

Accommodating for Taste and Variance Heterogeneity in Discrete Choice[†]

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Draft paper — June 15, 2011

Abstract

Understanding and accommodating heterogeneity in variance (also referred to as heteroscedasticity) and taste has become a major area of research within discrete choice analysis. Both scale and taste heterogeneity can be specified as continuous or discrete, the latter can be associated with socio economic characteristics (i.e. observed heterogeneity) or it can be derived probabilistically (i.e. unobserved heterogeneity). Within the context of the Mixed Logit models, unobserved heterogeneity can be represented by a continuous function, a discrete mixture or using a combination of both. This paper uses data from two recreational site choice studies (one elicited through stated preference methods and one through revealed preference methods) to compare various model specifications for accommodating both scale and preference heterogeneity. Results show that model fit, welfare estimates and choice predictions are sensitive to the manner in which both types of heterogeneity are accommodated.

Keywords: Scale heterogeneity; preference heterogeneity; latent class models; recreational site choice; willingness to pay space; welfare estimates.

[†] **Acknowledgments:** We gratefully acknowledge the funding by the Irish Department of Agriculture, Forestry and Fishing under the Stimulus fund and Teagasc under the Walsh Fellowship Scheme for the collection of stated preference dataset. We are also thank Stephen Hynes for supplying the revealed preference data.

1 Introduction

One of the central advances in discrete choice analysis has focused on developing models that can accommodate unobserved heterogeneity in estimation. For many years, the assumption of homogeneity in preferences dominated the literature on non-market valuation of recreational goods. In his seminal paper [Train \(1998\)](#) emphasized that the explicit recognition of taste heterogeneity is important in the estimation of recreational site-choice to avoid biased welfare results. As a consequence of this, the mixed logit (ML) model has been developed ([McFadden and Train, 2000](#)) and widely applied in recreational site choice analysis using both revealed ([Provencher et al., 2002](#); [Scarpa and Thiene, 2005](#); [Hynes et al., 2008](#); [Bujosa et al., 2010](#)) and stated preference methods ([Hanley et al., 2000](#); [Boxall and Adamowicz, 2002](#); [Brefle et al., 2011](#)).

A further important source of heterogeneity is scale heterogeneity (also referred to as heteroscedasticity), which refers to heterogeneity in variance associated with the random component of utility (e.g., [Louviere et al., 1999](#); [Louviere and Eagle, 2006](#)). Scale heterogeneity can be modelled with the Heteroscedastic Multinomial Logit (HMNL) model ([Swait and Adamowicz, 2001](#); [Swait, 2006](#)), which has been applied to capture differences in variance across respondents, as the specification of the deterministic component of utility works better for some respondents than for others ([Bradley and Daly, 1994](#); [Bhat, 1998, 2000](#); [Scarpa et al., 2003](#)) or it can be linked to the complexity of the choice task ([Swait and Adamowicz, 2001](#); [DeShazo and Fermo, 2002](#)). Most of previous studies accounted for observed scale heterogeneity (based on capturing differences in variance between pre-defined groups); however it is possible, within the context of ML models, to model unobserved scale heterogeneity (e.g., [Brefle and Morey, 2000](#)).

In this article we analyse site choice decisions for both stated preference (SP) and revealed preference (RP) recreational datasets comparing models that accommodate, either or both simultaneously, heterogeneity in taste and heterogeneity in scale. The motivation for this is that most of the analysis to date that incorporates taste heterogeneity ignores heteroscedasticity and viceversa.¹

¹Some of the early attempts to incorporate scale heterogeneity included/tested some sort of structure in the data: [Hu et al. \(2006\)](#) (Reference point effects in demand), [Cameron and Englin \(1997\)](#) (Experience in Contingent Valuation of Environmental Goods), [Brownstone et al. \(2000\)](#) (Revealed and Stated preferences in transport) and [Hanley et al. \(2005\)](#) (Price effects). More recent attempts at incorporating both scale and taste heterogeneity are: the Generalized RUM

As noted by [Thiene and Scarpa \(2010\)](#) addressing only preference or scale heterogeneity negates the fact that true choice behaviour is likely to be in some middle ground with some variation attributable to scale and some to taste. In our analysis we derive a series of models that we call Heteroscedastic Mixed Logit (HML) models and compare specifications within a latent class (LC) modelling framework and a random parameter logit (RPL) modelling framework, both reparameterised in Willingness to Pay space (WTP-space), to accommodate both types of heterogeneity. Specifically this paper examines how to account for heterogeneity in both taste and variance by combining the approach proposed by [Swait and Adamowicz \(2001\)](#); [Swait \(2006\)](#) and [Train and Weeks \(2005\)](#) with recent developments in latent class analysis such as the scale-adjusted Latent class ([Magidson and Vermunt, 2007](#); [Hensher et al., 2011](#)) and the discrete mixtures of continuous distributions ([Bujosa et al., 2010](#); [Greene and Hensher, 2010b](#)).²

This paper contributes to the debate on scale and taste heterogeneity by directly comparing different HML models (described by continuous distributions, finite mixtures or a combination of both) that allow to accommodate for both sources of heterogeneity. This comparison is feasible by parametrising our models in WTP-space ([Train and Weeks, 2005](#)). We are not aware of previous studies employing neither a scale-adjusted latent Class for accommodating observed heteroscedasticity within each homogeneous class nor a finite mixture of continuous distribution to accommodate for continuous unobserved scale heterogeneity and finite taste heterogeneity in WTP-space.

In our analysis we use two case-studies from the Republic of Ireland. The first is based on a stated preference study aimed at eliciting the public's preferences for attributes and alternatives of farmland walking trails using the discrete choice experiment methodology. The second is a revealed preference study based eliciting kayakers' preferences for attributes and alternatives of kayaking site choice obtained via the travel cost methodology.

In the next section we develop our modelling framework and highlight our modelling contributions to the simultaneous analysis of scale and preference het-

([Walker and Ben-Akiva, 2002](#)), on which is built the Generalised Mixed logit model ([Greene and Hensher, 2010a](#)), the scale-adjusted Latent class ([Magidson and Vermunt, 2007](#); [Hensher et al., 2011](#)); the WTP-space reparameterization ([Scarpa et al., 2008](#)).

²We acknowledge the fact that the Generalised Mixed Logit (GML— [Greene and Hensher, 2010a](#)) accommodates both types of heterogeneity simultaneously, however a comparison with such a model is beyond the scope of this paper. For a comparison between RPL in WTP-space and GML see [Thiene and Scarpa \(2010\)](#).

erogeneity. We then describe the design of our case-studies and selected results will be presented for both the SP and RP datasets respectively followed by a discussion and concluding remarks.

2 Methodology

2.1 The Heteroscedastic Multinomial Logit Model

The Random Utility Model (RUM) (McFadden, 1974) is based on the assumption that respondents choose their preferred alternative on the basis that it maximises their utility. When specifying the utility function the analyst has the option to parameterize it in the “preference space” (the most common approach) or in WTP-space (which is becoming more common). For convenience in our analysis we opt to parameterize our models in WTP-space.³ As highlighted by Thiene and Scarpa (2009) an important and beneficial feature of WTP-space is that, if one uses a continuous specification for the random parameter, it is possible to directly test the spread of the WTP distributions. A further advantage of using WTP-space models is that the welfare results are reported directly in the models. In addition, WTP space models have been shown to produce more reasonable estimates of the distribution of welfare estimates than models estimated in preference space (e.g., Train and Weeks, 2005; Scarpa et al., 2008; Balcome et al., 2009).

In the context of this paper specifying all the models in WTP-space allows us to directly compare estimates from continuous and discrete mixture representations of parameters as well as to demonstrate the importance of including the scale parameter even if utility is parameterised in WTP-space. Indeed, under this specification the WTP estimates are scale free, however it is important to note that the model itself is not. Therefore, if the analyst believes that the scale factor has an influence on the choice probabilities then this should be accommodated since it impacts on model estimates.⁴

In WTP-space, the utility function is represented as:

$$U_{nit} = -(\lambda\alpha) p_{nit} + (\lambda\alpha w)' X_{nit} + \varepsilon_{nit}, \quad (1)$$

³For a complete derivation of WTP-space we refer to Train and Weeks (2005), while for an application to recreation data see Scarpa et al. (2008).

⁴We refer interested readers to Swait and Louviere (1993), who provide a means to test for this.

where n denotes the respondent, i the chosen alternative in choice occasion t , p represents the cost coefficient, X is a vector of attributes, λ is the scale parameter, α is the cost coefficient, w is a vector of WTPs to be estimated and ε is a random error term (which is unobserved by the researcher) assumed to be an *iid* type I extreme value (EV1) distributed. The variance ($\pi^2/6$) in this specification is scale free.

Following the heteroscedastic MNL (HMNL) model as specified in [Swait and Adamowicz \(2001\)](#)⁵ and reparametrising it in WTP-space and accounting for the panel nature of the data, the probability of the sequence of choices made by individual n can be represented by the following HMNL model:

$$\Pr(y_n|Z_n, p_n, x_n) = \prod_{t=1}^{T_n} \frac{\exp[\mu_{ni}(Z_{ni}|\lambda_{ni}) \cdot V_{ni}(p_{ni}, X_{ni}|\alpha, \beta)]}{\sum_{j=1}^J \exp[\mu_{nj}(Z_{nj}|\lambda_{ni}) \cdot V_{nj}(p_{nj}, X_{nj}|\alpha, \beta)]}, \quad (2)$$

where y_n gives the sequence of choices over the T_n choice occasions for respondent n (i.e., $y_n = \langle i_{n1}, i_{n2}, \dots, i_{nT_n} \rangle$), $\mu_{ni}(Z_{ni}|\lambda)$ is the scale parameter (λ), which could depend on observed characteristics of either respondents (e.g. [Scarpa et al., 2003](#)) or choice situations (e.g. [DeShazo and Fermo, 2002](#)), and $V_{ni}(p_{ni}, X_{ni}|\alpha, \beta)$ is the observed part of the utility function in equation 1 ($-(\alpha) p_{nit} + (\alpha w)' X_{nit}$).

In their paper [Swait and Adamowicz \(2001\)](#) assume that tastes are constant and that only the scale parameter (λ) varies across the sample. In their conclusions they propose an extension based on an exploration of simultaneous representation of taste and scale heterogeneity. In what follows, given the important finding in [McFadden and Train \(2000\)](#) that any RUM can be approximated to any degree of accuracy by a ML with appropriate specification of variables and distributions for random coefficients, we describe and derive a series of model specifications that can accommodate heterogeneity across scale and preferences adopting a mix of discrete and continuous functions for describing both types of unobserved of heterogeneity as well as observed scale heterogeneity.

2.2 The Heteroscedastic Mixed Logit Model

In ML models the probability of the sequence of choices over the T_n choice occasions for respondent n is the integral of the multinomial logit probabilities over a

⁵For more comments and details on the derivation refer to [Swait \(2006\)](#).

density of parameters. It is therefore possible to describe a Heteroscedastic Mixed Logit (HML) model, in which the choice probability is described by a mix of homogeneity or observed and/or unobserved heterogeneity in scale and preferences, the latter being described by continuous and/or discrete mixing distributions:

$$Prob_{nit} = \prod_{t=1}^{T_n} \int L_{nit}(\kappa, \phi, \vartheta) f(\vartheta|\theta) d\vartheta, \quad (3)$$

where L_{nit} is the Logit formula for an HMNL (Equation 2) in which the preferences can also be heterogeneous. As it is well known, the ML probability is the weighted average of logits evaluated at different values of the parameters over a distribution (Train, 2009). In our parameterisation for the HML the choice probability is the weighted average of the logit formulas, having some parameters (κ) stable across the ML (to represent the homogeneity), evaluated at different values for ϕ (representing the observed heterogeneity in either taste, scale or both) and ϑ (representing the unobserved heterogeneity in either taste, scale or both) with the weights given by the density $f(\vartheta)$ (Train, 2009). In Equation 3 θ represents the parameters that describe the density function $f(\vartheta)$.

Depending on the distribution of the random coefficients chosen by the analyst, θ could represent the mean and standard deviation of the coefficients in models in which the unobserved part of the heterogeneity is described by continuous distributions (known as RPL models⁶) or different accumulation points in models where the unobserved part of the heterogeneity is described by discrete distributions (known as LC models). The specification of $L_{nit}(\kappa, \phi, \vartheta)f(\vartheta)$ gives rise to different forms of HML as described below.

Note that if, the model accounts for preference heterogeneity (including parameters for taste— α and β —preferences in ϑ), but the scale factor λ is considered constant (therefore included in κ), Equation 3 describes a ML model. On the other hand, if the parameters describing tastes (α and β) are included in κ , while heterogeneity in scale is accommodated (including the scale factor in ϑ), Equation 3 describes a HMNL model. In this context different possible specification of taste and scale coefficients are possible.

In case of non-heteroscedastic models, the scale factor is assumed to be constant and equal to one. In heteroscedastic models the heterogeneity in scale can be

⁶The ML models are all properly random parameters models described by continuous or discrete distributions.

assumed to be observed and linked to socio-economic characteristics of respondents, as in:

$$\mu_{g|y_n}(Z_{n|c}|\lambda_c) = 1 + \sum_{g=1}^G \gamma_g \eta_g, \quad (4)$$

or it can be associated to characteristic of choice situation (complexity or number of requested choices) as in:

$$\mu_{c|y_n}(Z_{n|c}|\lambda_c) = 1 + \sum_{c=1}^C \gamma_c \eta_c, \quad (5)$$

where γ is a dummy variable representing each group.

Furthermore, the heterogeneity in scale factor can be assumed to be unobserved and probabilistically described by a discrete mixing distribution, as in:

$$\mu_{n|y_n}(\lambda_{ni}) = 1 + \sum_{s=1}^S \pi_s \eta_s \quad \text{where} \quad \sum_{s=1}^S \pi_s = 1 \quad \text{and} \quad \pi_s > 0 \forall s, \quad (6)$$

or it can be described by a continuous distribution;

$$\mu_{n|y_n}(\lambda_{ni}) = \int \lambda f(\lambda) d\lambda. \quad (7)$$

Note that in the discrete specification of scale heterogeneity, we are interested in how the scale parameter differs in each group (class) from a baseline group (or class, for which the scale factor is fixed to one for avoiding specification problems). Therefore, we specify $\lambda = 1 + \eta$, subject to the constraint $\eta > -1$, and we estimate for each group (class) how its scale parameter differs from the baseline.

As previously mentioned, preferences can be described by allowing for different types and degrees of heterogeneity. Taste heterogeneity can be either assumed to be observed ((e.g., [Bhat, 2000](#))) or unobserved and the latter heterogeneity can be modelled with either continuous mixing distributions ((e.g., [Scarpa et al., 2008](#))), discrete mixing distributions ((e.g., [Boxall and Adamowicz, 2002](#))) or a mixture of both (e.g., [Bujosa et al., 2010](#)).

It is possible to obtain different HML models combining one of the Equations as described from 4 to 7 with one of specifications noted above to describe the taste parameters (whilst being mindful of potential identification problems and

confounding between scale and taste heterogeneity),

For the purposes of this paper, we decided to estimate a model accounting for unobserved taste heterogeneity as well as observed and unobserved scale heterogeneity (where the observed heterogeneity is associated with a characteristic of the respondents and not with choice task complexity). Therefore we used a LC specification accommodating only preference heterogeneity as a reference model and we compare it with a RPL model in WTP-space, two scale-adjusted LC models (one with observed and one with unobserved scale heterogeneity) and a mixture model of discrete and continuous distributions to accommodate scale and preference heterogeneity.

For both case-studies we estimate a number of different models based on the specifications outlined above. Since a central focus of this paper is to explore alternative methods to account for scale and preference heterogeneity within a LC specification, the models include a RPL in WTP-space and four LC specifications.⁷ The first LC specification (model LC) represents the standard LC model applied in the literature which accommodates between-class preference heterogeneity for the attributes and alternatives. The second LC model (model ObsHLC) accommodates between-class preference heterogeneity which we extend to allow observed scale heterogeneity between distinct groups within each class. The third LC model (Model probHLC) is an application of scale-adjusted LC specification (as used in [Magidson and Vermunt, 2007](#); [Hensher et al., 2011](#)) which accommodates probabilistic heterogeneity in scale within each class. Our final model (Model MLC) represents a discrete mixture of continuous distribution (mixed latent class specification), which enables intra-class variation in preferences through continuous random specifications for the scale heterogeneity confounded with the random within-class cost coefficients (directly comparable with the RPL in WTP-space).

The models were estimated with Pythonbiogeme (see [Bierlaire, 2003, 2009](#)) using maximum simulated likelihood (MSL) estimation procedures and the CF-SQP algorithm ([Lawrence et al., 1997](#)) with 500 quasi random draws derived using Latin hypercube sampling ([Hess et al., 2006](#)). In order to deal with the well known problem of local maxima in discrete mixture of parameters (LC models)

⁷Membership probability can be based only on a constant ([Scarpa and Thiene, 2005](#)) or be informed by socioeconomics covariates ([Boxall and Adamowicz, 2002](#)). In our paper we follow the former approach in order to facilitate a more direct comparison between RPL and LS models, and we leave to further research the specification of heteroscedastic LC models informed by socioeconomics covariates.

between 50-100 random starting values were used.⁸

3 Case study 1: Establishing preferences for farmland walking trails

3.1 Background to the study

The first case-study sought to establish preferences for the creation of farmland walking trails amongst Irish residents using the discrete choice experiment (DCE) methodology.

3.2 Survey design and data description

In the final version of the questionnaire, five attributes were decided upon to describe the walking trails based on qualitative interviews with key stakeholders and a series of focus group discussions with members of the general public. The first attribute, 'Length', indicated the length of time needed to complete the walk. This attribute was presented at three levels with the shortest length between 1–2 hours, the medium length between 2–3 hours and the longest length between 3–4 hours. The second attribute, 'Car Park', was a dummy variable denoting the presence of car parking facilities at the walking trail. The third attribute, 'Fence', was a dummy variable used to indicate if the trail was fenced-off from livestock. The fourth attribute, 'Path and Signage', was a dummy variable to distinguish if the trail was paved and signposted. These three attributes represented the infrastructural features that were deemed important and realistic for farmland walking trails based on findings from the qualitative part of the study. The final attribute, 'Distance', denoted the distance (in kilometres) that the walk is located from the respondent's home. This attribute was later converted to a 'Travel Cost' per trip using estimates of the cost of travelling by car from the Irish Automobile Association. Findings from focus group discussions indicated that this represented a realistic and acceptable payment mechanism. A labelled choice experiment, with the labels representing four main types of farmland walks namely 'Hill', 'Field', 'Bog' and 'River' was used. The attributes and levels applied to all alternatives,

⁸This was coded in 'PERL' and used in combination with Pythonbiogeme run under Ubuntu 10.04 LTS - the Lucid Lynx.

except in the case of the Fence attribute, which, following safety concerns raised in the focus group discussions, only applied to the Field and River alternatives.

In this paper we use a dummy variable to denote whether a respondent resides in a rural or urban location⁹ to explore discrete differences in scale between these subgroups. The reasons for focusing on rural-urban differences is as follows; in the context of making recreational choices related to specific recreational terrain such as farmland, differences between rural and urban respondents may manifest themselves because of differences in access, familiarity or perceptions of farmland walking trails. Indeed findings from the qualitative part of this study appeared to confirm these observations. In addition, evidence within the literature suggests that rural and urban respondents may differ in their preferences for outdoor recreation (e.g., [Airlinghus et al., 2008](#); [Shores and West, 2010](#)) and we explore whether these differences also manifest themselves through differences in scale heterogeneity.

3.3 Stated preference data results

In this section we compare results from a number of models that accommodate scale and/or preference heterogeneity. Table 1 compares model fit using a number of criteria across across the specifications outlines in Section 2.

[Table 1 about here.]

For the LC model it is important to determine the appropriate number of latent classes to characterise the data. In this case we explore model fit based on the criteria outlined in Table 1 for a number of different classes across the model specifications. Although we do not report the results from an MNL model we do present the model statistics associated with the MNL specifications for comparative purposes. In general the results suggest that the manner in which we accommodate scale and/or preference heterogeneity has implications for model identification. The standard LC model (which accommodates between class taste heterogeneity only) is not identified for models with more than four classes, similarly, the LC

⁹For the purpose of this case-study we define rural respondents as those who reside outside the main cities in Ireland and urban respondents as those who live in one of these cities. This classification reflects the ease with which respondents located outside the main cities can access farmland compared to their urban counterparts. The sample breakdown is 281 and 189 rural and urban respondents respectively.

model which accounts for unobserved scale heterogeneity is only identified for up to four classes, while the MLC model is not estimatable after three classes. For this data, the only specification that is identifiable for a large number of latent classes is the LC model which accommodates observed scale heterogeneity (obsHLC).

[Table 2 about here.]

Table 2 presents the result from the different model specifications. It is important to note that for all the models (including the RP data) the WTP estimates have been divided by 100 to ease estimation. For the RPL model we specify the WTP for the non-cost attributes as having univariate Normal distributions, since it is possible that that welfare estimates may span the distribution with both negative and positive welfare estimates. We specify the random cost coefficient to have a log-normal distribution so that scale is confounded with the cost coefficient, thus the estimated WTP for the attributes is scale free. The RPL model recovers a high degree of WTP heterogeneity for the random parameters, with statistically significant and large standard deviations. This result implies a high degree of dispersion as well as a sizeable share of respondents having a negative and positive WTP for the attributes.

To allow for direct comparisons with the RPL model, we hold the walk alternative constants fixed across classes in the LC models. Exploring the results from the first LC model there is evidence of three similar sized classes (containing 29, 28, 33 percent of respondents respectively). For this model class one could be characterised possibly as a class who have a preference for participating in the walk alternatives but do not care for facilities (such as car-parking or fencing) and appear to dislike any type of structured walking trails (given that the coefficient representing path and signage is negative and significant) so this class could represent those groups of walkers who prefer more natural walking experience. The second class is characterised mainly by non-significant coefficients at the five percent for the attributes (except for the coefficient representing travel cost) which suggests that a sizeable share of respondents do not care for the attributes considered in this study. Therefore similar to class one, this class may be characterised by walkers who prefer natural walking trails and while generally they do have a positive preference for the trail attributes, they do not have significant WTP estimates for these features. The final class is characterised as a class who has significant WTP estimates for the trail attributes. This class obviously has a preference

for walking trails and positively demands trails with facilities. They also dislike longer walking trails but are not WTP a large amount to avoid longer walks.

The second LC model (obs. HLC) has similar estimates compared to the LC model in terms of significance and value of WTP estimates as well as the estimated size of the latent classes. In this model we include a scale parameter for rural respondents in each of the latent classes, which is interpreted relative to the scale parameter for urban respondents which we have fixed to one for identification purposes. Across the three classes, the scale parameters are negative and significant suggesting that rural respondents have higher variance compared to the urban respondents who are probabilistically assigned into each of the same classes.

In the third model (prob. HLC) we follow the approach of [Magidson and Vermunt \(2007\)](#); [Hensher et al. \(2011\)](#) and allow for probabilistic within class scale heterogeneity. Within class one 39 percent of the sample are estimated to have a scale parameter equal to one, in class two approximately one quarter of respondents have a scale parameter equal to one, while in class three approximately nineteen percent of respondents have a scale parameter equal to one. The remaining respondents in each of the classes are estimated to have significantly lower variance. In general, the size and significance of the WTP estimates as well as the probabilities of class membership are highly similar across the reported LC models thus far.

The final model (MLC) in [Table 2](#) allows for a discrete-continuous representation for the cost coefficients in each class to enable intra-class heterogeneity for the cost coefficients, which are confounded with the scale parameter. As shown, each of the cost coefficients is associated with significant unobserved heterogeneity. In general, the estimated probabilities and the class specific coefficients are similar to the estimates retrieved from the previous LC models.

4 Case study 2: Revealed preferences on Kayak

4.1 The survey and the data

The data used for estimation has been previously applied in [Hynes et al. \(2008\)](#) and for a fuller description of the survey design and data statistics the reader is referred to either of these articles. The data includes information on eleven principle whitewater kayaking sites in Ireland. The site attributes chosen for this study

include quality of parking at the site, degree of expected crowding at the site, quality of the kayaking experience as measured by the star rating system used in the Irish Whitewater Guidebook, water quality, scenic quality, and reliability of water information. Information was also collected on individual's travel to a whitewater site and this was used to form the basis of the cost per trip for welfare estimation. With regard to the site attributes a subjective rating scale (except for travel cost attribute) was used. In this case each respondent was asked to rate each of the eleven sites in terms of the six attributes using a 1 to 5 likert scale system for each attribute. Respondents were asked to indicate how many trips they had made to each of the eleven whitewater sites in the previous year and were asked to rank the attributes for a site so long as they had previously visited the site.

We specify discrete differences in scale to be informed by the level of kayaking skill of the handler. This is included in our model as a dummy variable where one indicates an individual with advanced skill levels and zero indicates less skilled kayakers (for a discussion on how this grouping was identified, see [Hynes et al., 2007](#)). [Hynes et al. \(2008\)](#) outline why kayakers with different skill levels may be expected to exhibit differences in preferences for the site attributes used in this data set. In this paper we empirically investigate whether these differences also translate into differences in scale heterogeneity.

4.2 Results

Table 3 presents the model fit criteria statistics for the revealed preference kayaking data set. In general there are identification issues under some of the model specifications for this dataset also. The standard LC and observed HLC model are identifiable up to six classes, while the unobserved HLC (scale adjusted LC) and the MLC model are not estimable respectively after three and four classes. Similar to the SP results since some of the models are not identifiable after a certain number of classes we report model results for three classes across the LC specification, which can be estimated for all the model specifications.

[Table 3 about here.]

Table 4 reports the results from the various model specifications for the kayaking data. The group variables represent a combination of kayaking rivers in Ireland and these are interpreted relative to the River Liffey which is left out of the

model for estimation purposes, which we assume are fixed for all model specifications (including across the classes in the LC specifications).

For the RPL model we specify normal distributions for the WTP for attributes and cost is specified with a log-normal distribution. We also specify fixed WTP estimates for the river grouping variables. This is analogous to the preference space specification used in [Hynes et al. \(2008\)](#). In general, the majority of WTP values (including the mean and standard deviations of WTP for the attributes and the mean values for the grouping variables) are significant in the RPL model.

[Table 4 about here.]

Table 4 also presents a number of three class LC model specifications, these specifications are similar to those presented in Table 2 for the stated preference data. For the kayaking data, there are quite a number of significant WTP estimates across the three classes in the LC models. Generally across the LC models, class one has the highest number of respondents probabilistically assigned to it. In the first LC model presented in Table 4, almost half of respondents are probabilistically assigned to the first class.

Exploring the results from the obs. HLC model which accommodates observed scale heterogeneity based on kayakers handling skills (model observ LC). We find that for classes one and three, the advanced kayakers are estimated to have higher variance compared to the less advanced kayakers, whereas for class two the advanced kayakers are estimated to have lower variance. The third LC model (prob. HLC) has approximately two-thirds of respondents probabilistically assigned to class one. In general approximately 47 percent of respondents assigned to class one are estimated to have a scale parameter equal to one, while the remaining respondents are estimated to have a lower variance. It is peculiar, but perfectly acceptable, that within class 3 (accounting for approximately 15 percent) a proportion of about 25 percent of respondent is assigned to have scale equal to zero (their choices are not explained at all by the model or the class).

The final model in Table 4 is the MLC model, a discrete mixture of continuous distribution. For this data and number of classes we find generally good statistical significance across two over three classes, whilst class two (accounting only for about 10 percent) has few parameters significantly different from zero. To class three is probabilistically assigned the larger estimated share of respondents (about 58). The retrieved estimates suggest all the three classes have significant

heterogeneity associated with the cost coefficient within each class. Indeed, the estimated standard deviations associated with the cost coefficient (confounded with the scale) are significant and relatively large in magnitude.

5 Discussion and conclusions

This paper examined alternative ways of modelling heterogeneity in taste and scale for outdoor recreational goods in Ireland. We contrasted a number of modelling approaches, incorporating alternative specifications of the random parameters logit model and the latent class model. These models were used to reveal preferences of the Irish population for attributes and alternatives of farmland walks in a discrete choice experiment and of a sample of kayakers for attributes and alternatives related to whitewater site choice using the travel cost method. We conduct all the analysis in both datasets in willingness to pay space. This paper makes a number of contributions to the literature by extensively exploring alternate means to incorporate scale and/or preference heterogeneity adopting heteroscedastic mixed logit models into the analysis of DC data. Our main contributions relate to describing the heteroscedastic mixed logit model and comparing different specifications of LC, HLC and RPL models.

In the context of this paper, we find WTP space models provide a useful tool to accommodate scale and preference heterogeneity. In WTP space models, when cost is specified as random scale differences are confounded with the cost coefficient and hence, resulting WTP estimates are scale free. For models with fixed cost coefficients we highlight the importance of including a separate scale parameter. Indeed although the WTP is scale free, the model itself is not and scale can impact on results. Hence, if analysts believe scale heterogeneity may be present within their data it is important to accommodate this within WTP space models either through random cost coefficient or through separate scale parameter(s) for fixed coefficient models.

As with many studies that explore differences in representation of choice data across different models, which model provides the best description of the data is likely to be data dependent. In both the stated and revealed preference datasets employed within this paper the models that account for scale heterogeneity provided the best fit for the data. We show that observed heteroscedasticity is often associated with the possibility of estimating a larger number of classes. In our case, we see a number of benefits associated with using a observed HLC model

where it is beneficial to explore preference and scale heterogeneity for a particular study. Indeed, as previously noted LC models are highly advantageous due to the capability of identifying groups of respondents with particular demands, which in the case of recreational goods, is highly desirable. We believe that the observed HLC model due to its ability to identify the influence of class variability provides further useful information. This is because it can help better inform analysts on differences that exists between (groups of) individuals estimated to have homogeneous preferences (within class), and between individuals estimated to have heterogeneous preferences (between class), in the variance associated with their choice behaviour.

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Table 1: Comparing the models - SP data

Model	LogLik.	K	$\bar{\rho}^2$	χ^2	BIC	AIC	3AIC	crAIC
MNL	-6882.982	9	0.101	1561.084	13843.703	13783.964	13792.964	13784.316
HMNL	-6863.299	10	0.103	1600.450	13812.974	13746.598	13756.598	13747.067
HML	-6486.8	10	0.152	2353.448	13059.976	12993.600	13003.600	12994.069
RPL - Cost fixed	-6247.31	13	0.183	2832.428	12606.909	12520.620	12533.620	12521.591
2 cl. LC	-6181.596	15	0.191	2963.856	12492.757	12393.192	12408.192	12394.643
3 cl. LC	-5836.88	21	0.236	3653.288	11855.150	11715.760	11736.760	11719.544
4 cl. LC	-5796.234	27	0.240	3734.580	11825.684	11646.468	11673.468	11654.283
5 cl. LC*	-5735.933	33	0.247	3855.182	11756.908	11537.866	11570.866	11551.878
6 cl. LC*	-5638.028	39	0.259	4050.992	11612.924	11354.056	11393.056	11376.903
2 cl. obsHLC	-6180.8	17	0.191	2965.448	12508.440	12395.600	12412.600	12397.669
3 cl. obsHLC	-5825.869	24	0.237	3675.310	11859.041	11699.738	11723.738	11705.296
4 cl. obsHLC	-5702.303	31	0.252	3922.442	11672.373	11466.606	11497.606	11478.283
5 cl. obsHLC*	-5722.98	38	0.248	3881.088	11774.190	11521.960	11559.960	11543.131
6 cl. obsHLC	-5604.296	45	0.263	4118.456	11597.286	11298.592	11343.592	11333.382
2 cl. probHLC	-6030.245	19	0.211	3266.558	12224.605	12098.490	12117.490	12101.330
3 cl. probHLC	-5799.313	27	0.240	3728.422	11831.842	11652.626	11679.626	11660.441
4 cl. probHLC*	-5690.141	35	0.253	3946.766	11682.599	11450.282	11485.282	11466.923
5 cl. probHLC*	-5718.347	43	0.248	3890.354	11808.112	11522.694	11565.694	11553.128
6 cl. probHLC*	-5705.413	51	0.249	3916.222	11851.346	11512.826	11563.826	11563.141
RPL	-6034.622	14	0.211	3257.804	12190.171	12097.244	12111.244	12098.439
2 cl. MLC	-6068.022	17	0.206	3191.004	12282.884	12170.044	12187.044	12172.113
3 cl. MLC	-5802.464	24	0.240	3722.120	11812.231	11652.928	11676.928	11658.486
4 cl. MLC*	-5762.685	31	0.244	3801.678	11793.137	11587.370	11618.370	11599.047
5 cl. MLC*	-5661.866	38	0.256	4003.316	11651.962	11399.732	11437.732	11420.903
6 cl. MLC*	-5295.463	45	0.303	4736.122	10979.620	10680.926	10725.926	10715.716

Note: Observed heteroscedasticity is retrieved based on respondents from Rural and Urban (Baseline) areas
An * denotes an unidentified model.

Table 2: SP Estimations Results

	RPL		LC		ObsHLC		probHLC		MLC	
	est.	t-rat.	est.	t-rat.	est.	t-rat.	est.	t-rat.	est.	t-rat.
Class 1										
$-\ln(\beta_a)^*$	1.73	6.35	2.28	22.37	2.59	18.34	1.85	13.18	2.41	21.63
$\sigma_{a-\lambda}$	1.88	15.17	-	-	-	-	-	-	0.53	6.63
Wlength	-0.0931	14.88	-0.261	7.43	-0.244	8.38	-0.216	7.72	-0.224	7.33
σ_{length}	0.13	12.97	-	-	-	-	-	-	-	-
Wparking	0.0361	9.82	0.004	0.38	0.011	1.11	0.011	1.3	0.013	0.0132
σ_{parking}	0.0587	10.8	-	-	-	-	-	-	-	-
Wfence	0.00761	1.67	0.0117	0.89	0.0128	1.06	0.0065	0.63	0.0068	0.65
σ_{fence}	0.0345	3.76	-	-	-	-	-	-	-	-
Wtype	0.0552	7.24	-0.051	4.25	-0.0462	4.18	-0.047	4.7	-0.0458	4.64
σ_{type}	0.126	14.55	-	-	-	-	-	-	-	-
η_1^{**}	-	-	0	-	0	-	0	-	-	-
Prob η_1	-	-	-	-	-	-	0.393	-	-	-
η_2^{**}	-	-	0	-	-0.247	2.58	1.56	5.46	-	-
Prob η_2	-	-	-	-	-	-	0.607	-	-	-
Prob cls-1	1	-	0.286	-	0.289	-	0.309	-	0.313	-
Class 2										
$-\ln(\beta_a)^*$	-	-	-1.4	2.54	-1.41	2.3	-0.591	5.58	-1.64	2.33
$\sigma_{a-\lambda}$	-	-	-	-	-	-	-	-	0.399	4.77
Wlength	-	-	-0.813	1.52	-0.938	1.41	-1.32	1.37	-0.951	1.26
Wparking	-	-	1.59	1.76	1.75	1.59	2.89	2.79	2.14	1.41
Wfence	-	-	2.69	1.77	3.07	1.61	4.61	4.45	3.34	1.38
Wtype	-	-	2.78	1.82	3.11	1.64	4.67	4.39	3.56	1.43
η_1^{**}	-	-	0	-	0	-	0	-	-	-
Prob η_1	-	-	-	-	-	-	0.255	-	-	-
η_2^{**}	-	-	0	-	-0.295	3.61	2.65	0.86	-	-
Prob η_2	-	-	-	-	-	-	0.745	-	-	-
Prob cls-2	-	-	0.376	-	0.378	-	0.356	-	0.358	-
Class 3										
$-\ln(\beta_a)^*$	-	-	2.66	45.95	2.79	32.28	1.96	0.31	2.67	40.15
$\sigma_{a-\lambda}$	-	-	-	-	-	-	-	-	0.346	4.13
Wlength	-	-	-0.0201	3.1	-0.19	2.88	-0.0153	0.006	-0.014	2.27
Wparking	-	-	0.034	6.28	0.037	6.64	0.0354	0.054	0.0338	5.91
Wfence	-	-	0.0161	2.23	0.0175	2.41	0.0195	0.0071	0.0186	2.61
Wtype	-	-	0.055	6.69	0.0635	7.3	0.0633	0.00882	0.0634	6.92
η_1^{**}	-	-	0	-	0	-	0	-	-	-
Prob η_1	-	-	-	-	-	-	0.235	-	-	-
η_2^{**}	-	-	0	-	-0.19	2.21	1.42	2.23	-	-
Prob η_2	-	-	-	-	-	-	0.765	-	-	-
Prob cls-3	-	-	0.338	-	0.333	-	0.335	-	0.329	-
Fixed across classes										
W _{hill}	-	-	0.11	12.77	0.108	12.71	0.0979	10.66	0.101	10.8
W _{log}	-	-	0.082	9.21	0.0796	9.01	0.076	8.28	0.077	8.13
W _{field}	-	-	0.106	12	0.104	11.76	0.0971	10.61	0.099	10.26
W _{river}	-	-	0.139	15.96	0.135	15.68	0.127	14.33	0.129	14.17
$\mathcal{L}(\hat{\beta})$	-6034.622	-	-5836.880	-	-5825.869	-	-5799.313	-	-5802.464	-

* in RPL/MLC models - $\ln(\beta_a) = -\ln(\lambda \cdot \alpha)$. t-test is against 1. All random parameters are normal distributed, but the cost coefficient, which is Lognormal confounded with the scale parameter

** $\lambda = 1 + \eta_g$. In the det. Scaled LC $g = 1$ is urban, $g = 2$ is rural.

Table 3: Comparing the models - RP data

Model	LogLik.	K	$\bar{\rho}^2$	χ^2	BIC	AIC	3AIC	crAIC
MNL	-6521.625	12	0.214	3578.970	13141.059	13067.250	13079.250	13068.515
HMNL	-6456.013	13	0.222	3710.194	13017.986	12938.026	12951.026	12939.608
HML	-6182.588	13	0.255	4257.044	12471.136	12391.176	12404.176	12392.758
RPL - Cost fixed	-6117.901	18	0.262	4386.418	12382.516	12271.802	12289.802	12275.772
2 cl. LC	-6205.529	20	0.251	4211.162	12574.073	12451.058	12471.058	12456.424
3 cl. LC	-6136.662	28	0.258	4348.896	12501.545	12329.324	12357.324	12343.503
4 cl. LC	-6037.152	36	0.269	4547.916	12367.731	12146.304	12182.304	12175.835
5 cl. LC	-6021.998	44	0.270	4578.224	12402.629	12131.996	12175.996	12185.259
6 cl. LC*	-5965.833	52	0.276	4690.554	12355.505	12035.666	12087.666	12122.902
2 cl. obsHLC	-6179.361	22	0.254	4263.498	12538.039	12402.722	12424.722	12409.778
3 cl. obsHLC	-6086.171	31	0.264	4449.878	12425.015	12234.342	12265.342	12253.413
4 cl. obsHLC	-5997.526	40	0.274	4627.168	12321.082	12075.052	12115.052	12115.286
5 cl. obsHLC	-5995.339	49	0.273	4631.542	12390.065	12088.678	12137.678	12161.855
6 cl. obsHLC*	-5932.656	58	0.279	4756.908	12338.056	11981.312	12039.312	12101.876
2 cl. probHLC	-6092.658	24	0.264	4436.904	12380.934	12233.316	12257.316	12242.386
3 cl. probHLC	-6047.101	34	0.268	4528.018	12371.328	12162.202	12196.202	12187.186
RPL	-6003.015	19	0.275	4616.190	12160.894	12044.030	12063.030	12048.663
2 cl. MLC	-6112.195	22	0.262	4397.830	12403.707	12268.390	12290.390	12275.446
3 cl. MLC	-6010.504	31	0.273	4601.212	12273.681	12083.008	12114.008	12102.079
4 cl. MLC	-5971.242	40	0.277	4679.736	12268.514	12022.484	12062.484	12062.718

Note: Observed heteroscedasticity is retrieved based on respondents from Rural and Urban (Baseline) areas
An * denotes an unidentified model.

Table 4: RP Estimations Results

	RPL		LC		ObsHLC		probHLC		MLC	
	est.	t-rat.								
Class 1										
$-\ln(\beta_\alpha)^*$	1.760	14.844	2.380	34.937	2.440	32.727	1.490	7.313	2.190	9.370
$\sigma_{\alpha-\lambda}$	0.665	12.110	-	-	-	-	-	-	0.440	4.070
W_{crow}	0.007	1.070	0.024	5.670	0.033	6.910	0.004	1.000	0.026	3.440
σ_{crow}	0.083	10.750	-	-	-	-	-	-	-	-
W_{info}	0.029	4.420	-0.004	1.060	0.002	0.440	-0.013	2.740	-0.014	1.730
σ_{info}	0.041	8.200	-	-	-	-	-	-	-	-
$W_{parking}$	0.035	5.800	0.006	1.780	0.020	4.680	-0.013	2.960	0.029	3.300
$\sigma_{parking}$	0.083	12.150	-	-	-	-	-	-	-	-
W_{scen}	-0.003	0.630	0.003	0.660	-0.006	1.410	0.002	0.390	-0.028	3.380
σ_{scen}	0.010	1.920	-	-	-	-	-	-	-	-
W_{star}	0.070	7.080	0.029	3.900	0.046	6.320	0.016	2.230	-0.016	1.230
σ_{star}	0.108	11.980	-	-	-	-	-	-	-	-
W_{water}	-0.005	0.670	-0.006	1.120	0.007	1.300	-0.006	0.990	0.005	0.480
σ_{water}	0.016	2.710	-	-	-	-	-	-	-	-
η_1^{**}	-	-	0	-	0	-	0	-	-	-
$Prob_{\eta_1}$	-	-	-	-	-	-	0.476	-	-	-
η_2^{**}	-	-	0	-	-0.278	6.160	1.430	9.200	-	-
$Prob_{\eta_2}$	-	-	-	-	-	-	0.524	-	-	-
$Prob_{Cls-1}$	1.000	-	0.459	-	0.391	-	0.645	-	0.312	-
Class 2										
$-\ln(\beta_\alpha)^*$	-	-	1.260	5.108	0.898	1.333	0.910	0.814	-0.408	1.531
$\sigma_{\alpha-\lambda}$	-	-	-	-	-	-	-	-	0.677	2.740
W_{crow}	-	-	-0.003	0.290	-0.046	3.250	0.003	0.220	0.485	0.880
W_{info}	-	-	0.058	4.890	0.041	2.920	0.059	4.950	0.959	0.970
$W_{parking}$	-	-	0.054	4.480	0.061	4.120	0.155	7.650	1.460	1.080
W_{scen}	-	-	-0.046	4.200	-0.097	6.780	-0.062	5.310	-0.088	0.600
W_{star}	-	-	0.164	9.740	0.178	8.820	0.186	10.800	1.640	1.050
W_{water}	-	-	0.020	1.760	0.059	4.220	0.066	4.930	-0.772	0.920
η_1^+	-	-	0	-	0	-	0	-	-	-
$Prob_{\eta_1}$	-	-	-	-	-	-	0.208	-	-	-
η_2^+	-	-	0	-	0.547	3.240	1.860	6.210	-	-
$Prob_{\eta_2}$	-	-	-	-	-	-	0.792	-	-	-
$Prob_{Cls-2}$	-	-	0.355	-	0.267	-	0.208	-	0.103	-
Class 3										
$-\ln(\beta_\alpha)^*$	-	-	0.919	0.69	2.030	15.014	1.540	4.122	1.710	7.717
$\sigma_{\alpha-\lambda}$	-	-	-	-	-	-	-	-	0.700	8.730
W_{crow}	-	-	-0.072	3.220	-0.005	0.660	0.253	6.970	-0.013	2.170
W_{info}	-	-	-0.036	1.390	0.031	3.140	0.246	6.240	0.012	1.560
$W_{parking}$	-	-	-0.063	2.750	-0.051	4.860	-0.001	0.080	-0.017	3.320
W_{scen}	-	-	-0.000	0.000	0.054	4.360	0.071	3.680	0.015	2.600
W_{star}	-	-	-0.099	3.090	0.016	1.440	0.125	4.070	0.066	5.140
W_{water}	-	-	-0.059	2.380	-0.091	6.880	-0.127	3.970	-0.002	0.210
η_1^+	-	-	0	-	0	-	0	-	-	-
$Prob_{\eta_1}$	-	-	-	-	-	-	0.754	-	-	-
η_2^+	-	-	0	-	-0.683	22.960	-1.000	inf	-	-
$Prob_{\eta_2}$	-	-	-	-	-	-	0.246	-	-	-
$Prob_{Cls-3}$	-	-	0.185	-	0.342	-	0.147	-	0.585	-
Fixed across classes										
$W_{group 1}$	-0.248	10.520	-0.213	13.040	-0.216	13.040	-0.236	14.490	-0.208	11.480
$W_{group 2}$	-0.483	15.500	-0.374	15.560	-0.396	16.370	-0.434	16.370	-0.415	14.800
$W_{group 3}$	-0.122	5.690	-0.124	8.450	-0.131	8.380	-0.137	8.760	-0.119	7.730
$W_{group 4}$	-0.123	8.660	-0.100	10.560	-0.113	11.690	-0.126	11.800	-0.098	9.210
$W_{group 5}$	-0.024	0.860	-0.034	1.760	0.034	1.700	0.000	0.000	0.000	0.000
$\mathcal{L}(\hat{\beta})$	-6003.015	-	-6136.662	-	-6086.171	-	-6047.101	-	-6010.504	-

* in RPL/MLC models $-\ln(\beta_\alpha) = -\ln(\lambda \cdot \alpha)$. t-test is against 1. All random parameters are normal distributed, but the cost coefficient, which is Lognormal confounded with scale parameter

** $\lambda = 1 + \eta_g$. In the obs. HLC $g = 1$ is less advanced skilled kayakers and $g = 2$ is advanced skilled kayakers.