \frac{1}{p^* \bar{A}}$) where the marginal increase in value of the ex post reallocated rent $p^* \mu \bar{A}$ is lower than the marginal cost of rent seeking, otherwise agents would choose the maximum possible level of rent seeking. The major distinction between standard rent-seeking approaches and our ex post reallocation rent-seeking strategy is that the equilibrium permit price now determines the market value for the ex post reallocated rent.
Differentiating the aggregate rent-seeking strategy (14) with respect to the regulator’s optimal allocation choice, reveals, after some manipulation:

$$\frac{\partial S^*}{\partial \tilde{A}} = \frac{n-1}{n} \frac{1 + \varepsilon_p}{p^*(1 - p^* \mu \tilde{A})^2}$$

(15)

where $\varepsilon_p = \frac{\partial p^*}{\partial \tilde{A}}$ is the elasticity of the equilibrium price level based on a change in the regulator’s aggregate allocation choice.

In the standard rent-seeking literature, $\frac{\partial S^*}{\partial \tilde{A}}$ is unambiguously positive as the increase in rent increases wealth (Shaffer 2006). However, from (15), we see that the size of $\varepsilon_p$ will determine whether $\frac{\partial S^*}{\partial \tilde{A}}$ is positive or negative. Importantly, we find that increasing the total supply of permits has an ambiguous effect on rent seeking. When the equilibrium price is sensitive to changes in the regulator’s initial allocation choice, then $|\varepsilon_p| > 1$. In this case, from (15), one would expect that rent seeking is decreasing in the initial aggregate allocation choice of the regulator. However, it is perfectly feasible that $|\varepsilon_p| < 1$ may occur when $\frac{\partial p^*}{\partial \tilde{A}}$ in (7) is sufficiently small. This would suggest rent seeking is increasing in the initial allocation when many firms are emitting on a flat section of their marginal abatement curves. Although both scenarios are possible, in many existing schemes the permit price is extremely sensitive to changes in allocation. Therefore, we may expect $|\varepsilon_p| > 1$.

5  Stage one: regulator’s optimal choice of aggregate emissions

In stage one, the regulator selects a level of aggregate emissions $\tilde{A}^\ast$. As a consequence the resulting aggregate emissions level will be determined by expression (1) so that $A^\ast = \tilde{A}^\ast (1 + \mu \sum_{i=1}^{n} s^\ast_i)$ where $\sum_{i=1}^{n} s^\ast_i$ are the aggregate rent-seeking efforts from stage two. Let us initially assume that the regulator, such as the US EPA, is solely concerned about maximising social welfare in that region. We return to the case where the regulator benefits directly from rent seeking at the end of this section.

5.1 The regulator’s optimal choice of aggregate emissions

The regulator’s aim is to maximise the net welfare $W$ which consists of firms’ net profits from the tradable permit market $\sum_{i=1}^{n} \Pi_i(A)$ minus the damage from the
emissions and the cost of the (socially unproductive) rent-seeking effort. More formally, the regulator’s objective function is:

\[
\max_A W = \sum_{i=1}^{n} \Pi_i(A) - D(A) - S^* \tag{16}
\]

where \(D(A)\) is the damage caused by emissions where \(D'(A), D''(A) \geq 0\), and \(S^*\) is the aggregate rent seeking cost from all firms participating in the tradable permit market.

Using backward induction, the regulator knows the equilibrium rent-seeking effort by observing (14) that occurs in stage two, according to a given level of \(\mu\) and \(A\). In order to show the regulator’s optimal choice of allocation, it is important to compare this result to the socially optimal case when there exists no rent-seeking effort. That is, what aggregate allocation level would the regulator choose under the presence of zero rent seeking? As shown in Appendix A, when zero rent seeking occurs, the regulator selects an aggregate emissions cap:

\[
p^* = \frac{\partial D(A)}{\partial A} \tag{17}
\]

that is optimally solved for \(\tilde{A}^B\) where superscript \(B\) denotes the benchmark level (here we have \(\frac{\partial A}{\partial A} = 1\) hence \(\frac{\partial D(A)}{\partial A} = \frac{\partial D(A)}{\partial A}\)). This states that the regulator should set a level of aggregate emissions so that the marginal benefit (the equilibrium permit price) is equal to the marginal damage of emissions, a familiar first-best outcome.

In order to solve for the regulator’s optimal aggregate allocation in (16), we first sum over all firms’ profit functions, which gives \(-\sum_{i=1}^{n} c_i(e_i)\) and differentiating with respect to \(\tilde{A}\) yields:

\[
-\sum_{i=1}^{n} \frac{\partial c_i}{\partial e} \frac{\partial e^*}{\partial A} \frac{\partial A}{\partial A}. \tag{18}
\]

Using (3), this simplifies to:

\[
p^* \frac{\partial A}{\partial A} \sum_{i=1}^{n} \frac{\partial e_i}{\partial A} \tag{19}
\]

and noting that \(\sum_{i=1}^{n} \frac{\partial e_i}{\partial A} = 1\), this reduces to:

\[
\frac{\partial}{\partial A} \left( \sum_{i=1}^{n} \Pi_i(A) \right) = p^* \frac{\partial A}{\partial A}. \tag{20}
\]
Differentiating (16) with respect to $\tilde{A}$ and substituting in (20) yields the regulator’s first-order condition:

$$p^* \frac{\partial A}{\partial A} - \frac{\partial S^*}{\partial A} \frac{\partial A}{\partial A} - D'(A) \frac{\partial A}{\partial A} = 0. \tag{21}$$

Therefore $\tilde{A}^*$ is now chosen so that (21) holds. Note there are three influences on the regulator’s optimal choice of allocation. First there is an upward influence in the form of marginal increase in firms’ profit due to the increased aggregate allocation $p^* \frac{\partial A}{\partial A}$. Note that $\frac{\partial S^*}{\partial A} \frac{\partial A}{\partial A}$ shown in (15) has an ambiguous influence in terms of the marginal change in optimal aggregate rent-seeking effort and finally a downward influence due to the additional damage produced. Furthermore, we obtain an expression that allows analysis of the aggregate emissions level:

**Lemma 1** In the presence of rent-seeking effort, the regulator’s optimal choice of aggregate allocation $\tilde{A}^*$ is chosen so that:

$$p^* \Delta = \frac{\partial D(A)}{\partial A}. \tag{22}$$

where $\Delta = \left[ 1 - \frac{(n-1)}{n} \left( 1 + \mu S^* \left( 1 + \frac{[1+\varepsilon_p]}{[1-p^* \mu \tilde{A}]} \right) \right) \right] (1-p^* \mu \tilde{A})^2$.

**Proof.** See Appendix B.

Direct comparison of (17) and (22) show that aggregate emissions are only socially optimal when $\Delta = 1$, that is $|\varepsilon_p| = 1$, where the proportional change in the equilibrium permit price is equal to the proportional change in the regulator’s choice of initial allocation.

To observe the influence of $\varepsilon_p$ on the regulator’s choice of aggregate emissions, let us first analyse the case were the regulator is unresponsive to rent seeking $\mu = 0$. Given $\mu = 0$, the regulator’s choice of allocation becomes:

$$p^* \left[ 1 - \frac{(n-1)}{n} \left[ 1 + \varepsilon_p^0 \right] \right] = \frac{\partial D(A)}{\partial A}. \tag{23}$$

For $|\varepsilon_p^0| > 1$, aggregate emissions are larger than socially optimal. Emissions increase as benefits from the reduction in rent seeking are relatively larger than the costs of an increase in damages from additional emissions. Similarly, when

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9The second-order conditions are satisfied for the optimal value given a sufficiently small (absolute) $\frac{\partial^2 S^*}{\partial A^2}$. 

---
\( \frac{1}{n+1} < |\varepsilon_p^0| < 1 \), the change in equilibrium price is relatively unresponsive so that any increase in emissions will increase damages more than the benefit from reduced rent-seeking effort.

For the case of a responsive regulator, the analysis is similar. Let us consider the case when \( \Delta > 1 \). Notice from Lemma 1 that \( \Delta > 1 \), does not occur for \(|\varepsilon_p^1| < 1\). That is, if an inelastic \( \varepsilon_p \) occurs, the change (reduction) in rent seeking is small, therefore the reduction in rent seeking does not outweigh the damages of additional emissions. Instead let us concentrate on \(|\varepsilon_p^1| > 1\). To ensure that \( \Delta \) remains positive let us focus on elasticity levels of the range \( 1 < |\varepsilon_p^1| < |\tilde{\varepsilon}_p| \) where \(|\tilde{\varepsilon}_p|\) is defined by:\(^{10}\)

\[
[1 + \tilde{\varepsilon}_p] = -\frac{(1 - p^* \mu \tilde{A})}{(n-1)p^* \mu \tilde{A}} [n - p^* \mu \tilde{A}] < 0.
\]

Aggregate emissions are above the socially optimal (\( A^* > \tilde{A}^\beta \)) level as a relative responsiveness equilibrium permit price \( 1 < |\varepsilon_p^1| < |\tilde{\varepsilon}_p| \), results in relatively larger rent seeking reductions. Counter-intuitively, it is not the actual rent seeking that increases the aggregate emissions but the reduction in social costs associated with a reduction in rent seeking that encourages the regulator to issue additional permits. Similar analysis exists for \( \Delta \in (0, 1) \).

### 5.2 Regulatory responsiveness and welfare

From above, we were able to show that the regulator’s optimal choice of allocation depends on how responsive the equilibrium permit price is to changing allocations. Another type of responsiveness is that of the regulator towards the setting of the initial allocation. What effect will a different responsiveness have on social welfare? That is, how does a change in the cultural and governance relationships that link the regulator and firms alter welfare? These relationships are often fixed within a country, or at least within a piece of legislation. As such, one can interpret alternative responsiveness measures as different governments’ attitudes and incumbent constitutional frameworks that allow (or do not allow) for change in the aggregate emissions target. We are not concerned about what a regulator ought to do. Instead, we focus on the positive question of how welfare is altered given exogenous changes in responsiveness.

\(^{10}\)For \(|\varepsilon_p^1| > |\varepsilon_p|\), \( \Delta \) still may be positive but result in a lower aggregate allocation than socially optimal. In this case, the price is extremely sensitive, so much so that, the regulator’s optimal choice of aggregate emissions is chosen below the socially optimal level.
Proposition 1 The welfare change given by an increase in the regulator’s responsiveness is:
\[
\frac{dW}{d\mu} = \left[ p^* - D'(A^*) \right] \frac{\partial A^*}{\partial \mu} - \frac{\partial S^*}{\partial \mu}.
\]

Proof. See Appendix B.

From Proposition 1, two main factors determine whether increasing responsiveness changes welfare. The first term \( [p^* - D'(A^*)] \frac{\partial A^*}{\partial \mu} \) shows the distance away from the socially optimally level of allocation derived in Lemma 1. Under a socially optimal emissions cap, expression (17) shows that \( p^* - D'(A^*) = 0 \). However, Lemma 1 shows that, in most cases, marginal damage costs will not be set equal to the permit price. In fact, when \( p^* - D'(A^*) < 0 \), emissions are larger than socially optimal and tend to reduce welfare given a change in responsiveness. An additional increase in emissions away from the socially optimal level will reduce welfare, whereas when \( p^* - D'(A^*) > 0 \), an increase in responsiveness moves emissions closer to the socially optimal level and improves welfare. The second term shows that welfare is decreasing in marginal rent seeking. As rent seeking is socially wasteful, an increase in rent seeking will result in a reduction in social welfare.

It is clear from Proposition 1, that if a regulator’s responsiveness changes, then welfare may either increase or decrease. The change in welfare depends on which effect is offset. For example, welfare may increase when responsiveness results in a large reduction in rent seeking that offsets any increase in welfare loss from additional emissions. As Lemma 1 details whether the first term is either positive or negative, it is sufficient to investigate the determinants of the rent-seeking term.

Implicit differentiation of (14) with respect to \( \mu \) reveals:
\[
\frac{\partial S^*}{\partial \mu} = \frac{S^*}{p^* \left( 1 - p^* \mu A^* \right)} \left[ \frac{\partial p^*}{\partial \mu} + \tilde{A}(p^*)^2 \right].
\]

In terms of changing rent-seeking effort, (24) shows that as regulatory responsiveness increases, the change to social welfare is affected by two opposing factors. Using the chain rule we know \( \frac{\partial p^*}{\partial \mu} = \frac{\partial p^*}{\partial A^*} \frac{\partial A^*}{\partial \mu} \) and differentiation of (1) with respect to \( \mu \) yields:
\[
\frac{\partial A^*}{\partial \mu} = \tilde{A}^* \left( S^* + \mu \frac{\partial S^*}{\partial \mu} \right).
\]
Substituting (25) into (24) and collecting $\frac{\partial S^*}{\partial \mu}$ terms yields:

$$\frac{\partial S^*}{\partial \mu} = \left[ \frac{\partial p^*}{\partial A^*} S^* + (p^*)^2 \right] \frac{p^*}{S^* A^*} - \mu \frac{\partial p^*}{\partial A^*}$$

(26)

where $\frac{\partial p^*}{\partial A^*}$ is given by (7). The denominator of (26) is always positive therefore the sign of $\frac{\partial S^*}{\partial \mu}$ is determined by $\left[ \frac{\partial p^*}{\partial A^*} S^* + (p^*)^2 \right]$. The first effect $\frac{\partial p^*}{\partial A^*} S^*$ we denote as the price effect. When responsiveness increases, the equilibrium price per unit of emissions decreases, which reduces socially wasteful rent seeking and improves welfare (net of damages associated with increased emissions). This effect is due to the changing equilibrium permit price altering the value of the ex post reallocated rent. Second, the wealth effect $(p^*)^2 > 0$, has a dampening effect on social welfare. Increased responsiveness results in a larger supply of permits distributed to firms which increases rent-seeking effort and reduces social welfare. When the wealth effect dominates the price effect, marginal rent seeking effort is positive $\left( \frac{\partial S^*}{\partial \mu} > 0 \right)$ and vice versa.

### 5.3 The regulator and political contributions

Up to this point, we have considered a regulator that obtains no benefit from rent seeking activities. However, it is clear that regulators (politicians) may obtain a benefit in the form of political contributions which may alter the incentives to select the level of aggregate emissions (Hillman 1982; Grossman and Helpman 1994). In this subsection, we extend our model by allowing the regulator to optimise a function with additional (weighted) political contribution benefits.

To show this, let us assume that the regulator obtains political contributions from rent seekers given by $\beta S^*$, where $\beta > 0$ is an exogenous parameter. From the regulator’s payoff function in (16) we know that the net gain from rent seeking is given by $(\beta - 1)S^*$. When $0 < \beta < 1$, the net benefit from the political contributions is negative, and similar (but augmented) results are found to be the case when the regulator attains no political contributions. However, when $\beta > 1$, the net benefits of attaining political contributions are positive and additional results exist. In particular, the regulator’s choice of emissions is now determined by

$$p^* \left[ 1 + (\beta - 1) \frac{(n - 1)}{n} \frac{[1 + \epsilon_p]}{\left( 1 + \mu S^* \left( 1 + \frac{[1 + \epsilon_p]}{(1 - p^* \mu A^*)} \right) \right) (1 - p^* \mu A^* \mu)^2} \right] = D'(A).$$

(27)
In contrast to the previous case, the regulator has an incentive to increase emissions when the elasticity $\varepsilon_p$ (given a change in the allocation) is less than unity $|\varepsilon_p| < 1$. Intuitively, as the permit price is relatively unresponsive to changes in the total allocation, an increase in emissions results in only a small decrease in price and consequently rent-seeking effort continues to be relatively large, that in turn produces a large amount of political contributions for the regulator. A similar analysis can also be considered for $|\varepsilon_p| > 1$.

6 Modifications and extensions

In this section we relax a number of assumptions that were previously made in the analysis. In particular, we investigate the incentives for regulated firms to rent seek for a more stringent aggregate emissions target and discuss the implications of market power within the tradable permit market.

6.1 The choice of a stringent aggregate emissions target

If a firm is able to capture a large share of the initial allocation, it may experience tradeoffs between an increased number of permits but a lower equilibrium price at which the permits will be sold. Therefore, an incentive may exist to rent seek for a larger share of the initial allocation while simultaneously advocating a more stringent aggregate emissions target. Under what circumstances may firms rent seek for a smaller permitted aggregate level of emissions? To illustrate this, let the aggregate emissions cap be determined by:

$$A = \tilde{A} \cdot \left(1 + \mu \sum_{i=1}^{n} \alpha_i s_i\right)$$

(28)

where $\alpha_i$ is a constant. As before, rent-seeking effort can alter the regulator’s initial emissions cap choice. However, it is possible that $\alpha_i$ is negative, which represents the case where firm $i$ actively rent seeks to reduce the level of aggregate emissions. Note the previous analysis occurs for the special case $\alpha_i = 1 \ \forall \ i$. Assuming an interior Nash equilibrium exists, the first order condition for firm $i$ is given by substituting (28) into (10) and differentiating with respect to $s_i$. Setting to zero and rearranging for $\alpha_i$ yields:

$$\frac{\partial^2 A}{\partial s_i^2} + \frac{\partial p}{\partial s_i} \frac{\partial^2 s_i}{\partial s_i^2} < 0,$$

which we assume throughout. Note that $\alpha_i$ is not a choice variable under optimisation. Instead, the parameter allows investigation of cases where a firm may rent seek for a stringent aggregate emissions target.
\[ \alpha_i = \frac{1 - f'(s_i, s_{-i})\tilde{A}p^*(1 + \sum_{i \neq i}^{n} \alpha_{-i}s_{-i})}{\tilde{A}\mu(\frac{\partial p^*}{\partial A}(a_i - e^*_i) + p^*f(s_i, s_{-i}) + f'(s_i, s_{-i})p^*s_i)}. \]  \hspace{1cm} (29)

We concentrate on the case where firm \( i \) is a net seller and has an incentive to rent seek for a more stringent aggregate emissions target. Cases where firm \( i \) is a net demander of permits are similar in analysis and left out for clear exposition. The numerator of (29) shows the difference between the marginal cost and benefit of rent seeking. Here the marginal benefit is associated with the aggregate emissions cap determined by rivals’ rent-seeking efforts. The sign of the denominator is influenced by \( \frac{\partial p^*}{\partial A}(a_i - e^*_i) \) which shows the associated price change and the permits sold to the market by firm \( i \).

Let us initially assume that the marginal cost of rent seeking is larger than the marginal benefit so that the numerator is positive. Given \( \alpha < 0 \), then both the change in equilibrium permit price and the amount of permits sold in equilibrium must be large. This is intuitive. The less intuitive case occurs when the numerator of (29) is negative. An incentive to rent seek down a target continues to exist when either the firm is a small net seller or the price change with respect to allocation is small. Here, the value of the (marginal) allocation obtained by the firm must be large (e.g., large \( \tilde{A}\mu p^*f(s_i, s_{-i}) \)).

### 6.2 Market power in the pollution permit market

We now relax the assumption that all firms are price takers in the permit market. To do this, we follow Hahn (1984) and assume a dominant firm (denoted by firm 1) and a price-taking competitive fringe \( i = 2, \ldots, n \). To focus on the effects of market power, we assume \( \mu = 0 \) where aggregate emissions are fixed. As a result, there are only two stages. In stage one, each firm invests in rent-seeking effort in order to obtain pollution permits. In stage two, the dominant firm selects the equilibrium permit price and the competitive fringe select emissions. We solve using backward induction.

In stage two, the competitive fringe selects emissions so that \( p = -c'_i(e_i) \) for \( i = 2, \ldots, n \). As in Hahn (1984), the dominant firm selects an equilibrium permit price to maximise its payoff knowing the emissions choices of the competitive fringe. Formally, this is:

\[ \max_{p} p(a_1 - e_1) - c_1(e_1) \]  \hspace{1cm} (30)

where \( e_1 = A - \sum_{i=2}^{n} e_i \). The first-order condition is given by:
\[ (a_1 - A + \sum_{i=2}^{n} e_i) + \sum_{i=2}^{n} \frac{\partial e_i}{\partial p} (p + c'_i(\cdot)) = 0 \]  

(31)

which is solved for optimal \( p_D^* \). From (31), the tradable permit market is unlikely to be cost-efficient as marginal abatement costs are not equalised (unless the dominant firm obtains an initial allocation identical to its emissions in equilibrium). This is the standard first-order condition for the dominant firm in Hahn (1984).

In stage one, each firm invests in rent-seeking effort. For the competitive fringe, payoffs are:

\[ \max_{s_i} p_D^* (a_i - e_i^*) - c_i(e_i^*) - s_i \]  

(32)

where \( a_i = \frac{s_i}{\sum_{i=1}^{n} s_i} A \), and \( p_D^* \) and \( e_i^* \) are equilibrium price and optimal emissions chosen in stage two, respectively. Therefore, the competitive fringe rent-seeking effort is determined by:

\[ p_D^* \sum_{j \neq i}^{n} \frac{s_j}{(\sum_{i=1}^{n} s_i)^2} A = 1 \quad \forall i = 2, \ldots, n. \]  

(33)

Let us now turn to the dominant firm’s choice of rent-seeking effort. The dominant firm’s payoff is:

\[ \max_{s_1} p_D^* (a_1 - e_1^*) - c(e_1^*) - s_1 \]  

(34)

where \( a_1 = \frac{s_1}{\sum_{i=1}^{n} s_i} A \). Differentiating with respect to \( s_1 \) yields:

\[ \frac{\partial p_D^*}{\partial s_1} (a_1 - e_1) + p_D^* \left( \frac{\partial a_1}{\partial s_1} - \frac{\partial e_1^*}{\partial s_1} \right) + c'(\cdot) \left( \frac{\partial}{\partial s_1} \left( \sum_{i=2}^{n} e_i \right) \right) = 1. \]  

(35)

The dominant firm’s rent seeking will alter its share of permits and directly affect the permit price as \( \frac{\partial p_D^*}{\partial a_1} > 0 \) (Hahn 1984). Moreover, the change in price will also alter the emissions of the competitive fringe. Using (31), the first-order condition can be simplified to

\[ p_D^* \sum_{i=2}^{n} \frac{s_i}{(\sum_{i=1}^{n} s_i)^2} A = 1. \]  

(36)

Summing over the first-order conditions (31) and (36), yields a similar aggregate rent-seeking strategy to the basic model given in (14) (recalling that we have set \( \mu = 0 \)):

\[ S_D^* = p_D^* \frac{(n-1)}{n} A. \]  

(37)

However, two important features arise. First, under a market-power regime, the permit market is likely to be inefficient. Therefore, overall costs to society will
be larger. Second, the permit price, and hence the value of the contestable rent, is now partly determined by the actions of the dominant firm. What is clear is that dominance in the tradable permit market confers dominance within the rent-seeking contest. This occurs through the dominant firm having the ability to set the equilibrium permit price. Interestingly, this occurs even when all firms have the same rent-seeking effectiveness. That is, there is no explicit asymmetry in the effectiveness of rent-seeking within the contest.12

7 Discussion

In the majority of current tradable pollution permit markets, rent-seeking behaviour is a common occurrence. Both under the EU-ETS and U.S. legislation on climate change, such as the ‘Waxman-Markey’ Bill (HR. 2454), significant lobbying has been invested in order to capture rents. Although rent-seeking behaviour is socially wasteful, to what extent does this behaviour affect the consequences of implementing these schemes? This paper has attempted to model incentives for firm and regulator behaviour in a tradable permit market when rent seeking can influence the total supply of permits and their initial allocation to individual firms, when the value of each permit (and thus of rents) depends on \textit{ex post} permit trading.

An important contribution of this paper is to show that the incentive to rent seek for tradable pollution permit differs from traditional rents. This is a result of the market creating an equilibrium permit price which influences the rent-seeking incentives of firms and the selection of aggregate emissions by the regulator. Therefore, to draw meaningful conclusions about the social welfare consequences of this type of rent seeking, one must consider the effects on aggregate emissions and the level of rent seeking. Two opposing effects determine impacts on social welfare. Increased responsiveness (or simply increases in aggregate emissions) will create additional damages. However, the additional supply will decrease the market clearing price and thus reduce incentives for rent seeking. Counter-intuitively, it is thus possible that a regulator’s increased responsiveness may actually improve welfare. This occurs when the reduction in aggregate rent seeking creates larger benefits than the additional damages from increased emissions.

From (7) we can see that the change in the equilibrium permit price is based on the slopes of firms’ marginal abatement cost functions. That is, relatively steeper marginal abatement costs functions result in a more responsive equilibrium permit

\[ f_1(s_1,s_i) = \frac{\beta s_1}{\beta s_1 + \sum_{i=2}^s s_i} \] for a constant $\beta > 1$, where the dominant firm is the favourite whereas the competitive fringe are underdogs (Dixit 1987).

12 It may also be feasible that due to the dominant status of the firm that a greater rent seeking effectiveness may exist. In such a case, the dominant firm’s contest-success function can be altered to $f_1(s_1,s_i) = \frac{\beta s_1}{\beta s_1 + \sum_{i=2}^s s_i}$ for a constant $\beta > 1$, where the dominant firm is the favourite whereas the competitive fringe are underdogs (Dixit 1987).
price. Therefore, it follows that in markets where firms’ marginal abatement cost functions are steep and rent seeking is a significant and costly problem, welfare may increase when $\mu$ is large.\footnote{Steep marginal abatement cost may actually cause significant amounts of rent seeking as these firms tend to find investment in abatement relatively expensive and are more likely to consider investing in rent seeking as an alternative. Steeper marginal abatement costs may well characterise greenhouse gas control policy in the near future as increasingly ambitious targets are set for emission reductions, and simple, cheap abatement options get used up.}

## 8 Conclusion

A contentious and demanding aspect of a regulator’s role in a tradable permit market is the initial allocation of permits. In particular, the determination of the aggregate emissions cap and the distribution of permits among participants remain controversial issues. As with most valuable rents, a significant amount of rent-seeking effort tends to be employed in actual permit markets to influence both the size and the distribution of these rents. Yet in contrast to traditional contestable rents, tradable rents, such as tradable pollution permits, allow for \textit{ex post} reallocation. It is important to understand how this alters agents’ incentives to rent seek.

To do this, we introduce a contest where polluting firms in a tradable permit market have the option to invest in rent-seeking effort that has the potential to increase (i) their own permit allocation within the tradable permit market and (ii) the aggregate supply of permits from the regulator (i.e., political pressure to increase the aggregate level of emissions). The regulator selects a provisional aggregate emissions target but this can be influenced by firms’ rent-seeking effort. We analyse two cases, first where the regulator views rent seeking as ‘purely’ socially wasteful, and second where the regulator obtains political contributions from rent seekers. We show the incentives behind firms’ rent-seeking effort in a tradable permit market and compare this to a standard rent-seeking framework. We find individual rent seeking strategies depend on whether, in equilibrium, the firm is a net buyer or seller of the \textit{ex post} reallocated rent (initial allocation of permits). Regulator’s optimal allocation choices are shown to depend on the responsiveness of price to changes in allocations, whilst variations in the responsiveness of regulators to rent seeking (shown by the parameter $\mu$) are shown to have potentially positive or negative consequences for social welfare.

Due to the current political debate that surrounds the initial distribution of pollution permits to regulated firms, this framework has focused solely on rent seeking by firms. However, environmental groups also invest in rent seeking and often
try to influence draft legislation. It would be possible to extend this analysis to include rent seeking by environmental groups to alter the initial choice of aggregate emissions that regulated firms contest. Moreover, it would be interesting to consider a dynamic version of the model where permit markets go through a phase of initial allocations over time, as has happened, for example, with Phase 1 and Phase 2 of the EU ETS. As we have shown elsewhere (MacKenzie et al. 2008), allowing permit allocations in period \( t+1 \) to partly depend on holdings at the end of period \( t \) introduces complications to efficiency properties of tradable permit markets that would be relevant for the incentives to rent seek.

In conclusion, in a world where firms rent seek over both the distribution and the total supply of permits, the impacts of regulatory receptiveness to such lobbying behaviour are complex. We find instances where a zero degree of responsiveness to lobbying would not be welfare-maximising, due to a price effect of changes in aggregate permit supply. Moreover, we have not allowed for the fact that lobbying activity may generate useful information for the regulator, relating to the existence or emission levels of polluters (Montero et al. 2002).

**Appendix A**

**Proof of derivation for regulator’s benchmark case:**

**Proof** For zero rent seeking, firm \( i \) objective function is:

\[
\Pi_i = p^*(a_i^* - e_i^*) - c_i(e_i^*)
\]

for optimal choices \((e_i^*, a_i^*)\). The social welfare function is:

\[
\max_A W = \sum_{i=1}^n \Pi_i - D(A).
\]

Given that \(a_i^* = A/N = \bar{A}/N\) and \(\frac{\partial A}{\partial A} = 1\). The first-order condition is:

\[
\frac{\partial \Pi_i}{\partial A} = \frac{\partial p^*}{\partial A}(a_i^* - e_i^*) + p^*(1/N - \frac{\partial e^*}{\partial p^*} \frac{\partial p^*}{\partial A}) = \frac{\partial c_i}{\partial e} \frac{\partial e^*}{\partial p^*} \frac{\partial p^*}{\partial A}
\]

that can be simplified by using (3) so that:

\[
\frac{\partial \Pi_i}{\partial A} = \frac{\partial p^*}{\partial A}(a_i^* - e_i^*) + p^*/N.
\]

Summing over all firms yields:

\[
\sum_{i=1}^n \frac{\partial \Pi_i}{\partial A} = p^*.
\]
Substituting into the regulator’s welfare function yields:

\[ p^* = D'(A). \]

**Appendix B**

**Proof of Lemma 1:**

**Proof** Using (1) and (14), it is known that

\[ \frac{\partial A}{\partial A} = 1 + \mu S^* \left( 1 + \frac{1 + \varepsilon_p}{1 - p^* \mu A} \right) \]

and substituting (15) into (21) yields:

\[ (p^* - D'(A)) \left( 1 + \mu S^* \left( 1 + \frac{1 + \varepsilon_p}{(1 - p^* \mu A)} \right) \right) - \frac{n - 1}{n} p^* \frac{1 + \varepsilon_p}{(1 - p^* \mu A)^2} = 0 \]

rearranging yields:

\[
p^* \left[ 1 - \frac{(n - 1)}{n} \frac{1 + \varepsilon_p}{1 + \mu S^* \left( 1 + \frac{1 + \varepsilon_p}{(1 - p^* \mu A)} \right)} \right] = D'(A)
\]

**Proof of Proposition 1:**

**Proof** Totally differentiating (16) with respect to \( \mu \) yields:

\[
\frac{dW}{d\mu} = \frac{\partial W}{\partial \mu} + \frac{\partial W}{\partial A^*} \frac{dA^*}{d\mu}
\]

where \( \tilde{A}^* \) is the optimally chosen allocation level given by Lemma 1. Given the Envelope Theorem, this is simplified to

\[
\frac{dW}{d\mu} = \frac{\partial W}{\partial \mu} \bigg|_{\tilde{A}^* = \tilde{A}}
\]

where \( \tilde{A} \) is held fixed at the regulator’s optimal level \( \tilde{A}^* \). For the aggregate payoff for firms:

\[
\frac{\partial \Pi}{\partial \mu} = \frac{\partial p^*}{\partial A^*} \frac{\partial A^*}{\partial \mu} (a_i^* - e_i^*) + p^* \left( \frac{\partial A^*}{\partial \mu} \frac{\partial \Pi}{\partial A^*} - \frac{\partial p^*}{\partial \mu} \frac{\partial A^*}{\partial p^*} \right) - \frac{\partial c_i^*}{\partial \mu} \frac{\partial e_i^*}{\partial \mu} \frac{\partial p^*}{\partial \mu} \frac{\partial A^*}{\partial p^*}.
\]
which is simplified to:

\[
\frac{\partial \Pi_i}{\partial \mu} = p^* \frac{\partial A^*}{\partial \mu} (a_i^* - e_i^*) + p^* \frac{\partial a_i}{\partial \mu} \frac{\partial A^*}{\partial \mu}
\]

and summing over all firms:

\[
\sum_{i=1}^{n} \frac{\partial \Pi_i}{\partial \mu} = p^* \sum_{i=1}^{n} \frac{\partial A^*}{\partial \mu} = p^* \frac{\partial A^*}{\partial \mu}
\]

which is substituted to yield:

\[
\frac{dW}{d\mu} = p^* \frac{\partial A^*}{\partial \mu} - \frac{\partial S^*}{\partial \mu} - D'(A^*) \frac{\partial A^*}{\partial \mu}
\]

and rearranging gives:

\[
\frac{dW}{d\mu} = [p^* - D'(A^*)] \frac{\partial A^*}{\partial \mu} - \frac{\partial S^*}{\partial \mu}.
\]

**References**


